Calendar Spread Options for Storable Commodities

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Abstract

Many previous studies provide pricing models of options on futures spreads. However, none of them fully reflect the economic reality that spreads can stay near full carry for long periods of time. We suggest a new option pricing model that assumes that convenience yield follows arithmetic Brownian motion and is truncated at zero. An analytical solution of the new pricing model is obtained. We empirically test the new model by testing the truth of its assumptions. We determine the distribution of calendar spreads and convenience yield for Chicago Board of Trade corn calendar spread options. Panel unit root tests fail to reject the null hypothesis of a unit root and thus support our assumption of arithmetic Brownian motion as opposed to a mean-reverting process as is assumed in much past research. The assumption that convenience yield is a normal distribution truncated at zero is only approximate as the volatility of convenience yield never goes to zero and spreads tend to approach full carry, but rarely reach full carry.

Key words: calendar spreads, corn, futures, panel unit root tests, options

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1. Introduction

The Chicago Board of Trade (CBOT) offers trading of calendar spread options on futures in wheat, corn, soybean, soybean oil, and soybean meal and the New York Mercantile Exchange offers trading of calendar spread options on cotton and crude oil. Calendar spread options are a new risk management tool. For example, storage facilities can purchase a calendar spread call option to hedge the risk of futures spread narrowing or inverting. Grain elevators can use calendar spread options to partially offset the risk of offering hedge-to-arrive contracts.

Options on calendar spreads cannot be replicated by combining two futures options with different maturity dates. The reason is that calendar spread options are affected only by volatility and value of the price relationship while any strategy to replicate the spread using futures options is also sensitive to the value of the underlying commodity (CME Group). Despite such benefits, so far the volume of calendar spread options traded has been low. Table 1 presents the volume of CBOT futures, options, and calendar spread options on Aug 24, 2012. The daily volume across all agricultural calendar spread option markets was 324 contracts, compared to the volume in the corresponding futures contracts of 598,204. The small volume may at least be partly due to a lack of understanding of how to value such options.

Earlier papers model the relationship between spot and futures prices and assume a mean reverting convenience yield (Shimko; 1994, Schwartz; 1997). However, such an assumption is doubtful for storable agricultural commodities since convenience yield may not follow a mean reversion process. Gold does not have strong mean reversion (Schwartz 1997). Gold is typically stored continually with no convenience yield so its spreads tend to remain at full carry[1]. Spreads for agricultural markets will be close to full carry for long periods. Thus, there is a need to create a more suitable option formula on calendar spreads for storable commodities that takes account of all three factors: opportunity cost of interest, storage cost, and convenience yield.

In this article, we provide a calendar spread option pricing formula for storable commodities that accounts for the lower bound on calendar spreads due to imposing no arbitrage opportunities and also the assumptions and predictions of the model will be tested by determining the distribution of convenience yield and calendar spreads using historical data.

To do this, we suggest a two factor model where nearby futures prices follow a geometric Brownian motion and convenience yield is an arithmetic Brownian motion truncated at zero. The valuation problem is solved like an option bear spread by combining a long call option and a short call option with a strike price of zero. It is possible to test for distributional properties of futures spread and convenience yield since spread is observable and convenience yield can be estimated.

For the empirical test of model assumptions, daily CBOT corn futures prices are used from 1975 to 2012. We only consider post-harvest spreads because full carry is never hit until harvest. Using nonparametric regression, historical plots for corn show a downward trend after harvest. Three-month Treasury Bills and the Prime rate are considered for interest rate and storage costs are estimated using historical data on commercial storage rates between 1975 and 2012.

We estimated the convenience yield based on the theory of storage; convenience yield is equal to spread plus interest forgone and physical storage cost. We determine the distribution of

[1] The price difference between (futures) contracts with different maturity is prevented from exceeding the full cost of carrying the commodity. Carrying costs include interest, insurance and storage.

spread and convenience yield. As expected, the indicate that calendar spreads are not normally or log-normally distributed. The finding partially supports our assumption of truncated convenience yield at zero. The price difference between two futures is often limited at 80~90% of full carry. That is, full carry is rarely exceeded. Not quite reaching full carry can be explained by market participants having varying interest cost or physical storage costs or possibly lack of an incentive to take risks without some return.

Gibson and Schwartz (1990) develop a two-factor model taking account of stochastic convenience yield in order to price oil contingent claims. They assume a mean reverting convenience yield to explain an inverse relation between the level of inventory and the marginal convenience yield. Schwartz (1997) extends this model to a three-factor model including a stochastic interest rate and analyzes futures prices of copper, oil, and gold. He finds that copper and oil have strong mean reversion while gold has weak mean reversion. Note that almost all gold is stored, while long-term storage of copper and oil is less frequent.

Shimko (1994) derives a closed form solution to a futures spread option model, based on the framework of Gibson and Schwartz (1990). Nakajima and Maeda (2007) generalize Shimko's model by introducing stochastic interest rates via Heath-Jarrow-Morton interest rate model (Heath, Jarrow, and Morton 1992) as well as using the concept of future convenience yield.

Hinz and Fehr (2010) propose a commodity option pricing model considering no arbitrage in both futures and physical commodity trading. They derive an upper bound observed in the situation of contango limit by using an analogy between commodity and money markets. Their work represents an important theoretical contribution, however, their empirical work is based on using a shifted lognormal distribution and the Black-Scholes pricing formula. Their model does satisfy the no arbitrage condition created by the contango limit, but it does not reflect the economic reality that spreads will often stay near contango limit for long periods of time.

2. Theory

The theory of storage predicts the spread between futures and spot prices will be a function of the interest forgone, $S(t)R(t, T)$, the marginal storage cost, $W(t, T)$, and the marginal convenience yield, $C(t, T)$:

(1) $$F(t,T) - S(t) = S(t)R(t,T) + W(t,T) - C(t,T)$$

where $F(t, T)$ is the futures price at time t for delivery at time T and $S(t)$ denotes the spot price at t. Some studies argue that the commodity spot price is not readily observable and use the futures contract closest to maturity as a proxy for the spot price in empirical analysis for this reason (Brennan 1958; Gibson and Schwartz 1990; Schwartz 1997; Hinz and Fehr 2010). This is a strange argument since daily commodity spot prices are readily available. There are good reasons for using the nearby as a proxy for spot prices, but it is not because spot prices do not exist. Futures prices reflect the cheapest-to-deliver commodity and thus the spot price represented by futures contracts can change over time. Also, as Irwin et al. (2011) discuss, grain futures markets require the delivery of warehouse receipts or shipping certificates rather than the physical delivery of grain. During much of 2008-2011, the price of deliverable warehouse receipts (or shipping certificates) exceeded the spot price of grain and thus futures and spot prices diverged.

Inverse carrying charges have been observed in not only futures and spot prices but also prices of distant and nearby futures. In this point, we extend the relationship in the theory of storage from the futures and spot prices to the two futures prices. Nearby futures $F(t, T_1)$ with maturity T_1 is treated as the spot $S(T_1)$ at time T_1 and the periods for the interest rate, storage cost, and convenience yield are the difference between deferred time T_2 and near time T_1. Equation (1) is rewritten as:

$$(2) \qquad F(t,T_2) - F(t,T_1) = F(t,T_1)R(t,T_2 - T_1) + W(t,T_2 - T_1) - C(t,T_2 - T_1)$$

where $F(t, T_2)$ denotes the distant futures price at time t for delivery at T_2 and $F(t, T_1)$ is the nearby futures price. $R(t, T_2 - T_1)$, $W(t, T_2 - T_1)$, and $C(t, T_2 - T_1)$ denote the interest rate, the marginal storage cost, and the marginal convenience yield for the period $T_2 - T_1$ at time t, respectively.

The marginal convenience yield approaches zero as the difference between nearby and distant futures goes near full carry. Below full carry, the marginal convenience yield is positive which means the nearby futures price exceeds the distant futures price. Sometimes, spreads for agricultural markets remain near full carry for long periods. To explain this phenomenon, we assume that convenience yield is truncated at full carry. The truncated convenience yield can be represented as follows:

$$(3) \qquad \begin{aligned} C(t,T_2 - T_1) &= C^*(t,T_2 - T_1) && \text{if } C^*(t,T_2 - T_1) > 0 \\ &= 0 && \text{otherwise} \end{aligned}$$

We assume a calendar spread option model that takes account of the convenience yield being truncated at full carry and derive a formula for options on calendar spreads. The CBOT traded calendar spreads are defined as the nearby futures minus distant futures so that the spreads are negative under contango. Equation (2) is multiplied by negative one to match the CBOT definition of calendar spreads:

$$(4) \qquad F(t,T_1) - F(t,T_2) = -F(t,T_1)R(t,T_2 - T_1) - W(t,T_2 - T_1) + C(t,T_2 - T_1)$$

The calendar spread option is an option on the price difference between two futures prices on the same commodity with different maturities. When a calendar spread call option is exercised at expiration, the buyer receives a long position in the nearby futures and a short position in the distant futures. Consider a European calendar spread call option at time *t*. The call option expires at time $T \leq T_1$. i.e. the option expires prior to the delivery time of the nearby futures contract. The payoff of the calendar spread call option with exercise price *K* is defined for $t \leq T \leq T_1 \leq T_2$

(5) $$\max(F_1(t, T_1) - F_2(t, T_2) - K, 0)$$

As seen, the payoff of the call option is affected by the price difference between nearby and distant futures prices. The theory of storage shows spreads between two futures is equal to interest forgone plus storage costs minus convenience yield. We simplify the theory of storage by assuming that both interest rate and storage costs are constant. That is, we only consider nearby futures price and convenience yield to derive the payoff of the call option.

The European call price with the strike price *K* at maturity *T* is

(6) $$c(T, F_1, C^*) = \max(F_1(T) - K, 0)$$

The resulting model is a two factor model. The nearby futures price F_1 is assumed to follow geometric Brownian motion with drift μ and volatility σ_1. The convenience yield C^* follows an arithmetic Brownian motion that is truncated at zero. The drift of convenience yield is given by δ and its volatility is given by σ_2. Two standard Brownian motions have constant correlation ρ. The two stochastic factors can be expressed as:

(7) $$d\,F_1(t) = \mu\,F_1(t)dt + \sigma_1\,F_1(t)dZ_1(t)$$

(8) $$dC^*(t) = \delta dt + \sigma_2 dZ_2(t)$$

(9) $$dZ_1(t)dZ_2(t) = \rho dt$$

where $dZ_1(t)$ and $dZ_2(t)$ are standard Wiener process. The stochastic volatility model of Heston (1993) is one of the most popular option pricing models. Our model does not consider stochastic volatility but the approach to derive the call option follows steps similar to Heston's work; the partial differential equation of the call price is obtained from two stochastic processes, the similar solution required to solve the problem is provided by assuming specific functions for the two probabilities, and characteristic functions are used to obtain probability functions. The value of any asset $V(t, F_1, C^*)$ must satisfy the partial differential equation (PDE) under no arbitrage condition

(10) $$\frac{1}{2}\sigma_1{}^2 F_1{}^2 \frac{\partial^2 V}{\partial F_1{}^2} + \rho\sigma_1\sigma_2 F_1 \frac{\partial^2 V}{\partial F_1 \partial C^*} + \frac{1}{2}\sigma_2{}^2 \frac{\partial^2 V}{\partial C^{*2}} + rF_1 \frac{\partial V}{\partial F_1} + \delta \frac{\partial V}{\partial C^*} - rV + \frac{\partial V}{\partial t} = 0$$

A European call option satisfies the PDE (10) and a solution of pricing the call is

(11) $$c(t, F_1, C^*) = F_1 P_1 - Ke^{-r(T-t)} P_2$$

where the probability P_1 and P_2 are the conditional probability that the option is in-the-money Let $x = lnF_1$ and substitute the solution of (11) into the PDE (10). This leads to PDEs for P_1 and P_2

(12) $$\frac{1}{2}\sigma_1^2 \frac{\partial^2 P_1}{\partial x^2} + \rho\sigma_1\sigma_2 \frac{\partial^2 P_1}{\partial x \partial C^*} + \frac{1}{2}\sigma_2{}^2 \frac{\partial^2 P_1}{\partial c^{*2}} + \left(r + \frac{1}{2}\sigma_1^2\right)\frac{\partial P_1}{\partial x} + (\delta + \rho\sigma_1\sigma_2)\frac{\partial P_1}{\partial c^*} + \frac{\partial P_1}{\partial t} = 0$$

$$\frac{1}{2}\sigma_1^2 \frac{\partial^2 P_2}{\partial x^2} + \rho\sigma_1\sigma_2 \frac{\partial^2 P_2}{\partial x \partial C^*} + \frac{1}{2}\sigma_2{}^2 \frac{\partial^2 P_2}{\partial c^{*2}} + \left(r - \frac{1}{2}\sigma_1^2\right)\frac{\partial P_2}{\partial x} + \delta \frac{\partial P_2}{\partial c^*} + \frac{\partial P_2}{\partial t} = 0$$

The probabilities P_1 and P_2corresponding to the characteristic functions f_1 and f_2 are

(13) $$P_j = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty Re[\frac{e^{-i\emptyset lnK} f_j(\tau, x; \emptyset)}{i\emptyset}]d\emptyset$$

We guess the characteristic functions

(14) $$f_j(t, x, C^*; \emptyset) = \exp(A(\tau, \emptyset) + B(\emptyset)C^* + i\emptyset x)$$

where $$A(\tau, \emptyset) = r\emptyset i\tau + a\frac{1}{2}\sigma_2(b_j - \rho\sigma_1\emptyset i \pm d)\tau$$

$$B(\emptyset) = \frac{1}{2}\sigma_2(b_j - \rho\sigma_1\emptyset i \pm d)$$

$$d = \sqrt{(\rho\sigma_1\emptyset i - b_j)^2 - \sigma_1^2(2u_j\emptyset i - \emptyset^2)}$$

for *j*=1,2 and

$$u_1 = \frac{1}{2}, \qquad u_2 = -\frac{1}{2}, \qquad a = \delta, \qquad b_1 = -\rho\sigma_2, \qquad b_2 = 0\,, \qquad \tau = T - t$$

To handle convenience yield truncated at zero, we propose a second option for convenience yield which follows arithmetic Brownian motion (Bachelier; 1990) and has a strike price of zero. We specify the convenience yield $C_B^*(t)$ satisfying at time t

(15) $$dC_B^*(t) = \sigma_B dW(t)$$

For $0 \leq t \leq T$, where $dW(t)$ denotes standard Brownian Motion, subscript B stands for Bacheiler, and σ_B represents the volatility. The value of a European call option c_B at maturity T is

(16) $$c_B(T) = \max(C_B^*(T) - K, 0)$$

where K is the exercise price. Following the Bachelier framework, the convenience yield is normally distributed with mean $C^*(t)$ and variance $\sigma_B^2 T$. The call option at time $t = T$ is

(17) $$c_B(t) = (C^*(t) - K)\Phi\left(\frac{C^*(t)-K}{C^*(t)\sigma_B\sqrt{T}}\right) + C^*(t)\sigma_B\sqrt{T}\phi(\frac{C^*(t)-K}{C^*(t)\sigma_B\sqrt{T}})$$

where $\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$ is the standard normal density function. Our purpose is a call option with the strike price of zero. Substitute the strike price into zero.

(18) $$c_B(t)_{K=0} = C^*(t)\Phi\left(\frac{C^*(t)}{C^*(t)\sigma_B\sqrt{T}}\right) + C^*(t)\sigma_B\sqrt{T}\phi(\frac{C^*(t)}{C^*(t)\sigma_B\sqrt{T}})$$

$c_B(t)_{K=0}$ represents the second call option with the strike price of zero. By combining the long call option from equation (11) and the short call option from equation (18), a long calendar spread call option is

(19) $$c_{cs} = c(t, F_1(t), C^*) - c_B(t)_{K=0}$$

where c_{cs} is the calendar spread call option and $c(t, F_1(t), C^*)$ is the call option from the combined stochastic processes of changes in interest costs due to the change in the nearby price and convenience yield.

3. Data

The data are daily corn futures prices between 1975 and 2012 from the Chicago Board of Trade. Figures 1 through 5 depict the movement in Dec-Mar, Mar-May, July-Dec, and Dec-July corn futures spreads, respectively. Dec-Mar, Mar-May, and Dec-July corn futures spreads are mostly in contango in that corn futures spreads (nearby minus distant) are less than zero. The July-Dec corn futures spread is across crop years and is mostly in backwardation where the nearby is above the distant most of the time. Figure 1 is also used for visual estimated spread to see if spread is at full carry. Nonparametric regression is used to verify the trend. In Figure 6, Dec-Mar, May-July, and Dec-July spreads have a similar trend while Mar-May and July-Dec spreads have a similar curve. One common trend is that all five futures spreads decrease as maturity approaches. That is, historical corn spreads exhibit a downward trend during post-harvest. This downward trend might reflect a risk premium. We only consider post-harvest spreads for 100 days before expiration because full carry is never hit until harvest. Daily three-month Treasury Bills and the Prime rate are used for interest rate from the Federal Reserve System (FED) and annual storage costs are estimated using historical data on commercial storage rates (Franken, Garcia, and Irwin 2009). The storage cost data are converted from yearly to daily using the relevant SAS procedure. The sample period of storage costs and interest rate is 1975 - 2012. Table 2 summarizes the data for nearby and distant futures, calendar spread, interest rates, and storage costs for each spread. The price of distant futures is above that of nearby futures except July-Dec spread. Daily means of futures spread are between -20.01 and 5.2 where negative sign is because spread is defined as nearby futures minus distant futures price. As spread period is longer, the absolute mean of spread is larger. Absolute Dec-July spread is the largest mean of -20.1. The mean of the Prime rate (8.3%) is higher than that of three-mouth Treasury Bills (5.2%). The mean of convenience yield is between 1.2 and 25.7, which has a positive value even though the minimum of convenience yield has negative value. Figure 8 through 12 present the implicit convenience yield plots for each year using the Prime rate. Convenience yield is positive most of the time and negative convenience yield could occur from underestimating physical storage costs during these years.

4. Methodology

We estimate descriptive statistics and test distributions for both the calendar spreads and convenience yield. The distributional tests for spreads are conducted by tests of skewness ($\sqrt{\beta_1}$), kurtosis (β_2), and an omnibus test (K^2). Convenience yield is not directly observable. Convenience yield is estimated following equation (4) as

(20) $$C(t, T_2 - T_1) = \big(F(t, T_1) - F(t, T_2)\big) + F(t, T_1)R(t, T_2 - T_1) + W(t, T_2 - T_1)$$

To test the above calculation we also regress spread against interest forgone and storage costs

(21) $$F(t, T_1) - F(t, T_2) = \gamma + \alpha F(t, T_1)R(t, T_2 - T_1) + \beta W(t, T_2 - T_1)$$

We analyze the distributional properties of convenience yield to investigate whether the distribution of convenience yield is well approximated by a normal distribution that is truncated at zero employing historical data. Descriptive statistics and test for distributions are conducted for the estimated convenience yield.

We test for the presence of mean reversion in the spread and convenience yield for Dec-Mar, Mar-May, Mar-July, July-Dec, and Dec-July. If spread or convenience yield is stable it implies spread or convenience yield follows a mean reverting process as previous papers assume. The data are cross sectional time series, where the years are the cross section and the days to maturity is the time series. Stata provides several panel unit root tests such as Im-Pesaran- Shin (2003) and Fisher-type (Choi 2001) tests for unbalanced panels. All of the tests are used to diagnose a unit root. The null hypothesis is that the panels contain a unit root.

5. Empirical Results

We regress the spread against the interest forgone and storage costs with 3-month Treasury Bills as well as the Prime rate (Table 3). Dec-Mar and Mar-July spread regressions show the expected negative signs, but are mostly less than one in absolute value. The small coefficients could be due to attenuation bias due to measurement error as well as not including convenience yield in the regression. . The results show little consistent difference in the two interest rates.

Histograms for spread and convenience yield are presented in figure 7. For Dec-Mar, Mar-May, Mar-July, and Dec-July spreads, the skewness is close to that of a normal distribution while the relative kurtosis indicates a leptokurtic distribution. The histograms of convenience yield[2] show a right tail and skewness to the right. Especially, July-Dec histograms of spread and convenience yield more skew to right. Although the normality of convenience yield is rejected, it

[2] The convenience yield is computed by the Prime rate times nearby futures prices plus storage costs.

is shown that the shape of the distribution provides modest support for assuming truncation at zero once values less than zero are regarded as measurement noise. The empirical result shows that calendar spreads are only close to full carry most of sample period and thus the price differences between two futures are limited at 80~90% of full carry. It may be that convenience yield has measurement noise due to estimating storage and interest costs. But, it also could be that there is no economic incentive to run spreads all of the way to full carry. Table 4 reports normality tests for spreads and convenience yield. All omnibus tests reject the null of a normal distribution at the 5 % significance level in both five spreads and convenience yield.

Panel unit root test results are presented in table 6. Statistics for spreads and convenience yield range between -2.18 and 1.29. We cannot reject the null hypotheses of spreads and convenience yield having unit roots. This suggests that all the spreads and convenience yield do not follow mean reversion.

6. Summary and Conclusion

The theory of storage says that calendar spreads on a storable commodity are the sum of the opportunity cost of interest, the physical cost of storage, and convenience yield. We develop a new calendar spread option pricing model in which convenience yield follows arithmetic Brownian motion that is truncated at zero, nearby futures follow geometric Brownian motion, and interest rates and the physical cost of storage are held constant. An analytical solution for the two-factor model is obtained using steps similar to that used to derive the Heston stochastic volatility model although our model does not assume stochastic volatility. The premium of a call option on a calendar spread is then obtained as the sum of the premium of the two-factor model minus the premium of call option on the convenience yield that has a strike price of zero.

We compute the implicit convenience yield to determine whether spreads are at full carry, and regress spread against interest forgone and storage costs, and calculate convenience yield according to the theory of storage. The Prime rate appears to provide a better estimate of full carry than does U.S. Treasury Bill rates. Distributional tests are conducted for five calendar spreads and convenience yield using the Prime rate as interest rate. The null hypothesis of normality is rejected with both spread and convenience yield as expected. The histogram of spread, however, is somewhat similar to a normal distribution. The distribution of convenience yield is strongly skewed to the right which supports the assumption that full carry is acting as a lower bound. The variance of estimated convenience yield does not go to zero and convenience yield usually stops a little short of full carry. This result may reflect market participants that have varying physical cost of storage and varying interest rates. Most commercial elevators are likely net borrowers, but some producers may be net lenders. We conduct panel unit root tests for five calendar spreads and convenience yield. The null hypothesis of a unit root cannot be rejected and thus the results support our assumption of Brownian motion over the Gibson and Schwartz (1990) assumption of mean reverting convenience yield. Thus, the model represents a considerable improvement over past models. Future research will consider a four factor model where interest rates and physical costs are also random. Future study will also extend the test for other commodities and test the model using traded option premiums. The new calendar spread option pricing model developed here has the potential to allow traders to lower bid-ask spreads, which ultimately could increase volume in these markets much like has occurred with traders use of the Black-Scholes model.

Table 1. The volume of Chicago Board of Trade calendar spread options and futures

Type	Name	Daily Volume [a]	Monthly Volume [b]
Futures	Corn	217,347	6,825,321
Futures	Soybean	140,842	5,195,821
Futures	Soybean Meal	68,001	1,848,093
Futures	Soybean Oil	91,190	2,394,547
Futures	Wheat	80,824	2,404,137
	SUM	**598,204**	**18,667,919**
Options	Corn	151,021	3,537,600
Options	Soybean	53,127	2,460,585
Options	Soybean Meal	7,083	224,899
Options	Soybean Oil	16,359	208,483
Options	Wheat	18,858	620,709
	SUM	**246,448**	**7,052,276**
CSO	Consecutive Corn	235	6,981
CSO	Consecutive Soybean	0	0
CSO	Consecutive Soybean Meal	0	0
CSO	Consecutive Soybean Oil	50	265
CSO	Consecutive Wheat	0	3,225
CSO	Corn July-Dec	0	458
CSO	Corn Dec-July	0	511
CSO	Corn Dec-Dec	8	0
CSO	Soybean Jan-May	0	0
CSO	Soybean July-Nov	0	0
CSO	Soybean Aug-Nov	0	935
CSO	Soybean Nov-July	0	81
CSO	Soybean Nov-Nov	0	0
CSO	Soy Meal July-Dec	0	0
CSO	Soy Oil July-Dec	0	0
CSO	Wheat July-July	0	0
CSO	Wheat July-Dec	0	0
CSO	Wheat Dec-July	0	725
	SUM	**324**	**13,181**

a The daily data are from Aug 24, 2012.

b The monthly data are from July 2012.

Table 2. Summary statistics

Variable	Sample Period	Mean	Standard Deviation	Minimum	Maximum
<u>Dec-Mar CBOT Corn</u>					
Dec Futures (¢/bu)	10/10/1975-11/18/2011	279.0	101.9	161.5	775.3
Mar Futures (¢/bu)	10/10/1975-11/18/2011	289.2	102.9	173.0	787.3
Dec-Mar Spread (¢/bu)	10/10/1975-11/18/2011	-10.1	3.7	-19.5	3.8
ln(Dec/Mar) Spread (¢/bu)	10/10/1975-11/18/2011	0.0	0.0	-0.1	0.0
Dec-Mar Implicit Convenience Yield	10/10/1975-11/18/2011	1.5	4.1	-8.3	19.4
Three-month TB (%)	10/10/1975-11/18/2011	5.2	3.3	0.0	15.9
Prime rate(%)	10/10/1975-11/18/2011	8.3	3.3	3.3	20.5
Storage Costs (¢/bu)	10/10/1975-11/18/2011	6.2	1.0	4.5	9.0
<u>Mar-May CBOT Corn</u>					
Mar Futures (¢/bu)	11/12/1975-2/18/2012	287.5	101.8	142.8	712.8
May Futures (¢/bu)	11/12/1975-2/18/2012	294.2	102.5	150.8	723.0
Mar-May Spread (¢/bu)	11/12/1975-2/18/2012	-6.7	2.9	-13.8	2.5
ln(Mar/May) Spread (¢/bu)	11/12/1975-2/18/2012	0.0	0.0	-0.1	0.0
Mar-May Implicit Convenience Yield	11/12/1975-2/18/2012	1.2	3.1	-5.2	12.6
Three-month TB (%)	11/12/1975-2/18/2012	5.2	3.4	0.0	17.1
Prime rate(%)	11/12/1975-2/18/2012	8.3	3.5	3.3	21.5
Storage Costs (¢/bu)	11/12/1975-2/18/2012	4.1	0.7	3.0	6.0
<u>Mar-July CBOT Corn</u>					
Mar Futures (¢/bu)	11/12/1975-2/16/2012	287.5	101.8	142.8	712.8
July Futures (¢/bu)	11/12/1975-2/16/2012	298.7	102.7	155.3	726.8
Mar-July Spread (¢/bu)	11/12/1975-2/16/2012	-11.2	5.9	-24.3	6.3
ln(Mar/July) Spread (¢/bu)	11/12/1975-2/16/2012	0.0	0.0	-0.1	0.0
Mar-July Implicit Convenience Yield	11/12/1975-2/16/2012	4.7	6.9	-10.0	31.6
Three-month TB (%)	11/12/1975-2/16/2012	5.2	3.4	0.0	17.1
Prime rate(%)	11/12/1975-2/16/2012	8.3	3.5	3.3	21.5
Storage Costs (¢/bu)	11/12/1975-2/16/2012	8.2	1.4	6.0	12.0
<u>July-Dec CBOT Corn</u>					
July Futures (¢/bu)	3/12/1976-6/19/2012	307.1	119.6	160.8	787.0
Dec Futures (¢/bu)	3/12/1976-6/19/2012	301.9	105.2	171.8	780.0
July-Dec Spread (¢/bu)	3/12/1976-6/19/2012	5.2	30.5	-34.3	159.3
ln(July/Dec) Spread (¢/bu)	3/12/1976-6/19/2012	0.0	0.1	-0.1	0.4
July-Dec Implicit Convenience Yield	3/12/1976-6/19/2012	25.7	32.3	-9.1	186.6
Three-month TB (%)	3/12/1976-6/19/2012	5.2	3.5	0.0	17.0
Prime rate(%)	3/12/1976-6/19/2012	8.3	3.7	3.3	20.5
Storage Costs (¢/bu)	3/12/1976-6/19/2012	10.4	1.8	7.5	15.0
<u>Dec-July CBOT Corn</u>					
Dec Futures (¢/bu)	8/12/1976-11/16/2011	279.0	101.7	161.5	775.3
July Futures (¢/bu)	8/12/1976-11/16/2011	299.1	103.2	182.0	794.0
Dec-July Spread (¢/bu)	8/12/1976-11/16/2011	-20.1	8.9	-42.0	11.0
ln(Dec/July) Spread (¢/bu)	8/12/1976-11/16/2011	-0.1	0.0	-0.1	0.0
Dec-July Implicit Convenience Yield	8/12/1976-11/16/2011	7.2	10.4	-18.8	47.8
Three-month TB (%)	8/12/1976-11/16/2011	5.2	3.3	0.0	15.9
Prime rate(%)	8/12/1976-11/16/2011	8.3	3.3	3.3	20.5
Storage Costs (¢/bu)	8/12/1976-11/16/2011	14.4	2.3	10.5	21.0

Note: Implicit convenience yield is computed by the Prime rate times nearby futures prices plus storage costs. All data are daily.

Figure 1. Plots of CBOT Dec-Mar corn spread

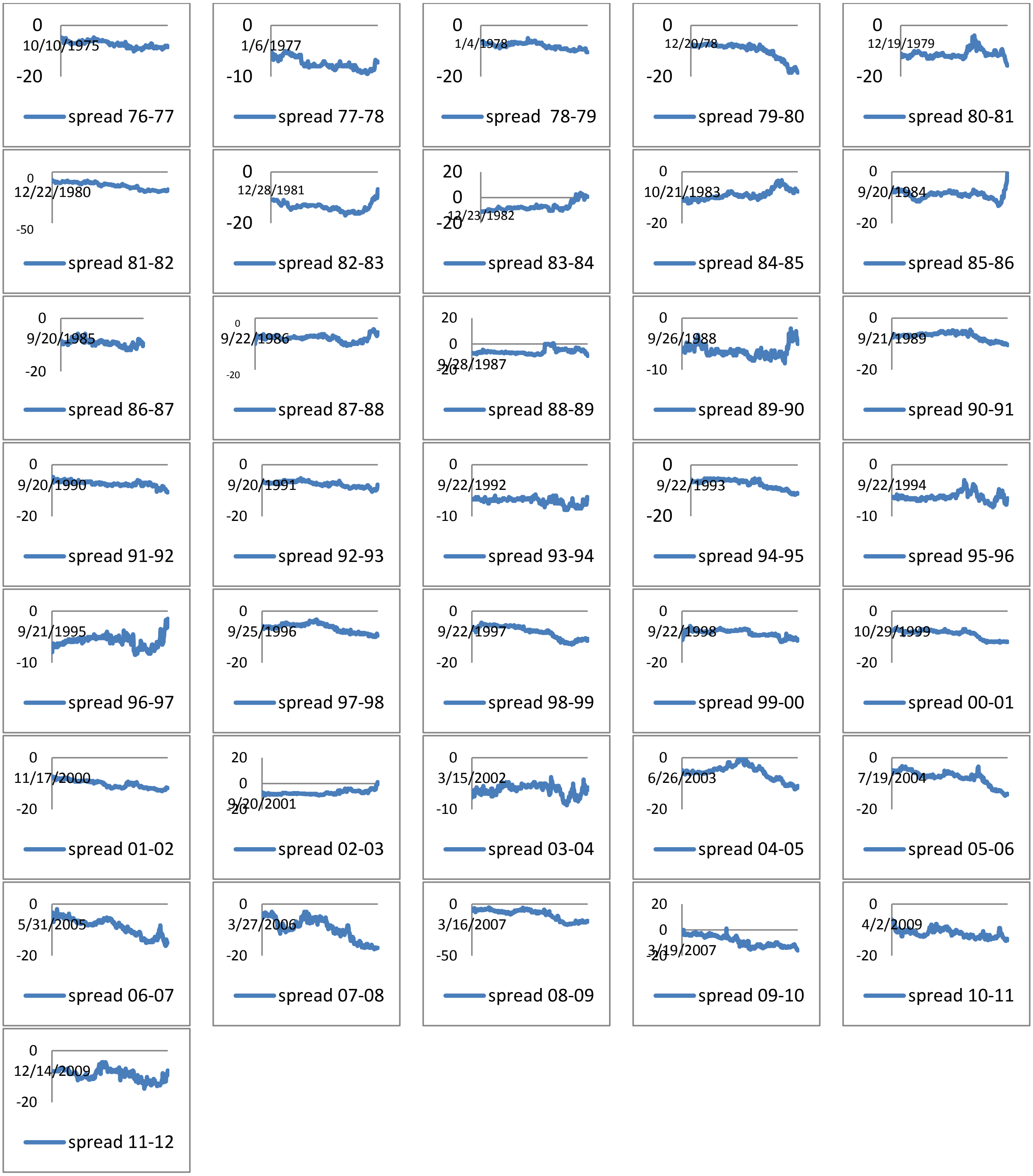


Figure 2. Plots of CBOT Mar-May corn spread

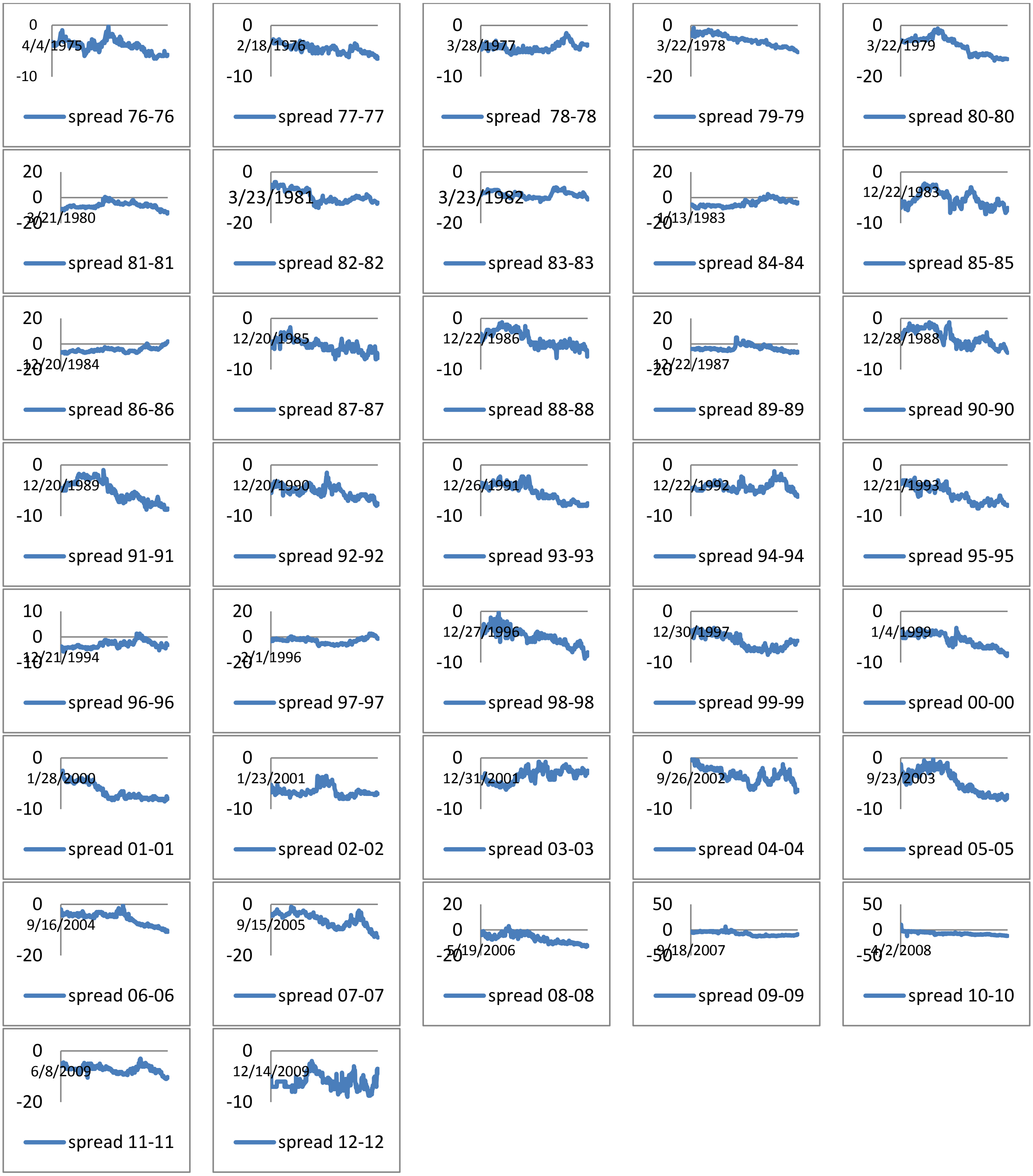


Figure 3. Plots of CBOT Mar-July corn spread

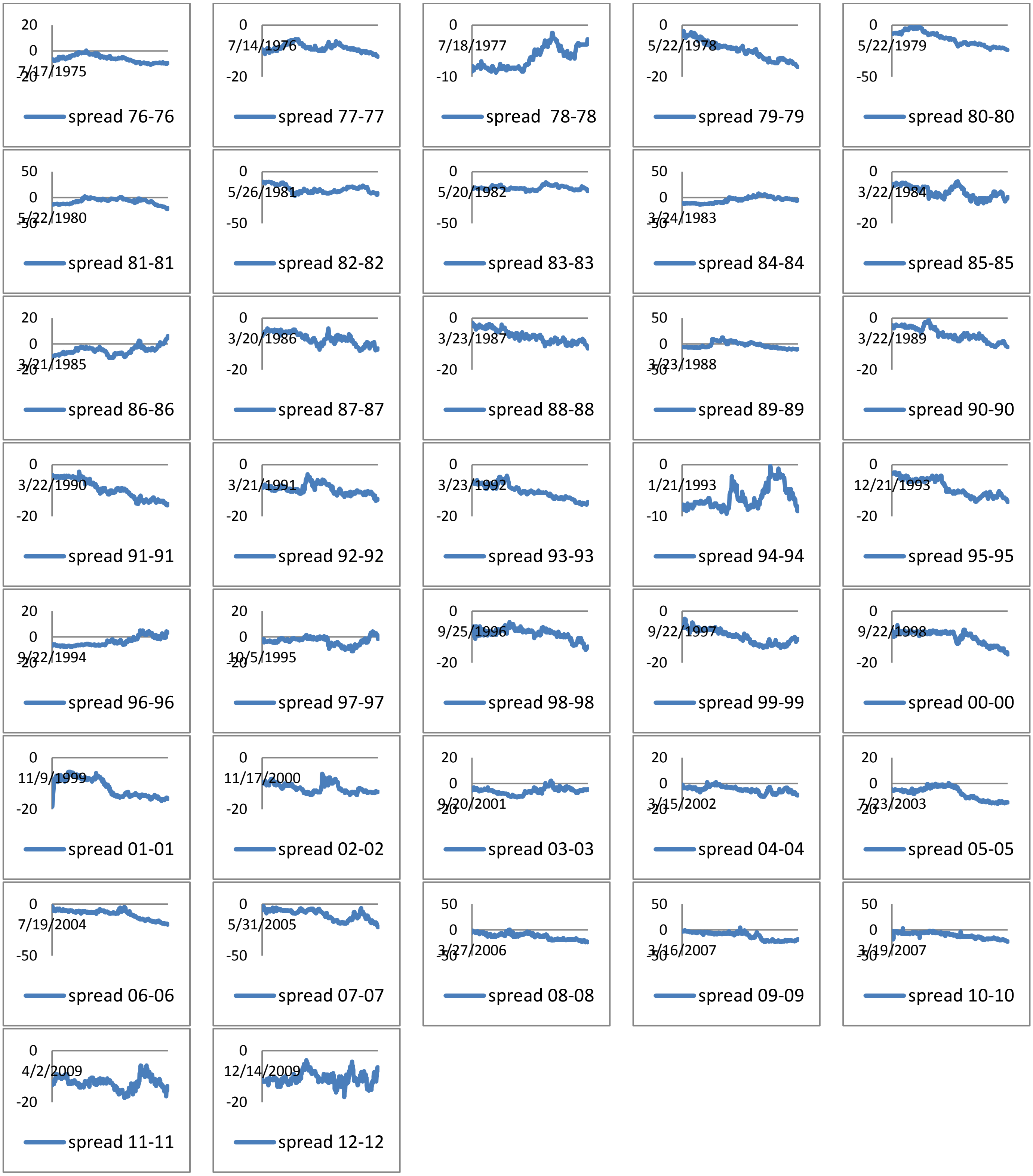


Figure 4. Plots of CBOT July-Dec corn spread

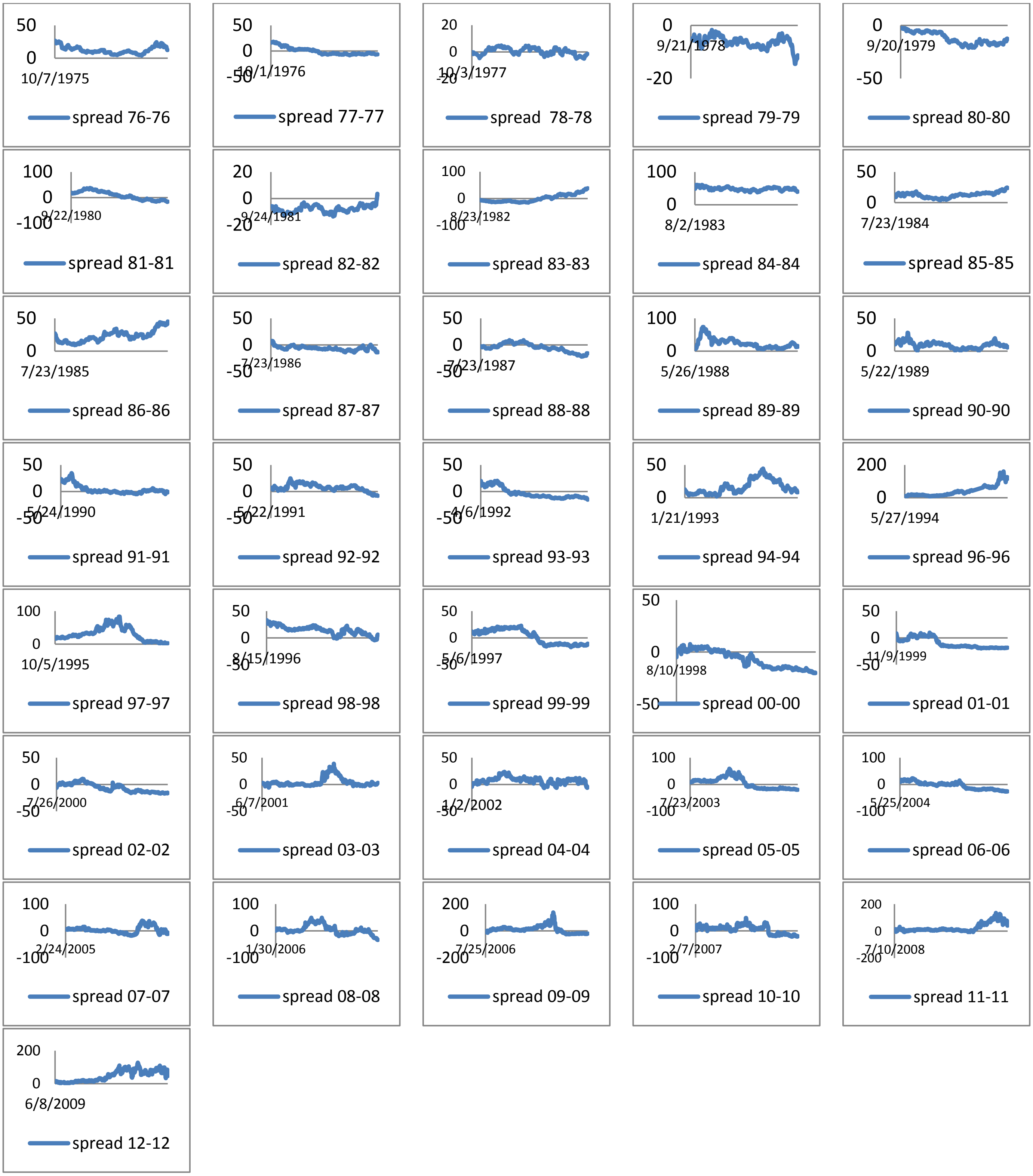


Figure 5. Plots of CBOT Dec-July corn spread

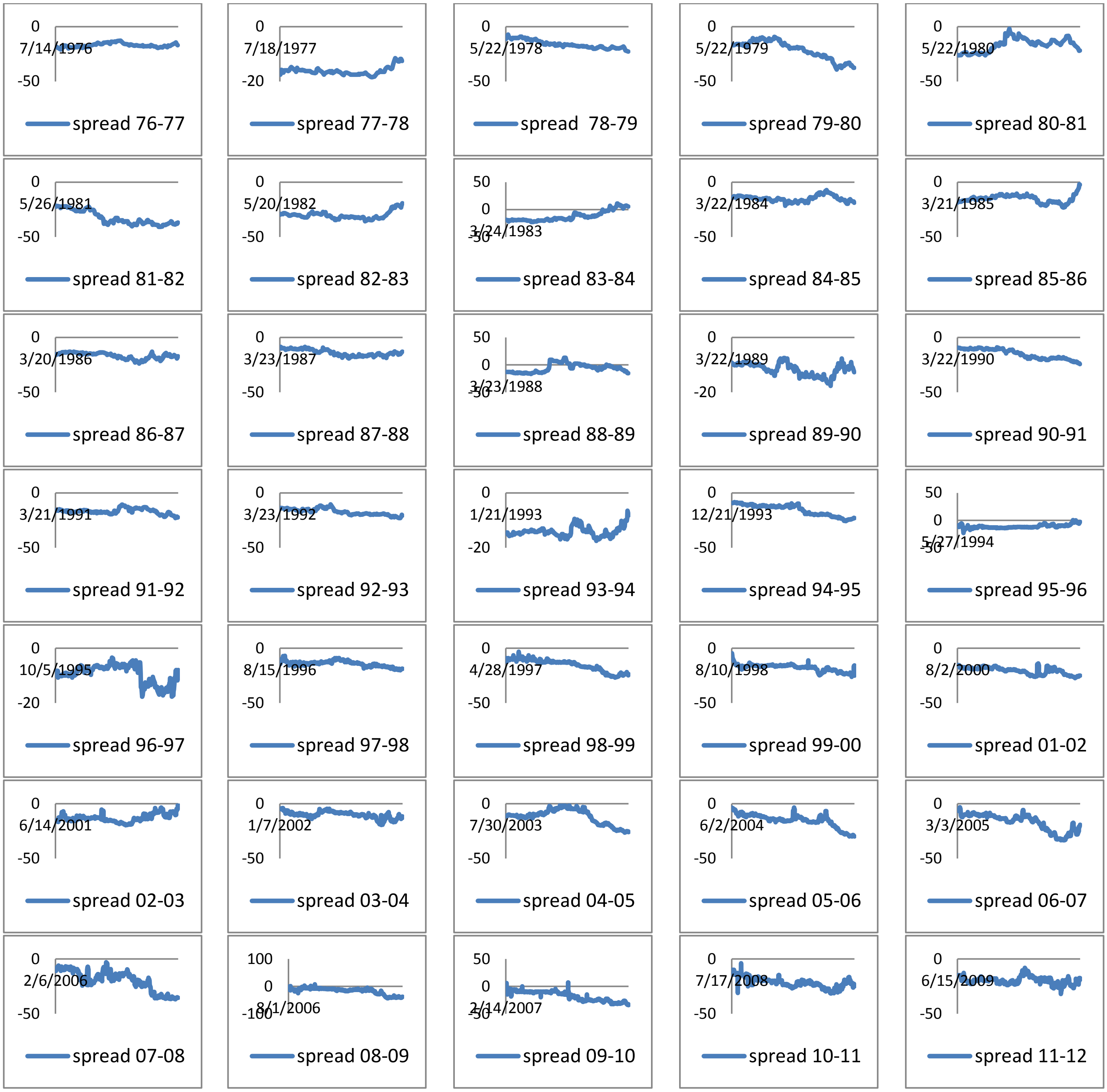


Figure 6. Nonparametric regression of corn spread versus days to expiration

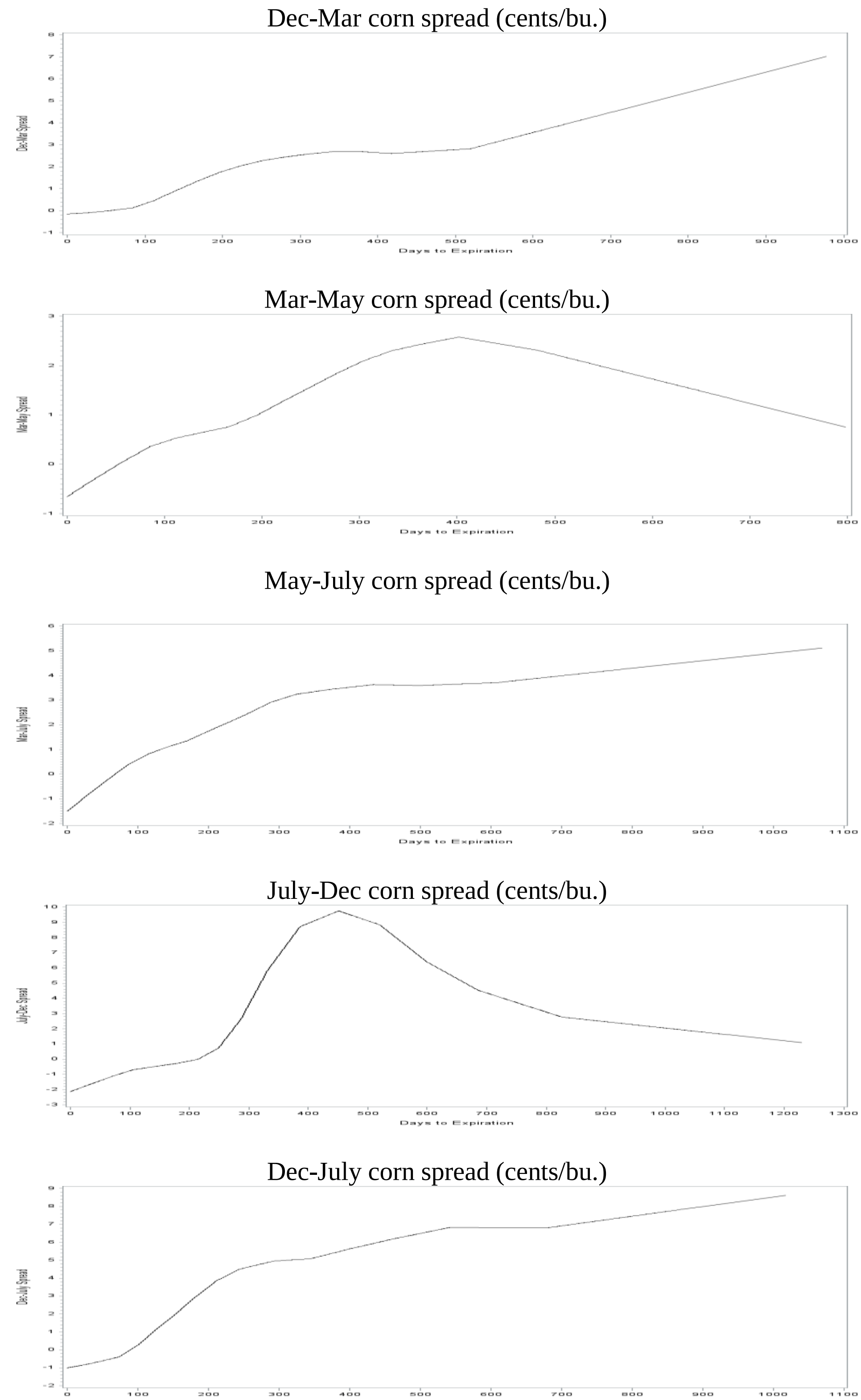


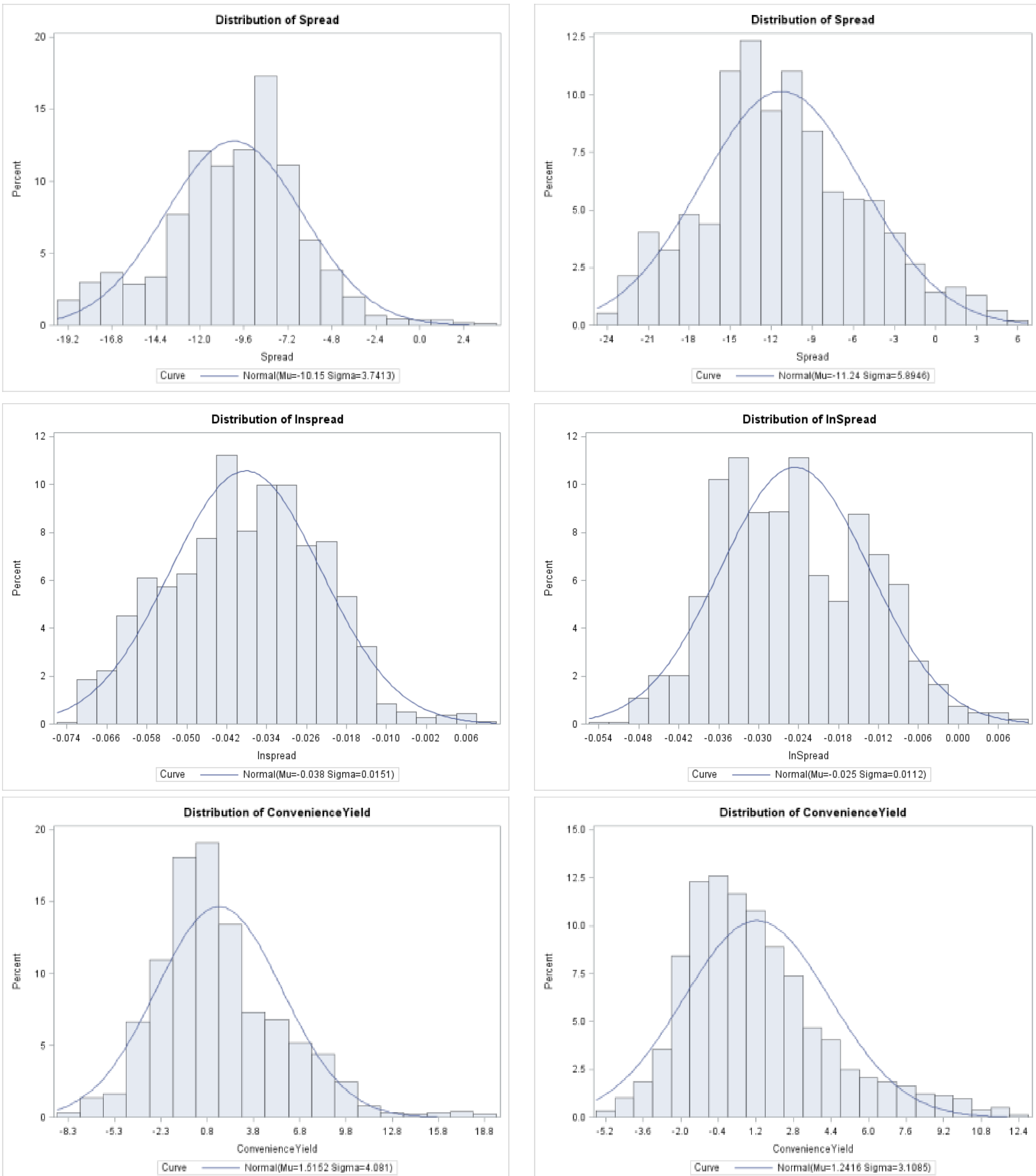
Table 3. Regression of spread on interest forgone and storage costs

Used interest rate	Γ	α	β	R^2
Dec-Mar				
TB3	-5.81* (0.535)	-0.07* (0.031)	-0.66* (0.079)	0.03
Prime	-4.57* (0.488)	-0.28* (0.027)	-0.66* (0.072)	0.06
Mar-May				
TB3	-1.29* (0.381)	-0.23* (0.033)	-1.18* (0.084)	0.08
Prime	-1.20* (0.354)	-0.30* (0.028)	-1.06* (0.079)	0.09
Mar-July				
TB3	-3.78* (0.781)	0.037* (0.033)	-0.93* (0.086)	0.05
Prime	-2.93* (0.734)	-0.04* (0.029)	-0.96* (0.082)	0.05
July-Dec				
TB3	-56.32* (3.883)	0.94* (0.129)	5.36* (0.336)	0.09
Prime	-57.28* (3.578)	1.23* (0.112)	4.81* (0.313)	0.11
Dec-July				
TB3	-15.49* (1.299)	0.11* (0.032)	-0.37* (0.082)	0.02
Prime	-11.67* (1.206)	-0.10* (0.029)	-0.48* (0.076)	0.02

Note: γ is intercept. α is coefficient of interest forgone. β is coefficient of storage costs. * indicates significance at the 5% level.

Figure 7. Histograms of spread and convenience yield, (1975-2012)

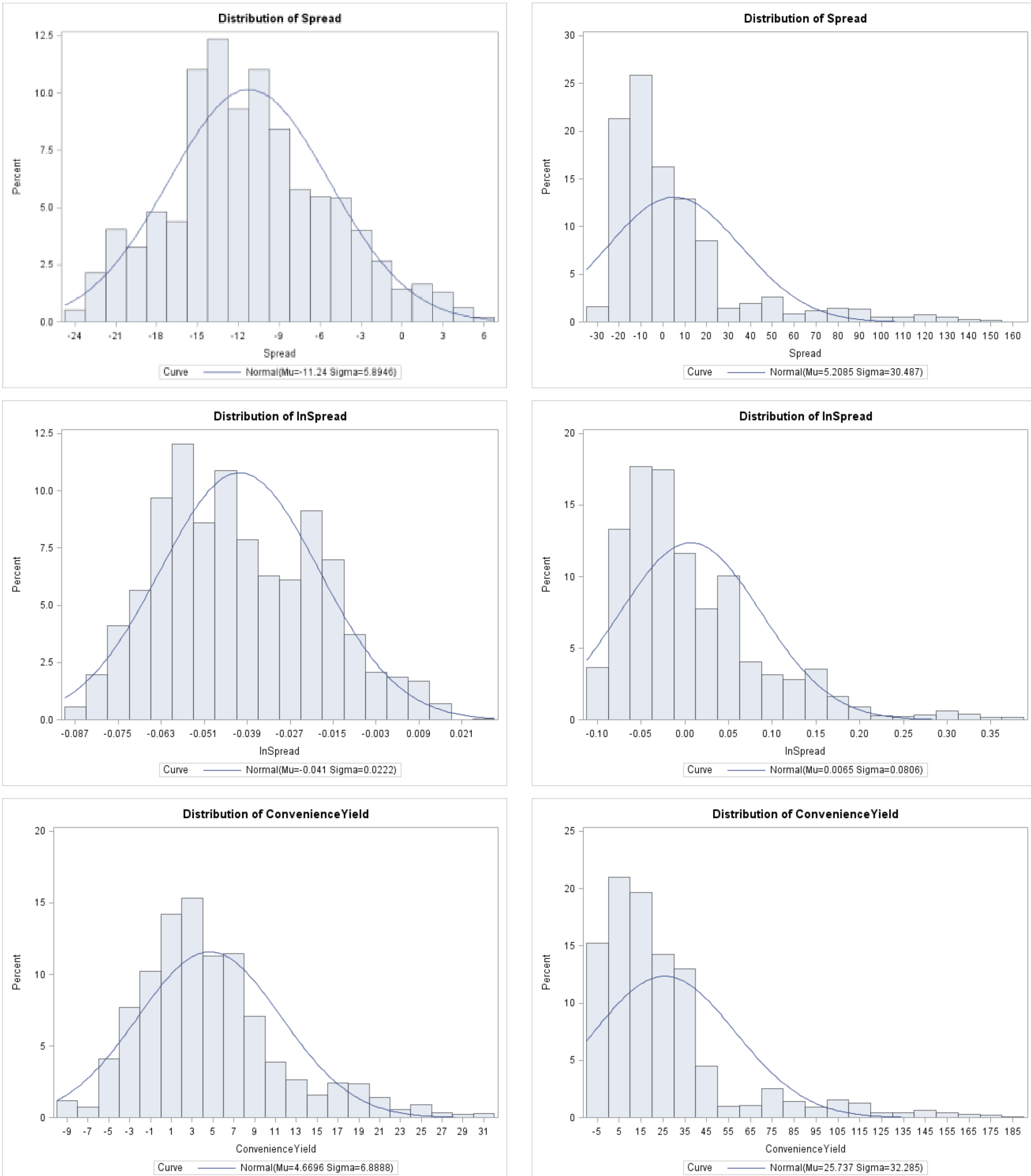
Dec-Mar **Mar-May**



Note: Convenience yield is computed as the spread minus the Prime rate times nearby futures price and also minus storage costs.

Figure 7. Histograms of spread and convenience yield, (1975-2012) continued

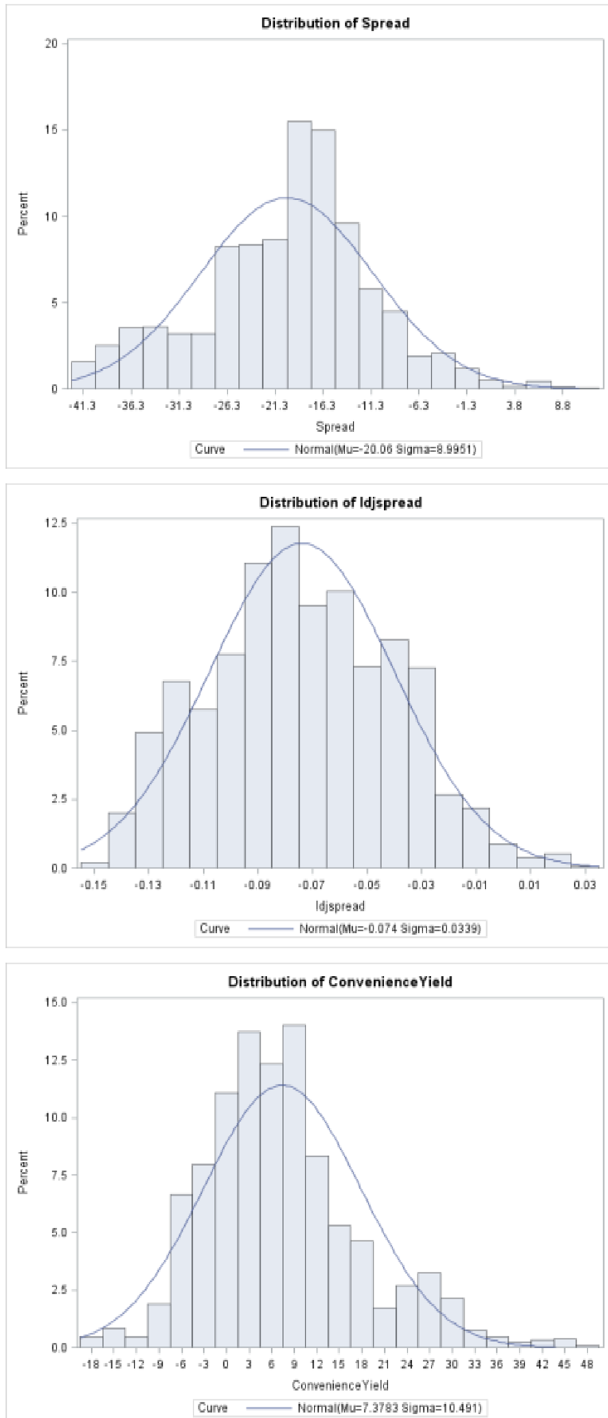
Mar-July **July-Dec**



Note: Convenience yield is computed as the spread minus the Prime rate times nearby futures price and also minus storage costs.

Figure 7. Histograms of spread and convenience yield, (1975-2012) continued

Dec-July



Note: Convenience yield is computed as the spread minus the Prime rate times nearby futures price and also minus storage costs.

Table 4. Distribution for corn futures spread and convenience yield

	Obs.	Skewness	Kurtosis	Kolmogorov-Smirnov	Cramer-von Mises	Anderson-Darling
Dec-Mar						
Dec-Mar Spread (c/bu)	2552	-0.12	0.45	0.01*	0.005*	0.005*
ln(Dec/Mar) Spread (¢/bu)	2552	0.03	0.0002	0.01*	0.005*	0.005*
Dec-Mar Implicit Convenience Yield	2552	0.88	1.48	0.01*	0.005*	0.005*
Mar-May						
Mar-May Spread (¢/bu)	2500	0.18	-0.06	0.01*	0.005*	0.005*
ln(Mar/May) Spread (¢/bu)	2500	0.26	-0.48	0.01*	0.005*	0.005*
Mar-May Implicit Convenience Yield	2500	0.96	0.92	0.01*	0.005*	0.005*
Mar-July						
Mar-July Spread (¢/bu)	2500	0.31	-0.18	0.01*	0.005*	0.005*
ln(Mar/May) Spread (¢/bu)	2500	0.32	-0.61	0.01*	0.005*	0.005*
Mar-July Implicit Convenience Yield	2500	6.89	1.27	0.01*	0.005*	0.005*
July-Dec						
July-Dec Spread (¢/bu)	2587	2.16	5.02	0.01*	0.005*	0.005*
ln(July/Dec) Spread (¢/bu)	2587	1.46	2.62	0.01*	0.005*	0.005*
July-Dec Implicit Convenience Yield	2587	2.09	4.71	0.01*	0.005*	0.005*
Dec-July						
Dec-July Spread (¢/bu)	2479	-0.13	0.23	0.01*	0.005*	0.005*
ln(Dec/July) Spread (¢/bu)	2479	0.15	-0.49	0.01*	0.005*	0.005*
Dec-July Implicit Convenience Yield	2479	0.77	0.87	0.01*	0.005*	0.005*

Note: Implicit convenience yield is computed as the spread minus the Prime rate times nearby futures price and also minus storage costs. * indicates rejection of the null hypothesis of normality at the 5% level.

Figure 8. Dec-Mar convenience yield plots by year, (1975-2012)

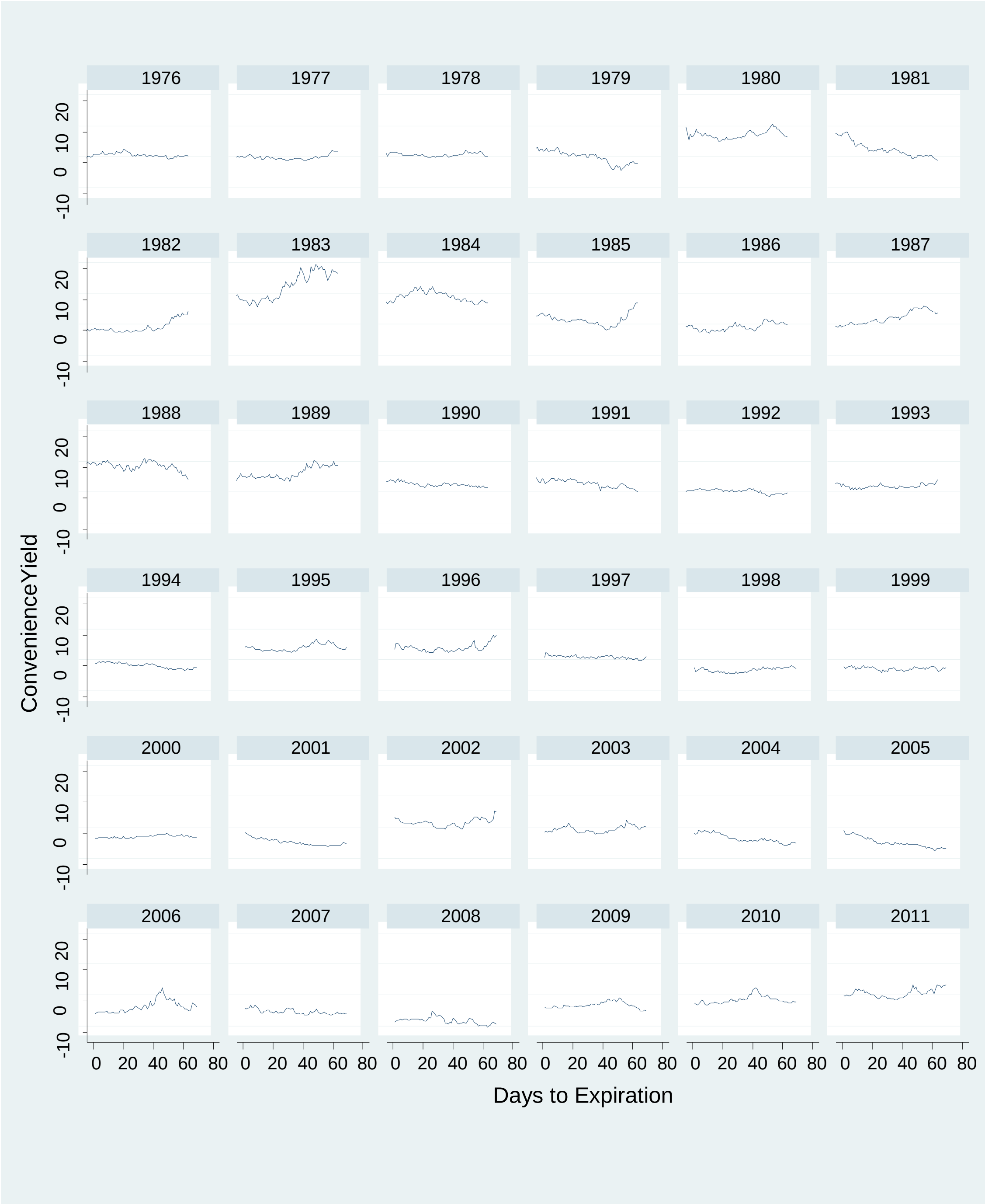


Figure 9. Mar-May convenience yield plots by year, (1975-2012)

Figure 10. Mar-July convenience yield plots by year, (1975-2012)

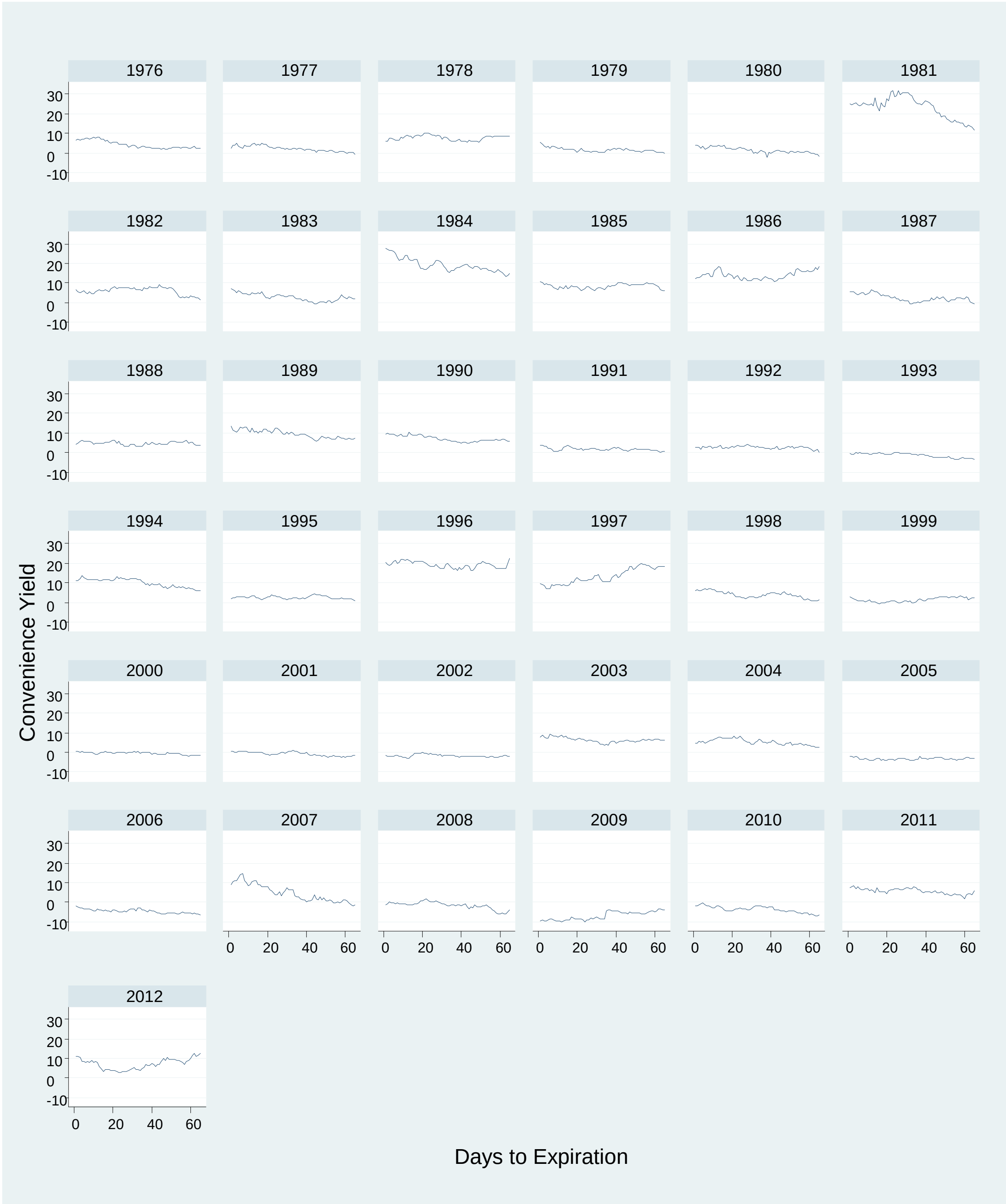


Figure 11. July-Dec convenience yield plots by year, (1975-2012)

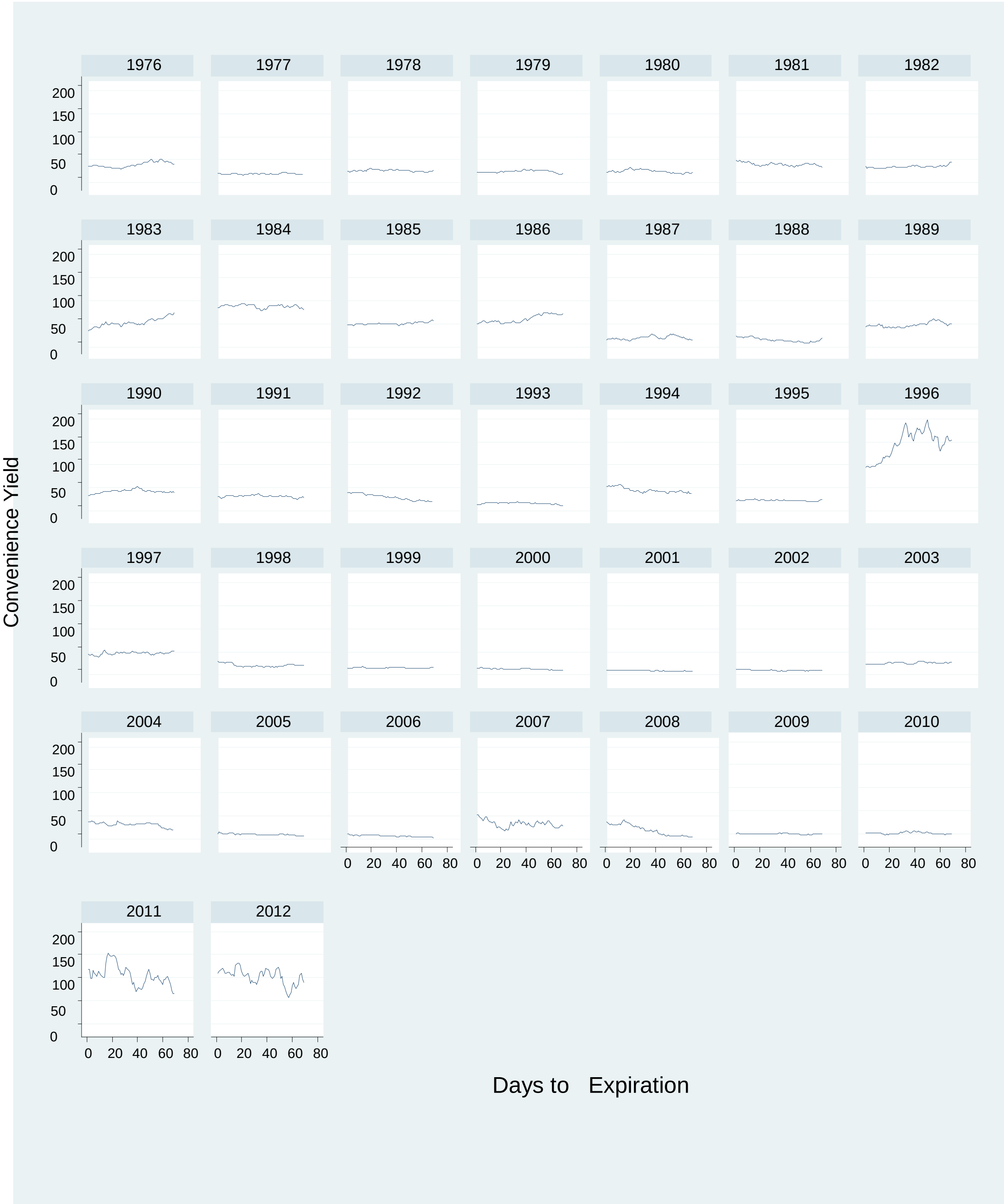


Figure 12. Dec-July Convenience Yield Plots by year, (1975-2012)

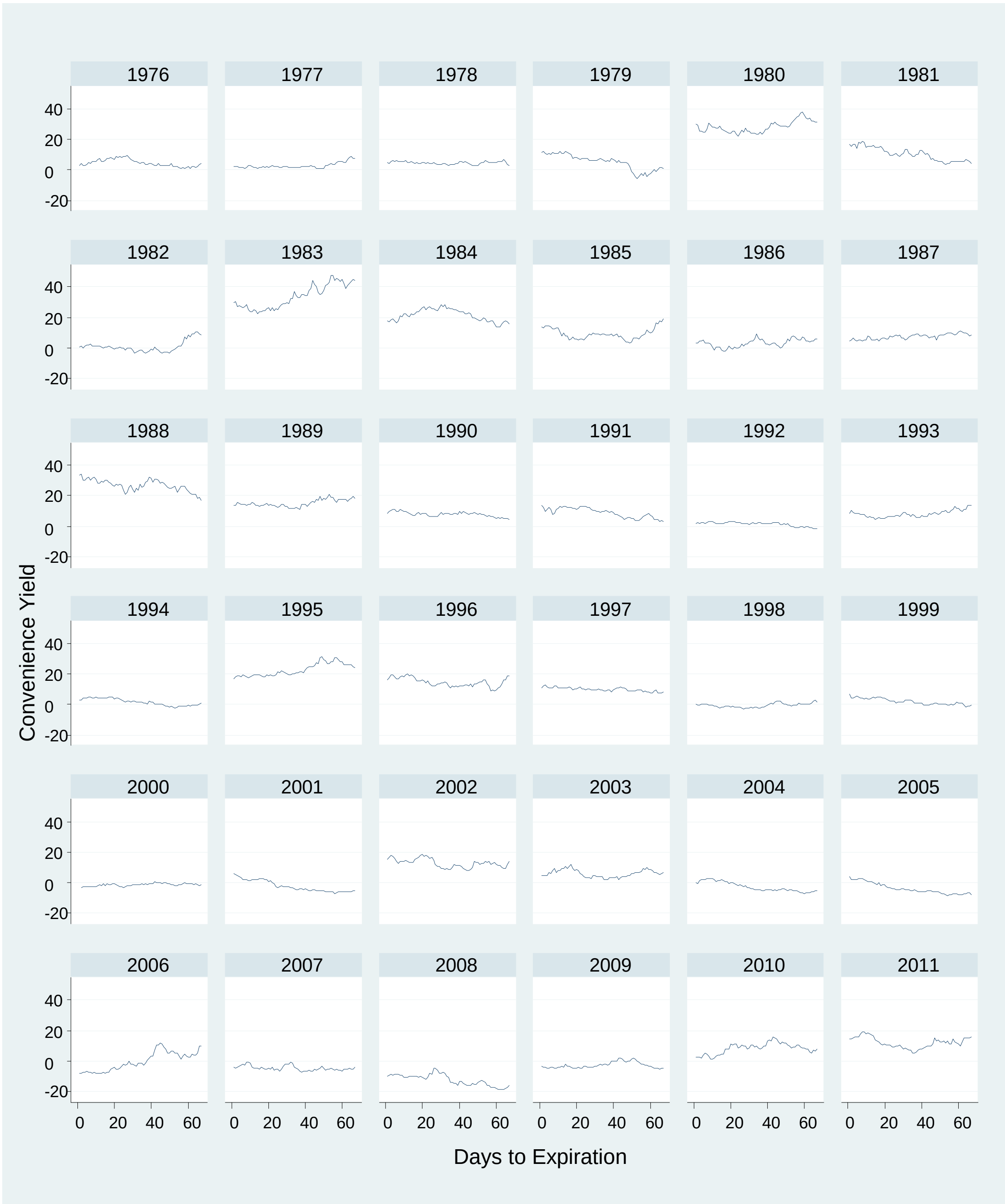


Table 6. Panel Unit Root Tests in Corn Dec-Mar Futures Spread and Convenience Yield, (1975-2006)

Variable	3-Month Treasury Bill (TB)	Prime
Dec-Mar Futures Spread		
Im-Pesaran-Shin Test	-1.39 (0.08)	-1.40 (0.08)
Fisher-type unit-root test	-0.40 (0.35)	0.39 (0.35)
Dec-Mar Implicit Convenience Yield		
Im-Pesaran-Shin Test	0.91 (0.18)	-1.17 (0.12)
Fisher-type unit-root test	-0.28 (0.61)	-0.35 (0.64)
Mar-May Futures Spread		
Im-Pesaran-Shin Test	-	-
Fisher-type unit-root test	0.31 (0.38)	-0.83 (0.79)
Mar-May Implicit Convenience Yield		
Im-Pesaran-Shin Test	-	-
Fisher-type unit-root test	1.36 (0.09)	0.33 (0.37)
Mar-July Futures Spread		
Im-Pesaran-Shin Test	-1.50 (0.07)	-
Fisher-type unit-root test	-0.68 (0.75)	-0.68 (0.75)
Mar-July Implicit Convenience Yield		
Im-Pesaran-Shin Test	-0.36 (0.36)	-0.50 (0.31)
Fisher-type unit-root test	-0.64 (0.74)	-0.45 (0.67)
July-Dec Futures Spread		
Im-Pesaran-Shin Test	-0.12 (0.45)	-0.12 (0.45)
Fisher-type unit-root test	1.28 (0.09)	1.28 (0.09)
July-Dec Implicit Convenience Yield		
Im-Pesaran-Shin Test	-0.06 (0.48)	-0.31 (0.38)
Fisher-type unit-root test	0.85 (0.19)	1.29 (0.09)
Dec-July Futures Spread		
Im-Pesaran-Shin Test	0.22 (0.59)	-0.03 (0.49)
Fisher-type unit-root test	-0.57 (0.72)	-0.48 (0.68)
Dec-July Implicit Convenience Yield		
Im-Pesaran-Shin Test	0.95 (0.83)	0.78 (0.78)
Fisher-type unit-root test	-1.72 (0.96)	-1.43 (0.92)

Note: The null hypothesis is that panels contain a unit root and thus the null hypothesis is not rejected using any of the tests. Numbers in parentheses indicate p-values.

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