Explaining Production Heterogeneity By Contextual Environments: Two-Stage DEA Application to Technical Change Measurement

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Abstract: One of the most important objectives in efficiency analysis is to investigate the relationships between production decisions and their contextual environments like geographical regions, production time periods, modes of production, or policies and regulations. Using the measurement of technical change as a template, the study presents a general framework to better understand varying production decisions under different time periods by showing how such production heterogeneity can be attributable to the differences in time-specific technological frontiers at industry level and the differences in the prevalence of technical inefficiency at producer level. In DEA, a leading non/semi-parametric frontier estimation method, these differences can be analyzed through decomposing Malmquist productivity index (MPI) into technical change (TC) and technical efficiency change (TEC) respectively. The decomposition approach falls into the non-Hicks-neutral TC estimation as the mean distance measures among time-specific frontiers, which is generally less restrictive than the Hicks-neutral TC estimation as an intertemporal-shift component of the frontier specification under fixed substitution patterns across time periods. The method is more generally applicable to the comparisons between any two different contextual environments, including before and after a policy intervention, by which a sample can be partitioned. To make the existing method more empirically accessible and appealing, the study proposes a regression-based MPI decomposition that overcomes its limitations, or the need of balanced panel data and the lack of control for potentially confounding non-production factors. The proposed methodology is demonstrated with an empirical application using data from the Schedule F Tax returns of 62 dairy farmers in Maryland during 1995-2009. For conventional, confinement dairy operations, the preliminary results under preferred specifications show a 26.4%/decade expansion in technological frontier, accompanied by a 6.3%/decade decline in the mean technical efficiency levels (i.e. increases in the prevalance of technical *inefficiencies*). The indicators for farm ownership and off-farm income are associated with a 4.5% increase and a 5.8% decrease in technical efficiency respectively. Higher seasonal rainfalls and temperatures, except for winter rainfall and summer temperature, are associated with larger technical feasibility in a given year.

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1 Introduction

One of the most important objectives in efficiency analysis is to investigate the relationships between production decisions and their contextual environments like geographical regions, production time periods, modes of production, or certain phases of policies and regulations. For example, production decisions may systematically differ between two time periods due to the difference in technical feasibility at industry level and the difference in the prevalence of technical inefficiency at producer level. In data envelopment analysis (DEA),[1] these differences can be analyzed through decomposing Malmquist productivity index (MPI) into technical change (TC) and technical efficiency change (TEC) respectively. The method is more generally applicable to the comparisons between any two different contextual environments, including before and after a policy intervention, by which a sample can be partitioned. Despite the variety of issues in which this method may offer new insights into the heterogeneity among production decisions, such applications are rare.

To enhance its appeal, the current study considers an alternative to the standard MPI decomposition technique. A major drawback of the existing method is the requirement for having balanced panel data in order to compute MPI and its components at individual producer level before summarizing them for industry-level averages. Balanced panel dataset is often unavailable for micro-data on production decisions, and if any, it may not be representative since it does not account for entries and exits of firms over time. To address this limitation, the study proposes a regression-based MPI decomposition (akin to ANOVA), in which the mean estimates of MPI, TC, and TEC can be obtained using unbalanced panel data or repeated cross-sections data. Intuitively, since TEC measures intertemporal change in inefficiency, the mean TEC can be obtained as the difference in regression-means where the regression estimates the time-specific mean prevalences of technical inefficiency. Also, since MPI measures intertemporal change in *productivity*, the mean MPI is similarly obtained when using pseudo-technical inefficiency measured against a common, enveloping frontier (i.e. a meta-frontier) of all time periods. The difference in the mean MPI and the mean TEC then isolates the mean intertemporal shift in technological frontier, or the mean TC. Moreover, the regression-based MPI decomposition allows the researcher to control for the intertemporal trends in producer-specific characteristics like compositions in age groups or education levels of employees and the time-specific shocks in weather or market outcomes that may confound the estimates of TEC and TC respectively.[2]

[1] DEA (e.g. Charnes, Cooper, and Rhodes, 1978) is the leading nonparametric approach to efficiency analysis.

[2] In the standard MPI decomposition defined at producer level, the measurement of TC is independent of producer-specific characteristics while MPI and TEC are not. The standard MPI, TEC, and TC measures all do not account for time-specific characteristics.

The mechanism is very simple and intuitive; the method compares efficiency and frontier levels for their time-specific means while controlling for producer-specific characteristics, and those means can be further orthogonally projected against time-specific factors.

The proposed method can be seen as a variant of the popular, second stage statistical analysis that investigates the determinants of estimated technical inefficiency. In the two-stage DEA procedure, observed input-output bundles are first evaluated for technical inefficiency, and then predicted technical inefficiencies are analyzed for the systematic correlations with so-called environmental factors (i.e. shifters for the underlying distribution of technical inefficiency). The current method builds on this procedure. In a first-stage DEA, context-specific technological frontiers and the enveloping meta-frontier categorize observed production heterogeneity into frontier gaps and technical inefficiencies at observation level. In a second-stage regression analysis, the efficiency estimates from the first stage are sorted out for the mean differentials at sample level. Viewing multiple time periods as different contextual environments, the study estimates the mean MPI, TEC, and TC measures while controlling the confounding influences of non-production factors on productivity, inefficiency, and technological frontier gaps respectively.

To ascertain a sound foundation for the proposed methodology, the study provides an alternative interpretation for the two-stage DEA procedure, which is often criticized for the lack of a clear relationship to the data generating process (DGP), and as such for the poor statistical inferences of its coefficients estimates in the second stage. Today, its leading statistical interpretation is found in Simar and Wilson (2007), yet the use of the two-stage procedure is still cautioned (e.g. Simar and Wilson, 2011b). The discussions in appendix A[3] explore its statistical coherence with the DGP using the framework of Kuosmanen (2008), in which DEA frontier is cast as a sign-constrained convex nonparametric least squares (CNLS) for the single-output case. It is shown that feasible estimation procedure and the corresponding assumption for the DGP generally depend on the functional relationship between inefficiency measurement (i.e. either additive or multiplicative) and the determinants of inefficiency. An additive inefficiency structure with linear marginal effects (of environmental factors) and a DEA frontier can be jointly estimated by quadratic programming since the inefficiency and frontier specifications are both linear in parameters. On the other hand, multiplicative inefficiency structure with proportional marginal effects can be coherently estimated in above two-stage DEA procedure; provided that the marginal effects dissipate toward zero at the frontier (i.e. at zero-inefficiency), the non-linear

[3]For the one-output, multiple-input production decisions case, appendix A contrasts two major approaches to frontier modeling, or SFA and DEA, with additive and multiplicative technical inefficiency measurements to highlight functional relationships in the frontier modeling. Its contents share many aspects with the main article but are discussed using a different framework to better elaborate implications from various specification assumptions.

estimation can be replaced with two separate, sequential linear estimations for the frontier and inefficiency structures (with a log-transformation of inefficiency scores between the two). Statistical inferences in these deterministic frontier models differ from those of the stochastic frontier models where the DGP is directly modeled based on the presumed distributional structure. However, they can be legitimately viewed in light of the goodness of fit in the least squares framework.

The proposed methodology is demonstrated in an empirical application with Maryland dairy production data during 1995-2009. Two groups of dairy operations are separately analyzed: conventional confinement operations and intensive (rotational) grazing operations. The preliminary results under preferred specifications show 26.4%/decade and 19.2%/decade expansions in the technological frontiers of confinement and grazing operations respectively, accompanied by 6.3%/decade and 14.4%/decade declines in the mean technical efficiency levels (i.e. increases in the prevalance of technical *inefficiencies*). Among confinement dairies, the indicators for farm ownership and off-farm income are associated with a 4.5% increase and a 5.8% decrease in technical efficiency respectively. Higher seasonal rainfalls and temperatures, except for winter rainfall and summer temperature, are associated with larger technical feasibility in a given year. In measuring technical change in agriculture, it is essential to control for random factors like weather conditions that could account for shifts in a technological frontier.

Lastly, the study briefly elaborates a broader scope for the proposed methodology to integrate contextual information in an efficiency analysis. In general, the measurement of technical change can serve as a template for a general conceptual framework for utilizing the information on production environments through two distinctive channels, or technical feasibility and technical efficiency. That is, in relation to specific contextual information like different phases of policies and regulations, observed heterogeneity in production decisions can be analyzed through the average shifts in context-specific frontiers and the changes in the prevalence of technical inefficiency. The current study contributes to this perspective by making the concept of MPI decomposition empirically more accessible and appealing for the targeted distance measurement in the input-output space.

The rest of the study proceeds as follows. Section 2 discusses how the current methodology fits to the general context of measuring technical change. After introducing notations and preliminary concepts, section 3 describes the proposed methodology as a regression-based MPI decomposition.[4] The methodology is demonstrated with an empirical application to the data on

[4] More general discussion on the joint modeling of a technological frontier and technical inefficiency is provided in section Appendix A. Also, a simple extension to group-specific MPI decompositions is shown in section B.

dairy production in Maryland in section **??** and discussed for its broader applications in section 5. Finally, section 6 concludes the study.

2 Measurement of Technical Change

Technical change (TC) refers to intertemporal shifts of a technological frontier. There are two major classifications to define its measurement, depending on the restrictions placed on such shifts.[5] Under Hicks-neutral (hereafter "Hicksian") TC, it is assumed that the intertemporal shifts do not alter the marginal rates of transformations (MRTs) among inputs and outputs.[6] The assumption allows a direct specification of the intertemporal frontier shifts as an integral part of the frontier estimation. On the other hand, without Hicks-neutrality ("non-Hicksian"), separate time-specific frontiers specification allows the predicted substitution patterns to vary across time periods. In below, it is shown that summary non-Hicksian TC measures can be obtained in a second-stage regression analysis as the mean distances among the estimated time-specific frontiers.

Figure 1 illustrates such Hicksian and non-Hicksian TC in the space of two inputs x_1, x_2 where a frontier (as a solid curve) represents the MRT between the two inputs holding outputs and other inputs constant. Given a frontier F_{t0} of time period $t0$, the postulates of the corresponding frontier F_{t1} in period $t1$ under Hicksian and non-Hicksian TC are depicted in relation to the substitution patterns of F_{t0}. In this example, the Hicksian TC takes the form of an proportional contraction from the origin while the non-Hicksian TC represents a contraction with unrestricted changes in the substitution patterns. The direct measurement of Hicksian TC is obtained (in the estimation process) by restricting local distances between the time-specific frontiers to be at a constant proportion of one another along the frontier curvature. Meanwhile, the indirect non-Hicksian measurement is obtained (after the frontier estimations) by taking the mean of such local distances as a summary of intertemporal frontier relationships.

In parametric frontier models like stochastic frontier analysis (SFA: Aigner, Lovell, and Schmidt, 1977; Meeusen and Broeck, 1977), the researcher typically estimates a frontier model with Hicksian TC. The main advantage of Hicksian TC is the immediate interpretation of the frontier shifts as an integral part of a statistical model for the data generating process. The

[5] Aside from the literature focusing on technical change, there is a large literature on the changes in productivity (and sources of growth) Bartelsman and Doms (e.g. see 2000); Syverson (e.g. see 2011). In the context of MPI decomposition, the study on the determinants of productivity can be seen as a special case under full technical efficiency. The two strands of literature are complementary with offering important insights to one another, yet the MPI decomposition into TC and TEC may yield more-detailed diagnosis on the change in productivity Jerzmanowski (e.g. 2007).

[6] Strictly speaking, there are several types of Hicksian TC, depending on the technological, homothetic restrictions among inputs, outputs, and a time index. See Chambers and Fre (1994) for its details.

assumption of time-invariant substitution patterns can be easily tested as a hypothesis on parametric restrictions. If the time-invariance is rejected, the researcher should employ the indirect measures of technical change (see Appendix A). Such a hypothesis testing and frontier estimation must proceed with caution, for the model identification heavily relies on the complex mutual interactions between the assumed functional forms (i.e. technological frontier, technical inefficiency, and their intertemporal-shift structures) and the assumed distribution for the composite error (i.e. technical inefficiency and stochastic noise components). With the lack of economic theory to guide specification choices on all these accounts, the chance of misspecification can be prohibitively high.

One way to mitigate this concern is to measure TC in Data Envelopment Analysis (DEA), a leading methodology for non/semi-parametric frontier modeling. Its variable returns to scale (VRS) frontier is solely built on the monotonicity and convexity of technical feasibility without any presumed functional form or arbitrary distributional assumption on technical inefficiency. For non/semi-parametric frontier models like DEA, frontier estimation under Hicksian TC is generally infeasible. It is because in the absence of distributional assumptions, the proportional shifts in the technological frontier (i.e. TC) and the proportional shifts in the prevalence of technical inefficiency (i.e. technical efficiency change :TEC) are indistinguishable from each other.[7] Instead, indirect, non-Hicksian TC measure can be obtained through Malmquist Productivity Index (MPI) decompositions (Nishimizu and Page, 1982).[8] MPI (Caves, Christensen, and Diewert, 1982a,b) is a generalization of Törnqivist index by allowing technical inefficiency (Fare et al., 1994)[9] and closely related to other important productivity indices like Fisher's productivity index and its variants (e.g. see Grosskopf, 2003).[10] Once time-specific frontiers are estimated, the researcher can calculate MPI and its decomposition into TC and TEC at producer level, which can be averaged for the whole sample. The most well-known applications include Fare et al. (1994), Kumar and Russell (2002), Timmer and Los (2005), and Fare, Grosskopf, and Margaritis (2006). As previously noted, the major drawbacks of the existing procedure are the requirement for balanced panel data and the lack of control for potentially confounding non-production fac-

[7]In DEA, Hicksian TC may be implemented only under certain functional forms of intertemporal structures in TC and TEC.

[8]In the context of the growth accounting literature, Nishimizu and Page (1982) first introduced the MPI decomposition into TC and TEC (, equivalently technical catch-up,) as a mechanism to generalize a Solow's model with a new source of growth that a less developed economy catches up to more advanced economies through adopting the latter group's technological innovations and social institutions.

[9]Under CRS assumption and full technical (and allocative) efficiency, MPI and Törnqivist index, a discrete approximation to a continuous Divisia index, are equivalent.

[10]The ratio of a Malmquist output quantity index to a Malmquist input quantity index yields a comparable measure to Törnqivist, Fisher, and its variants. These indices are suited for isolating input and output contributions to productivity change. See also Fare, Grosskopf, and Russell (1998).

tors. The proposed methodology below overcomes these drawbacks through a regression-based MPI decomposition.

3 The Model

3.1 Preliminaries

A production *technology* is a set of feasible input-output bundles, denoted by $F = \{(\boldsymbol{x}, \boldsymbol{y}) \in \mathbb{R}_+^L \times \mathbb{R}_+^M$: inputs $\boldsymbol{x}$ can produce outputs $\boldsymbol{y}\}$. The boundary of set F is called a *technological frontier* or *production function*. Technical efficiency is measured with respect this boundary, so that technology F and its boundary can be interchangeably referenced throughout the study. The technology specific to each time period $t \in \{1,..,T\}$ is denoted by $F_t = \{\forall(\boldsymbol{x}, \boldsymbol{y}) \in \mathbb{R}_+^L \times \mathbb{R}_+^M$: $\boldsymbol{x}$ can produce $\boldsymbol{y}$ in time $t\}$. Each time-specific technology F_t is assumed to satisfy the following properties: (a) feasible inaction $((\mathbf{0}, \mathbf{0}) \in F_t)$, (b) monotonicity ($(\boldsymbol{x}, \boldsymbol{y}) \in F_t$, $(-\boldsymbol{x}', \boldsymbol{y}') \leq (-\boldsymbol{x}, \boldsymbol{y}) \Rightarrow (\boldsymbol{x}', \boldsymbol{y}') \in F_t$), and (c) convexity $((\boldsymbol{x}, \boldsymbol{y}), (\boldsymbol{x}', \boldsymbol{y}') \in F_t$, $\lambda \in [0,1] \Rightarrow \lambda(\boldsymbol{x}, \boldsymbol{y}) + (1-\lambda)(\boldsymbol{x}', \boldsymbol{y}') \in F_t)$. The collection of such time-specific technologies is referred to as meta-technology $F = \cup_t F_t$, or a hypothetical technology that envelops subsample-specific technologies (e.g. Bhattacharjee, 1955; Griliches, 1964; Salter, 1966; Krueger, 1968; Hayami and Ruttan, 1970).

To represent an empirical case, consider data set $\{(\boldsymbol{x}_{it}, \boldsymbol{y}_{it})\}_{it \in IT}$ where subscript it denotes an index of observations $it \in \mathbb{IT} = \{11, ..1T, .., IT\}$ for producer $i \in \mathbb{I} = \{1, .., I\}$ and time $t \in \mathbb{T} = \{1, .., T\}$. Observations partitioned by time periods are referred to by time-specific subsample index $\mathbb{IT}(k) = \{it |\, t = k\}$ that contains the observations in period k. The number of observations in $\mathbb{IT}(k)$ is denoted by N_k, which sums to the original N observations across time periods (i.e. $\sum_k N_k = N$). Then, the time-specific technology F_k of time k is constructed from a *subsample* of observations $it \in \mathbb{IT}(k)$. For example, given assumptions (a)-(c) on technologies F_t, $\forall t$, the DEA approximation to technology F_t under non-increasing returns to scale (NIRS) is the following free-disposable convex hull, including the origin;[11]

$$\begin{aligned} \forall k,\ \widehat{F}_k = &\{(\boldsymbol{x}', \boldsymbol{y}') \in \mathbb{R}_+^L \times \mathbb{R}_+^M : \sum_{j \in \mathbb{IT}(k)} \lambda_j \leq 1, \\ &\sum_{j \in \mathbb{IT}(k)} \lambda_j \boldsymbol{x}_j \leq \boldsymbol{x}',\ \sum_{j \in \mathbb{IT}(k)} \lambda_j \boldsymbol{y}_j \geq \boldsymbol{y}',\ \boldsymbol{\lambda} \in \mathbb{R}_+^{N_k}\}, \end{aligned} \tag{1}$$

[11] In the absence of assumption (a), variable returns to scale (VRS) is commonly used under $\lambda_j = 1$. However, NIRS is desirable here for assessing output-oriented technical inefficiencies to avoid undefined technical inefficiency measures under hypothetical production contexts (for example, see figure 3). Alternatively, constant returns to scale (CRS) may be used by setting $\sum \lambda_j \in \mathbb{R}$.

for which the approximation to meta-technology F is given by;

$$\begin{aligned}\widehat{F} =&\{(\boldsymbol{x}', \boldsymbol{y}') \in \mathbb{R}^L_+ \times \mathbb{R}^M_+ : \forall k = 1, .., T, \sum_{j\in \mathbb{IT}(k)} \lambda_j \leq 1, \\ &\sum_{j\in \mathbb{IT}(k)} \lambda_j \boldsymbol{x}_j \leq \boldsymbol{x}', \sum_{j\in \mathbb{IT}(k)} \lambda_j \boldsymbol{y}_j \geq \boldsymbol{y}', \ \boldsymbol{\lambda} \in \mathbb{R}^N_+\}. \end{aligned} \tag{2}$$

Technology approximations $\widehat{F}_k$, $\widehat{F}$ are obtained from a subsample of N_k observations and the whole sample of N observations respectively.[12]

For a given input-output combination $(\boldsymbol{x}_0, \boldsymbol{y}_0)$, the output-oriented radial efficiencies against time-specific technology F_t and against meta-technology F are defined by the distance functions of Farrell (1957);[13]

$$\begin{aligned}\phi(\boldsymbol{x}_0, \boldsymbol{y}_0; t) &= \inf\{\phi : \ (\boldsymbol{x}_0, \boldsymbol{y}_0/\phi) \in F_t\}, \\ \phi(\boldsymbol{x}_0, \boldsymbol{y}_0; \mathbb{T}) &= \inf\{\phi : \ (\boldsymbol{x}_0, \boldsymbol{y}_0/\phi) \in F\} \end{aligned} \tag{3}$$

where technical efficiency (TE) score ϕ represents the maximal expansion of outputs within the technical feasibility; the higher a TE score is, the higher the evaluation for the observed output level is relative to what is technically feasible. Technical efficiency ϕ takes a value in $(0, 1]$ if $(\boldsymbol{x}_0, \boldsymbol{y}_0)$ is technically feasible, with $\phi = 1$ being fully technically-efficient. Formally, the efficiency ϕ should be set at $-\infty$ if $(\boldsymbol{x}_0, \boldsymbol{y}_0)$ is infeasible, yet the current study allows for ϕ to be greater than one in the efficiency assessments under hypothetical production contexts; for example, production decisions observed in later time periods can outperform what is technically feasible at that time.[14] Such treatment is used in calculating the standard Malmquist productivity index (MPI) (see below), in which the production decisions of one period are evaluated against the time-specific frontier of another period. Substituting technologies F_t, F with their estimates $\widehat{F}_t$, $\widehat{F}$ like those in (1), (2) yields empirical TE measures $\widehat{\phi}(\boldsymbol{x}_0, \boldsymbol{y}_0; t)$, $\widehat{\phi}(\boldsymbol{x}_0, \boldsymbol{y}_0)$. Note that the time-specific and meta- technologies estimated by equations (1), (2) imply the empirical relationship $\widehat{\phi}(\boldsymbol{x}_i, \boldsymbol{y}_i; \mathbb{T}) = \inf_t\{\widehat{\phi}(\boldsymbol{x}_i, \boldsymbol{y}_i; t)\}$.[15]

[12]Under the constant returns to scale (CRS) assumption, meta-technology $\hat{F}$ is simply the standard free-disposable convex cone applied to pooled observations. Under CRS, the sum of weights (e.g. $\sum_j \lambda_j$) is unrestricted; $F_k = \{\forall(\boldsymbol{x}', \boldsymbol{y}') : \sum_{j\in\{IT\}(k)} \lambda_j \boldsymbol{x}_j \leq \boldsymbol{x}', \ \sum_{j\in\{IT\}(k)} \lambda_j \boldsymbol{y}_j \geq \boldsymbol{y}', \ \boldsymbol{\lambda} \in \mathbb{R}^{N_k}_+\}$, $F = \{\forall(\boldsymbol{x}', \boldsymbol{y}') : \sum_{j\in\{IT\}} \lambda_j \boldsymbol{x}_j \leq \boldsymbol{x}', \ \sum_{j\in\{IT\}} \lambda_j \boldsymbol{y}_j \geq \boldsymbol{y}', \ \boldsymbol{\lambda} \in \mathbb{R}^N_+\}$.

[13]Farrell's measure is a special case of the directional distance function (Luenberger, 1992; Chambers, Chung, and Fare, 1996) where technical efficiency relates to observed input-output decision multiplicatively, and its direction and unit is taken to be proportional to the observed output vector. The radial distance measure is well-suited for capturing the most common notion of technical change, or proportional output growth.

[14]The exception is that in the case of output-oriented efficiency, no efficiency evaluation is available (e.g. $\phi = -\infty$) if the input level takes an extreme value and lies outside of the specified technology.

[15]Substituting $\widehat{F} = \cup_t \widehat{F}_t$ implies $\hat{\phi}(\boldsymbol{x}_0, \boldsymbol{y}_0; \mathbb{T}) = \inf\{\phi : \ (\boldsymbol{x}_0, \boldsymbol{y}_0/\phi) \in \cup_t \widehat{F}_t\} = \inf_t\{\inf\{\phi : \ (\boldsymbol{x}_0, \boldsymbol{y}_0/\phi) \in \widehat{F}_t\}\} =$

The ratio of the two efficiency measurements in (3) defines a technology gap ratio (TGR) that measures the difference in technical feasibilities between the meta-frontier and the subsample-specific frontier(s) (Battese, 2002; Battese, Rao, and ODonnell, 2004).[16] The local measure of TGR for time t at point $(\boldsymbol{x}_0, \boldsymbol{y}_0)$ is given by;

$$TGR(\boldsymbol{x}_0, \boldsymbol{y}_0; t) = \phi(\boldsymbol{x}_0, \boldsymbol{y}_0; \mathbb{T})/\phi(\boldsymbol{x}_0, \boldsymbol{y}_0; t), \tag{4}$$

which represents the pseudo-technical efficiency of subsample-specific frontier F_t relative to meta-frontier F along the ray $(\boldsymbol{x}_0, \lambda\boldsymbol{y}_0)$, $\lambda \in \mathbb{R}$. Figure 2 depicts how point A at $(\boldsymbol{x}_0, \boldsymbol{y}_0)$ is projected to meta- and time-specific technologies $\widehat{F}$, $\widehat{F}_t$, where the projected points are labeled as B, C respectively. The similar projection to the X-axis is labeled as point Q. Given the radial efficiency measures in (3), TE is estimated by $\widehat{\phi}(\boldsymbol{x}_0, \boldsymbol{y}_0; t) = AQ/CQ$ relative to $\widehat{F}_t$, and $\widehat{F}_t$ is evaluated by $\widehat{TGR}(\boldsymbol{x}_0, \boldsymbol{y}_0; t) = CQ/BQ$ relative to $\widehat{F}$.

The central idea in this study is to exploit the conceptual similarities between TGR and technical change (TC) as measurements of between-frontier distances. Pseudo-technical efficiency $\phi(.; \mathbb{T})$ measured against the meta-frontier can be viewed as a productivity measure that is commonly applicable to the observations of different time periods. Comparing the time-specific means of this productivity is analogous to calculating the Malmquist productivity index (MPI). Then, analogously to the MPI decomposition into technical efficiency change (TEC) and TC, in below productivity measure $\phi(.; \mathbb{T})$ are decomposed into within-time efficiency $\phi(.; t)$, or the distance between the observed decision and time-specific frontier, and between-time frontier gap $\phi(.; \mathbb{T})/\phi(.; t)$, or the distance between the time-specific and meta- frontiers. Then, comparing the time-specific means of efficiencies $\phi(.; t)$ and frontier gaps $\phi(.; \mathbb{T})/\phi(.; t)$ across time periods yields alternative measures of TEC and TC respectively. These comparisons of means are carried out in a second-stage regression framework on the estimated pseudo-technical and technical efficiency scores, in which the researcher can control for potentially confounding non-production factors. The proposed model is formally introduced in the next section after a belief description of the standard MPI calculation and its decomposition.

$\inf_t\{\hat{\phi}(\boldsymbol{x}_0, \boldsymbol{y}_0; t)\}$.

[16] Recent applications of TGR in empirical contexts include the productivity comparisons of aggregate agricultural outputs across 97 countries (ODonnell, Rao, and Battese, 2008), banking industries in China and Taiwan during 1993-2007 (CHEN and SONG, 2008), and farm-level dairy production in Argentina, Chile, and Uruguay (Moreira and Bravo-Ureta, 2010). While most of these applications use SFA, ODonnell, Rao, and Battese (2008) formalize the concept of meta-technology with distance function and apply it with both DEA and SFA.

3.2 Regression-Based Productivity Index and Technical Change

One common measure of productivity growth is Malmquist productivity index (MPI) by Caves, Christensen, and Diewert (1982a,b), which compares the efficiency measurements of observations from two different time periods, say $\{t0,\, t1\}$, using the frontier of either period as a base time period. The calculation involves efficiency assessments under hypothetical production contexts in the sense that observation $(\boldsymbol{x}_{t1}, \boldsymbol{y}_{t1})$ in period $t1$ is evaluated against the technology of period $t0$ and *vice versa*. The MPI measures of productivity growth with base time period $t \in \{t0,\, t1\}$ are defined as;

$$\begin{aligned} MPI_{t0}(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}, \boldsymbol{x}_{t1}, \boldsymbol{y}_{t1}) &= \phi(\boldsymbol{x}_{t1}, \boldsymbol{y}_{t1}; t0)/\phi(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}; t0) \\ MPI_{t1}(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}, \boldsymbol{x}_{t1}, \boldsymbol{y}_{t1}) &= \phi(\boldsymbol{x}_{t1}, \boldsymbol{y}_{t1}; t1)/\phi(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}; t1). \end{aligned} \tag{5}$$

In place of an arbitrary choice between MPI_{t0} and MPI_{t1}, researchers often use the geometric mean of the two as suggested by Fare et al. (1994);

$$MPI_{t0,t1}(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}, \boldsymbol{x}_{t1}, \boldsymbol{y}_{t1}) = \left[MPI_{t0}(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}, \boldsymbol{x}_{t1}, \boldsymbol{y}_{t1}) \cdot MPI_{t1}(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}, \boldsymbol{x}_{t1}, \boldsymbol{y}_{t1})\right]^{1/2}. \tag{6}$$

A common use of MPI is to consider a decomposition into technical efficiency change (TEC) and technical change (TC).[17] The standard MPI decomposition into TEC and TC are given by;

$$\begin{aligned} MPI_{t0,t1}(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}, \boldsymbol{x}_{t1}, \boldsymbol{y}_{t1}) &= TEC_{t0,t1}(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}, \boldsymbol{x}_{t1}, \boldsymbol{y}_{t1}) \cdot TC_{t0,t1}(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}, \boldsymbol{x}_{t1}, \boldsymbol{y}_{t1}) \\ TEC_{t0,t1}(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}, \boldsymbol{x}_{t1}, \boldsymbol{y}_{t1}) &= \phi(\boldsymbol{x}_{t1}, \boldsymbol{y}_{t1}; t1)/\phi(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}; t0) \\ TC_{t0,t1}(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}, \boldsymbol{x}_{t1}, \boldsymbol{y}_{t1}) &= \left(\frac{\phi(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}; t0)}{\phi(\boldsymbol{x}_{t1}, \boldsymbol{y}_{t1}; t1)} \frac{\phi(\boldsymbol{x}_{t1}, \boldsymbol{y}_{t1}; t0)}{\phi(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}; t1)}\right)^{1/2}. \end{aligned} \tag{7}$$

TEC is the ratio of technical efficiency measurements for two observed decisions in the two time periods, where each decision is evaluated against the corresponding time-specific frontier. TC is (the geometric mean of) the relative distance between the two frontiers along two rays $(\boldsymbol{x}_{t0}, \lambda_0 \boldsymbol{y}_{t0})$, $(\boldsymbol{x}_{t1}, \lambda_1 \boldsymbol{y}_{t1})$, $\forall \lambda_0,\, \lambda_1 \in \mathbb{R}_+$.

Figure 3 illustrates these measurements. Observations $A : (\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0})$, $A' : (\boldsymbol{x}_{t1}, \boldsymbol{y}_{t1})$ from two time periods $t0$, $t1$ are projected to two time-specific frontiers $\widehat{F}_{t0}$, $\widehat{F}_{t1}$, where the projected points are labeled as B, C for point A and B', C' for point A' respectively. Then, the measures

[17] MPI decomposition may include scale efficiency change (SEC), which is discussed in the following section.

of MPI, TEC, and TC for points A, A' are;

$$\widehat{MPI}_{t0,t1} = \left[\frac{A'Q'/C'Q'}{AQ/CQ}\frac{A'Q'/B'Q'}{AQ/BQ}\right]^{1/2}, \ \widehat{TEC}_{t0,t1} = \frac{A'Q'/B'Q'}{AQ/CQ}, \ \widehat{TC}_{t0,t1} = \left[\frac{BQ}{CQ}\frac{B'Q'}{C'Q'}\right]^{1/2}. \tag{8}$$

The calculations help visualize that TEC and TC are the intertemporal differences in technical efficiencies and technological frontiers respectively.

At sample level, MPI, TC, and TEC are commonly summarized as the means of producer-level estimates. For example, given a balanced panel data set with producer index $j = 1, .., J$ for J producers, the sample-mean technical change from time $t0$ to time $t1$ is defined as; $E[TC_{t0,t1}] = \frac{1}{J}\sum_j TC_{t0,t1}(\boldsymbol{x}_{j,t0}, \boldsymbol{y}_{j,t0}, \boldsymbol{x}_{j,t1}, \boldsymbol{y}_{j,t1})$. Note that such estimates require balanced panel data to initially calculate producer-level estimates for two time periods $\{t0, t1\}$.

Alternatively, sample-level estimates can be obtained in a second-stage statistical analysis on estimated technical efficiencies. For instance, consider regression-average MPI decompositions using the pseudo-technical efficiencies measured against a meta-frontier and technical efficiencies measured against time-specific frontiers. The case under two time periods $t \in \{t0, t1\}$ can be estimated in the following specifications for technical efficiency measurements $\widehat{\phi}(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}; \mathbb{T})$, $\widehat{\phi}(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}; t)$, and TGR $\widehat{\phi}(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}; \mathbb{T})/\widehat{\phi}(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}; t)$ without constant terms;

$$\begin{aligned}
\ln \widehat{\phi}(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}; \mathbb{T}) &= \tau_t^M + \varepsilon_{it}^M, \\
\ln \widehat{\phi}(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}; t) &= \tau_t^s + \varepsilon_{it}^s, \quad \text{and mechanically} \\
\ln(\widehat{\phi}(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}; \mathbb{T})/\widehat{\phi}(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}; t)) &= \tau_t^{M-s} + \varepsilon_{it}^{M-s} \\
\text{with } \tau_t^{M-s} = &\tau_t^M - \tau_t^s, \ \varepsilon_{it}^{M-s} = \varepsilon_{it}^M - \varepsilon_{it}^s
\end{aligned} \tag{9}$$

where time-fixed effects $\tau_t^M, t \in \{t0, t1\}$ are implicit coefficients for time period indicators $\mathbb{1}_t(t = s)$ that take the value of one if $t = s$ and zero otherwise (i.e. $\tau_t^M \equiv \tau_{t0}^M \mathbb{1}_t(t = t0) + \tau_{t1}^M \mathbb{1}_t(t = t1)$). Superscripts M, s denote "meta" and "subsample" equations respectively, yielding $M - s$ equation as their difference. Note that estimating above equations involving only indicator variables are equivalent to estimating group-specific means under the analysis of variance (ANOVA). Specific assumptions on the error terms and method of statistical inferences are discussed later in the section. For each period $t \in \{t0, t1\}$, parameter τ_t^s represents the time-t mean technical efficiency measured against the corresponding time-specific frontier, and similarly parameter τ_t^{M-s} represents the time-t mean TRG measured against the meta-frontier. Due to the natural logarithm transformation on the dependent variables, parameters τ_t^s, τ_t^M predict a proportional marginal effect for a given output $\boldsymbol{y}_{it}$, and parameter τ_t^{M-s} similarly predicts a proportional effect for a given projected frontier-output $\boldsymbol{y}_{it}/\widehat{\phi}(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}; t))$.

Then, the simple differences in these regression-means of technical efficiency measurements yield regression-based MPI and its components;

$$\begin{aligned}
\ln E[MPI_{t0,t1}] &\equiv \tau_{t1}^M - \tau_{t0}^M \\
\ln E[TEC_{t0,t1}] &\equiv \tau_{t1}^s - \tau_{t0}^s \\
\ln E[TC_{t0,t1}] &\equiv \tau_{t1}^{M-s} - \tau_{t0}^{M-s}
\end{aligned} \tag{10}$$

where $E[.]$ is the expectation operator over relevant observations. Thus, the difference in the mean pseudo-technical efficiencies between two periods is interpreted as a regression-based MPI; the difference in the mean technical efficiencies between two periods yields a regression-based TEC; and the difference in the mean frontier gaps between two periods corresponds to a regression-based TC. These alternative estimates for mean MPI, TEC, and TC correspond to the ratio of means whereas the standard, sample-averages of producer-level estimates are based-on the means of ratios.[18] The two sets of sample-level estimates measure the same distance concepts in the same units among production decisions and frontiers and differ in the method of aggregation to sample-level estimates. As previously noted, one advantage over the conventional mean-of-ratios-estimators is that the ratio-of-means-estimators in equations (10) do not require balanced panel data since the time-specific means can be calculated without referencing to a particular producer.

More generally, second-stage analysis (9) can be specified for multiple time periods $t \in \{1, .., T\}$ with controlling for producer/observation-specific factors $\boldsymbol{z}_{it}$;[19]

$$\begin{aligned}
\ln \widehat{\phi}(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}; \mathbb{T}) &= \tau_t^M + \boldsymbol{z}_{it}\, \boldsymbol{\alpha}^M + \varepsilon_{it}^M, \\
\ln \widehat{\phi}(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}; t) &= \tau_t^s + \boldsymbol{z}_{it}\, \boldsymbol{\alpha}^s + \varepsilon_{it}^s, \text{ and mechanically} \\
\ln(\widehat{\phi}(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}; \mathbb{T})/\widehat{\phi}(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}; t)) &= \tau_t^{M-s} + \boldsymbol{z}_{it}\, \boldsymbol{\alpha}^{M-s} + \varepsilon_{it}^{M-s}, \\
\text{with } \boldsymbol{\tau}^{M-s} = \boldsymbol{\tau}^M - \boldsymbol{\tau}^s,\ \ &\boldsymbol{\alpha}^{M-s} = \boldsymbol{\alpha}^M - \boldsymbol{\alpha}^s,\ \ \varepsilon_{it}^{M-s} = \varepsilon_{it}^M - \varepsilon_{it}^s,
\end{aligned} \tag{11}$$

for which MPI decomposition for any two periods is similarly defined as in equations (10). In this way, compared to the standard MPI decomposition in (7), the regression-based approach

[18] If the true values of MPI, TEC, and TC are uniform across different segments of the frontiers, the two approaches to the sample-level estimators are both consistent, and their estimators approach to the same true values.

[19] The influences of time-specific factors $\boldsymbol{W}_t$ varying with time t cannot be distinctly identified from those of time-fixed effects τ_t in these regressions. Their influences on TC measurements are treated in a subsequent analysis (see below).

allows the researcher to control for producer characteristics like age, education, and other factors that may vary at observation level. The equation s in (11) represents a version of the commonly-used two-stage DEA procedure where estimated technical efficiency is regressed on so-called environmental factors $\boldsymbol{z}_{it}$. In such an analysis, coefficients $\boldsymbol{\alpha}^s$ are interpreted as the determinants of technical efficiency. Analogous interpretations for coefficients $\boldsymbol{\alpha}^M$, $\boldsymbol{\alpha}^{M-s}$ would be the determinants of productivity (i.e. pseudo-technical efficiency measure) and time-specific frontiers respectively.

The estimation in (11), like any other two-stage DEA procedures, is built on a so-called separability assumption, or the separability of "environmental" factors $\boldsymbol{z}_{it}$ from the production possibility F_t. To put it in econometrics terms, variables $\boldsymbol{z}_{it}$ are assumed to shift the underlying distribution of technical efficiency $\phi(\boldsymbol{x}, \boldsymbol{y}; t)$ without influencing time-specific technical feasibility F_t. An ideal specification is to use all variables $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$ simultaneously in a joint estimation of a technological frontier and technical efficiency. However, such a specification is difficult to estimate as it nonlinearly combines a non/semi-parametric piecewise-linear DEA frontier estimation with a parametric technical efficiency specification (see Appendix A[20]). Instead, the current two-stage procedure sequentially estimates the frontier F_t for input-output bundle $(\boldsymbol{x}, \boldsymbol{y})$ and then a structure of the estimated technical efficiency $\widehat{\phi}(\boldsymbol{x}, \boldsymbol{y}; t)$ with respect to variables $\boldsymbol{z}$. For the two stages of estimations to be coherent with the underlying data generating process (DGP) for variables $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$ and for the whole model to be statistically consistent, (1) the first-stage estimate of frontier F_t needs to remain consistent without using variables $\boldsymbol{z}_{it}$, and (2) the second-stage model of technical efficiency must conform with item (1) in an implicit joint model of variables $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$. The separability assumption could be reasonable only when these conditions are met.

To maintain item (1), it merits to clarify the variables that can be appropriately admitted as "environmental" factors $\boldsymbol{z}$ in the current two-stage DEA procedure. The term "production environment" often represents a catch-all for anything that may affect production decisions (and production possibilities) but are not under direct control of the producer in the way traditional production inputs are. It is safe to assume that some variables like producer age or education can affect production decisions but cannot affect production possibilities that are usually assumed to be given at industry level. Variables that may shift production possibilities like weather conditions in agriculture are ideally incorporated as a part of technology specification (e.g. non-discretionary inputs), but it is not always possible. This happens precisely in situations where

[20] In fact, combining the two components entirely in a linear fashion allows a feasible joint estimation by quadratic programming.

those variables do not vary across producers in a given context (like a given time period or given phase of a policy). In essence, the current methodology is a way to overcome this lack of producer/observation-level variation in non-production factors (to be directly incorporated in a technology specification); instead, some of those variables can be used to define distinct contextual environments for production decisions, and others can be used to characterize them. That is, the current estimation strategy is to identify context-specific frontiers and characterize their relative performances, as opposed to imposing *a priori* functional relationships between a technological frontier and contextual environmental variables. In the case of technical change, it estimates time-specific frontiers and efficiencies and then studies the effects of time-specific variables (e.g. weather outcomes varying across time periods but not across producers) by regressing the estimated time specific-means τ_t^q's in (11) on those variables (e.g. see below).

Item (2) calls for the coherent, underlying relationships among all variables $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$ in the model. In the first stage, variables $\boldsymbol{z}$ are assumed to potentially influence production decisions $(\boldsymbol{x}, \boldsymbol{y})$ through technical efficiency $\phi(.;t)$ but not technical feasibility F_t, implying that time-specific frontier F_t is considered independent of variables $\boldsymbol{z}$. Then, one way to ensure a coherent estimation in the second stage to require the functional relationship between estimated technical efficiency $\widehat{\phi}(.;t)$ and variables $\boldsymbol{z}$ to diminish at the estimated frontier. This happens at the full technical efficiency, or $\widehat{\phi}(.;t) = 1$. The current specification in (11) conforms with such a functional relationship by employing the *proportional* marginal effects of environmental factors $\boldsymbol{z}$ on technical efficiency where all the marginal effects are identically zero locally at $\widehat{\phi}(.;t) = 1$.[21]

Now, some time-specific factors, which cannot be directly incorporated in a technology estimation, can be indirectly used to refine the estimates of mean MPI, TEC, and TC. That is, in order to better interpret the intertemporal differences in equations (10) as mean MPI, TEC, and TC, it is desirable to purge these estimates from time-specific shocks in weather, markets, and regulatory environments that might have influenced the estimated time-specific frontiers and efficiencies. With a modest number of time periods, one can adjust estimates $\boldsymbol{\tau}^M$, $\boldsymbol{\tau}^s$, and $\boldsymbol{\tau}^{M-s}$ for the mean-level shocks associated with time-specific factors $\boldsymbol{W}_t$. For example, adjusted estimates $\tilde{\tau}_t^M$, $\tilde{\tau}_t^s$, and $\tilde{\tau}_t^{M-s}$ are obtained as the residuals in the following linear regressions;

$$
\begin{aligned}
q \in \{M, s, M-s\}, \quad \tau_t^q = \boldsymbol{W}_t \boldsymbol{\gamma}^q \, + \, \tilde{\tau}_t^q \\
\forall t, \;\; \gamma^{M-s} = \gamma^M \, - \, \gamma^s, \;\; \tilde{\tau}_t^{M-s} = \tilde{\tau}_t^M \, - \, \tilde{\tau}_t^s
\end{aligned} \tag{12}
$$

[21] Another approach is to introduce adjustment/shift to the estimated frontier(s) by using the second-stage estimates (and use a truncated regression for the second-stage estimation). In this way, Simar and Wilson (2007) provide a statistical model for the two-stage DEA procedure. See Appendix A for some discussion on the method.

where τ_t^M, τ_t^s, and τ_t^{M-s} are the estimates from the second-stage analysis in equations (11). Purged parameters $\tilde{\tau}_t^M$, $\tilde{\tau}_t^s$, and $\tilde{\tau}_t^{M-s}$ define new estimates of MPI, TEC, and TC in (10), conditionally on both producer-specific factors $\boldsymbol{z}_{it}$ and time-specific factors $\boldsymbol{W}_t$.

Statistical inferences for parameters $\hat{\boldsymbol{\theta}} \equiv [\hat{\boldsymbol{\tau}} \quad \hat{\boldsymbol{\alpha}} \quad \hat{\boldsymbol{\gamma}}]$ in (11), (12) are made, for example, by bootstrap estimates of their confidence intervals. Each cycle of bootstrapping begins with drawing (with replacement) $\{\varepsilon_{it}^{*q}\}$ for equations $q \in \{M, s, M-s\}$ from empirical distributions of the error terms and adding them to the predicted technical efficiencies. Parameter estimates $\hat{\boldsymbol{\theta}}^{*b}$ are calculated for bootstrapping samples $b = 1, ..., B$. For each parameter $\hat{\theta}_j \in \hat{\boldsymbol{\theta}}$, let $\hat{\theta}_{j,x}^*$ denote the x-percentile value in each bootstrap distribution $\{\hat{\theta}_j^{*b}\}_{b=1}^B$. Then, the $1-a$ confidence interval for $\hat{\theta}_j$ is estimated by $[\hat{\theta}_{j,a/2}^*, \quad \hat{\theta}_{j,1-a/2}^*]$, assuming that $1 - a = Prob[\hat{\theta}_{j,a/2}^* \leq \hat{\theta}_j - \theta_j \leq \hat{\theta}_{j,1-a/2}^*] \approx Prob[\hat{\theta}_{j,a/2}^* \leq \hat{\theta}_j^{*b} - \hat{\theta}_j \leq \hat{\theta}_{j,1-a/2}^*]$. The bootstrap distributions of $\hat{\boldsymbol{\gamma}}^{*b}$ can be obtained using the bootstrap estimates $\hat{\boldsymbol{\tau}}^{*b}$ as dependent variables.

In the current application, the following assumptions are made for the distributions of the error terms $\varepsilon_{it}^q, q \in \{M, s, M-s\}$, all assumed to be distributed with zero-mean and time-specific standard deviation σ_t^q. For each equation $\varepsilon_{it}^q, q \in \{M, s, M-s\}$, time-specific variances σ_t^q are estimated by $\hat{\sigma}_k^q = \sum_{it \in \mathbb{IT}(\mathbb{k})} (\varepsilon_{it}^q)^2/(N_k - 1)$, $\forall k \in \mathbb{T}$ and implemented by randomly sampling from the error distributions $\{\varepsilon_{it}^q\}_{it \in \mathbb{IT}(\mathbb{k})}$, $\forall k \in \mathbb{T}$ with replacement.The procedure accounts for the heteroskedasticity across time periods, which are introduced in equations M, $M-s$ by the interdependent error structures with equation s. While the current study makes rather conservative statistical inferences, it is also possible to mechanically utilize such interdependence to further account for inter-equation correlations.[22]

3.3 Frontier-Based TC Measurements

At the basis of TC measurement, defining a distance between two frontiers involves selecting the *points*, between which the distance is measured. These points are usually chosen at places where an observed input-output decision is projected to the frontiers. In the equation $M-s$ of (11), each data point $(\boldsymbol{x}_{it}, \boldsymbol{y}_{it})$ is projected to meta-frontier $\widehat{F}$ and time-specific frontier $\widehat{F}_t$, yielding the relative between-frontier distance measure $\widehat{\phi}(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}; \mathbb{T})/\widehat{\phi}(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}; t)$. However, it turns out that it is reasonable to expand each data point into multiple "observations" of

[22]For example, the three bootstrap error terms are sequentially constructed based on the bootstrapping error for equation s. First, error term ε_{it}^{*s} for producer i, period t is randomly drawn from empirical distribution $\{\varepsilon_{it}^s\}_{in \in \mathbb{IT}(\mathbb{t})}$ with replacement. Second, error term ε_{it}^{*M} is defined given the bootstrapped efficiency error terms ε_{it}^{*s}, $\forall t \in \mathbb{T}$; by the relationship $\phi(.; \mathbb{T}) = \min_k\{\phi(.; k)\}$, we have $\varepsilon_{it}^{*M} = \min_k\{\hat{\tau}_k^s + \varepsilon_{ik}^{*s}\} - \tau_t^M$. Third, the difference between the two defines the last error term; $\varepsilon_{it}^{*M-s} = \varepsilon_{it}^{*M} - \varepsilon_{it}^{*s}$. This bootstrapping procedure accounts for the interdependence in the error terms between equation s and equation M (and $M-s$). It implements implicit statistical inferences for error terms $\varepsilon_{it}^q \sim (0, \sigma_t^q), \forall t \in \mathbb{T}$, $q \in \{M, s, M-s\}$ without directly estimating standard deviations σ_t^q's.

distance measurements $\widehat{\phi}(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}; \mathbb{T})/\widehat{\phi}(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}; k)$ for $k = 1, .., T$ corresponding to the hypothetical, pseudo-technical efficiency assessments $\widehat{\phi}(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}; k)$ at each time-specific technology $\widehat{F}_k,\ k = 1, .., T$.

To see why such an operation is sensible, it is useful to recall the radial distances of MPI, TC, and TEC in equations (7). These are all point-based distance measurements, taken at discrete data points/rays, rather than the integrals of the differences along those curves. The MPI calculation uses technical and pseudo-technical efficiencies measured from each production decision $(\boldsymbol{x}_t, \boldsymbol{y}_t), t \in \{t0, t1\}$ (for a given producer) to each time-specific technology $\widehat{F}_t, t \in \{t0, t1\}$, regardless whether it corresponds to actual production time period. In effect, such an operation creates hypothetical "observations" where decisions in certain time periods are compared to the time-specific frontiers of different periods.

To illustrate the concept of these hypothetical observations, let us consider TGR calculations at three points $\{A, A', A''\}$ respectively observed in time $t \in \{t0, t1, t2\}$. In figure 4, their output-oriented projections to the frontiers of three technologies $\widehat{F}_t, t \in \{t0, t1, t2\}$ are depicted with the corresponding projections labeled as $\{D, C, B\}$ for point A, $\{D', C', B'\}$ for A', and $\{D'', C'', B''\}$ for A''. In this example, meta-frontier is given by $\widehat{F} = \cup_t \widehat{F}_t = \widehat{F}_{t2}$, to which TGR for each time-specific frontier can be calculated. The set of TGRs defined between these points and the technologies of the corresponding time periods is $\{DQ/BQ, C'Q'/B'Q', B''Q''/B''Q''\}$, which consists of single distance measurements along a ray from each point in $\{A, A', A''\}$. Frontier gaps between two periods are $\widehat{TGR}_{t0} = DQ/BQ$, $\widehat{TGR}_{t1} = C'Q'/B'Q'$, and $\widehat{TGR}_{t2} = B''Q''/B''Q''(= 0)$ with implied technical change $\widehat{TC}_{t0,t1} = \widehat{TGR}_{t0} - \widehat{TGR}_{t1}$ and $\widehat{TC}_{t1,t2} = \widehat{TGR}_{t1} - \widehat{TGR}_{t2}$. These TC measures correspond to those described in the previous subsection and are now referred to as efficiency-based TC (E.TC) measures. In contrast, the set of TGRs defined between the three points and all technologies at time periods $t =\in \{t0, t1, t2\}$ is $\{BQ/BQ,\ CQ/BQ,\ DQ/BQ,\ B'Q'/B'Q', C'Q'/B'Q',\ D'Q'/B'Q',\ B''Q''/B''Q'',\ C''Q''/B''Q'',\ D''Q''/B''Q''\}$, which consists of three distance measurements along a ray from each point in $\{A, A', A''\}$. In this case, mean frontier gaps are, for example, the geometric means of these distances; $\widehat{TGR}_{t0} = (DQ/BQ \cdot D'Q'/B'Q' \cdot D''Q''/B''Q'') \wedge (1/3)$, $\widehat{TGR}_{t1} = (CQ/BQ \cdot C'Q'/B'Q' \cdot C''Q''/B''Q'') \wedge (1/3)$, and $\widehat{TGR}_{t2} = (BQ/BQ \cdot B'Q'/B'Q' \cdot B''Q''/B''Q'') \wedge (1/3) = 1$ with implied technical change $\widehat{TC}_{t0,t1} = \widehat{TGR}_{t0} - \widehat{TGR}_{t1}$ and $\widehat{TC}_{t1,t2} = \widehat{TGR}_{t1} - \widehat{TGR}_{t2}$. The latter distance measures represent the point-based distances among the frontiers more comprehensively and are now referred to as frontier-based TC (F.TC) measures.

The following example illustrates the important roles of sampling points in frontier comparisons. In figure 5, input decisions $\{A, B, C, D\}$ are located in the two-input space x_1-x_2 for a given output level. Suppose that decisions $\{A, B\}$ are observed in time $t0$, $\{A, C\}$ in time $t1$, and $\{A, D\}$ in time $t2$, which correspondingly form time-specific frontiers F_{t0}, F_{t1}, and F_{t2} such that $F_{t0} \subset F_{t1} \subset F_{t2} = F$ (meta-frontier). In this example, mean technical efficiency measures for time $t \in \{t0, t1, t2\}$ are all at 1. Then, for the comparison between frontiers F_{t0}, F_{t1}, efficiency-based technical change (E.TC) from period $t0$ to period $t1$ can be measured by comparing the mean MPI's of the two periods. Albeit the exact values of MPI's depend on the directions and units of technical efficiency measurements, by inspection it is very likely that the mean MPI for period $t0$ (using points $\{A, B\}$) is higher than that of period $t1$ (using points $\{A, C\}$), implying negative technological change (e.g. technological regress) from F_{t0} to F_{t1}. Note that this is not consistent with the true relationship $F_{t0} \subset F_{t1}$. On the other hand, frontier-based technical change (F.TC) using the common set of sampling points $\{A, B, A, C\}$ is surely to yield a higher MPI for period $t1$ than period $t0$. As this example illustrates, for the purpose of frontier comparisons, it is maintained that F.TC measures are preferred to E.TC measures.

Then, such F.TC measures improve the TGR equation $M - s$ in the second-stage analysis (11).[23] TGR measurement $\widehat{\phi}(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}; \mathbb{T})/\widehat{\phi}(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}; k)$ is defined for each production decision $\{\boldsymbol{x}_{it}, \boldsymbol{y}_{it}\}_{it \in \cup_k \mathbb{I}\mathbb{T}(k)}$ for each time-specific frontier $\widehat{F}_k$, $k = 1, .., T$. Variables $\boldsymbol{z}_{it}$ are no longer useful as these hypothetical observations are reduced to mere "sampling points" for between-frontier distance measurements as opposed to the decisions made by individual producers with associated characteristics $\boldsymbol{z}_{it}$ (i.e. Assigning all indicator variables $\mathbb{1}_t(t = k)$ for time-specific frontier $k = 1, .., T$ to each observation it makes these indicator variables orthogonal to any variables $\boldsymbol{z}_{it}$).

The mean TGR calculations under frontier-based measures are;

$$\ln(\widehat{\phi}(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}; \mathbb{T})/\widehat{\phi}(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}; k)) = \tau_k^{M-s} + \varepsilon_{itk}^{M-s} \tag{13}$$

where subscript itk represents the unit of observation at the combination of each production decision it (observed in time t) and assigned time-specific frontier F_k of time k. The corresponding TC measures, or F.TC's, are defined in equation (7) using either the coefficients $\boldsymbol{\tau}^{M-s}$ in (13) or the orthogonal projections $\tilde{\boldsymbol{\tau}}^{M-s}$ of time-specific characteristics $\boldsymbol{W}_t$ in linear model (12). For statistical inferences, bootstrap error term ε_{itk}^{M-s*} for each period $k \in \mathbb{T}$ can be defined by random draws from $\{\varepsilon_{itk}^{M-s}\}_{\mathbb{I}\mathbb{T}(\mathbb{k})}$ with replacement, assuming the time-specific (e.g. frontier-specific)

[23]Due to the current system of independent linear equations (11), the first and second equations for $\ln \widehat{\phi}(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}; \mathbb{T})$, $\ln \widehat{\phi}(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}; t)$ can be omitted. The two equations do not add any new information since the first one is redundant under the hypothetical observations, and the second one no longer bears the interpretation for the determinants of *technical efficiency*.

distribution $\varepsilon_{itk}^{M-s} \sim (0, \sigma_k^{M-s}), \forall k \in \mathbb{T}$.

Thus, for non-Hicksian TC two versions of TC measures are obtained, depending on how targeted distances are measures in the input-output space. The mean levels of time-specific frontiers (e.g. mean output levels given inputs) can be measured across the directions of input-output decisions in the *corresponding* time periods (i.e. E.TC measures) or the directions of input-output decisions in *all* time periods (i.e. F.TC measures). The standard MPI decomposition belongs somewhere between the two TC measures[24] but appears rather closer to the latter F.TC measure in spirit. In comparison, there is no need to distinguish the two versions of TC measures for Hicksian TC, including a typical joint estimation for the technological frontier and its shift structure under SFA.

In summary, the common MPI measure of productivity changes and its decompositions can be seen as the comparisons of various between- and within-subsample mean-efficiency measurements. The existing approach of ratio-based measurements are capable for obtaining technical change decompositions for individual producers. On the other hand, the regression approach seems more versatile at sample level to conduct an analysis in the absence of balanced panel data and control for confounding producer-specific and time-specific factors. By imitating how the standard MPI calculation utilizes pseudo-technical efficiency assessments with hypothetical time periods, a second stage DEA analysis on the sample of such pseudo-efficiency estimates yields frontier-based TC measures that compares between-frontier distances more thoroughly than the efficiency-based frontier comparisons. Given that the unit of observations in equations (11) is at observed input-output decisions while the unit of observations in equation (13) is at the interactions between those decisions and time periods, it seems that TEC, TC are better measured respectively in the efficiency-based comparisons in (11) and the frontier-based comparisons in (13).

4 Application

The proposed methodology is demonstrated with a brief analysis on Maryland dairy operations. The dataset contains unbalanced panel data of revenues and expenses of 63 dairy farms during 1995-2009. Readers interested in detailed data descriptions and simple statistical analyses are directed to Hanson et al. (2013).[25] There are two types of dairy systems in the

[24]Two given frontiers are compared in the directions of input-output decisions that always belong to either one of the two time periods.

[25]The sample consists of typical dairies in the Northeast region for the herd size of 200 or less but does not contain dairies of very large scale operations.

data; traditional confinement operations and management-intensive (rotational) grazing dairies. The intensive grazing system is characterized with smaller scales of operations relative to the confinement system. The relative profitabilities of the two diary systems largely depend on the factor prices in markets and the technical efficiencies of individual producers. While the two systems are comparable for budget analyses, production inputs (e.g. breeds of cows) may not be homogeneous enough for the purpose of production analyses. For this reason, each system is independently analyzed for systematic differences across calender years.

Milk production is modeled with four inputs[26]: the number of cows, capital equivalent (i.e. the total expense of production deflated by a farm production cost index), and crop and pasture acreages.[27] Statistical properties of these inputs and output are summarized in table 1, along with their averages by dairy system and calender year in table 2. Given the relatively small sample size, no technical regression (, or cumulative reference frontiers,) is imposed.[28] This is done by constructing reference technologies from observations of the concurrent and previous time periods (i.e. Time-specific index set in the previous section is modified to $\mathbb{IT}(k) = \{it|t \leq k\}$ for time $t \leq k$.)

In table 1, one can discern major trends in production decisions for the two dairy systems. In terms of milk output, average confinement dairy has nearly doubled its output from 15,338 (cwt) in 1995 to 30,399 (cwt) in 2009, for which the increase mostly comes from the increased scale of operation (e.g. number of cows) with a slight increase in milk output per cow from 183 (cwt/cow) to 199 (cwt/cow). The increase in production has been accompanied by a similar increase in capital equivalent input without much changes in land acreage. In contrast, during the same period, the milk output for an average grazing operation has remained at around 13,000 (cwt). While the average grazing operation has become slightly larger in terms of herd size, its milk output per cow has declined from 183 (cwt/cow) to 124 (cwt/cow) along with some reductions in land acreage. Note that the changes in production decisions over time may neither take the form of equiproportional expansions of feasible production set nor proceed gradually at a constant rate as commonly assumed in empirical specifications. Using the method proposed in the previous section, the current application examines technical efficiency of the production decisions and relative performances of year-specific frontiers, controlling for producer-specific characteristics such as indicators for farm ownership and the presence of off-farm income and time-specific variables like seasonal-average precipitations and temperatures.

[26]See Appendix for the results under alternative input-output specifications.

[27] Milk revenues account for more than 85% of income shares in the sample. The analyses for the total revenue from milk, cattle, and crop sales yield qualitatively similar results.

[28]See appendix C for the results under the standard specification that allows the possibility of technological regression.

Table 3 shows the summary of DEA results for efficiency scores measured against year-specific frontiers, pseudo-efficiency scores against enveloping meta-frontiers, and TGR's as their ratios.[29] These efficiency scores are calculated separately for confinement and grazing operations under non-increasing returns to scale (NIRS). Throughout the empirical analyses, parallel specifications under constant returns to scale (CRS) obtain qualitatively very similar results. Under NIRS, the median technical efficiency is found at about 0.90 for confinement and 0.85 for grazers, indicating that for given inputs, respectively the 90% and 85% of the maximum output levels relative to year-specific frontiers are achieved by the producers of the median efficiency levels. Similarly, the median TGR's for the time-specific frontiers of confinement and grazers are 0.95 and 1.00 respectively, implying that for given inputs, the 95% and 100% of the maximum output levels relative to the meta-frontiers are feasible at the medians of the frontier gaps across sampling points.

The second stage analysis examines the within-subsample differentials in technical efficiencies and the between-subsample frontier gaps among time-specific frontiers. The current application focuses on the intertemporal differentials in productivity changes, frontier shifts, and efficiency improvements. As noted previously, these estimates are summarized through the regression-based Malmquist productivity index (MPI), technical change (TC), and technical efficiency change (TEC). For the regression-based MPI decomposition in (10) using regressions (11), table 4 shows the estimated coefficients taking year 1995 as the base level (i.e. at 1.000) under the main model that controls for producer-specific factors.[30] [31] The regression coefficients are treated as point estimates, and bootstrapping from the empirical distribution of the error term is used for statistical inferences. For confinement operations, the results overall indicate positive productivity changes (i.e. MPI) that can be decomposed into negative technical efficiency changes (TEC) and positive technical changes (TC). For example, in 2009 mean productivity was 13.3% higehr, mean technical efficiency 7.4% lower, and technological possibilities frontier 20.3% higher respectively relative to their 1995 levels. While the overall trends are clear, the year-to-year estimates for MPI, TEC, and TC are somewhat imprecise due to the relatively small sample size in a given year. For grazers, the model shows some negative productivity changes, which decomposes into negative TEC's and positive TC's; in 2009 mean productivity was 13.3% lower

[29] For those observations with zero crop acreage, pseudo-technical efficiency measurement under year-specific frontiers can be infeasible if those frontiers cannot be defined at the crop acreage of zero. Such infeasible scores are imputed at full efficiency values. For the most part, these imputed values become irrelevant when taking the minimum efficiency scores across year-specific frontiers for the purpose of meta-level efficiency calculations.

[30] Exponentials of regressions coefficients (and their differences from 1995 level coefficient) give MPI, TEC, and TC in their usual ratio-based unit with the reference to base-year at 1.000.

[31] Under the cumulative reference frontiers, the effects of time-specific factors may not be accurately estimated in (12) and need further considerations; some results from the specifications that use these variables are presented in tables but not interpreted in the current version of the draft.

(statistically insignificant), mean technical efficiency 18.9% lower, and technological possibilities frontier 6.8% higher respectively relative to their 1995 levels.

Table 5 shows TC estimates from models under frontier-based TC (F.TC) measurements in (13) along with the variant of the previous model in table 4 (, which uses efficiency-based TC (E.TC) measurements in (11)). As noted previously, F.TC measures represent more comprehensive frontier comparisons. The estimated F.TC indicate consistently higher technical changes, compared to the previous E.TC estimates in table 4. The specifications without controlling for time-specific characteristics (in the middle column) indicate that the frontiers for confinement and grazing dairies are 39.3% and 41.0% more efficient in 2009 respectively, compared to their 1995 counterparts.[32] Since technical changes have taken place non-proportionally (i.e. non-Hicksian TC) with respect to the earlier-year input-output mixes, frontier comparisons can be substantially different, depending on whether the distances of year-specific frontiers from the meta-frontiers are measured for a set of year-specific points/rays (i.e. E.TC measures) or a set of all observed data points (i.e. F.TC measures). While technical efficiency here is measured in the direction of a single output,[33] non-proportional shifts of a technological frontier still arise from the non-proportional changes in the best input mixes or the best input-output mixes of the time. If all technical changes consist of parallel/proportional shifts of a frontier, the E.TC and F.TC measures share the same expected values. The large discrepancy between the two in the current application seems to suggest the importance of non-Hicksian TC measurements in a dynamic industry experience structural changes with many entries and exits.

Combining the preferred estimates for efficiency-based TEC and frontier-based TC estimates, table 6 summarizes the current empirical application. The first and last 5-year-averages of the estimated coefficients during 1995-2009, averaging out the year-to-year fluctuations in efficiency and frontier levels, are reported as the preferred summary measures, between which the differences provide average TEC and TC estimates. Under NIRS, confinement and grazing systems have experienced 26.4%/decade and 19.2%/decade positive TC's along with 6.3%/decade and 14.4%/decade negative TEC's respectively. The widening efficiency differences and positive technical changes in both systems suggest that some producers have successfully adopted new technologies and improved their management compared to the 1995-level while others have been struggling to keep up with these changes. The difference between the two systems, however, is that overall productivity gain seems to have accrued to the majority of confinement operations

[32]Note that the possibility of technological regression is eliminated in the F.TC measures without time-specific factors (in the middle column in table 5).

[33] Note that the direction of efficiency measure can be chosen independently from estimating a piecewise-linear DEA frontier.

but only to a minority of grazing counterparts. This is consistent with the view that confinement dairy operations are more likely to follow fairly standardized production techniques of the industry while intensive grazing involves very localized production practices, depending on local soil and micro-climate conditions that require some experimentations by individual producers.

The estimated marginal effects for producer-specific and time-specific variables are reported in tables 7, 8 respectively. The producer-specific characteristics include two indicator variables for the farmland ownership for dairy operation and the presence of off-farm income. Additionally, the base model accounts for seasonal-average rainfalls and temperatures for four seasons. The coefficient estimates for equation "M," "s," and "M-s" that correspond to those in equations (11) represent the systematic correlations with pseudo-technical efficiencies against meta-frontiers (i.e. productivities), technical efficiencies against year-specific frontiers, and technological gaps between those frontiers (i.e. TGR's) respectively. Focusing on equation "s" where the coefficients are most naturally interpreted as the determinants of technical efficiencies, among the confinement dairies farm ownership and off-farm income are associated with 4.48 percentage points higher and 5.78 percentage points lower technical efficiencies respectively relative to time-specific frontiers. Among grazers, the corresponding marginal effects are positive 10.13 percentage points and negative (and insignificant) 5.59 percentage points. The results are in line with certain economic theory; potential principal-agent problems may reduce technical efficiency if a farmer does not own the farm property he manages, and a higher opportunity cost of the producer, indicated by the presence of off-farm income, would also reduce his commitment to dairy operation.

Table 8 reports coefficient estimates for time-specific factors for the same equations "M," "s," and "M-s" in equations (11) and for the equation "M-s" from frontier-based TC measure in (13). The weather variables are found to shift time-specific frontiers without much influences on the mean levels of technical efficiencies (e.g. mostly insignificant coefficient estimates for equation "s"). For confinement operations, seasonal rainfalls and temperatures, except for the summer temperature, are positively correlated with TGR's of time-specific frontiers relative to the meta-frontier. These relationships between the weather variables and time-specific frontiers are likely better captured in the coefficient estimates from the frontier-based TC model. These estimates show stronger relationships for confinement and find similar relationships for grazers; the changes in winter rainfall, summer rainfall, winter temperature, and summer temperature by one standard deviation are associated with -3.78, 2.57, 3.24, and -6.91 percentage point changes in the frontier output level among confinement operations and correspondingly -3.35, -0.08(insignificant), 2.03, and -8.44 percentage changes among grazers.

Lastly, the results from the common two-stage DEA estimation are presented in table 9. The estimations follow the algorithm in Simar and Wilson (2007), which is based on the truncated normal regression on estimated technical inefficiency scores and uses a certain bias-correction technique (using the initial/tentative second-stage coefficient estimates) to account for the downward bias in the first-stage technical inefficiency estimates. In the current application, observations are pooled across years with controlling for a quadratic time trend.[34] Results show positive correlations between farm ownership and technical efficiency while the coefficient estimates (, reported for the coefficient estimates on technical *efficiency*,) are rather imprecise compared to the estimates for equation "s" in tables 7, 8. Most of the marginal effects are relatively small and insignificant, compared to the year-to-year fluctuations in technical efficiency levels.

It should be noted that above empirical exercises are conducted with a relatively small sample of unbalanced panel data. Admittedly, the crude input-output specification particularly for grazers (e.g. not accounting for changes in the land use, pasture quality, or herd composition etc.) might have led to model misspecification. The results are preliminary at best and need to be interpreted with cautions. A more rigorous analysis using a much larger dataset is desired.

5 Comparisons of Frontiers & Mean Efficiency Levels

The section discusses the use of contextual variables for frontier and efficiency comparisons in DEA and elaborates the broader applicability of the proposed method beyond an application to technical change.

The relationships between production and its contexts are typically treated in one of the two major frameworks, based on the two channels, or technical feasibility and technical efficiency, through which those factors may influence production decisions.[35] Through the first channel, technical feasibility can vary with time, regional specificity, or disruptions to production at industry level *theoretically* by shifting the technical possibilities frontier. Also, policies in agriculture and natural environment or atypical market conditions can alter the technical feasibility *observationally* by shifting optimal production decisions in the input-output space. More generally, heterogeneity in production decisions can be analyzed for collective differences in technically-efficient production decisions by production contexts. Like the measure of mean tech-

[34] Alternatively, with further assumptions, one could construct more complex models to account for different error structures across years. Also, if more observations were available, observations in each year could have been separately analyzed.

[35] Besides what is mentioned, another use of contextual/environmental variables is to account for different production environments in the direction of efficiency measurement. For example, with the techniques of Banker and Morey (1986) or Reinhard, Knox Lovell, and Thijssen (2000), environmental factors can serve as non-discretionary inputs or outputs that are used in defining production feasibility yet excluded from the radial efficiency assessment.

nical change using time-specific frontiers, comparisons of context-specific frontiers yield mean context-induced frontier shifts that are distinguished from the changes in mean efficiency levels.

Through the second channel, non-production variables can be systematically correlated to the prevalence of technical inefficiencies. Numerous applications in the literature have sought to relate estimated inefficiencies to so-called environmental factors, including the form of business ownership, labor union status, geographical locations, government regulations, and just about anything outside the producers's control.[36]

The proposed method for measuring technical change in the previous sections can serve as a template for a general empirical framework for assessing the influences of production contexts through the two distinctive channels. Contextual information can be used to partition a sample and construct subsample-specific technologies in light of the first channel. Using a meta-frontier as a common basis for normalization, the between-frontier differences and the within-frontier differences in production decisions can be seen as between- and within-subsample inefficiencies (or heterogeneity) respectively. Those efficiency measurements can be statistically analyzed for the systematic relationships with the contextual factors in light of the second channel. Thus, by actively integrating the two channels of relationships to production heterogeneity into a research design, one can study the mean influences of production contexts on production decisions as well-defined distance concepts in input-output decisions. In particular, the impacts of policies and regulations can be studied through the average shifts in frontiers and the changes in the prevalence of technical inefficiency. For a policy evaluation, secondary analysis may be implemented in conjunction with other econometric techniques such as instrumental variable (IV) to deal with endogenously determined policy participation or difference-in-differences (DID) to remove confounding time trends.

In dairy production, for example, consider marketing orders, which regulate uniform minimum milk prices for dairy farmers, set minimum product-specific milk prices for processing plants (to purchase milk from dairies), and rectifies the resulting disparities in milk prices among various processing plants for different milk usage. The policy effectively sets price floors for dairies and barriers to vertical integrations that could have discouraged exits of uneconomically small dairies and prevented large dairies from integrating into processing plants respectively. Then, the policy might have contributed to the increased prevalence and extent of inefficiencies and slowed technical change toward more intensive use of the inputs that are characterized by high availability and small diminishing returns – like high-protein feeds and artificial growth hormone that make high-volume milk production possible. On the other hand, if inefficient dairies were

[36] See some discussion in Coelli (2005), for example.

more likely to exit and free up their land, intensive grazing might have attracted more entries and increased the chances of innovations to more efficiently manage small-to-medium scale dairies. While such an analysis may not be feasible under the existing level of variations in marketing order policies across time and regions, in theory policy influences could be analyzed for observed frontier shifts and changes in efficiency level, for which different policy responses could be prescribed depending on the trade-offs between policy objectives and market distortions.

In another example, regulating water pollutants may require, in the future, dairy operations to implement a cleaner manure handling procedure. To comply, producers will need to allocate more resources to the task, which can lead to a contraction of technical feasibility.[37] Such a policy can also reduce the extent of inefficiencies if more efficient producers, who tend to operate larger and more highly-confined dairies, are more severely affected by the regulation compared to their peers; the policy may reduce production heterogeneity by the narrowed production gaps between the technically-efficient and the less technically-efficient.

In some cases, research interests may lie in the influences of time-specific factors on production decisions. For example, the proposed methodology can be used to study region-specific impacts of climate change in agricultural production. Region-specific impacts may differ from the inter-regional average effects found in a national level study that exploits variations in weather and production outcomes across time, regions, and producers. Analysis with some panel or repeated cross sections data on production may find the region-specific linkages between weather outcomes and production outcomes. With little variation in weather outcomes across producers in a relatively small geographic, however, it is often difficult to econometrically identify the influences of weather conditions on a technological frontier and technical efficiency level while controlling for their intertemporal trends. Given that the unknown functional forms for technical change and the interactions between the production frontier and weather variables, specifications like Hicks-neutral frontier shifts and similarly parsimonious functional relationships for those interactions would significantly increase the risk of model misspecification. In such a circumstance, the proposed methodology offers a conservative approach that non/semi-parametrically estimates these unknown functional relationships in the first-stage DEA and parametrically summarizes their mean relationships in the second-stage regressions.

In addition, distance measures in the input-output space may be refined by the use of direc-

[37] Strictly speaking, it depends on the way technology is defined. If the negative externality from manure is a part of output specification, the policy would simply induce shifts of production decisions within the constant, underlying technical feasibility. In fact, the true feasibility would be rarely affected by policies and regulations when the input-output space is finely defined. On the other hand, if the input-output space contains monetary components like revenues and expenses rather than physical quantities of inputs and outputs, policies can easily alter the technical feasibility through the changes in effective prices for the factors of production.

tional distance measurement. Namely, consider the directional distance function of Chambers, Chung, and Fare (1996) and Luenberger (1994);

$$D(\boldsymbol{x}, \boldsymbol{y}; \boldsymbol{d}_x, \boldsymbol{d}_y) = \max\{a : (\boldsymbol{x} - a\boldsymbol{d}_x, \boldsymbol{y} + a\boldsymbol{d}_y) \in F\}, \tag{14}$$

which measures the maximal distance from point $(\boldsymbol{x}, \boldsymbol{y})$ to the frontier of technology F in direction $(-\boldsymbol{d}_x, \boldsymbol{d}_y)$. By choosing a specific direction in which an input-output decision is projected to frontiers and frontier differences are compared, relative importance of inputs and outputs may be specified for the particular technical inefficiency measurement. For example, one can focus on measuring frontier shifts in the direction of a particular input. Moreover, one can easily transform the additive measure to a multiplicative inefficiency measure (i.e. the proportional unit). For instance, once directional inefficiency is additively measured in the direction of l-th input $(-\boldsymbol{d}_x, \boldsymbol{d}_y) = ([\mathbf{0}\ -x_l\ \mathbf{0}], \mathbf{0})$, one can convert it to multiplicative technical efficiency measure under transformation $1/(1 + D(\boldsymbol{x}, \boldsymbol{y}; [\mathbf{0}\ \ -x_l\ \mathbf{0}], \mathbf{0}))$, which can be used as a dependent variable in above second-stage regression analysis.

Lastly, the current approach may be compared to a relatively new, probabilistic approach to efficiency measurement that shares some non-parametric treatment for the influences of environmental factors. Conditional free disposal hull (CFDH) estimators proposed by Daraio and Simar (2005, 2007) use similarities in production ensvironments to select a relevant subsample in which efficiency is measured.[38] Building on the distributional description of input-output bundles of Cazals, Florens, and Simar (2002), its efficiency concept is based on the expected probability of non-dominance in a randomly-selected sample from the observed production decisions (e.g. order-m efficiency is computed from an order-m frontier based on m randomly drawn observations.). CFDH takes a step further to account for the differences in production environments by taking a subsample conditionally on environmental factors; consequently, it jointly utilizes the survival distribution for free disposal hull (FDH) technologies and the non-parametric distributions of environmental factors, which together determine a relevant subsample for efficiency measurement.[39] Note that in this probabilistic-efficiency approach, environmental factors are assumed to influence both technical feasibility and efficiency without clear distinction. In contrast, the focus of the current approach is precisely on such distinctions to classify and compare the influences of contextual environmental factors on production decisions.

[38]Strictly speaking, environmental factors are used to give weights across observations depending on similarities. But, under fairly general conditions, it reduces to the dichotomous weight of including in or excluding from a subsample.

[39]The basic idea is to obtain a subsample to which the observation $(\boldsymbol{x}_0, \boldsymbol{y}_0)$ is compared in a FDH technology, conditionally on that production environments are sufficiently close to $\boldsymbol{z}_0$ for some bandwidth h; e.g. peer observation j is included in the subsample if $||\boldsymbol{z}_j - \boldsymbol{z}_0|| \leq h$. Asymptotic properties for this estimator are established in Jeong, Park, and Simar (2008), and some refinement for optimal band width is proposed in Badin, Daraio, and Simar (2010).

6 Conclusions

The study proposes a systematic treatment of contextual variables in comparing technical efficiencies and technological frontiers. Focusing on the measurement of technical change, it shows that regression-average Malmquist productivity index and its decompositions can be obtained from a second-stage analysis on estimated efficiency scores. Unlike the standard MPI decomposition technique, the proposed method can be applied to repeated cross sections data and allows the model to account for additional covariates. The regression-based method summarizes production heterogeneity by production contexts in the form of mean relationships between context-specific frontiers and associated efficiency levels.

Study of production heterogeneity is no simple matter. Empirical production analysis typically examines observed input-output decisions through the lens of a technological frontier and technical inefficiencies. Incorporating non-production factors or information on production contexts in such an estimation model introduces tremendous amount of complications in econometric specifications. Consequently, researchers often rely on strong, simplifying assumptions to confine their analyses to specific functional relationships among frontiers, inefficiencies, and non-production factors under presumed distributional properties. The current study shows that it is possible to avoid many of such arbitrary assumptions. Despite the complexity of the problem, the proposed methodology employs simple estimation techniques based on fairly conservative assumptions. In turn, it requires careful considerations for the specifications of production variables and clear concepts of distance measurements in the input-output space in order to draw precise implications of the study. That is, it forces the researcher to be conscious about the input-output space under study and explicit about which production heterogeneity is in focus and how it is measured. Only then can the relationships between the heterogeneity and non-production factors, including information on contextual environments, be fully investigated.

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Table 1: Summary Statistics

Variable	Mean	S.D.	Min	Max
Confinement (Obs. 314)				
Output Equiv.	369,033	289,506	56,331	1,917,846
Milk (cwt)	24,145	17,577	3,761	110,668
Cow	122	76	22	468
Capital Equiv.	416,037	308,346	70,637	1,780,881
Total Acre	338	160	90	845
Crop Acre	289	155	60	704
Pasture Acre	338	160	90	845
Grazers (Obs. 161)				
Output Equiv.	199,108	85,553	59,487	696,891
Milk (cwt)	12,442	5,573	2,670	42,955
Cow	87	29	37	195
Capital Equiv.	204,625	91,698	58,246	645,498
Total Acre	283	134	115	700
Crop Acre	132	108	0	600
Pasture Acre	283	134	115	700

1. Output equivalent is the gross income deflated by the price indices weighted for its components at the observation level. Capital equivalent is the total expense of production, deflated by a farm production cost index.

2. Total land (acres) add up to land used for crop production and land used for pasture at observation level.

Table 2: Average Production Decisions By Dairy System and Year

Year	N.Obs	Output Equiv.	Milk (cwt)	Cow	Capital Equiv.	Tot.Acre	Cro.Acre	Pas.Acre
		Sample Means						
Confinement								
1995	21	222173	15,338	85	255,522	328	273	55
1996	22	213511	16,249	90	272,876	332	277	55
1997	20	248605	18,389	97	306,554	302	254	48
1998	22	276714	21,729	112	384,388	314	272	42
1999	22	321742	22,711	115	402,671	314	272	42
2000	21	358828	24,649	121	403,807	340	292	48
2001	22	341851	23,730	117	415,356	328	282	47
2002	22	395945	24,782	122	418,442	328	282	47
2003	22	366650	24,958	128	419,106	358	309	49
2004	22	349620	25,282	129	448,014	358	307	51
2005	22	397284	27,628	137	498,483	348	297	51
2006	18	460286	29,202	144	505,202	356	304	52
2007	19	420043	29,734	146	534,688	351	300	51
2008	20	439283	29,518	144	505,584	356	302	53
2009	19	534353	30,399	150	504,675	369	316	53
Grazers								
1995	4	183251	13,534	75	207,129	368	195	173
1996	7	185215	13,584	79	207,818	298	133	165
1997	8	200431	13,115	79	210,913	331	151	180
1998	9	180634	13,214	80	211,340	326	153	174
1999	9	196184	13,069	85	224,744	326	153	174
2000	11	225735	13,270	85	215,347	295	130	164
2001	11	191660	12,536	85	204,831	295	130	164
2002	11	197800	12,058	86	212,984	295	130	164
2003	12	179603	11,536	87	191,500	285	126	159
2004	12	183328	12,375	90	193,782	288	130	159
2005	12	181581	11,076	84	181,681	254	109	144
2006	14	204991	12,021	86	194,577	244	118	126
2007	15	190465	11,899	88	213,488	235	110	125
2008	14	196874	12,232	92	201,761	260	133	127
2009	12	244684	13,168	101	210,822	273	138	135

Table 3: Summary of DEA Efficiency and TGR Scores

	Sampling	Dairy	Summary Statistics					
	RTS	Type	Min	25th	Median	Mean	75th	Max
Efficiency-Based Estimates								
A. Efficiency at year-specific frontiers								
(1)	NIRS	Confinement	0.465	0.807	0.899	0.884	0.978	1.000
(2)	NIRS	Grazers	0.362	0.715	0.852	0.822	0.951	1.000
(3)	CRS	Confinement	0.465	0.799	0.887	0.876	0.967	1.000
(4)	CRS	Grazers	0.362	0.695	0.801	0.802	0.933	1.000
B. Efficiency at meta-frontiers								
(5)	NIRS	Confinement	0.408	0.764	0.820	0.827	0.902	1.000
(6)	NIRS	Grazers	0.362	0.698	0.797	0.796	0.927	1.000
(7)	CRS	Confinement	0.408	0.749	0.808	0.815	0.891	1.000
(8)	CRS	Grazers	0.362	0.686	0.777	0.789	0.919	1.000
C. TGR using meta-frontiers								
(9)	NIRS	Confinement	0.704	0.903	0.954	0.937	0.988	1.000
(10)	NIRS	Grazers	0.723	0.959	1.000	0.971	1.000	1.000
(11)	CRS	Confinement	0.678	0.899	0.935	0.931	0.987	1.000
(12)	CRS	Grazers	0.723	1.000	1.000	0.986	1.000	1.000
Frontier-Based Estimates								
D. Efficiency at year-specific frontiers								
(1)	NIRS	Confinement	0.408	0.764	0.820	0.827	0.902	1.000
(2)	NIRS	Grazers	0.362	0.698	0.797	0.796	0.927	1.000
(3)	CRS	Confinement	0.408	0.749	0.808	0.815	0.892	1.000
(4)	CRS	Grazers	0.362	0.686	0.777	0.789	0.919	1.000
E. TGR using meta-frontiers								
(5)	NIRS	Confinement	0.117	0.884	0.950	0.918	0.991	1.000
(6)	NIRS	Grazers	0.213	0.922	0.993	0.935	1.000	1.000
(7)	CRS	Confinement	0.310	0.886	0.934	0.918	0.989	1.000
(8)	CRS	Grazers	0.213	0.945	1.000	0.948	1.000	1.000

1. Technical efficiencies are measured against year-specific frontiers as well as against a meta-frontier. Technology gap ratio (TGR) is the ratio of those efficiency measurements at the observation level.

2. Frontier-based measures include the pseudo-technical efficiency scores that evaluate observed decisions against frontiers of different time periods.

Table 4: Estimates of Regression-Based MPI, TEC, and TC

	MPI			TEC			TC		
Year	Point Est.	95% CI	S.Sig.	Point Est.	95% CI	S.Sig.	Point Est.	95% CI	S.Sig.
Confinement									
1996	0.996	(0.926, 1.069)		0.983	(0.912, 1.057)		1.013	(0.995, 1.031)	
1997	1.037	(0.963, 1.117)		1.000	(0.931, 1.081)		1.037	(1.018, 1.055)	***
1998	1.012	(0.940, 1.087)		0.944	(0.880, 1.017)		1.073	(1.054, 1.091)	***
1999	1.017	(0.952, 1.091)		0.924	(0.862, 0.992)	**	1.101	(1.082, 1.119)	***
2000	1.071	(1.004, 1.148)	**	0.935	(0.872, 1.002)	*	1.145	(1.126, 1.167)	***
2001	1.047	(0.971, 1.126)		0.914	(0.851, 0.990)	**	1.145	(1.124, 1.167)	***
2002	1.072	(0.993, 1.154)	*	0.935	(0.866, 1.006)	*	1.147	(1.127, 1.167)	***
2003	1.045	(0.969, 1.121)		0.910	(0.843, 0.979)	***	1.149	(1.128, 1.168)	***
2004	1.037	(0.965, 1.124)		0.898	(0.833, 0.974)	***	1.154	(1.130, 1.177)	***
2005	1.061	(0.984, 1.145)		0.907	(0.839, 0.980)	**	1.169	(1.148, 1.192)	***
2006	1.072	(0.999, 1.157)	*	0.905	(0.840, 0.983)	**	1.184	(1.162, 1.206)	***
2007	1.069	(0.982, 1.147)		0.895	(0.828, 0.960)	***	1.194	(1.172, 1.215)	***
2008	1.080	(1.002, 1.157)	**	0.901	(0.833, 0.974)	***	1.199	(1.177, 1.222)	***
2009	1.113	(1.036, 1.190)	***	0.926	(0.855, 0.997)	**	1.203	(1.178, 1.225)	***
Grazers									
1996	0.968	(0.790, 1.209)		0.987	(0.797, 1.230)		0.982	(0.914, 1.051)	
1997	0.932	(0.759, 1.146)		0.923	(0.745, 1.150)		1.010	(0.944, 1.067)	
1998	0.988	(0.802, 1.213)		0.991	(0.790, 1.226)		0.997	(0.931, 1.045)	
1999	0.911	(0.744, 1.113)		0.921	(0.750, 1.133)		0.990	(0.924, 1.040)	
2000	0.982	(0.789, 1.184)		0.966	(0.779, 1.176)		1.017	(0.955, 1.069)	
2001	0.966	(0.802, 1.188)		0.913	(0.753, 1.135)		1.058	(0.986, 1.109)	
2002	0.871	(0.710, 1.034)		0.831	(0.670, 1.004)	*	1.049	(0.979, 1.101)	
2003	0.882	(0.727, 1.059)		0.845	(0.692, 1.020)	*	1.044	(0.974, 1.097)	
2004	0.927	(0.756, 1.135)		0.904	(0.728, 1.108)		1.026	(0.958, 1.080)	
2005	0.919	(0.766, 1.104)		0.894	(0.729, 1.097)		1.029	(0.966, 1.082)	
2006	0.878	(0.710, 1.074)		0.831	(0.669, 1.010)	*	1.057	(0.984, 1.106)	
2007	0.828	(0.685, 1.005)	*	0.781	(0.641, 0.942)	***	1.060	(0.993, 1.110)	*
2008	0.837	(0.685, 1.008)	*	0.785	(0.631, 0.957)	***	1.066	(0.992, 1.119)	*
2009	0.867	(0.706, 1.059)		0.811	(0.645, 1.000)	**	1.068	(0.991, 1.126)	*

1. Statistical significance for the deviation from one, based on 400 bootstraps applied to the empirical distribution of the year-specific error term in the second-stage regression: *** $\alpha = 0.01$, ** $\alpha = 0.05$, * $\alpha = 0.1$.

2. Regression-based MPI, TEC, and TC are calculated relative to the coefficients for 1995.

3. The model controls for producer-specific factors ("$\boldsymbol{z}_{it}$"; indicator variables for the farmland ownership for dairy operation and the presence of off-farm income).

Table 5: Alternative TC Estimates: Efficiency-based TC (E.TC) and Frontier-based TC (F.TC) Estimates

Model	E.TC (w Time-var.)			F.TC (w/o Time-var.)			F.TC (w Time-var.)		
Year	Point Est.	95% CI	S.Sig.	Point Est.	95% CI	S.Sig.	Point Est.	95% CI	S.Sig.
Confinement									
1996	1.052	(1.038, 1.064)	***	1.043	(1.019, 1.071)	***	1.109	(1.092, 1.128)	***
1997	1.065	(1.054, 1.077)	***	1.082	(1.055, 1.113)	***	1.107	(1.092, 1.125)	***
1998	1.089	(1.077, 1.099)	***	1.210	(1.184, 1.239)	***	1.190	(1.175, 1.207)	***
1999	1.075	(1.062, 1.088)	***	1.242	(1.212, 1.275)	***	1.168	(1.151, 1.188)	***
2000	1.112	(1.102, 1.124)	***	1.299	(1.268, 1.335)	***	1.207	(1.193, 1.223)	***
2001	1.140	(1.124, 1.156)	***	1.305	(1.275, 1.340)	***	1.292	(1.270, 1.314)	***
2002	1.112	(1.096, 1.128)	***	1.312	(1.283, 1.349)	***	1.241	(1.217, 1.267)	***
2003	1.103	(1.092, 1.114)	***	1.313	(1.282, 1.349)	***	1.210	(1.192, 1.229)	***
2004	1.057	(1.043, 1.069)	***	1.314	(1.282, 1.350)	***	1.116	(1.100, 1.136)	***
2005	1.100	(1.088, 1.110)	***	1.355	(1.326, 1.394)	***	1.193	(1.179, 1.208)	***
2006	1.194	(1.174, 1.211)	***	1.379	(1.347, 1.415)	***	1.379	(1.353, 1.409)	***
2007	1.110	(1.096, 1.123)	***	1.382	(1.346, 1.421)	***	1.200	(1.182, 1.220)	***
2008	1.185	(1.166, 1.204)	***	1.385	(1.354, 1.422)	***	1.355	(1.329, 1.387)	***
2009	1.087	(1.076, 1.097)	***	1.393	(1.365, 1.437)	***	1.172	(1.157, 1.190)	***
Grazers									
1996	1.008	(0.969, 1.049)		1.198	(1.163, 1.246)	***	1.125	(1.103, 1.154)	***
1997	1.042	(1.010, 1.071)	***	1.242	(1.203, 1.286)	***	1.121	(1.101, 1.144)	***
1998	1.031	(0.994, 1.061)		1.253	(1.218, 1.298)	***	1.168	(1.150, 1.194)	***
1999	1.008	(0.972, 1.040)		1.280	(1.243, 1.329)	***	1.130	(1.109, 1.157)	***
2000	1.046	(1.015, 1.071)	***	1.325	(1.285, 1.373)	***	1.161	(1.143, 1.184)	***
2001	1.059	(1.006, 1.093)	**	1.332	(1.293, 1.384)	***	1.239	(1.215, 1.273)	***
2002	1.047	(0.996, 1.087)	*	1.336	(1.298, 1.386)	***	1.218	(1.188, 1.256)	***
2003	1.046	(1.009, 1.074)	**	1.336	(1.295, 1.390)	***	1.164	(1.145, 1.191)	***
2004	1.016	(0.978, 1.048)		1.336	(1.296, 1.378)	***	1.138	(1.116, 1.166)	***
2005	1.042	(1.002, 1.071)	**	1.338	(1.300, 1.388)	***	1.186	(1.168, 1.207)	***
2006	1.084	(1.019, 1.134)	**	1.389	(1.348, 1.444)	***	1.342	(1.310, 1.389)	***
2007	1.041	(0.998, 1.074)	*	1.394	(1.354, 1.444)	***	1.196	(1.171, 1.225)	***
2008	1.063	(1.011, 1.105)	**	1.406	(1.365, 1.459)	***	1.303	(1.270, 1.345)	***
2009	1.040	(1.009, 1.070)	**	1.410	(1.369, 1.462)	***	1.149	(1.130, 1.174)	***

1. Statistical significance for the deviation from one, based on 400 bootstraps applied to the empirical distribution of the year-specific error term in the second-stage regression: *** $\alpha = 0.01$, ** $\alpha = 0.05$, * $\alpha = 0.1$.

2. The TC estimates on the left are additionally control for time-specific factors. Time-specific factors ("$\boldsymbol{W}_t$") are seasonal-average rainfall and temperatures. Those in the center and right columns are frontier-based TC measures that compare distances between frontiers in the directions/rays of all observed input-output decisions (regardless their observation years).

Table 6: Summaries of TEC, TC Estimates

	Confinement		Grazers	
RTS Specification	Efficiency	Frontier	Efficiency	Frontier
A. Without Controlling for Time-specific Variables				
NIRS Frontiers				
1995-1999	0.970	1.115	0.964	1.195
2005-2009	0.907	1.379	0.820	1.387
Difference: TEC, TC	-0.063	0.264	-0.144	0.192
CRS Frontiers				
1995-1999	0.967	1.079	0.957	1.210
2005-2009	0.905	1.282	0.823	1.379
Difference: TEC, TC	-0.062	0.203	-0.134	0.169
B. With Controlling for Time-specific Variables				
NIRS Frontiers				
1995-1999	0.960	1.115	0.938	1.109
2005-2009	0.927	1.260	0.856	1.235
Difference: TEC, TC	-0.033	0.145	-0.082	0.126
CRS Frontiers				
1995-1999	0.962	1.139	0.940	1.110
2005-2009	0.927	1.351	0.869	1.285
Difference: TEC, TC	-0.035	0.212	-0.071	0.175

1. The first and last 5-year averages of estimated coefficients are reported as summary measures of TEC and TC during 1995-2009. Efficiency-based technical efficiency change (E.TEC) and frontier-based technical change (F.TC) calculations are used.

Table 7: Marginal Effects of Producer-Specific Characteristics

	Confinement			Grazers		
	M.E. (Percentage Points)			M.E. (Percentage Points)		
Variables	Point Est.	95% CI	S.Sig.	Point Est.	95% CI	S.Sig.
Equation M						
1(Farm ownership)	5.66	(2.33, 8.66)	***	10.17	(4.80, 16.63)	***
1(Off-farm income)	-5.73	(-10.36, -0.19)	**	-6.30	(-12.18, 0.20)	*
Equation s						
1(Farm ownership)	4.48	(1.06, 8.03)	***	10.13	(4.28, 16.31)	***
1(Off-farm income)	-5.78	(-10.61, -0.19)	**	-5.59	(-12.54, 0.88)	
Equation M-s						
1(Farm ownership)	1.18	(0.39, 1.97)	***	0.04	(-1.84, 1.73)	
1(Off-farm income)	0.05	(-1.20, 1.46)		-0.71	(-2.48, 1.23)	

1. Statistical significance, based on 400 bootstraps applied to the empirical distribution of the year-specific error term in the second-stage regression: *** $\alpha = 0.01$, ** $\alpha = 0.05$, * $\alpha = 0.1$.

2. The specification under CRS obtains qualitatively the same results.

3. Producer-specific indicators for farm ownership and off-farm income have the means of 0.77 and 0.07 respectively among confinement and 0.71 and 0.21 among grazers.

Table 8: Marginal Effects of Time-Specific Characteristics

	Confinement			Grazers		
	S.D.*M.E. (Percentage Points)			S.D.*M.E. (Percentage Points)		
Variables	Point Est.	95% CI	S.Sig.	Point Est.	95% CI	S.Sig.
Equation M						
Rainfall winter	-1.79	(-3.46, 0.17)	*	1.02	(-0.42, 2.30)	
Rainfall spring	1.31	(-1.02, 3.47)		-0.67	(-1.93, 0.74)	
Rainfall summer	0.16	(-2.75, 3.02)		1.42	(-0.51, 3.14)	
Rainfall autumn	1.25	(-0.50, 2.72)		-1.02	(-1.86, -0.18)	***
Temp. winter	1.00	(-1.56, 3.71)		0.67	(-1.86, 3.44)	
Temp. spring	0.24	(-1.37, 2.14)		0.81	(-1.19, 2.86)	
Temp. summer	-1.73	(-4.68, 1.53)		1.86	(-2.49, 6.56)	
Temp. autumn	1.06	(-0.72, 2.72)		-2.13	(-5.00, 0.74)	
Equation s						
Rainfall winter	0.78	(-0.87, 2.73)		1.47	(0.06, 2.86)	**
Rainfall spring	0.11	(-2.37, 2.32)		-1.04	(-2.40, 0.39)	
Rainfall summer	-0.89	(-3.66, 2.21)		1.84	(-0.10, 3.60)	*
Rainfall autumn	-0.65	(-2.46, 0.91)		-1.14	(-2.02, -0.33)	***
Temp. winter	-0.05	(-2.85, 2.63)		1.34	(-1.22, 4.01)	
Temp. spring	-0.93	(-2.65, 1.07)		0.72	(-1.36, 2.87)	
Temp. summer	1.64	(-1.20, 5.06)		1.76	(-2.48, 6.62)	
Temp. autumn	-2.44	(-4.23, -0.67)	***	-2.55	(-5.67, 0.46)	*
Equation M-s						
Rainfall winter	-2.57	(-3.08, -2.17)	***	-0.45	(-0.82, -0.10)	**
Rainfall spring	1.20	(0.57, 1.77)	***	0.37	(0.01, 0.81)	**
Rainfall summer	1.04	(0.25, 1.76)	***	-0.42	(-0.88, 0.10)	
Rainfall autumn	1.90	(1.52, 2.36)	***	0.13	(-0.13, 0.35)	
Temp. winter	1.05	(0.31, 1.93)	***	-0.66	(-1.37, 0.02)	*
Temp. spring	1.17	(0.70, 1.61)	***	0.09	(-0.54, 0.66)	
Temp. summer	-3.37	(-4.16, -2.66)	***	0.10	(-1.06, 1.58)	
Temp. autumn	3.51	(3.09, 3.94)	***	0.42	(-0.48, 1.25)	
Equation M- s (Frontier-based)						
Rainfall winter	-3.78	(-4.34, -3.13)	***	-3.35	(-4.21, -2.56)	***
Rainfall spring	1.41	(0.53, 2.43)	***	0.55	(-0.55, 1.60)	
Rainfall summer	2.57	(1.53, 3.67)	***	-0.08	(-1.53, 1.26)	
Rainfall autumn	3.21	(2.60, 3.84)	***	2.93	(2.13, 3.67)	***
Temp. winter	3.24	(2.26, 4.33)	***	2.03	(0.89, 3.32)	***
Temp. spring	2.12	(1.44, 2.75)	***	-0.29	(-1.12, 0.55)	
Temp. summer	-6.91	(-8.06, -5.73)	***	-8.44	(-9.78, -7.00)	***
Temp. autumn	6.81	(6.11, 7.41)	***	4.41	(3.54, 5.18)	***

1. Statistical significance, based on 400 bootstraps applied to the empirical distribution of the year-specific error term in the second-stage regression: *** $\alpha = 0.01$, ** $\alpha = 0.05$, * $\alpha = 0.1$.

2. Marginal effects are shown for the unit change of each variable by one standard deviation. Estimates for the constant term are omitted from this table.

3. Time-specific weather variables of seasonal rainfall (inches) and temperatures (Degrees Fahrenheit) for winter, spring, summer, and autumn have the means (s.d.) of 44.3 (8.8), 36.3 (2.6), 54.0 (1.7), 75.2 (1.6), and 57.3 (1.4) during 1995-2009 respectively. Marginal effects are calculated with the corresponding statistics during 1981-2010, which are 9.3 (2.9), 11.1 (3.5), 10.8 (3.3), and 10.7 (3.9) for rainfalls and 35.1 (2.4), 53.4 (1.9), 74.9 (1.7), 56.8 (1.4).

Table 9: Determinants of Technical Efficiency (Truncated Regressions)

	Confinement			Grazers		
	Estimated Coefficients			Estimated Coefficients		
Variables	Point Est.	95% CI	S.Sig.	Point Est.	95% CI	S.Sig.
Intercept	-0.919	(-3.840, 2.220)		-8.388	(-22.240, 9.990)	
1(Farm ownership)	0.090	(0.030, 0.150)	***	0.333	(0.100, 0.590)	**
1(Off-farm income)	-0.070	(-0.170, 0.030)		-0.124	(-0.360, 0.070)	
Year	0.017	(-0.020, 0.050)		-0.095	(-0.260, 0.110)	
Year Squared	0.000	(0.000, 0.000)		0.003	(-0.010, 0.010)	
Rainfall winter	-0.004	(-0.020, 0.010)		0.016	(-0.030, 0.070)	
Rainfall spring	0.002	(-0.020, 0.020)		-0.008	(-0.080, 0.050)	
Rainfall summer	0.001	(-0.020, 0.020)		0.070	(-0.010, 0.150)	*
Rainfall autumn	-0.002	(-0.010, 0.010)		-0.019	(-0.060, 0.020)	
Temp. winter	0.005	(-0.020, 0.030)		0.037	(-0.060, 0.140)	
Temp. spring	-0.001	(-0.020, 0.020)		0.063	(-0.050, 0.150)	
Temp. summer	0.001	(-0.040, 0.040)		0.017	(-0.170, 0.190)	
Temp. autumn	-0.012	(-0.040, 0.020)		0.019	(-0.100, 0.120)	

1. Statistical significance, based on 400 bootstraps applied to the assumed truncated normal distribution in the second-stage regression: *** $\alpha = 0.01$, ** $\alpha = 0.05$, * $\alpha = 0.1$.

2. It follows Simar and Wilson (2007)'s truncated normal regression on technical inefficiency (without log-transformation) with bias-corrections.

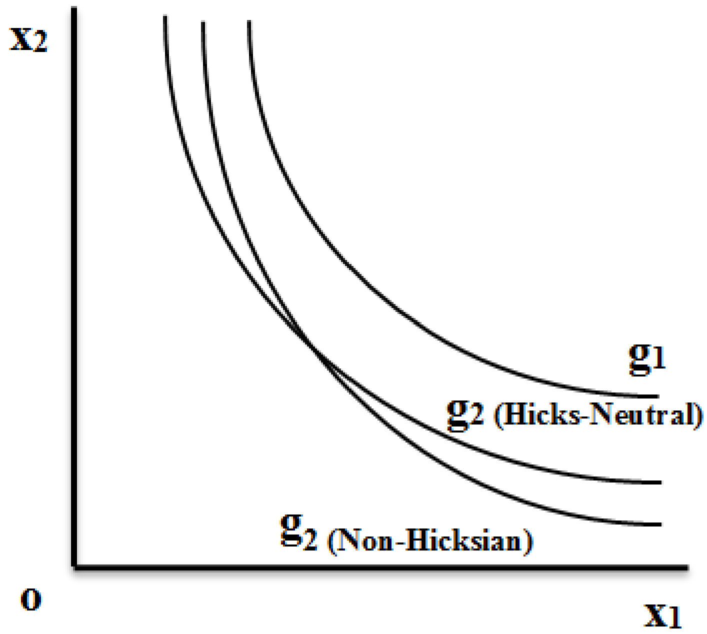

Figure 1: Technical Change and Hicks-Neutrality

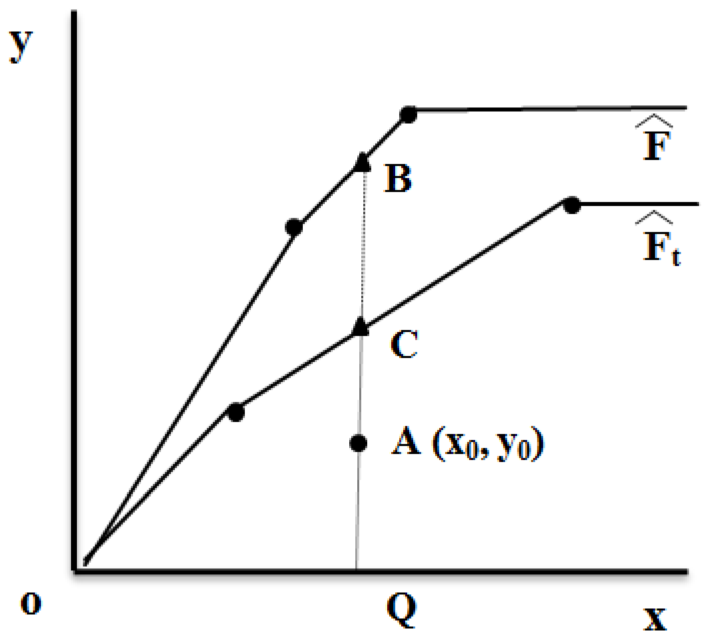

Figure 2: Technological Gap Ratio

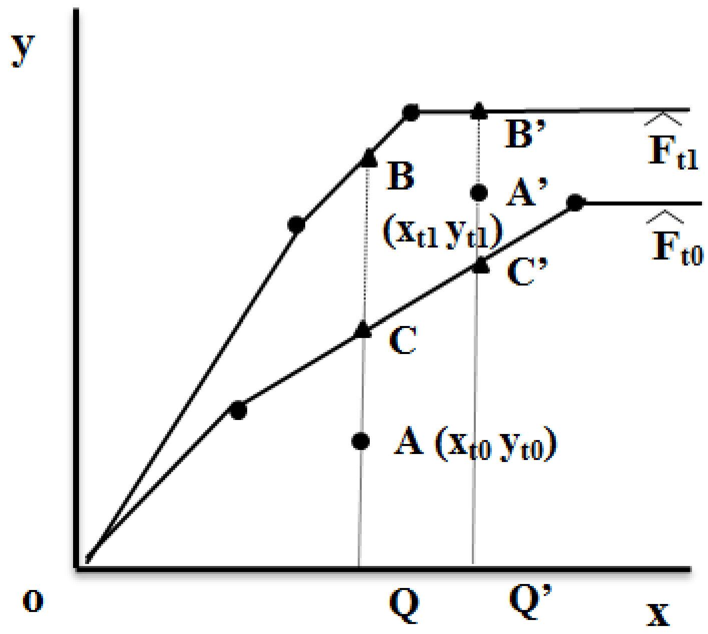

Figure 3: MPI Decomposition

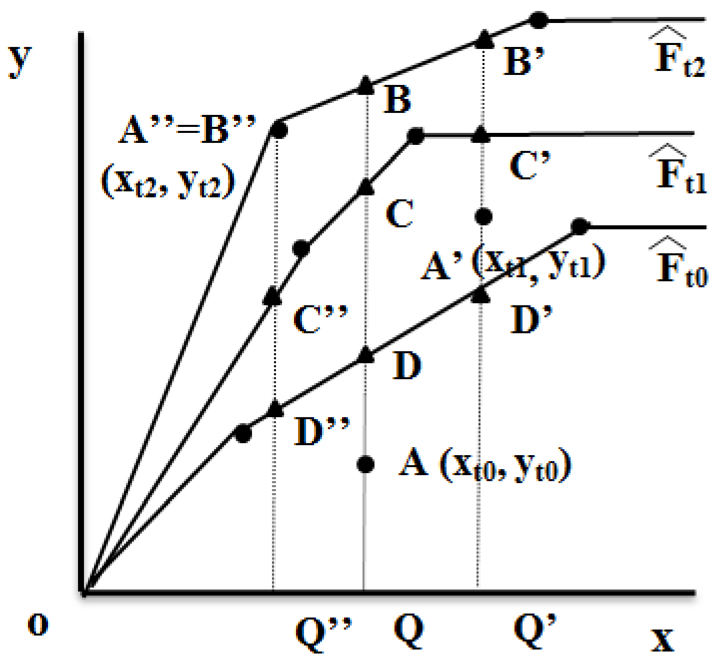

Figure 4: Multiple Between-Frontier Distances

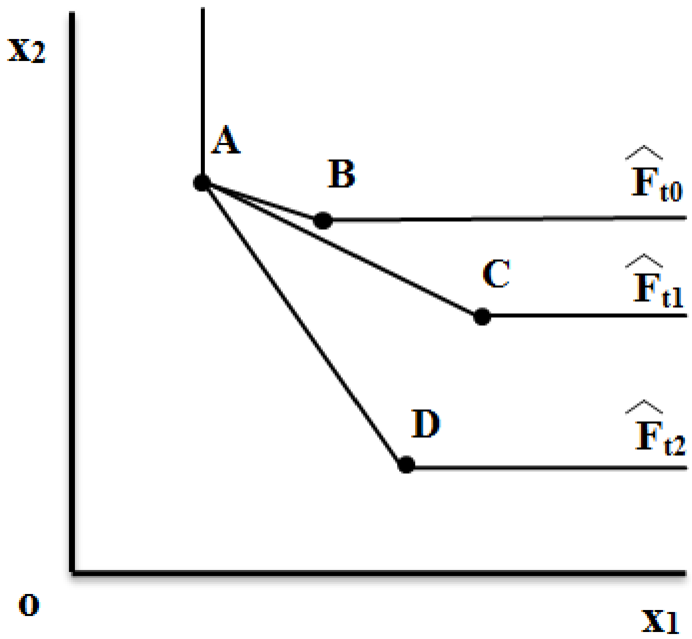

Figure 5: Role of Sampling Points in Frontier Comparisons

A Discussions on Single-Stage SFA and Two-Stage DEA Models

Focusing on the output-oriented technical inefficiency for single-output case, this section discusses (1) two competing approaches to technological frontier estimation, or DEA and SFA, (2) their extensions to estimating technical inefficiency as a function of so-called environmental factors, and (3) the measurement of technical change as an intertemporal shift of a technological frontier.

A.1 Deterministic and Stochastic Approaches to Estimating Production Frontiers

Economic theory of production is built on the transformation function that describes the technically feasible choice of input-output combinations. For given transformation function $g(\boldsymbol{x}, \boldsymbol{y})$, input-output bundle $(\boldsymbol{x}, \boldsymbol{y}) \in \mathbb{R}_+^L \times \in \mathbb{R}_+^M$ is technically feasible if and only if

$$g(\boldsymbol{x}, \boldsymbol{y}) \leq 0, \tag{A.1}$$

for which the equality holds at the boundary of transformation function $g(\boldsymbol{x}, \boldsymbol{y})$, or the technological *frontier*. Typical axiomatic properties for a technology include (a) feasibility of inaction $g(\mathbf{0}, \mathbf{0}) \leq 0$, (b) monotonicity/free-disposability ($g(\boldsymbol{x}, \boldsymbol{y}) \leq 0$, $(-\boldsymbol{x}, \boldsymbol{y}) \geq (-\boldsymbol{x}', \boldsymbol{y}') \Rightarrow g(\boldsymbol{x}', \boldsymbol{y}') \leq 0$), and (c) convexity ($g(\lambda \boldsymbol{x} + (1-\lambda)\boldsymbol{x}', \lambda \boldsymbol{y} + (1-\lambda)\boldsymbol{y}') \leq \lambda g(\boldsymbol{x}, \boldsymbol{y}) + (1-\lambda) g(\boldsymbol{x}, \boldsymbol{y})$ for $\lambda \in [0, 1]$).

The distance from technically feasible decision $(\boldsymbol{x}_i, \boldsymbol{y}_i)$ to frontier $g(\tilde{\boldsymbol{x}}_i, \tilde{\boldsymbol{y}}_i) = 0$ is referred to technical inefficiency, or a relative production performance measure compared to some projected point $(\tilde{\boldsymbol{x}}_i, \tilde{\boldsymbol{y}}_i)$ on the frontier. Output-oriented technical inefficiency measures how far observed outputs $\boldsymbol{y}_i$ can be expanded to reach the frontier, keeping inputs $\boldsymbol{x}_i$ fixed (i.e. $\tilde{\boldsymbol{x}}_i = \boldsymbol{x}_i$, $\tilde{\boldsymbol{y}}_i \geq \boldsymbol{y}_i$). Two important distance measures are the multiplicative, radially-proportional distance θ_i^M of Farrell (1957)[40] and the additive, radially-constant distance θ_i^A of Chambers, Chung, and Fare (1996) respectively defined as;

$$\begin{aligned} \theta_i^M &= \max\{\theta : g(\boldsymbol{x}_i, \theta \boldsymbol{y}_i) = 0\} \in [1, \infty) \\ \theta_i^A &= \max\{\theta : g(\boldsymbol{x}_i, \boldsymbol{y}_i + \theta \boldsymbol{d}_y) = 0\} \in [0, \infty) \end{aligned} \tag{A.2}$$

where θ_i^M measures the maximal radial output expansion in direction $\boldsymbol{y}_i$ while θ_i^A measures the maximal constant output expansion of outputs in given direction $\boldsymbol{d}_y$.[41] Technical inefficiency

[40] The counterpart for input-oriented, radial maximal contraction of inputs is known as Shephard (1970)'s input distance function.

[41] In the general case, the directional distance function of Chambers, Chung, and Fare (1996) (based on Luenberger (1994)) is

$$D(\boldsymbol{x}, \boldsymbol{y}; \boldsymbol{d}_x, \boldsymbol{d}_y) = \max\{a : (\boldsymbol{x} - a\boldsymbol{d}_x, \boldsymbol{y} + a\boldsymbol{d}_y) \in F\}$$

which measures the maximal distance from point $(\boldsymbol{x}, \boldsymbol{y})$ to the frontier of technology F in direction $(-\boldsymbol{d}_x, \boldsymbol{d}_y)$.

achieves the lower bound at $\theta_i^M = 1$, $\theta_i^A = 0$ if and only if decision $(\boldsymbol{x}_i, \boldsymbol{y}_i)$ is technically efficient. In single-output case where there is only one direction of output expansion to be considered, the two measurements are identical in the sense that $\theta_i^M = 1 + \theta_i^A / y_i$. The discussions below focus on this single-output case, in which the functional relationships between a technological frontier and technical inefficiency can be compared under the equivalent concepts of technical inefficiency.

The single-output case also offers an econometrically tractable platform for a frontier specification. In particular, the equations in (A.2) can be rewritten as the model for output y_i in terms of frontier function $f(\boldsymbol{x}_i)$ and its residual $u_i \in \{u_i^A,\ u_i^M\}$ that is interpreted as technical inefficiency;

$$y_i = f(\boldsymbol{x}_i) - u_i^A,\ \ y_i = f(\boldsymbol{x}_i)\, exp(-u_i^M),\ \ u_i^A,\ u_i^M \in [0, \infty) \tag{A.3}$$

where multiplicative and additive inefficiencies u_i^A, u_i^M are related to each other by $y_i / exp(-u_i^M) = y_i + u_i^A$, referring to the identical frontier projection. Decision $(\boldsymbol{x}, y)$ is technically feasible if and only if $f(\boldsymbol{x}) \geq y$.

Historically, the pioneering work of Farrell (1957) first introduced the concept of industry production *frontier* in empirical contexts. Let his parametric programming (PP) approach to frontier specification (A.3) be denoted by;

$$y_i = f^{PP}(\boldsymbol{x}_i; \boldsymbol{\beta}) - u_i^A,\ \ y_i = f^{PP}(\boldsymbol{x}_i; \boldsymbol{\beta})\, exp(-u_i^M),\ \ u_i^A,\ u_i^M \in [0, \infty) \tag{A.4}$$

where $f^{PP}(\boldsymbol{x}_i, \boldsymbol{y}_i; \boldsymbol{\beta})$ takes some presumed parametric functional structure like a Cobb-Douglas production function with parameters $\boldsymbol{\beta}$. One-sided (i.e. sign-constrained) residual $u_i \geq 0$ marked an important departure from the average response function characterized by the then-conventional two-sided (i.e. sign-unconstrained) residual. Early studies in deterministic PP frontier models include Aigner and Chu (1968), Timmer (1971), Afriat (1972), and Richmond (1974). The deterministic modeling meant that the deviation from the estimated parametric structure were attributable to technical inefficiency while the average response modeling regarded the deviation a stochastic noise arising from unobserved factors in the production process or parametric misspecification. In some cases, probabilistic modeling of the constraint (A.4) (e.g. $Pr[y_i = f^{PP}(\boldsymbol{x}_i; \boldsymbol{\beta}) - u_i^A] \leq \pi$ or $Pr[y_i = f^{PP}(\boldsymbol{x}_i; \boldsymbol{\beta})\, exp(-u_i^M) = 0] \leq \pi$ for some externally fixed probability π) was suggested as a complementary tool to mitigate the influences of outliers, which also helped to bridge the gaps in the early empirical studies where the dramatic shift in the interpretation of the model residual took place.

These models were, however, criticized for being algebraic rather than being statistical. For example, Schmidt (1976) pointed out that the estimation based on least squares was appropriate to test the hypothesis regarding parameters $\boldsymbol{\beta}$ but not the hypothesis of the frontier itself. The author instead hinted that under a particular distributional assumption on the error term, maximum likelihood estimation (MLE) would provide a more coherent, statistical approach to modeling production frontiers.[42]

Shortly, Aigner, Lovell, and Schmidt (1977) and Meeusen and Broeck (1977) have independently proposed what came to known as stochastic frontier approach (SFA). SFA views constraint (A.3) to hold in a probabilistic sense through the estimated "noise-to-signal" (i.e. randomness-to-inefficiency) ratio. With the composite error term of one-sided technical inefficiency $u_i \in \mathbb{R}_+$ and two-sided statistical noise $v_i \in \mathbb{R}$, let the SFA frontier approach be denoted by; [43]

$$y_i = f^{SFA}(\boldsymbol{x}_i; \boldsymbol{\beta}) - u_i^A + v_i^A, \;\; y_i = f^{SFA}(\boldsymbol{x}_i; \boldsymbol{\beta})\, exp(-u_i^M + v_i^M), \;\; u_i^A, \; u_i^M \in [0, \infty) \tag{A.5}$$

which corresponds to the model of likelihood $Pr[y_i = f^{SFA}(\boldsymbol{x}_i; \boldsymbol{\beta})]$ for a presumed joint distribution of the two random variables u_i, v_i. SFA cannot separate out technical inefficiency u_i from noise component v_i for each observation i but instead intends to identify their joint distribution based on the the skewness of one-sided distribution of inefficiency u_i in relation to the symmetric distribution of stochastic noise v_i. Under MLE, equation (A.5) can be interpreted as the most-likely statistical model for the data generating process (DGP), or the underlying input-output probability density function, say $h(\boldsymbol{x}, y)$. The stochastic representation implies that a part of observed decisions lie beyond the technical feasibility (A.3) due to unobserved random factor v_i.

On the other hand, building on the traditional, deterministic frontier representation, data envelopment analysis (DEA) (Charnes, Cooper, and Rhodes, 1978; Banker, Charnes, and Cooper, 1984) and its variants have established the non-parametric approach to the production feasibility (A.3). Let the deterministic, non/semi-parametric DEA approach be denoted by;

$$y_i = f^{DEA}(\boldsymbol{x}_i) - u_i^A + \xi_i^A, \;\; y_i = f^{DEA}(\boldsymbol{x}_i)\, exp(-u_i^M + \xi_i^M), \;\; u_i^A, \; u_i^M, \; \xi_i^A, \; \xi_i^M \in [0, \infty) \tag{A.6}$$

where f^{DEA} represents a non/semi-parametric DEA frontier and u_i^A, u_i^M additive and multiplicative technical inefficiencies and $\xi_i^A \equiv f^{DEA}(\boldsymbol{x}_i, \boldsymbol{y}_i) - f(\boldsymbol{x}_i, \boldsymbol{y}_i) \geq 0$, $exp(\xi_i^M) \equiv f(\boldsymbol{x}_i)/f^{DEA}(\boldsymbol{x}_i) \geq 1$ are the corresponding one-sided bias terms. Bias ξ_i arises when a piecewise-linear DEA frontier is constructed by connecting non-dominated decisions since the most productive decisions in the universe, locating at the boundary of true technological feasibility, are likely left unobserved in

[42] In the case of deterministic frontier modeling, ML interpretation is inappropriate since the range of a random variable such as an output depends on the model parameters (e.g. The range of an output is bounded from above by the parametrized frontier). By using the composite error, SFA avoids this problem.

[43] Under certain assumptions, SFA model can be constructed for the more general case of multiple-output, multiple-input.

a finite sample. Under certain distributional assumptions, one may try to mitigate this bias by estimating ξ_i^A or ξ_i^M through the bootstrapping method suggested in Simar and Wilson (2000) or Simar and Wilson (2007);

$$f^{CDEA}(\boldsymbol{x}) = f^{DEA}(\boldsymbol{x}) + \hat{\xi}^A,\ \ f^{CDEA}(\boldsymbol{x}) = f^{DEA}(\boldsymbol{x})\, exp(\hat{\xi}^M),\ \ \hat{\xi}^A,\ \hat{\xi_i^M} \geq 0 \qquad \text{(A.7)}$$

where f^{CDEA} denotes a bias-corrected DEA frontier.[44] Note that regardless of such bias mitigation, the interpretation of model (A.6) fundamentally remains algebraic rather than statistical since the underlying frontier relation is still viewed deterministic, and so is technical inefficiency obtained as a residual.[45]

In most DEA applications, researchers leave the bias term ξ_i untreated on the ground that $\xi_i \to 0$ as the number of observations tends to infinity (e.g. Banker and Maindiratta, 1992) and maintain that model (A.6) provides consistent estimates for frontier f and the technical inefficiency u_i. The implicit presence of bias ξ_i can be denoted by rewriting DEA approximation (A.6) as

$$y_i = f^{DEA}(\boldsymbol{x}_i) - u_i^{*A},\ \ y_i = f^{DEA}(\boldsymbol{x}_i)\, exp(-u_i^{*M}),\ \ u_i^{*A},\ u_i^{*M} \in [0, \infty) \qquad \text{(A.8)}$$

where $u_i^{*A} = u_i^A - \xi_i^A$, $u_i^{*M} = u_i^M - \xi_i^M$ are measured relative to estimable DEA frontier $f^{DEA}(\boldsymbol{x}_i) \geq y_i$, or an empirically-relevant subset of the true technical feasibility that suffices to represent production feasibility and patterns of input substitutability to draw policy implications. The unobserved gap between the true and estimable technologies are feasible but are likely characterized with relatively low probability densities.

In short, the two dividing approaches, represented by SFA and DEA, to the empirical model of production frontiers have evolved under the two different interpretations of technical feasibility (A.3). In the stochastic modeling, technological frontier envelopes only a part of observed data points and regards the deviation from the frontier as a mixture of random noise and technical inefficiency. It is assumed that the boundary relation (A.3) to lie somewhere between the observed data points. Its statistical inferences are based on a statistical model of the sampling process under certain distributional assumptions. On the other hand, in the deterministic modeling, it is assumed that the boundary relation (A.3) lies at the (most-outward) non-dominated data points, which falls short to trace the true boundary but may be assumed to provide an empirically-relevant representation of the true frontier. Its statistical inferences are based on the

[44] Similarly, by accounting for this bias, statistical inference on individual, technical inefficiency estimate u_i is suggested in Kneip, Simar, and Wilson (2008); Simar and Wilson (2011b); Simar and Vanhems (2012); Simar, Vanhems, and Wilson (2012) where the distribution of u_i is locally approximated to the asymptotic distribution for data point $(\boldsymbol{x}_i, \boldsymbol{y}_i)$. Simar and Wilson (2010) also suggests a similar statistical inference for the technical inefficiency estimate in SFA.

[45] Note that the deterministic interpretation of technical inefficiency u_i means that u_i is assumed identified in contrast to the stochastic interpretation like SFA where u_i is only jointly identified with stochastic error as $u_i - v_i$.

goodness of fit and are free of distributional assumptions.

In the absence of clear theoretical guidance, the decisions to model a technological frontier and its deviation are generally up to the researcher. Typical decisions include;

- Deterministic or stochastic interpretation of transformation function $g(\boldsymbol{x}, \boldsymbol{y}) \leq 0$
- Frontier structure in a certain parametric or non-parametric functional form
- Objects of interests or hypotheses

As seen in above discussions, historically the decisions on the first two accounts are often bundled together. However, the long division between the SFA and DEA approaches is increasingly blurred by recent developments in the non/semi-parametric frontier modeling combined with a SFA-like composite error structure (e.g. Fan, Li, and Weersink, 1996; Park, Sickles, and Simar, 1998, 2003; Park, Simar, and Zelenyuk, 2008; Kumbhakar et al., 2007; Simar and Zelenyuk, 2011; Kuosmanen and Kortelainen, 2012). The closer relationships between the two approaches suggest that the first two choices may serve as an integral part of the third item, or hypothesis testing. That is, the most appropriate specification and associated statistical concepts, now seem to depend on the applicability and credibility of the hypothesis testing regarding the specific aspects of a production process in focus.

A.2 Joint Model of Technological Frontier and Technical Inefficiency

One of the most popular hypotheses in frontier analysis is regarding the structure of technical inefficiency u_i. In particular, the researcher is often interested in how non-production factors, or so-called environmental variables $\boldsymbol{z}_i \in \mathbb{R}^R$, may have influenced production outcomes through technical inefficiency. In the literature, the relationships with environmental factors are interpreted as the determinants of technical inefficiency that shift the distribution of technical inefficiency without affecting the technical feasibility. One of the earliest examples is Timmer (1971), who investigated the correlations between the estimated technical inefficiency and variables like geographical region, agricultural policy, and producer characteristics. As the author has noted, such correlations require careful interpretations, for these relationships might have been driven by the underlying measurement errors in inputs and outputs rather than idiosyncratic constraints or distortions in the production process.

By adding some parametric model of technical inefficiency $u(\boldsymbol{z}_i; \boldsymbol{\alpha})$, stochastic frontier equation (A.5) can be rewritten as;

$$\begin{aligned} y_i &= f^{SFA}(\boldsymbol{x}_i; \boldsymbol{\beta}) - u_i^A + v_i^A = 0, \;\; u_i^A = u^A(\boldsymbol{z}_i; \boldsymbol{\alpha}) + \eta_i^A \geq 0, \\ y_i &= f^{SFA}(\boldsymbol{x}_i; \boldsymbol{\beta})\, exp(-u_i^M + v_i^M) = 0, \;\; u_i^M = u^M(\boldsymbol{z}_i; \boldsymbol{\alpha}) + \eta_i^M \geq 0, \end{aligned} \tag{A.9}$$

for which the joint distribution of technical inefficiency and stochastic noise can be estimated by maximum likelihood (e.g. Battese and Coelli, 1992, 1995).[46] Added component $u(\boldsymbol{z}_i;\boldsymbol{\alpha})$ may help maintain the assumption of homoskedasticity for the residual part of technical inefficiency η_i and stochastic error component v_i, for which the identification is sensitive to the violation of distributional assumptions(e.g. Caudill and Ford, 1993; Florens and Simar, 2005).

Similarly, DEA equation (A.8) can be rewritten as;

$$\begin{aligned} y_i &= f^{DEA}(\boldsymbol{x}_i;\boldsymbol{\beta}) - u_i^{*A} = 0,\;\; u_i^{*A} = u^{*A}(\boldsymbol{z}_i;\boldsymbol{\alpha}) + \eta_i^A \geq 0 \\ y_i &= f^{DEA}(\boldsymbol{x}_i;\boldsymbol{\beta})\, exp(-u_i^{*M}) = 0,\;\; u_i^{*M} = u^{*M}(\boldsymbol{z}_i;\boldsymbol{\alpha}) + \eta_i^M \geq 0, \end{aligned} \tag{A.10}$$

representing a simultaneous estimation of the technological frontier and the determinants of technical inefficiency that accounts for the underlying joint distribution, say $h(\boldsymbol{x}, y, \boldsymbol{z})$. While the standard DEA frontier f^{DEA} is not compatible with such a model, it helps to view the equations in (A.10) in the framework of convex nonparametric least squares (CNLS), or a close variant of DEA frontiers.

As shown in Kuosmanen (2008), the DEA frontier (for the single-output case) can be seen as a special case of the sign-constrained CNLS. The general CNLS problem is to find function f^{CNLS} from the family of continuous, monotonically increasing, and globally concave functions $\mathbb{f}$ by minimizing the sum of square residuals;[47]

$$f^{CNLS} = \underset{f}{\text{argmin}}\{\sum_{i=1}^{N} \varepsilon_i^2 : \forall i,\; y_i = f(\boldsymbol{x}_i) + \varepsilon_i,\;\; f \in \mathbb{f},\; \boldsymbol{\varepsilon} \in \mathbb{R}^N\}. \tag{A.11}$$

The author shows that the piecewise linear production function under DEA can be interpreted as a variant of the CNLS frontier using the system of Afriat inequalities (Afriat, 1967, 1972). Let $\mathbb{f}^{DEA}$ denote the family of possible frontier functions under the variable returns to scale (VRS) DEA technology[48];

$$\mathbb{f}^{DEA} = \{f : \forall i,\; f(\boldsymbol{x}_i) = c_i + \boldsymbol{x}_i\boldsymbol{\beta}_i : \forall i,j,\; c_i + \boldsymbol{x}_i\boldsymbol{\beta}_i \leq c_j + \boldsymbol{x}_i\boldsymbol{\beta}_j,\;\; \boldsymbol{c} \in \mathbb{R}^N,\; \boldsymbol{\beta} \in \mathbb{R}_+^{NL}\}. \tag{A.12}$$

In frontier family $\mathbb{f}^{DEA}$, the contributions of inputs to output are assumed to be positive and linear (i.e. $\boldsymbol{\beta} \geq 0$). In addition, the system of supporting hyperplanes defined by the Afriat inequalities imposes the concavity of the function. Then, additive and multiplicative technical inefficiency measures under DEA frontier $f^{DEA} \in \mathbb{f}^{DEA}$ can be found in the solutions to the

[46]The pioneering work of Battese and Coelli (1992, 1995) has modeled technical efficiency as a function of observation-specific characteristics (i.e. time period of production decision) under SFA.

[47]Frontier (A.11) is a infinite-dimensional problem, and as of today such a CNLS frontier can be solved only for the single-output single-input case.

[48]For constant, increasing, and decreasing returns to scale (CRS, IRS, DRS), set $c = 0$, $c \leq 0$, and $c \geq 0$ respectively.

following problem;

$$\{u_i^{*A}\}_{i=1}^N = \underset{\boldsymbol{u}}{\text{argmin}}\{\sum_{i=1}^N u_i^2 : \forall i,\ y_i = f(\boldsymbol{x}_i) - u_i,\ \ f \in \mathbb{f}^{DEA},\ \boldsymbol{u} \in \mathbb{R}_+^N\}$$
$$= \sum_{i=1}^N \underset{u_i}{\text{argmin}}\{u_i : \forall i,\ y_i = f(\boldsymbol{x}_i) - u_i,\ \ f \in \mathbb{f}^{DEA},\ \boldsymbol{u} \in \mathbb{R}_+^N\} \tag{A.13}$$
$$\{u_i^{*M}\}_{i=1}^N = \underset{\boldsymbol{u}}{\text{argmin}}\{\sum_{i=1}^N u_i^2 : \forall i,\ y_i = f(\boldsymbol{x}_i)\, exp(-u_i),\ \ f \in \mathbb{f}^{DEA},\ \boldsymbol{u} \in \mathbb{R}_+^N\}$$
$$= \sum_{i=1}^N \underset{u_i}{\text{argmin}}\{u_i : \forall i,\ y_i = f(\boldsymbol{x}_i)\, exp(-u_i),\ \ f \in \mathbb{f}^{DEA},\ \boldsymbol{u} \in \mathbb{R}_+^N\}. \tag{A.14}$$

The second line in (A.13) follows from the fact that each square residual u_i^2 can be minimized by independently minimizing u_i in an independent linear programming problem. By the identity $f(\boldsymbol{x}_i) = y_i + u_i^{*A} = y_i/exp(-u_i^{*M})$, the parallel structure arguments holds for multiplicative counterpart (A.14). Problem (A.13) for an additive inefficiency measure can be seen as a sign-constraint variant of the standard CNLS problem (i.e. $\boldsymbol{u} \geq 0$) while problem (A.14) is the counterpart for a multiplicative inefficiency measurement. The coefficients for f^{DEA}, though generally not uniquely solved in above problems, provide a local first-order Taylor series approximation to unknown function f in the neighborhood of points $\{\boldsymbol{x}_i\}_{i=1}^N$. In this light, Kuosmanen and Johnson (2008) have noted that DEA frontier f^{DEA} is a non-parametric generalization of parametric programming frontier f^{PP} of Aigner and Chu (1968) and Timmer (1971).

Adding a simple parametric model of technical inefficiency to (A.13), (A.14) and having the objective minimize the least squares of residuals yields;

$$\{u_i^{*A}\}_{i=1}^N = \underset{\boldsymbol{u}}{\text{argmin}}\{\sum_{i=1}^N \eta_i^2 : \forall i,\ y_i = f(\boldsymbol{x}_i) - u_i,$$
$$\forall i,\ u_i = \alpha_0 + \boldsymbol{z}_i\boldsymbol{\alpha} + \eta_i,\ \ f \in \mathbb{f}^{DEA},\ \boldsymbol{u} \in \mathbb{R}_+^N,\ \alpha_0 \in \mathbb{R},\ \boldsymbol{\alpha} \in \mathbb{R}^R,\ \boldsymbol{\eta} \in \mathbb{R}^N\} \tag{A.15}$$
$$\{u_i^{*M}\}_{i=1}^N = \underset{\boldsymbol{u}}{\text{argmin}}\{\sum_{i=1}^N \eta_i^2 : \forall i,\ y_i = f(\boldsymbol{x}_i)\, exp(-u_i),$$
$$\forall i,\ u_i = \alpha_0 + \boldsymbol{z}_i\boldsymbol{\alpha} + \eta_i,\ \ f \in \mathbb{f}^{DEA},\ \boldsymbol{u} \in \mathbb{R}_+^N,\ \alpha_0 \in \mathbb{R},\ \boldsymbol{\alpha} \in \mathbb{R}^R,\ \boldsymbol{\eta} \in \mathbb{R}^N\} \tag{A.16}$$

where environmental factors $\boldsymbol{z}_i$ are assumed to influence output y_i through technical inefficiency u_i with constant effects in problem (A.15) and proportional effects in problem (A.16). The solutions for technical inefficiencies $\{u_i^{*A}\}_{i=1}^N$, $\{u_i^{*M}\}_{i=1}^N$ in problems (A.15), (A.16), generally differ from those in problems (A.13), (A.14).[49]

[49] While any common parametric structure shared across observations is dropped from Afriat inequalities, or the local contribution of inputs defined relative to all the observed input-output bundles, the solution to non/semi-parametric frontier $f(\boldsymbol{x}_i)$ in (A.12) is altered by the additional, parametric structure $\alpha_0 + \boldsymbol{z}_i\boldsymbol{\alpha}$.

When there is no factors $\boldsymbol{z}_i$ but constant α_0, problem (A.15) becomes a version of the corrected convex non-parametric least squares (C2NLS) proposed by Kuosmanen and Johnson (2008). C2NLS is obtained by estimating problem (A.13) without constraints $\boldsymbol{u} \geq 0$ and then making post-estimation adjustments to both the frontier and technical inefficiency measurements such that the minimum technical inefficiency is set at zero, or $c_i^{C2NLS} = c_i + \min_j\{u_j\}$ and $u_i^{C2NLS} = u_i - \min_j\{u_j\} \geq 0$. Similarly, constant parameter α_0 in problem (A.15) would nullify the constraint $\boldsymbol{u}^* \geq 0$.[50] The use of the post-estimation transfer of some constant from technical inefficiency to the frontier shows that problem (A.15) and C2NLS are nonparametric counterparts to the corrected OLS (COLS) (Greene, 1980). Problem (A.15) can be solved by quadratic programming.

On the other hand, estimating problem (A.16) would be challenging. The nonlinear conditional mean, or $E_\eta[y_i|\boldsymbol{x}_i, \boldsymbol{z}_i] = f(\boldsymbol{x}_i)\, exp(-\alpha_0 - \boldsymbol{z}_i\boldsymbol{\alpha})$ (where $E_a[b|c]$ denotes the expected value of b with respect to a conditionally on c), is by itself not a problem and can be accommodated in the methods of moments or maximum likelihood estimation. But, the non-linearity cannot be readily handled in the quadratic programming used for estimating non-parametric function $f \in \mathbb{f}^{DEA}$. One feasible approach is to estimate problem (A.16) in two stages; technical inefficiency is obtained in DEA, and then the second stage estimation approximates its relationships with environmental factors $\boldsymbol{z}_i$. This leads to the two-stage DEA procedure of the form;

$$\{u_i^{*M}\}_{i=1}^N = \underset{\boldsymbol{u}}{\operatorname{argmin}}\{\sum_{i=1}^N u_i^2 : \forall i,\ y_i = f(\boldsymbol{x}_i)\, exp(-u_i),\ \ f \in \mathbb{f}^{DEA},\ \boldsymbol{u} \in \mathbb{R}_+^N\}$$
$$E[u_i^{*M}|\boldsymbol{z}_i] = \underset{\alpha_0 + \boldsymbol{z}_i\boldsymbol{\alpha}}{\operatorname{argmin}}\{\sum_{i=1}^N \eta_i^2 : \forall i,\ u_i^{*M} = \alpha_0 + \boldsymbol{z}_i\boldsymbol{\alpha} + \eta_i,\ \ \alpha_0 \in \mathbb{R},\ \boldsymbol{\alpha} \in \mathbb{R}^R,\ \boldsymbol{\eta} \in \mathbb{R}^N\}. \quad \text{(A.17)}$$

The first stage is simply the standard DEA estimation for technical inefficiency, which is legitimate even though the solution may slightly differ from the C2NLS-like estimation in problem (A.16). The second stage is to parametrize the estimated inefficiency for its conditional mean given environmental factors $\boldsymbol{z}_i$. To make statistical inferences for second-stage parameters $\boldsymbol{\alpha}$, bootstrapping techniques can be employed instead of the standard inferences based on limiting distributions.

On may substitute problem (A.16) with two-stage problem (A.17) under the so-called separability assumption that environmental factors $\boldsymbol{z}_i$ shift the distribution of technical inefficiency

[50] Consequently, constant terms $\{c_i\}_{i=1}^N$, α_0 in (A.15) cannot be uniquely identified. Applying the same post-estimation adjustments as C2NLS to problem (A.15) fixes the minimum level of inefficiency at zero and reduces to the C2NLS estimator.

u^{*M} without affecting frontier f^{DEA}. Denote the separability condition as;

$$E_{u^{*M}}[f^{DEA}|y_i, \boldsymbol{x}_i] = E_{u^{*M}}[f^{DEA}|y_i, \boldsymbol{x}_i, \boldsymbol{z}_i]. \tag{A.18}$$

At conceptual level, in many situations technological frontier f is not determined by the environmental factors $\boldsymbol{z}$ of producer-specific characteristics like age, experience, and idiosyncratic resource endowments or constraints. Thus, $f(\boldsymbol{x}) = f(\boldsymbol{x}|\boldsymbol{z})$ even if the underlying DGP has a relationship $h(\boldsymbol{x}, y) \neq h(\boldsymbol{x}, y|\boldsymbol{z})$. Empirically, however, the assumption may appear contradictory with the DGP; while observations $\{(\boldsymbol{x}_i, y_i, \boldsymbol{z}_i)\}_{i=1}^{N}$ as a whole are drawn from the conditional density $h(\boldsymbol{x}, y|\boldsymbol{z})$, the subset supporting empirical frontier f^{DEA} at the full technical efficiency are assumed to be drawn independently from $\boldsymbol{z}$, or $h(\boldsymbol{x}, y|\boldsymbol{z}) = h(\boldsymbol{x}, y)$. Hence, the separability assumption can be sensible only under certain kinds of DGP.

One situation where a DGP supports separability is that the influence of environmental factors $\boldsymbol{z}$ in conditional density $h(\boldsymbol{x}, y|\boldsymbol{z})$ dissipates toward zero as output y approaches to frontier $f(\boldsymbol{x}|\boldsymbol{z})$. In the multiplicative model with *proportional* effects of environmental factors $\boldsymbol{z}$, or $y_i = f^{DEA}(\boldsymbol{x}_i)\, exp(-u_i^{*M})$, such influences, represented by the second-stage marginal effects of $\boldsymbol{z}$ on $exp(-u^{*M})$, approach zero at frontier $y_i = f^{DEA}(\boldsymbol{x}_i)$ (i.e. $exp(-u_i^{*M}) = 1$), implying that the model specification admits separability condition (A.18). Then, given DEA estimates for multiplicative technical inefficiency $\theta_i^M \equiv 1/exp(-u^{*M}) \geq 1$, regressing the log-transformed inefficiency estimate $\ln \theta_i^M (\equiv u^{*M})$ on $\boldsymbol{z}$ yields a separability-consistent two-stage DEA procedure. Note that this is purely due to the assumption regarding the functional relationship between the frontier and technical inefficiency and is not always the case for the two-stage DEA procedure in general.

Alternatively, the two-stage DEA procedure may be rationalized by the statistical interpretation proposed by Simar and Wilson (2007) and reiterated in Simar and Wilson (2011a). The authors view the second stage estimation as a part of the underlying DGP for joint distribution $h(\boldsymbol{x}, y, \boldsymbol{z})$ that is given as a series of conditional distributions; $h(\boldsymbol{x}, y, \boldsymbol{z}) = h(\boldsymbol{x}, y|u, \boldsymbol{z})h(u|\boldsymbol{z})h(\boldsymbol{z})$. That is, in a DGP random variables are sequentially drawn; first, environmental factors $\boldsymbol{z}$ according to distribution $h(\boldsymbol{z})$, second, technical inefficiency u according to $h(u|\boldsymbol{z})$, and finally input-output decision $(\boldsymbol{x}, y)$ according to $h(\boldsymbol{x}, y|u, \boldsymbol{z})$.[51] In this formulation, conditional distribution $h(u|\boldsymbol{z})$ corresponds to the second-stage analysis estimating the determinants of technical inefficiencies.

Since their interpretation casts the two-stage DEA in the reverse order from the sequential DGP, the authors suggest to correct for the sampling bias of the first-stage DEA estimates by

[51] Strictly speaking, the authors describe conditional distribution of input-output $h(\boldsymbol{x}, y|u, \boldsymbol{z})$ with y described by two parts in polar coordinates. Also, their model is not restricted to the single-output case.

using the information of the second-stage model. By modifying the notations in (A.7), the bias-correction proposed in Simar and Wilson (2007) is denoted as;

$$f^{CDEA}(\boldsymbol{x}_i|\boldsymbol{z}_i) = f^{DEA}(\boldsymbol{x}_i) + \hat{\xi}^A(\boldsymbol{z}_i),\ \ \hat{\xi}^A(\boldsymbol{z}_i) \geq 0,$$
$$f^{CDEA}(\boldsymbol{x}_i|\boldsymbol{z}_i) = f^{DEA}(\boldsymbol{x}_i)\, exp(\hat{\xi}^M(\boldsymbol{z}_i)),\ \ \hat{\xi_i^M}(\boldsymbol{z}_i) \geq 0. \tag{A.19}$$

f^{CDEA} is obtained using bootstrapping from the tentative second-stage model of DEA technical inefficiency u_i^* that accounts for the influences of environmental factors $\boldsymbol{z}_i$.[52] Once adjustments for the influences of $\boldsymbol{z}$ are made, bias-corrected technology f^{CDEA} deterministically approximates the true technology f, and re-estimating the second-stage model approximates density $h(u|\boldsymbol{z})$.

Simar and Wilson (2011b) reiterate the following points[53]: (a) the conditional distribution $h(u|\boldsymbol{z})$ must be modeled by a truncated regression, truncated at the full technical efficiency level (which excludes fully efficient observations from the second stage analysis), (b) statistical inferences in the second-stage estimation are made by a certain bootstrapping procedure(s), and (c) the separability assumption should be tested for its validity (e.g. as described in (Daraio, Simar, and Wilson, 2010)).

To this date, the statistical interpretation of Simar and Wilson (2007) represents the most coherent view on the two-stage DEA procedure. As such, it merits noting some observations on its framework. First, there are some odd aspects to the sequential nature of the assumed DGP that draws technical inefficiency u prior to input-output decision $(\boldsymbol{x}, y)$. It reflects the view that there is some pre-existing, absolute measurement in managerial ability that exactly leads to the corresponding technical inefficiency level. However, in empirical contexts the concept of technical inefficiency seems to represent more of an *ex post* (i.e. post-production) relative evaluation like the residual of a frontier estimation equation (e.g. recall the initial interpretation in equation (A.4)) than an *ex ante* absolute-scale factor that dictates input-output decisions. In addition, the sequential DGP leads to a somewhat circular argument in the proposed procedure for finite-sample bias correction and re-estimation of parameters.[54]

[52] The bias correction utilizes a truncated-regression estimate of $\boldsymbol{\alpha}$ that fits better toward inefficient decisions, compared to the parameter estimates based on regressing all observations on variables $\boldsymbol{z}_i$ in a non-truncated manner. Applying these parameters to efficient (out of sample) observations allows to predict a smaller technical feasibility on average (of bootstrap replications). In essence, the gap between the observed and predicted frontiers yields the bias term estimate.

[53] The authors discuss the separability condition in the examples where functional forms are defined over *separate* variables for technical inefficiency u and environmental factor z. Then, they point out that the separability may not hold unless special cases of DGP. This is somewhat misleading; the frontier obtained by letting $u \to 0$ can be easily independent of z when the two are related in the form $u(z)$.

[54] The authors suggest to use the initial model for the determinants of technical inefficiency $h(u|\boldsymbol{z})$ to generate pseudo-observations in the input-output space via bootstrapping, predict the extent of the underestimated technical feasibility $\xi(\boldsymbol{z})$ for the first stage, and finally reestimate the second stage equation $h(u|\boldsymbol{z}, \xi(\boldsymbol{z}))$. While the idea may be sound, for implementation it is unclear whether this sort of correction should be completed in just a single cycle or continued for multiple cycles. Without any theoretical fixed points or pre-determined convergence criteria, such a circular argument leaves an impression that it would be more appropriate to use a simultaneous estimation for joint distribution $h(\boldsymbol{x}, y, \boldsymbol{z})$ as in model (A.15).

Second, with regarding item (a), it is not necessary for the second-stage analysis to be estimated by a truncated regression. In their view, the purpose of the second-stage regression is to model the DGP of technical inefficiency $h(u|\boldsymbol{z})$ while maintaining the distributional assumptions for the separability condition. It is assumed that the conditional distribution $h(u|\boldsymbol{z})$ is truncated at the full technical efficiency level. However, such truncation is unnecessary if direct distributional assumption via MLE is avoided. The truncation may rather create distortions to the underlying relationships between u and $\boldsymbol{z}$. More importantly, the MLE of technical inefficiency $h(u,|\boldsymbol{z})$ cannot bear the interpretation of DGP under a truncated regression where the range of random variable u depends on model parameter $\boldsymbol{\alpha}$. Early models like (A.4) share the same problem.

Third, with regarding item (c), the apparent need of separability test arises due to the use of two-stage estimation procedure but only under a certain case of presumed functional relationships. Under the linear functional relationship with the constant effects of environmental factors $\boldsymbol{z}$ like model (A.15), one can avoid a two-stage procedure and simply employ a single-step simultaneous estimation for the frontier and technical inefficiency.

The current study maintains that under certain functional forms like (A.17), the two-stage DEA procedure can be implemented coherently with the underlying DGP. OLS regression in the second stage consistently characterizes the underlying relationship between technical inefficiency u_i^* and environmental factors $\boldsymbol{z}_i$, given that u_i^* is consistently estimated in the first stage. The OLS coefficients, however, do not bear the interpretation for causality since some unobserved characteristic like inherent managerial ability may be systematically correlated with both factors $\boldsymbol{z}_i$ and technical inefficiency u_i^*. In certain situations, second-stage model $E[u^*|\boldsymbol{z}_i]$ may predict a more-than-full technical efficiency level if the model is poorly fit. Some may suspect this as incongruence with the DGP, but this is simply a result of poor parametrization. Also, in the semi-parametric estimation, the hypotheses regrading the parametric part may be tested using the model residual. That is, model residual η_i can provide statistical inferences for the parameters $\boldsymbol{\alpha}$ in the second stage without making inferences for the (exactly-identified) frontier structure f^{DEA} or individual technical inefficiency u_i^*.[55] Bootstrapping method using an empirical distribution of error term η_i seems most appropriate for such statistical inferences.[56]

To summarize, a joint model of technological frontier and technical inefficiency requires considerations for a frontier representation, a direction of technical inefficiency measurement, and a specification for the determinants of inefficiency. Joint estimation equation helps interpret the

[55] Note that the deviations from the presumed parametric structure allow hypothesis testing on the parameters in the early deterministic parametric models in (A.4).

[56] If desired, the probabilistic/stochastic characterization of technical feasibility (A.3) can be adopted in conjunction with non/semi-parametric model structure (e.g. Fan, Li, and Weersink, 1996; Park, Sickles, and Simar, 1998; Kumbhakar et al., 2007; Kuosmanen and Kortelainen, 2012), in which different statistical inferences may be devised based on the distributional properties of the stochastic noise and the model residual.

underlying relationships among variable as a frontier model with accounting for a heterogeneous error structure. Single-stage SFA approach can be improved if the additional information on the error structure helps maintain its distributional assumptions. Similar single-stage DEA-based approach is feasible for the additive measure of technical inefficiency (with linear, additive inefficiency shifts by environmental factors). Two-stage DEA approach provides a feasible estimation method under the multiplicative measure of technical inefficiency (with multiplicative, proportional inefficiency shifts by environmental factors). The statistical concept for the SFA approach is based on the distributional assumptions of the DGP while that of the DEA-based approach is based on the sample-level goodness of fit (e.g. least squares) for a given parametric structure of technical inefficiency.

A.3 Extension to the Model of Technical Change

The intertemporal shifts in a technological frontier and technical efficiency provide the measures of technical change (TC) and technical efficiency change (TEC) respectively. These shifts can be directly estimated as a part of the frontier and efficiency parametric structures or indirectly estimated as the summary relationships among the separate time-specific frontiers and associated technical inefficiencies. The direct approach typically employs Hicks-neutral TC where intertemporal changes are restricted to a single-dimensional intertemporal shift structure under the time-invariant substitutions of inputs and outputs. On the other hand, the indirect approach is based on a second-stage statistical analysis on the estimated time-specific frontiers that summarizes their relationships into mean TC measures. The latter TC measures make use of more flexible frontier estimations without a priori parametric restrictions of Hicks-neutrality yet carry the interpretations similar to their Hicks-neutral counterparts. The following discussion shows that such indirect TC measures are derived in a variant of the two-stage DEA procedure, which can be regarded as a regression-based Malmquist Productivity Index (MPI) decomposition. Similar two-stage TC measures can be also derived for parametric frontier models like SFA.

The family of piecewise linear DEA frontiers (A.12) is now extended as follows. When a time-specific frontier is constructed from a subsample of observations that are observed in time period t, the family of possible frontiers for time $t = \{1, .., T\}$ is given by;[57]

$$\begin{aligned} \mathbb{f}_t^{DEA} = \{ f : \forall i,\ f(\boldsymbol{x}_{it}) = c_{it} + \boldsymbol{x}_{it}\boldsymbol{\beta}_{it} : \\ \forall i, j,\ c_{it} + \boldsymbol{x}_{it}\boldsymbol{\beta}_{it} \le c_{jt} + \boldsymbol{x}_{it}\boldsymbol{\beta}_{jt},\ \boldsymbol{c} \in \mathbb{R}^N,\ \boldsymbol{\beta} \in \mathbb{R}_+^{NL} \}. \end{aligned} \tag{A.20}$$

[57]If the frontier parameters for input substitution are time-invariant (i.e. $\boldsymbol{\beta}_{it} = \boldsymbol{\beta}_i$), the frontiers in (A.21) reduces to the set of Hick-neutral frontier specifications.

Given these time-specific frontiers, consider constructing two types of meta-frontiers; one connecting the most productive segments of these frontiers, and the other is the convex combination of those.[58] The former can be non-convex while the latter is always convex. Let $\mathbb{f}_{M1}^{DEA}$, $\mathbb{f}_{M2}^{DEA}$ be the set of such potentially-non-convex and convex meta-frontiers where

$$\begin{aligned} \mathbb{f}_{M1}^{DEA} &= \{f : f = \max_t f_t,\ f_t \in \mathbb{f}_t^{DEA}\} \\ \mathbb{f}_{M2}^{DEA} &= \{f : f = \sum \mu_t f_t,\ \sum \mu_t = 1,\ f_t \in \mathbb{f}_t^{DEA},\ \boldsymbol{\mu} \in \mathbb{R}^T\}. \end{aligned} \tag{A.21}$$

Now, consider estimations for technical inefficiency measurement $u_{it}^{*(q)}$ for estimation equation $q = \{s, M\}$ where equation $q = s$ represents the technical inefficiency evaluation under the time-specific frontiers $\mathbb{f}_t^{DEA}$ in (A.20), and equation $q = M$ represents the counterpart under the meta-frontier $\mathbb{f}_{M1}^{DEA}$ or $\mathbb{f}_{M2}^{DEA}$ in (A.21). Since the latter is a pseudo-technical inefficiency measurement defined given the former, they are obtained sequentially: first, estimating the model for time-specific frontier inefficiency $\{u_{it}^{*(s)}\}_{it}$ and then meta-frontier inefficiency $\{u_{it}^{*(M)}\}_{it}$ using those frontier esitmates.

By adding the intertemporal dimension, additive model (A.15) is extended to;

$$\begin{aligned} \{u_{it}^{*(s)A}\}_{it} = \underset{\boldsymbol{u}}{\operatorname{argmin}}\{\sum_i \sum_t \eta_{it}^2 :\ &\forall i, t,\ y_{it} = f_t(\boldsymbol{x}_{it}) - u_{it},\ \ u_{it} = \boldsymbol{z}_{it}\boldsymbol{\alpha} + \tau_t + \eta_{it}, \\ &\forall t,\ f_t \in \mathbb{f}_t^{DEA},\ \ \boldsymbol{u} \in \mathbb{R}_+^{NT},\ \boldsymbol{\alpha} \in \mathbb{R}^R,\ \boldsymbol{\tau} \in \mathbb{R}^T,\ \boldsymbol{\eta} \in \mathbb{R}^{NT}\}. \end{aligned} \tag{A.22}$$

It is desirable to have similar numbers of observations across time periods for comparable measures of time fixed effects τ_t (i.e. the magnitude of finite-sampling bias ξ_{it} for frontier $f_t \in \mathbb{f}_t^{DEA}$ tends to decrease as the number of observations in period t increases). Similarly, feasible multiplicative model (A.17) is extended to;

$$\begin{aligned} \{u_{it}^{*(s)M}\}_{it} &= \underset{\boldsymbol{u}}{\operatorname{argmin}}\{\sum_i \sum_t u_{it}^2 : \\ &\qquad \forall i, t,\ y_{it} = f_t(\boldsymbol{x}_{it})\, exp(-u_{it}),\ \ \forall t,\ f_t \in \mathbb{f}_t^{DEA},\ \ \boldsymbol{u} \in \mathbb{R}_+^{NT}\}, \\ E[u_{it}^{*(s)M} | \boldsymbol{z}_{it}, t] &= \underset{\boldsymbol{z}_{it}\boldsymbol{\alpha} + \tau_t}{\operatorname{argmin}}\{\sum_i \sum_t \eta_{it}^2 : \\ &\qquad \forall i, t,\ u_{it}^{*(s)M} = \boldsymbol{z}_{it}\boldsymbol{\alpha} + \tau_t + \eta_{it},\ \ \boldsymbol{\alpha} \in \mathbb{R}^R,\ \boldsymbol{\tau} \in \mathbb{R}^T,\ \boldsymbol{\eta} \in \mathbb{R}^{NT}\} \end{aligned} \tag{A.23}$$

where the separability assumption requires that each time-specific frontier estimate f_t^{DEA} is independent of environmental factors $\boldsymbol{z}_{it}$.

Next, consider technical inefficiency measurement under a meta-frontier. By the definition of

[58] The technology implied under the convex meta-frontier is the free-disposal convex hull of non-dominated decisions among all observations pooled across producers and time periods.

meta-frontier in (A.21), one can easily back out its pseudo technical inefficiency measurements $\{u_{it}^{*(M)}\}_{it}$. Notationally, let $f_t^{DEA}(\boldsymbol{x}_k; j) = c_{jt} + \boldsymbol{x}_k \boldsymbol{\beta}_{jt}$ denote the predicted output level for input $\boldsymbol{x}_{it}$ for the segment around input $\boldsymbol{x}_k \in \{\boldsymbol{x}_{it}\}_{i=1}^N$ of time-t frontier f_t^{DEA}. Then, the pseudo technical inefficiencies for $f_M \in \mathbb{f}_{M1}^{DEA}$ or $f_M \in \mathbb{f}_{M2}^{DEA}$ can be backed out in the following operations;

$$
\begin{aligned}
f_M \in \mathbb{f}_{M1}^{DEA}: \ u_{it}^{*(M)A} &= \max_{s,k}\{f_s^{DEA}(\boldsymbol{x}_{it}; k)\} - y_{it}, \ u_{it}^{*(M)M} = \ln[\max_{s,k}\{f_s^{DEA}(\boldsymbol{x}_{it}; k)\}/y_{it}] \\
f_M \in \mathbb{f}_{M2}^{DEA}: \ u_{it}^{*(M)A} &= \max_{k}\{\sum \mu_s f_s^{DEA}(\boldsymbol{x}_{it}; k) - y_{it}, \ \sum_s \mu_s = 1, \ \boldsymbol{\mu} \in \mathbb{R}_+^T\}, \\
u_{it}^{*(M)M} &= \ln[\max_{k}\{\sum \mu_s f_s^{DEA}(\boldsymbol{x}_{it}; k), \ \sum_s \mu_s = 1, \ \boldsymbol{\mu} \in \mathbb{R}_+^T\}/y_{it}]. \qquad \text{(A.24)}
\end{aligned}
$$

The second-stage estimation for the determinants of pseudo-technical inefficiency $u_{it}^{*(M)d}$ (e.g. some productivity measure common to all time periods) defined parallel to (A.22), (A.23) yields;

$$
\begin{aligned}
E[u_{it}^{*(M)d} | \boldsymbol{z}_{it}, t] = \underset{\boldsymbol{z}_{it}\boldsymbol{\alpha} + \tau_t}{\operatorname{argmin}}, \{&\sum_i \sum_t \eta_{it}^2 : \\
&\forall i, t, \ u_{it}^{*(M)A} = \boldsymbol{z}_{it}\boldsymbol{\alpha} + \tau_t + \eta_{it}, \ \boldsymbol{\alpha} \in \mathbb{R}^R, \ \boldsymbol{\tau} \in \mathbb{R}^T, \ \boldsymbol{\eta} \in \mathbb{R}^{NT}\} \qquad \text{(A.25)}
\end{aligned}
$$

where direction $d \in \{A, M\}$ denotes additive or multiplicative technical inefficiency specification.

Finally, regression-based, sample-average MPI and its decomposition are defined using the two sets of parameterizations for technical inefficiency in (A.23) (or (A.22)) and pseudo-technical inefficiency in (A.25). Let τ_t^q for equation $q \in \{s, M\}$ denote the parameters for the time-specific fixed effect in technical inefficiency that are obtained in problems (A.23) (or (A.22)), (A.25) respectively. In addition, let $\tau_t^{M-s} = \tau_t^M - \tau_t^s$ denote the difference in τ_t^q across these estimation equations. Then, the mean MPI, TEC, and TC for two time periods $t0$, $t1$ are defined as (without natural logarithm for additive measures);

$$
\begin{aligned}
\ln MPI_{t1,t0} &= \tau_{t1}^M - \tau_{t0}^M \\
\ln TEC_{t1,t0} &= \tau_{t1}^s - \tau_{t0}^s \\
\ln TC_{t1,t0} &= \tau_{t1}^{M-s} - \tau_{t0}^{M-s}. \qquad \text{(A.26)}
\end{aligned}
$$

Using coefficients τ's from multiplicative model (A.23) (or additive model (A.22)) provides the mean multiplicative (or additive) shifts of productivity, technical inefficiency, and technological frontier between two periods.

With a modification to equations (A.26), one can define the measures of mean MPI, TEC, and TC that are purged from the influences of time-specific shocks $\boldsymbol{W}_t$. For example, using OLS residual $\tilde{\tau}^q$, $q \in \{M, s, M-s\}$ can net out the influences of $\boldsymbol{W}_t$ for mean MPI, TEC, and TC measures in equations (A.26) where

$$\tau_t^q = \boldsymbol{W}_t \boldsymbol{\gamma}^q + \tilde{\tau}_t^q. \tag{A.27}$$

The associated interpretation is that *ex post* technological frontiers $f_t(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}|\boldsymbol{W}_t)$ given realization $\boldsymbol{W}_t$ are collectively related to the *ex ante* meta-frontier $f_t(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}) = \max_{\boldsymbol{W}_t} f_t(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}|\boldsymbol{W}_t)$.[59] Time-specific factors $\boldsymbol{W}_t$ may include weather conditions, market shocks, supply disruptions, or other events that fall outside the individual producer's control and potentially affect both technological frontier and technical inefficiency.

For parametric frontier models like SFA, the measure of TC can be directly estimated under Hicks-neutrality. For notational simplicity, let us focus on the more common, multiplicative technical inefficiency model in equation (A.9). A joint estimation for a time-invariant frontier with augmenting intertemporal shift function $\tau^f(t)$ and some technical inefficiency structure (with its intertemporal shift function $\tau^u(t)$) can be written as;

$$y_{it} = f^{SFA}(\boldsymbol{x}_{it}, \boldsymbol{W}_t; \bar{\boldsymbol{\beta}}, \tau^f(t))\, exp(-u_{it} + v_{it}) = 0, \;\; u_{it} = u(\boldsymbol{z}_{it}, \boldsymbol{W}_t; \boldsymbol{\alpha}, \tau^u(t)) + \eta_{it} \geq 0. \tag{A.28}$$

Replacing time-invariant frontier parameters $\bar{\boldsymbol{\beta}}$ with time-variant counterparts , $\forall t$, $\boldsymbol{\beta}_t = \bar{\boldsymbol{\beta}} + \tilde{\boldsymbol{\beta}}_t$ allows a test of Hicks-neutral TC with the null hypothesis of parameter restrictions $H_0 : \tilde{\boldsymbol{\beta}}_t = \boldsymbol{0}$.

When the hypothesis is rejected, the researcher may employ indirect mean TC measures (along with mean MPI and TEC measures) similarly to the non/semi-parametric counterparts above. For example, a joint estimation for time-specific frontiers (with separate coefficients $\boldsymbol{\beta}_t$, $\forall t$) and a linear technical inefficiency structure with time fixed effects τ_t^s and the marginal effects of environmental variables $\boldsymbol{\alpha}$ is given by;

$$y_{it} = f^{SFA}(\boldsymbol{x}_{it}; \boldsymbol{\beta}_t)\, exp(-u_{it}^s + v_{it}^s) = 0, \;\; u_{it}^s = \boldsymbol{z}_{it}\boldsymbol{\alpha}^s + \tau_t^s + \eta_{it}^s \geq 0. \tag{A.29}$$

When the meta-frontier is defined as a union of these time-specific frontiers (i.e. $f^{SFA}(\boldsymbol{x}_{it}; \boldsymbol{\beta}_M) = \cup_s f^{SFA}(\boldsymbol{x}_{it}; \boldsymbol{\beta}_s)$), the composite error of pseudo-technical inefficiency u_{it}^M and stochastic noise v_{it}^M against the meta-frontier can be backed out by;

$$u_{it}^M - v_{it}^M = \ln[\max_s \{f^{SFA}(\boldsymbol{x}_{it}; \boldsymbol{\beta}_s)\}/y_{it}] \tag{A.30}$$

where u_{it}^M and v_{it}^M can be only jointly obtained. This allows to define (composite) local frontier

[59]Though the terms "*ex ante*" and "*ex post*" are used, for simplicity the current paper abstracts away from modeling decisions under uncertainty in the sense that production process does not allow the producer to prepare for the contingent states of nature. That is, uncertain event realizations are regarded exogenous to producer decisions.

gap $u_{it}^{M-s} - v_{it}^{M-s}$;

$$u_{it}^{M-s} - v_{it}^{M-s} \equiv u_{it}^{M} - v_{it}^{M} - (u_{it}^{s} - v_{it}^{s}) = \ln[f^{SFA}(\boldsymbol{x}_{it}; \boldsymbol{\beta}_M)/f^{SFA}(\boldsymbol{x}_{it}; \boldsymbol{\beta}_t)]. \tag{A.31}$$

Then, one can parameterize the structures of the meta-frontier composite error $u_{it}^{M} - v_{it}^{M}$ and/or the composite frontier gap $u_{it}^{M-s} - v_{it}^{M-s}$. For example, using the OLS regression on the frontier gaps and backing out the parametric structure for the meat-frontier error yields;

$$\begin{aligned} u_{it}^{M-s} - v_{it}^{M-s} &= \boldsymbol{z}_i \boldsymbol{\alpha}^{M-s} + \tau_t^{M-s} + \eta_{it}^{M-s} \\ \tau^{M} &\equiv \tau^{M-s} + \tau^{s}, \;\; \alpha^{M} \equiv \alpha^{M-s} + \alpha^{s}. \end{aligned} \tag{A.32}$$

The estimates for $\boldsymbol{\tau}^{M}$, $\boldsymbol{\tau}^{s}$, and $\boldsymbol{\tau}^{M-s}$ can be then differenced as in equations (A.26) to define the mean MPI, TEC, and TC measures. Additionally, these coefficients may be purged for the time-specific shocks $\boldsymbol{W}_t$ in a simple method like equation (A.27). Note that statistical concepts for $\boldsymbol{\alpha}$'s and $\boldsymbol{\tau}$'s are based on the goodness of fit (with inferences obtained by some bootstrap method) as opposed to a part of the direct statistical model for the DGP.[60] Unlike the two-stage DEA procedure, the separability condition is not needed in this case since the frontier estimation in equation (A.29) jointly accounts for the structure of technical inefficiency.

[60] If desired, one can measure mean TC using hypothetical observations for pseudo-composite error $u_{itk}^{s} - v_{itk}^{s}$ under different time-specific frontiers $f^{SFA}(\boldsymbol{x}; \boldsymbol{\beta}_k)$, $k = 1, .., T$ can be obtained as

$$u_{itk}^{s} - v_{itk}^{s} \equiv \ln f^{SFA}(\boldsymbol{x}_{it}; \boldsymbol{\beta}_k).$$

B Extensions to Meta- and Group-Specific Technologies (and Scale Efficiency)

The section extends the previous concepts of between- and within-subsample technical efficiency comparisons to the situation where subsamples can be constructed with two dimensions of production contexts. Specifically, building on the context of MPI and its decomposition, subsamples in below are defined over given categorical groups and time periods. Thus, technical efficiency comparisons are extended to those for within-group/group-specific MPI and its decompositions and those for between-group counterparts. The main advantage of the regression approach is that it simplifies the measurements of such distances while keeping its concepts consistent with the existing methodology.

Notationally, suppose that observations can be partitioned by group variable $g \in \mathbb{G} = \{1, .., G\}$ (e.g. geographical region) and time period $t \in \mathbb{T} = \{1, .., T\}$. Let subsample $s(.)$ be the mapping of the two variables g, t into the index of mutually exclusive subsamples that represent all (existing) combinations of group g and time t. Accordingly, let the index of observations $i(g, t)$ reflect such partitioning into subsamples, so that subsample-specific frontiers are defined as described in the previous section.[61] Similarly, let the meta-technology be defined as the union of such subsample-specific technologies, or $F = \cup_{s(g,t)} F(s(g, t))$. Following the conventional terminologies in the literature, consider the meta-frontier-level productivity change (meta-fronter MPI: "MMPI") and its decomposition into TEC ("MTEC") and TC ("MTC") and the group-level, group-specific MPI ("GMPI^g") and its decompositions into TEC^g and TC^g for each group g. The relationship between MMPI and GMPI^g can be described with pure technological catch-up (PTCU^g) and frontier catch-up (FCU^g) (Chen and Yang, 2011); for each $g \in \{1, .., G\}$[62]

$$
\begin{aligned}
GMPI^g_{t0,t1} &= TEC^g_{t0,t1} \cdot TC^g_{t0,t1} \\
MMPI^g_{t0,t1} &= MTEC^g_{t0,t1} \cdot MTC^g_{t0,t1} = GMPI^g_{t0,t1} \cdot PTCU^g_{t0,t1} \cdot FCU^g_{t0,t1} \\
\text{where } TEC^g_{t0,t1} &= \widehat{\phi}(\boldsymbol{x}_{t1}, \boldsymbol{y}_{t1}; s(g, t1))/\widehat{\phi}(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}; s(g, t0)) \\
TC^g_{t0,t1} &= \left(\frac{\widehat{\phi}(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}; s(g, t0))}{\widehat{\phi}(\boldsymbol{x}_{t1}, \boldsymbol{y}_{t1}; s(g, t1))} \frac{\widehat{\phi}(\boldsymbol{x}_{t1}, \boldsymbol{y}_{t1}; s(g, t0))}{\widehat{\phi}(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}; s(g, t1))} \right)^{1/2} \\
PTCU^g_{t0,t1} &= TGR(\boldsymbol{x}_{t1}, \boldsymbol{y}_{t1}; s(g, t1))/TGR(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}; s(g, t0)) \\
FCU^g_{t0,t1} &= \left(\frac{TGR(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}; s(g, t0))}{TGR(\boldsymbol{x}_{t1}, \boldsymbol{y}_{t1}; s(g, t1))} \frac{TGR(\boldsymbol{x}_{t1}, \boldsymbol{y}_{t1}; s(g, t0))}{TGR(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}; s(g, t1))} \right)^{1/2} = \frac{MTC_{t0,t1}}{TC^g_{t0,t1}}
\end{aligned}
\tag{B.1}
$$

[61]For example, let $i(g, t)$ be an index for observation i with $t_i = t$, $g_i = g$ and $I(g, t) = \{I | g_i = k, g_i = t\}$ the subset of index set containing N_{gt} observations of $i(g, t)$'s for group g and time t (with $\sum_{gt} N_{gt} = N$).

[62]The current definition of meta-frontier differs from that of Chen and Yang (2011), who consider the meta-frontiers across groups at different time periods.

with MTC, MTEC defined equivalently as TC and TEC in (6) respectively. Thus, conceptually PTCU^g is the intertemporal change in TGR , or the catch-up of the group-specific frontier to the meta-frontier while FCU^g is the relative technical changes between the meta-frontier and group-specific frontier. All of these components of productivity change are calculated on a pair-of-points basis, for instance, at two particular points $(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0})$, $(\boldsymbol{x}_{t1}, \boldsymbol{y}_{t1})$ as discussed in the previous section.

Now, consider a regression-based secondary analysis on efficiencies that sorts out such decompositions for the pooled observations across groups. In particular, the specification parallel to (11) is given by;

$$
\begin{aligned}
\ln \widehat{\phi}(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}; \mathbb{GT}) &= \sum_{g=1}^{G}\sum_{k=1}^{T} \big(\tau^{M}_{g,k} \mathbb{1}_{it}(g_{it}=g)\mathbb{1}_t(t=k)\big) + \boldsymbol{z}_{it}\,\boldsymbol{\alpha}^{M} + \varepsilon^{M}_{it},\\
\ln \widehat{\phi}(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}; s(g,t)) &= \sum_{g=1}^{G}\sum_{k=1}^{T} \big(\tau^{s}_{g,k} \mathbb{1}_{it}(g_{it}=g)\mathbb{1}_t(t=k)\big) + \boldsymbol{z}_{it}\,\boldsymbol{\alpha}^{s} + \varepsilon^{s}_{it}\\
\ln \widehat{\phi}(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}; \mathbb{GT})/\widehat{\phi}(\boldsymbol{x}_{it}, \boldsymbol{y}_{it}; s(g,t) &= \sum_{g=1}^{G}\sum_{k=1}^{T} \big(\tau^{M-s}_{g,k} \mathbb{1}_{it}(g_{it}=g)\mathbb{1}_t(t=k)\big) + \boldsymbol{z}_{it}\,\boldsymbol{\alpha}^{M-s} + \varepsilon^{M-s}_{it},\\
\forall g = 1,..,G,\ t=1,..,T,\ \tau^{M-s}_{g,t} &= \tau^{M}_{g,t} - \tau^{s}_{g,t},\ \ \boldsymbol{\alpha}^{M-s} = \boldsymbol{\alpha}^{M} - \boldsymbol{\alpha}^{s},\ \ \varepsilon^{M-s}_{it} = \varepsilon^{M}_{it} - \varepsilon^{s}_{it}
\end{aligned}
\tag{B.2}
$$

where group $g_{it} \in \{1,..,G\}$ is the group index for each observation it. For g-th group in time t, parameters $\tau^{M}_{g,t}, \tau^{s}_{g,t}$ capture the mean technical efficiencies at the meta-level and within-subsample level respectively, and parameter $\tau^{M-s}_{g,t}$ measures the mean frontier gap between the meta-frontier and subsample-specific frontier. Those coefficients define the regression-average decomposition of MMPI in (B.1); for a given group g and two time periods $\{t0, t1\}$

$$
\begin{aligned}
\ln E[MTEC^{g}_{t0,t1}] &\equiv \tau^{M}_{g,t1} - \tau^{M}_{g,t0}\\
\ln E[MTC^{g}_{t0,t1}] &\equiv 0\\
\ln E[TEC^{g}_{t0,t1}] &\equiv \tau^{s}_{g,t1} - \tau^{s}_{g,t0}\\
\ln E[TC^{g}_{t0,t1}] &\equiv \tau^{M-s}_{g,t1} - \tau^{M-s}_{g,t0} = \ln E[PTCU^{g}_{t0,t1}] = -\ln E[FCU^{g}_{t0,t1}].
\end{aligned}
\tag{B.3}
$$

The second line says that the meta-frontier does not vary over time in our application simply by its definition, and the definitions and interpretations of $MTEC^g$, TEC^g, and TC^g are parallel to those in (10). Since the meta-frontier is constant across time periods, $PTCU^g$ and $-FCU^g$ simply coincide with TC^g. With the definitions in (B.3), the decomposition of MMPI is consistent with (B.1) in the following sense;

$$\begin{aligned}\ln E[MMPI^g_{t0,t1}] &= \ln E[MTC^g_{t0,t1}] + \ln E[MTEC^g_{t0,t1}] = \tau^M_{g,t1} - \tau^M_{g,t0} \\ &= \ln E[TEC^g_{t0,t1}] + \ln E[TC^g_{t0,t1}] + \ln E[PTCU^g_{t0,t1}] + \ln E[FCU^g_{t0,t1}]. \qquad \text{(B.4)}\end{aligned}$$

Finally, if desired, additional layer of decomposition may be added regarding the assumed returns to scale (RTS) structure of the estimated frontier. In DEA, the priori assumption on RTS yields different frontier approximations. In particular, the ratio of the two technical efficiency scores estimated under CRS and VRS frontiers is often referred to scale efficiency (SE). Under the assumption that the CRS frontier is constructed based on the most efficient scale (MES) of operation, a technically-efficient decision under the VRS frontier can be regarded scale-inefficient by the inefficient choice of operation scale other than the MES. When this layer of decomposition for scale efficiency is added to the previous model, all the components of (B.1) are calculated separately under VRS and CRS, and new components regarding scale efficiency change (SEC) are introduced to explain the gaps between the technologies under the CRS and VRS assumptions.[63] Correspondingly, a part of (B.1) is now rewritten as;

$$\begin{aligned}GMPI^{g,C}_{t0,t1} &= TEC^{g,V}_{t0,t1} \cdot TC^{g,V}_{t0,t1} \cdot SEC^g_{t0,t1} \\ MMPI^C_{t0,t1} &= MTEC^V_{t0,t1} \cdot MTC^V_{t0,t1} \cdot MSEC_{t0,t1} \\ &= GMPI^{g,C}_{t0,t1} \cdot PTCU^{g,V}_{t0,t1} \cdot FCU^{g,V}_{t0,t1} \cdot MSEC_{t0,t1}/SEC^g_{t0,t1} \\ SEC^g_{t0,t1} &= \left(\frac{SE(\boldsymbol{x}_{t1}, \boldsymbol{y}_{t1}; s(g,t0))}{SE(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}; s(g,t0))} \frac{SE(\boldsymbol{x}_{t1}, \boldsymbol{y}_{t1}; s(g,t1))}{SE(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}; s(g,t1))} \right)^{1/2} \\ MSEC_{t0,t1} &= \left(\frac{SE(\boldsymbol{x}_{t1}, \boldsymbol{y}_{t1}; s(g,t0))}{SE(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}; s(g,t0))} \frac{SE(\boldsymbol{x}_{t1}, \boldsymbol{y}_{t1}; s(g,t1))}{SE(\boldsymbol{x}_{t0}, \boldsymbol{y}_{t0}; s(g,t1))} \right)^{1/2} \\ &\text{where} \quad SE(\boldsymbol{x}_t, \boldsymbol{y}_t; s(g,t')) = \widehat{\phi}^{CRS}(\boldsymbol{x}_t, \boldsymbol{y}_t; s(g,t'))/\widehat{\phi}^{VRS}(\boldsymbol{x}_t, \boldsymbol{y}_t; s(g,t')) \in (0,1], \qquad \text{(B.5)}\end{aligned}$$

for which superscripts C, V denote CRS and VRS assumptions on the technology, and scale efficiency (SE) takes the ratio of the efficiency under CRS technology to the efficiency under VRS.

The assumption on returns to scale (RTS) wraps the whole model of meta-technology F in (2) (and subsample-specific technologies in (1)) in the sense that $F^{VRS} \subset F^{CRS}$.[64] This reflects the fact that efficiency measurements, including those against meta-technologies, are calculated by taking scale-environment $RTS \in \{VRS, CRS\}$ as given. Statistical estimations like (11) can be carried out either pooled or separately for each scale-environment, depending on the assumptions on statistical errors. Finally, all the regression-average decompositions like (B.3)

[63] Fare et al. (1994) have integrated the concept of SEC in the MPI decomposition.
[64] If NIRS is added, $F^{VRS} \subset F^{NIRS} \subset F^{CRS}$.

are calculated from efficiency measurements under VRS except for new SEC components given by;

$$\begin{aligned}
\ln E[MSEC^g_{t0,t1}] &= (\tau^{M,C}_{g,t1} - \tau^{M,C}_{g,t0}) - (\tau^{M,V}_{g,t1} - \tau^{M,V}_{g,t0}) \\
\ln E[SEC^g_{t0,t1}] &= (\tau^{g,C}_{g,t1} - \tau^{g,C}_{g,t0}) - (\tau^{g,V}_{g,t1} - \tau^{g,V}_{g,t0})
\end{aligned} \tag{B.6}$$

where C, V in the superscripts denote CRS and VRS for the assumed RTS structure. Thus, SEC is obtained as the difference in mean-level efficiencies across time in the difference between the two scale assumptions, resembling a difference-in-differences estimator.

C Alternative Input-Output Specifications

Estimations are replicated for the following alternative specifications of outputs and reference technologies

- A. Output: Milk (cwt), Reference: obs. in concurrent years
- B. Output: Output equivalent, Reference: obs. in all previous and concurrent years
- C. Output: Output equivalent, Reference: obs. in concurrent years
- (Base model: Milk (cwt), Reference: obs. in all previous and concurrent years)

where output equivalent is the gross income deflated by the price indices weighted for its components at the observation level. Summary statistics and estimation results are presented in below. More details on the results in these specifications are available upon request.

Table C.1: Summary of DEA Efficiency and TGR Scores

	RTS	Subsample	Summary Statistics Min	25th	Median	Mean	75th	Max
A. Milk, Concurrent Reference Frontiers								
a. Efficiency at year-specific frontiers								
(1)	NIRS	Confinement	0.489	0.840	0.930	0.909	1.000	1.000
(2)	NIRS	Grazers	0.419	0.815	0.973	0.900	1.000	1.000
(3)	CRS	Confinement	0.489	0.832	0.918	0.903	1.000	1.000
(4)	CRS	Grazers	0.419	0.789	0.936	0.881	1.000	1.000
b. TGR using meta-frontiers								
(5)	NIRS	Confinement	0.724	0.895	0.934	0.927	0.967	1.000
(6)	NIRS	Grazers	0.579	0.858	0.907	0.899	0.964	1.000
(7)	CRS	Confinement	0.692	0.887	0.920	0.919	0.957	1.000
(8)	CRS	Grazers	0.579	0.863	0.922	0.909	0.975	1.000
B. Output Equiv., Cumulative Reference Frontiers								
a. Efficiency at year-specific frontiers								
(1)	NIRS	Confinement	0.511	0.794	0.866	0.866	0.960	1.000
(2)	NIRS	Grazers	0.538	0.764	0.887	0.863	1.000	1.000
(3)	CRS	Confinement	0.511	0.785	0.863	0.863	0.956	1.000
(4)	CRS	Grazers	0.475	0.750	0.868	0.847	0.990	1.000
b. TGR using meta-frontiers								
(5)	NIRS	Confinement	0.642	0.816	0.928	0.902	0.985	1.000
(6)	NIRS	Grazers	0.564	0.728	0.810	0.820	0.928	1.000
(7)	CRS	Confinement	0.633	0.821	0.920	0.897	0.980	1.000
(8)	CRS	Grazers	0.554	0.705	0.806	0.816	0.922	1.000
C. Output Equiv., Concurrent Reference Frontiers								
a. Efficiency at year-specific frontiers								
(1)	NIRS	Confinement	0.532	0.842	0.923	0.906	1.000	1.000
(2)	NIRS	Grazers	0.561	0.865	1.000	0.927	1.000	1.000
(3)	CRS	Confinement	0.532	0.839	0.918	0.904	1.000	1.000
(4)	CRS	Grazers	0.475	0.840	1.000	0.914	1.000	1.000
b. TGR using meta-frontiers								
(5)	NIRS	Confinement	0.643	0.806	0.895	0.878	0.948	1.000
(6)	NIRS	Grazers	0.542	0.672	0.742	0.768	0.830	1.000
(7)	CRS	Confinement	0.649	0.799	0.884	0.869	0.940	1.000
(8)	CRS	Grazers	0.527	0.662	0.738	0.762	0.829	1.000

Table C.2: Comparison of TC Estimates Across Models

Model	A. Milk, Concurrent			B. Output Eq., Cumulative			C. Output Eq., Concurrent		
Year	Point Est.	95% CI	S.Sig.	Point Est.	95% CI	S.Sig.	Point Est.	95% CI	S.Sig.
Confinement									
1996	1.069	(1.052, 1.087)	***	1.126	(1.104, 1.146)	***	1.031	(1.012, 1.051)	***
1997	1.078	(1.064, 1.097)	***	1.170	(1.154, 1.190)	***	1.133	(1.117, 1.155)	***
1998	1.142	(1.126, 1.159)	***	1.227	(1.206, 1.247)	***	1.171	(1.154, 1.193)	***
1999	1.131	(1.115, 1.153)	***	1.171	(1.150, 1.193)	***	1.127	(1.108, 1.149)	***
2000	1.167	(1.154, 1.184)	***	1.288	(1.273, 1.306)	***	1.261	(1.244, 1.281)	***
2001	1.226	(1.207, 1.249)	***	1.369	(1.343, 1.395)	***	1.330	(1.307, 1.361)	***
2002	1.186	(1.164, 1.209)	***	1.317	(1.291, 1.348)	***	1.266	(1.241, 1.300)	***
2003	1.173	(1.157, 1.189)	***	1.280	(1.262, 1.302)	***	1.277	(1.257, 1.298)	***
2004	1.078	(1.060, 1.098)	***	1.144	(1.122, 1.167)	***	1.062	(1.046, 1.083)	***
2005	1.142	(1.128, 1.158)	***	1.255	(1.237, 1.274)	***	1.183	(1.168, 1.203)	***
2006	1.279	(1.254, 1.309)	***	1.524	(1.489, 1.561)	***	1.401	(1.373, 1.439)	***
2007	1.146	(1.129, 1.166)	***	1.282	(1.260, 1.306)	***	1.181	(1.163, 1.205)	***
2008	1.271	(1.248, 1.301)	***	1.478	(1.447, 1.515)	***	1.363	(1.333, 1.396)	***
2009	1.127	(1.112, 1.144)	***	1.214	(1.196, 1.234)	***	1.174	(1.159, 1.196)	***
Grazers									
1996	1.074	(1.054, 1.104)	***	1.121	(1.097, 1.149)	***	1.031	(1.011, 1.059)	**
1997	1.042	(1.025, 1.064)	***	1.145	(1.125, 1.172)	***	1.107	(1.087, 1.128)	***
1998	1.057	(1.038, 1.081)	***	1.193	(1.169, 1.220)	***	1.102	(1.080, 1.124)	***
1999	1.036	(1.013, 1.062)	***	1.091	(1.068, 1.121)	***	0.974	(0.954, 0.997)	**
2000	1.080	(1.062, 1.101)	***	1.191	(1.171, 1.214)	***	1.130	(1.111, 1.151)	***
2001	1.069	(1.046, 1.099)	***	1.292	(1.262, 1.325)	***	1.188	(1.162, 1.216)	***
2002	1.021	(0.996, 1.055)		1.193	(1.163, 1.227)	***	1.011	(0.985, 1.040)	
2003	1.023	(1.004, 1.047)	**	1.164	(1.142, 1.190)	***	1.051	(1.031, 1.073)	***
2004	1.034	(1.014, 1.060)	***	1.096	(1.074, 1.122)	***	0.968	(0.948, 0.991)	***
2005	1.078	(1.061, 1.101)	***	1.215	(1.195, 1.236)	***	1.132	(1.113, 1.152)	***
2006	1.160	(1.132, 1.202)	***	1.450	(1.409, 1.493)	***	1.342	(1.306, 1.381)	***
2007	1.116	(1.094, 1.147)	***	1.216	(1.190, 1.243)	***	1.120	(1.097, 1.147)	***
2008	1.144	(1.112, 1.182)	***	1.306	(1.275, 1.347)	***	1.145	(1.114, 1.179)	***
2009	1.083	(1.065, 1.110)	***	1.240	(1.215, 1.269)	***	1.251	(1.227, 1.276)	***

1. Statistical significance for the deviation from one, based on 400 bootstraps applied to the empirical distribution of the year-specific error term in the second-stage regression: *** $\alpha = 0.01$, ** $\alpha = 0.05$, * $\alpha = 0.1$.

2. The table shows the results from NIRS specifications with time-specific variables.

Table C.3: Summaries of TEC, TC Estimates

	Confinement		Grazers	
Output, Ref. Specification	Efficiency	Frontier	Efficiency	Frontier
I. Without Controlling for Time-specific Variables				
A. Milk, Concurrent Reference Frontiers				
1995-1999	0.979	1.100	0.994	1.134
2005-2009	0.950	1.291	0.925	1.198
Difference: TC, TEC	-0.029	0.191	-0.069	0.064
B. Output Equiv., Cumulative Reference Frontiers				
1995-1999	0.975	1.086	0.956	1.137
2005-2009	0.905	1.490	0.882	1.489
Difference: TC, TEC	-0.070	0.404	-0.074	0.352
C. Output Equiv., Concurrent Reference Frontiers				
1995-1999	1.012	1.045	1.000	1.059
2005-2009	0.965	1.378	0.942	1.354
Differencz: TC, TEC	-0.047	0.333	-0.058	0.295
II. With Controlling for Time-specific Variables				
A. Milk, Concurrent Reference Frontiers				
1995-1999	0.978	1.084	0.99	1.042
2005-2009	0.961	1.193	0.939	1.116
Difference: TC, TEC	-0.017	0.109	-0.051	0.074
B. Output Equiv., Cumulative Reference Frontiers				
1995-1999	0.958	1.081	0.962	1.106
2005-2009	0.924	1.199	0.948	1.221
Difference: TC, TEC	-0.034	0.118	-0.014	0.115
C. Output Equiv., Concurrent Reference Frontiers				
1995-1999	0.993	1.092	1.001	1.043
2005-2009	0.974	1.260	0.986	1.198
Difference: TC, TEC	-0.019	0.168	-0.015	0.155

1. The first and last 5-year averages of estimated coefficients are reported as summary measures of TEC and TC during 1995-2009. Efficiency-based technical efficiency change (E.TEC) and frontier-based technical change (F.TC) calculations are shown.

Table C.4: Marginal Effects of Producer-Specific Characteristics

	Confinement			Grazers		
	M.E.			M.E.		
A. Milk, Concurrent Reference Frontiers						
Equation s						
$\mathbb{1}$(Farm ownership)	4.25	(1.15, 7.02)	**	7.42	(0.01, 13.62)	**
$\mathbb{1}$(Off-farm income)	-6.45	(-11.55, -1.13)	**	-6.40	(-13.79, 0.17)	*
Equation M- s						
$\mathbb{1}$(Farm ownership)	1.10	(-0.04, 2.15)	*	2.02	(-1.58, 5.12)	
$\mathbb{1}$(Off-farm income)	0.04	(-1.91, 1.80)		0.99	(-1.87, 4.49)	
B. Output Equiv., Cumulative Reference Frontiers						
Equation s						
$\mathbb{1}$(Farm ownership)	3.55	(-0.03, 7.27)	*	8.13	(0.50, 15.17)	**
$\mathbb{1}$(Off-farm income)	-4.79	(-10.66, 1.18)	*	-11.22	(-20.05, -3.08)	***
Equation M-s						
$\mathbb{1}$(Farm ownership)	0.32	(-0.76, 1.52)		-0.89	(-5.40, 2.55)	
$\mathbb{1}$(Off-farm income)	1.62	(-0.18, 3.37)	*	-10.43	(-14.59, -6.17)	***
C. Output Equiv., Concurrent Reference Frontiers						
Equation s						
$\mathbb{1}$(Farm ownership)	2.54	(-0.71, 6.10)		4.62	(-2.54, 11.53)	
$\mathbb{1}$(Off-farm income)	-4.64	(-10.37, 1.31)		-10.52	(-17.55, -1.82)	**
Equation M- s						
$\mathbb{1}$(Farm ownership)	1.10	(-0.36, 2.34)		1.99	(-1.94, 5.88)	
$\mathbb{1}$(Off-farm income)	1.30	(-1.06, 3.67)		-10.76	(-15.64, -5.96)	***

1. Statistical significance, based on 400 bootstraps applied to the empirical distribution of the error term in the second-stage regression: *** $\alpha = 0.01$, ** $\alpha = 0.05$, * $\alpha = 0.1$.

2. NIRS is assumed.

3. Producer-specific indicators for farm ownership and off-farm income have the means of 0.77 and 0.07 respectively among confinement and 0.71 and 0.21 among grazers.

Table C.5: Marginal Effects of Time-Specific Characteristics

	Confinement			Grazers		
	S.D.*M.E. (Percentage Points)			S.D.*M.E. (Percentage Points)		
Variables	Point Est.	95% CI	S.Sig.	Point Est.	95% CI	S.Sig.
A. Milk, Concurrent Reference Frontiers						
Equation M- s (Frontier-based)						
Rainfall winter	-2.47	(-3.11, -1.75)	***	-1.86	(-2.74, -0.79)	***
Rainfall spring	0.65	(-0.22, 1.42)		0.32	(-1.07, 1.60)	
Rainfall summer	4.76	(3.75, 5.84)	***	-1.30	(-2.76, 0.50)	
Rainfall autumn	2.95	(2.28, 3.55)	***	0.46	(-0.38, 1.34)	
Temp. winter	5.99	(4.91, 7.05)	***	1.16	(-0.24, 2.68)	*
Temp. spring	2.44	(1.78, 3.09)	***	-0.45	(-1.44, 0.46)	
Temp. summer	-6.50	(-7.60, -5.38)	***	-6.11	(-7.82, -4.49)	***
Temp. autumn	5.59	(4.89, 6.26)	***	0.26	(-0.58, 1.32)	
B. Output Equiv., Cumulative Reference Frontiers						
Equation M- s (Frontier-based)						
Rainfall winter	-5.62	(-6.34, -4.98)	***	-6.43	(-7.40, -5.48)	***
Rainfall spring	3.13	(2.29, 4.16)	***	5.46	(4.15, 6.91)	***
Rainfall summer	4.58	(3.51, 5.63)	***	-0.56	(-1.95, 0.93)	
Rainfall autumn	4.16	(3.57, 4.80)	***	6.03	(5.16, 6.83)	***
Temp. winter	3.24	(2.14, 4.50)	***	1.41	(-0.10, 2.91)	*
Temp. spring	3.08	(2.31, 3.91)	***	1.43	(0.42, 2.46)	**
Temp. summer	-6.23	(-7.47, -4.94)	***	-8.87	(-10.40, -7.10)	***
Temp. autumn	8.25	(7.55, 8.95)	***	6.33	(5.32, 7.19)	***
C. Output Equiv., Concurrent Reference Frontiers						
Equation M- s (Frontier-based)						
Rainfall winter	-5.17	(-5.82, -4.47)	***	-7.19	(-8.11, -6.37)	***
Rainfall spring	3.27	(2.29, 4.19)	***	6.00	(5.04, 7.09)	***
Rainfall summer	7.75	(6.52, 8.92)	***	2.31	(1.09, 3.87)	***
Rainfall autumn	4.17	(3.53, 4.78)	***	6.25	(5.52, 7.10)	***
Temp. winter	6.97	(5.80, 8.05)	***	4.87	(3.70, 6.32)	***
Temp. spring	2.72	(2.02, 3.40)	***	0.49	(-0.31, 1.34)	
Temp. summer	-4.96	(-6.13, -3.80)	***	-9.10	(-10.81, -7.69)	***
Temp. autumn	6.72	(6.01, 7.49)	***	2.96	(2.18, 3.85)	***

1. Statistical significance, based on 400 bootstraps applied to the empirical distribution of the year-specific error term in the second-stage regression: *** $\alpha = 0.01$, ** $\alpha = 0.05$, * $\alpha = 0.1$.

2. NIRS is assumed.

3. Marginal effects are shown for the unit change of each variable by one standard deviation. Estimates for the constant term are omitted from this table.

4. Time-specific weather variables of annual rainfall (inches) and seasonal temperatures (Degrees Fahrenheit) for winter, spring, summer, and autumn have the means (s.d.) of 44.3 (8.8), 36.3 (2.6), 54.0 (1.7), 75.2 (1.6), and 57.3 (1.4) during 1995-2009 respectively.

Table C.6: Truncated Regressions for Specification A: Milk, Concurrent Reference Frontiers

	Confinement			Grazers		
	Estimated Coefficients			Estimated Coefficients		
Variables	Point Est.	95% CI	S.Sig.	Point Est.	95% CI	S.Sig.
Intercept	-0.771	(-3.760, 2.180)		-8.036	(-21.620, 10.340)	
$\mathbb{1}$(Farm ownership)	0.091	(0.020, 0.160)	***	0.334	(0.110, 0.580)	**
$\mathbb{1}$(Off-farm Income)	-0.070	(-0.170, 0.020)		-0.126	(-0.430, 0.090)	
Year	0.018	(-0.010, 0.050)		-0.098	(-0.250, 0.090)	
Year Squared	0.000	(0.000, 0.000)		0.003	(-0.010, 0.010)	
Rainfall winter	-0.005	(-0.020, 0.010)		0.015	(-0.040, 0.060)	
Rainfall spring	0.002	(-0.010, 0.020)		-0.009	(-0.070, 0.050)	
Rainfall summer	0.000	(-0.020, 0.020)		0.068	(0.000, 0.140)	*
Rainfall autumn	-0.001	(-0.010, 0.010)		-0.019	(-0.050, 0.020)	
Temp. winter	0.005	(-0.020, 0.030)		0.036	(-0.070, 0.140)	
Temp. spring	-0.001	(-0.020, 0.020)		0.061	(-0.050, 0.140)	
Temp. summer	-0.001	(-0.040, 0.040)		0.017	(-0.160, 0.170)	
Temp. autumn	-0.011	(-0.040, 0.020)		0.016	(-0.090, 0.120)	

1. Statistical significance, based on 400 bootstraps applied to the assumed truncated normal distribution in the second-stage regression: *** $\alpha = 0.01$, ** $\alpha = 0.05$, * $\alpha = 0.1$.

2. It follows Simar and Wilson (2007)'s truncated normal regression on technical inefficiency (without log-transformation) with bias-corrections.

3. NIRS is assumed.