A Dynamic Stochastic Programming Framework for Modeling Large Scale Land Deals in Developing Countries

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A Dynamic Stochastic Programming Framework for Modeling Large Scale Land Deals in Developing Countries

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Abstract

The attractiveness of agricultural land available in developing countries has markedly increased in the last few years. Driven by rising and highly volatile prices for agricultural commodities, large land acquisitions have been undertaken by foreign investors. We formalize the discussion surrounding such large scale land deals through a dynamic stochastic programming model. Within this framework, we first determine the value of a land development project under uncertainty about prices for agricultural commodities, political risk and irreversible capital investment. Second, given an exogenously set corporate tax rate, we determine, in both a cooperative and a non-cooperative setting, the optimal land rental payment. We show that 1) the optimal policy scheme is equivalent to a risk-sharing contract, 2) trading off rental payment with tax revenue is detrimental for both total project value and domestic benefits and 3) taxation has a neutral impact on long-run the land development pace. We complete our study by illustrating our results through an empirical application based on observed individual land deals from Ethiopia and simulations for a specific crop in a selected region that has recently been targeted by foreign investments.

KEYWORDS: FOREIGN DIRECT INVESTMENT, LAND LEASING, REAL OPTIONS, NASH BARGAINING.

JEL CLASSIFICATION: C61, D81, F23, Q24, Q58.

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1 Introduction

Foreign direct investment (hereafter FDI) in the agricultural land of developing and least developed countries is an ongoing trend\(^1\) which seems to be led by the rising and increasingly volatile prices of agricultural commodities (see e.g. von Braun and Meinzen-Dick 2009; Collier and Venables, 2011; Deininger et al., 2011). It remains subject to ongoing research if and to what extent national biofuel policies may add to and boost these market trends (Franco et al., 2010), and if the rising number of large scale foreign land deals indeed may reflect increasing scarcity of productive farmland,\(^2\) since opponents of this perspective claim that the rising number of large-scale land deals rather reflects investors’ expectations of future developments for food and biofuel markets.\(^3\)

Regardless of what is driving the rising number of large scale foreign land acquisitions,\(^4\) this type of FDI may be beneficial for host countries since it may lead to infrastructure development and job opportunities in rural areas (see e.g. von Braun and Meinzen-Dick, 2009). Indeed, for many land deals in Africa it can be observed that the host countries actually negotiate with the investors for such commitments. However, Cotula et al. (2009) find only weak evidence that such benefits, even though negotiated, would actually get delivered.

From an economic perspective, there are several reasons why this delivery may fail. Some authors blame investors’ exploitative intention (see e.g. Borras and Franco, 2010). Official reports of international organizations instead rather tend to attribute this failure to the institutional difficulties and governance problems encountered by foreign investors in certain host countries (von Braun and Meinzen-Dick, 2009; Cotula et al., 2009). Furthermore, several large-scale land deals in Africa have, taking the host country’s perspective, been disappointing in that investors have either completely failed or were observed to only gradually take the newly acquired land under cultivation.

In this paper, we formalize the discussion surrounding such large scale land deals through a dynamic stochastic programming model. This model represents many aspects of the typical bargaining situation between host country and investor. In fact, as for many large-scale land deals in Africa, our model involves a foreign investor willing to invest in land development and corresponding agricultural activities, and a host country which is potentially willing to provide access to land on the basis of a long-term leasing contract. Access to land is, however, costly. The foreign investor must pay a fixed rent to the host country which is negotiated among the parties. In addition, taxes on the investor’s profit may be levied.

Once signed the lease contract, the investor has full control on the land development

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\(^1\)Visser and Spooren (2011) report large land acquisitions by multinational corporations, foreign investors and foreign governments taking place mainly in Least Developed Countries (LDCs, hereafter) Large acquisitions in Sub-Saharan Africa, for instance, concern projects requiring more than 1000 ha. Examples of such projects include a 452,500 ha biofuel project in Madagascar, a 150,000 ha livestock project in Ethiopia, and a 100,000 ha irrigation project in Mali (see Cotula et al., 2009).

\(^2\)On the competition between energy and food crops see Chakravorty et al., (2009).

\(^3\)Among the potential drivers for land acquisition, Hall (2011) mentions also the increasing gains from carbon emission trading and the recent financial crisis. The author notices in fact that, for instance, vast areas of Savannah-land in Africa may qualify not only as potential farmland but also as carbon sink. The crisis instead may have been triggering investors’ interest toward secure assets such as farmland.

\(^4\)Even though in Africa land deals tend to be long-term leases rather than purchases.
process. Land development decisions must, however, be taken accounting for 1) uncertainty about global market conditions for agricultural products, 2) risk of adverse institutional or political events in the host country, and 3) a fixed sunk capital cost for the activation of land as input for the production of agricultural goods. We model the stochastic evolution of profits by considering a shifting random component evolving according to a geometric Brownian motion. Political risk is accounted by attaching a loss in terms of project value to the occurrence of politically critical events. We model the occurrence of such events by using a Poisson process.\footnote{Note that the occurrence of other catastrophic events (draught, flood, etc.) may be introduced in the model in a similar way.}

We solve the underlying land development problem by determining the optimal time trajectory for land conversion and the value of the land development project. It is found that in expected terms land development proceeds slowly when expected agricultural profit growth is slow and characterized by high volatility. This result is clearly in line with findings in the literature using a real option approach. Political risk raises the implicit interest rate which should be used in order to evaluate the economic sense of investing in land development and has a negative impact on land development speed. This makes sense considering that the investor is aware that the value generated by additional investment is under political threat and may be later seized. We then study the effect of taxation on short-run land dynamics and show that profit taxation has a negative impact on expected land conversion speed. Clearly, taxation by reducing the profit attached to land development makes more prudent the investor who prefers to wait for higher profit realizations in order to fairly compensate the investment effort. In contrast, studying the long-run dynamics, we find that the long-run rate of land development is not affected by tax considerations. In the long-run, what will only matters are profit expected growth and volatility and the magnitude of decreasing return to scale characterizing the production technology.

In the next step, once the value of a hypothetical land development project has been assessed, we determine the optimal rental payment the host country should require. This is done by considering two possible settings, namely a cooperative and a non-cooperative one.

We show that under both settings the level of the rental payment has no impact on the land development policy. In fact, the foreign investor has, by signing the contract, committed to this payment irrespectively of the destination that s/he would give to leased land during the contract duration. In contrast, we find that the level of taxation levied by the host country on future corporate profits may impact on the extent of land which may be developed in the short-run.

In the cooperative scenario, we view the two parties as engaged in a cake splitting game which may be solved applying the Nash bargaining solution concept. Each party is characterized by a certain bargaining power and negotiates over the amount to be paid as rental payment. Disagreement payoffs are null given that without an agreement the land development project is not activated. We solve the underlying game and then determine the optimal rental payment. It is found that 1) the parties share the total value generated by land development project on the basis of their relative bargaining strength, 2) the optimal rental payment should be such that, once added to tax revenues, the value accruing to the host country is equal to his/her share of the total value, and, 3) the lease contract is equiv-
alent to the definition of a risk-sharing contract between the parties. In this respect, we notice in fact that the host country’s revenues include a certain component represented by the rental payment and a volatile one represented by taxes on the uncertain profits earned by the foreign investor. This implies that, for instance, by setting higher taxes and a lower rental payment the host country assume a more risky position. In contrast, the foreign investor would reduce the risk of the project by obtaining, through a reduction in the (certain) rental payment, an implicit subsidy. We then study the effect that corporate taxation has on the total value of the project and on the shares accruing to both parties. We show that a complete tax exemption would maximize both total value and parties’ payoffs. In this case, the parties would not share the risk of the project. In fact, the host country receives only the rental payment while the foreign investor bears the entire risk characterizing the land development project. It is worth to highlight that, taking the investor’s perspective, the potential benefits from risk-sharing are more than balanced by the negative effect that taxes would have on the optimal definition of the land development strategy. Hence, no taxes, by increasing the total value of the project, make also the host country better off in that a higher rental payment is paid.

We then consider a non-cooperative setting in which host country and the foreign investor are engaged in a two-stage game. In the first stage the host country sets the rental payment. His/her objective is the maximization of domestic benefits. In the second stage, given the rental payment set by the host country, the foreign investor considers the opportunity of accepting the proposed contract, and, if accepted, activates the land development project. We show that in this case the host country would be able to extract the entire value of the project generated by the initiative undertaken by the foreign investor. It would suffice in fact to set the rental payment equal to the expected value of the after-tax earnings attached to the project by the foreign investor. As above, we notice that by setting a zero tax rate the host country would be better off. Basically, given the negative effect of taxes, s/he would be able to fully exploit a more valuable project.

Comparison between cooperative and non-cooperative settings shows that the host country would always prefer to lead the game and impose his/her will on the foreign investor. However, given that in reality host countries may compete with each other in order to attract FDI, the bargaining power of the foreign investors increases and the two parties may reach an agreement only by negotiating in a cooperative frame.

In the empirical part of this article, the model is calibrated to a large scale land contract that has been signed between the government of Ethiopia and the Indian "Whitefields" company; the investment concerns 10 000 hectares of cotton in the Ethiopian district of Dasenech Nebremus kebele. We use response surface design to evaluate the sensitivity of the model with respect to certain parameters that may vary in certain plausible ranges. The last section of this article then discusses and concludes about the applicability of the proposed framework for the evaluation of future and past large scale land deals, which might not only be in the interest the two negotiating partners, but may also provide a concerned public with a versatile information tool.

The remainder of this paper is organized as follows: Section 2 summarizes the literature on previous modeling as far as these works constitute the roots of our proposed modeling framework. Section 3 explains how the project value is derived and what role the timing of the land conversion process has in this respect. In section 4 we derive implications for
the optimal rental payment and the optimal profit taxation, respectively, under alternative bargaining situations. Section 5 introduces our empirical illustration, and the remaining sections discuss and conclude implications of our analysis.

2 Related literature

The model developed in this article is originally combining two strands of literature: on the one hand, the theory of optimal investment under irreversibility and uncertainty\footnote{See Dixit and Pindyck (1994) for a complete illustration.} is applied to the inter-temporal allocation of land under alternative uses (e.g., Capozza and Li, 1994; Bulte et al., 2002, Schatzki, 2003; Isik et al., 2004, Song et al., 2011). On the other hand, our analysis builds on previous work about FDI in developing countries (Pennings, 2005; Di Corato, 2013). A common aspect in these two strands of literature is the effect that uncertainty and irreversibility have on investment decisions: irreversible decisions under uncertainty about future net benefits may and later points in time be regretted, and therefore the option that arises from postponing such an irreversible decision may constitute a considerable value, the option value of waiting. The decision to invest occurs within this view as soon as the value of future prospects becomes higher than the cost of investment and the option value taken together.

In the context of this first strand of literature, Capozza and Li (1994) were the first to take a real options approach to the problem of land development in the presence of an option to adjust the land-capital ratio. In Bulte et al. (2002), a social planner manages forestland and must determine a socially optimal conversion plan. This is done by trading off potential profits accruing from agriculture and the uncertain value of a vector of environmental goods and services provided by the forest if kept in its pristine state. Both Schatzki (2003) and Isik and Yang (2004) investigate the decision of setting aside land under the Conservation Reserve Program. Schatzki (2003) shows that hysteresis may characterize the decision of switching to permanent land uses; the authors discuss how not accounting for this effect may influence the outcome of conservation policies. Isik and Yang (2004) study the decision to participate to the Conservation Reserve Program under uncertainty about agricultural profits and set-aside payments. It is shown that the probability of program participation may be importantly affected by option value considerations. Song et al. (2011) adopt a standard entry-exit model in the spirit of Dixit (1989) in order to study land allocation between two competitive destinations, namely traditional cultivation of food crops and energy crops. They allow for the possibility of switching back and forth between food and energy crops and show how regime reversibility may have an effect on land allocation decisions.

As part of the second line of scientific literature, Pennings (2005) studies the decision of a foreign monopolist who may either export or set-up capacity in a host country and feed the local market. The decision must be taken under uncertainty about future demand and by considering that an irreversible investment is required in order to move abroad. The host country maximizes local welfare by offering a policy package which includes initial investment subsidies and a scheme for profit taxation. It is found that the domestic benefits are maximized when the initial foreign investment is strongly subsidized and the tax rate is set such that benefits exceeding the gains from exporting are fully absorbed. Taking
a global welfare perspective, it is also shown that, in the absence of subsidies, domestic welfare maximization induces underinvestment. In Di Corato (2013), a foreign investor contemplates the opportunity of investing in a project for the extraction of a natural resource in a developing country. FDI is compensated by a share on the profits accruing from such a mining project. Residual profits are used to reward the host country for providing access to the resource. Since investment is sunk, the investment decision is analyzed accounting not only for market uncertainty but also for the threat of a successive nationalization. In this setting, it is shown under which conditions a Nash bargaining may lead to a profit distribution not deterring FDI and maximizing the joint venture surplus.

3 The Model: Basic set-up

Consider a risk-neutral host country (hereafter, HC) where a certain surface, \( \mathcal{L} \), of land still in its pristine state, e.g. savannah, forestland, wetland, etc., is available. Assume that HC is financially-constrained and cannot fund a project for the development of this land for agriculture.\(^7\) A risk-neutral foreign investor (hereafter, FI) is willing to invest in such a project if conveniently rewarded. Suppose that at a generic time period \( t \) the two parties can reach a bilateral agreement for the lease of \( \mathcal{L} \) hectares of land. On the basis of such agreement, HC leases land to FI in front of a fixed and certain total rental payment, \( R \geq 0 \).\(^8\) FI has then the right to develop land and destine it to agriculture. A corporate income tax, \( s \in (0, 1) \), must be paid over each unit of profit accruing from land once developed.

Denoting by \( A_t \) the hectares of land developed and under agriculture and by \( L_t \) the extent of land still in its pristine state, at each \( t \geq 0 \) land is allocated as follows,

\[
A_t + L_t = \mathcal{L}, \quad \text{with } A_0 = 0
\]  

Assume that land under cultivation guarantees the following profit flow:\(^9\)

\[
\pi(\theta_t, A_t) = \theta_t A_t^{1-\phi}/(1 - \phi)
\]

where \( 0 < \phi < 1 \) is constant term representing the degree of decreasing return to scale and \( \theta_t \) is a random variable shifting profits, \( \pi(\theta_t, A_t) \), over time.

Let \( \theta_t \) evolve according to the following diffusion:

\[
d\theta_t = \mu \theta_t dt + \sigma \theta_t dZ_t, \quad \text{with } \theta_0 = \theta
\]  

\(^7\)One may equivalently consider ranching, cultivation of energy crops, commercial forestry, etc.

\(^8\)This amount may be thought as the net present value (hereafter, NPV) of a periodic rental payment, \( r \), per hectare paid over the entire contract duration. So, assuming that the contractual agreement has a term sufficiently long that can be approximated by infinity, we may set \( R = (r/\rho)\mathcal{L} \) where \( \rho \) is the discount rate. This can be done at no loss in terms of generality for our results considering also that contractual agreements for the lease of land have generally a long duration.

\(^9\)Our profit function is consistent with a standard setting such as a price-taking farm whose production technologies show decreasing returns to scale (see Appendix). Note also that it may apply to the case of a monopolist using a constant returns to scale technology and facing a demand curve with \(-1/\phi\) as constant elasticity and a multiplicative shocks \( \theta_t \). See for instance Hartman and Hendrikson (2002).
where $\mu$ and $\sigma$ are drift and volatility parameters and $dW_t$ is the increment of a Wiener process with $E[dZ_t] = 0$ and $E[dZ_t^2] = dt$.

Using (2) and (3), we can express profit dynamics as follows:

$$
d\pi(\theta_t, A_t) = (\partial \pi(\theta_t, A_t)/\partial \theta_t)d\theta_t + (\partial \pi(\theta_t, A_t)/\partial A_t)dA_t
$$

$$
= [(\mu dt + \sigma dZ_t) + (1 - \phi)(dA_t/A_t)] \pi(\theta_t, A_t)
$$

(3.1)

where first term represents the marginal effect of changes in $\theta_t$ while the second captures the marginal effect due to additional land conversion.\(^{10}\)

Finally, we complete our set-up by including the following assumptions:

1. Land development is costly and irreversible. In particular, we assume that land development requires a sunk investment in capital costing $k$ per hectare.

2. Land development is undertaken under country specific political risk. In this respect, our definition of political risk includes all political decisions and events reducing the profitability of the land development project initiated by FI. We regulate their occurrence by a Poisson process\(^{11}\) with intensity $\lambda \in (0, \infty)$ and denote by $\omega \in (0,1]$ the percentage of project’s value lost. This means that at each generic $t$, for each $\$ of project’s value, a loss equal to $\omega$ may occur with probability $\omega dt$.

### 4 Project value and optimal land conversion policy

In this section, we view FI as holding the option to develop land and study the optimal land development policy to be followed once signed the contract. Once determined the value attached to the land development project, we will move backward and compare it to the total rental payment, $R$. This will allow us to assess the economic convenience of having signed a contractual agreement with HC in the first place. As one may easily see, once signed the contract, the opportunity of developing land does not depend on the rental payment.\(^{12}\) In contrast, it does depend on 1) the random fluctuating relative convenience of agriculture with respect to land conservation and, 2) the threat of political events seizing the value of the development project.

Suppose that at the generic time period $t$ a surface $A_t \leq \overline{L}$ is developed while the remaining area, $L_t = \overline{L} - A_t$, is still undeveloped. Hence, assuming that $\pi(\theta_t, A_t)$ is such that the optimal policy is to maintain the current land allocation, the value of the development project is given by the following Bellman equation:

$$
V^{FI}(\theta_t, A_t) = (1 - s)\pi(A_t)dt + (1 - \omega \lambda dt)E[V^{FI}(\theta_t + d\theta_t, A_t)]
$$

$$
= 1 + \rho dt
$$

(5)

\(^{10}\)Note in fact that $\partial \pi(\theta_t, A_t)/\partial A_t = (1 - \phi)\pi(\theta_t, A_t)/A_t = \theta_t A_t^{-\phi}$.

\(^{11}\)See for instance Clark (1997).

\(^{12}\)In this respect, we remember that $R$ (or the periodic per hectare rental payment $r$) must be paid irrespectively of the land destination set by FI, i.e. land conserved in its pristine state or land converted to agriculture.
where $\rho(> \mu)$ is the discount rate.$^{13}$

Solving the maximization problem, we show in the appendix that

**Proposition 1** FI develops land ($dA_t > 0$) every time the process $\{\theta_t : t \geq 0\}$ reaches the barrier

$$\theta^*(A_t) = \frac{\beta}{\beta - 1} \frac{k}{1 - s}(\delta - \mu)A_t^\phi$$  \hspace{1cm} (6.1)

or, rearranged in terms of profit, whenever current profit, $\pi(A_t)$, reaches the critical threshold profit level

$$\pi^*(A_t) = \frac{\beta}{\beta - 1} \frac{k}{1 - s}(\delta - \mu) \frac{A_t}{1 - \phi}$$  \hspace{1cm} (6.2)

where $\delta = \rho + \omega \lambda$ and $\beta(> 1)$ is the positive root of the equation $\Phi(\beta) = (\sigma^2/2)\beta(\beta - 1) + \mu\beta - \delta = 0$.

**Proof.** See Appendix. ■

The critical profit threshold, $\pi^*(A_t)$, is linearly increasing in $A_t$. That is, the larger is the surface under agriculture the higher should be the agricultural profit inducing additional land conversion. This implies that the expected timing for the development of the next marginal unit of land increases as land is developed. This makes intuitive sense considering that agricultural profits are concave in the degree of (decreasing) return to scale. Note also that $\partial\pi^*(A_t)/\partial \phi > 0$. That is, the lower the degree of (decreasing) return to scale ($\phi \to 0$), the earlier land development occurs in expected terms. As expected, the critical threshold in (6.2) is also increasing in $s$, which means that the higher the corporate tax rate, $s$, the slower is land development. A further element deterring conversion is represented by higher capital investment costs, $k$, $\partial\pi^*(A_t)/\partial k > 0$.

Let’s now briefly discuss the impact of a change in remaining parameters $\sigma$, $\mu$, and $\delta$.$^{14}$ In order to do it, we rearrange (6.2) as follows:

$$(1 - s)\theta^*(A_t)A_t^\phi = [(\sigma^2/2)\beta + \delta]k$$  \hspace{1cm} (6.3)

On the LHS of (6.3) we find the marginal net benefit from developing a hectare of land while on RHS the relative marginal cost. Note that the marginal cost is represented by the rental cost of a unit of capital, $\delta$, adjusted by adding the term, $(\sigma^2/2)\beta$, to account for market uncertainty. The impact of expected profit growth, $\mu$, and profit volatility, $\sigma$, is in line with findings in the real options literature. In particular, we note that as future agricultural net returns becomes more volatile FI postpones land conversion, i.e., $\partial\theta^*(A_t)/\partial \sigma^2 > 0$. In contrast, the higher the expected profit growth rate then the lower is the critical threshold which triggers additional land conversion, i.e., $\partial\theta^*(A_t)/\partial \mu < 0$. Note also that $\lim_{\sigma \to 0}[(\sigma^2/2)\beta + \delta] = \delta$. That is, as market uncertainty vanishes then land conversion occurs whenever marginal profits covers the rental cost of capital, $\delta k$. Finally, a higher discount induces delayed land conversion, i.e., $\partial\theta^*(A_t)/\partial \delta > 0$. This result deserves further comment considering each of

$^{13}$Note that $\rho > \mu$ is needed in order to guarantee that the discounted stream of profits converges. In addition, note also that, in order to account for risk aversion, one may use CAPM and calculate a risk-adjusted discount rate. See Harrison and Kreps (1979).

$^{14}$Note that $\partial\beta/\partial \delta > 0$, $\partial\beta/\partial \mu < 0$ and $\partial \beta/\partial \sigma^2 < 0$. See section A.3 in the appendix.
specific components of the discount rate $\delta$. An higher $\rho$ implies an higher rental cost for the capital, $\rho k$, while an higher $\lambda$ and $\omega$ a more likely loss in the project value and a larger loss due to political events, respectively. It is immediate to see that all these considerations lead to a more prudent land development policy for FI.

Now, let’s determine the values of the land development project for both parties. In the Appendix we show that

**Proposition 2** For any land allocation $A \leq \bar{L}$, FI and HC’s value function are given, respectively, by

$$V^{FI}(\theta, A) = \frac{k}{\beta - 1} \int^\bar{L}_A \left( \frac{\theta}{\theta^*(\xi)} \right)^{\beta} d\xi + (1 - s) \frac{\pi(\theta, A)}{\delta - \mu}$$

(7.1)

and

$$V^{HC}(\theta, A) = \frac{\beta}{\beta - 1} \frac{s}{1 - s} k \int^\bar{L}_A \left( \frac{\theta}{\theta^*(\xi)} \right)^{\beta} d\xi + s \frac{\pi(\theta, A)}{\delta - \mu}$$

(7.2)

where $\theta^*(\xi) = \frac{\beta}{\beta - 1} \frac{k}{1 - s} (\delta - \mu) \xi^\phi$.

**Proof.** See Appendix. ■

In (7.1) the first term represents the value of the option to develop the surface $\bar{L} - A \geq 0$ while the second term represents the expected present value of the project if the current land allocation $A \leq \bar{L}$ is kept forever. A similar interpretation can be given to the terms in (7.2). However, it is worth to highlight that the main difference between the two parties is that only FI has control over the development process. In fact, while FI, on the basis of the contractual agreement, keeps under its own control the land development process, $dA$, the HC may attach to the surface potentially developable only the expected value of the potential earnings which can be obtained through the taxation of the profits. Note that the term $(\theta/\theta^*(\xi))^\beta$ is a stochastic discount factor which discounts future potential earnings accruing from the future development of the surface $\bar{L} - A$.

Finally, let’s conclude this section by studying the factors determining the dynamic of land development in the long-run. Using (6.1) and denoting by $E[d\ln A]/dt$ the long-run average growth rate of land development we can prove that

**Proposition 3** For any land allocation $A \leq \bar{L}$ the expected long-run growth rate of land development is given by

$$\frac{1}{dt} E[d\ln A] \simeq \begin{cases} 
\frac{\mu - \sigma^2/2}{\phi} & \text{for } \mu > \sigma^2/2 \\
0 & \text{for } \mu \leq \sigma^2/2 
\end{cases}$$

(8)

**Proof.** See Appendix. ■

It is worth to highlight that expected profit growth must be strong enough in order to have a positive long-run average development rate, i.e., $\mu > \sigma^2/2$. Otherwise, due to the deterring effect of profit volatility, the rate is null. In line with these considerations, note that the long-run rate development rate is increasing $\mu$ and decreasing $\sigma^2$. We notice also that, as one may expect, land development speed is decreasing in the degree of (decreasing) return to scale, $\phi$. Finally, by (15), an immediate consideration is that the expected land development rate is independent on the rate of corporate tax, $s$. This in turn implies that for what concerns long-run dynamics, HC’s fiscal policy has a neutral impact.
5 The optimal rental payment

The value of the project for both parties depends on the timing of land development. This is in turn dictated by the optimal development trigger, $\pi^*(A)$, which, as highlighted above, is set by the party having control over the development process, i.e., FI. It is, however, important to stress the crucial role that other two crucial aspects have on the development process: 1) the rental payment, $R$, to be paid by FI in order to have access to the exploitation of land surface $T$, and 2) the tax rate, $s$, set by HC on FI’s profits.

First, concerning $R$, as one may immediately see, the start of the land development project is conditional on the two parties’ agreement upon the terms of the contract. Once such agreement is reached, the contract is signed and the project can start. In this respect, setting $R$ is crucial. The rental payment must in fact be set in order to satisfy a basic set of participation constraints. That is, at $t = e_t$ where $e_t$ is the time at which the contract agreement is reached, the following conditions must hold:

$$W^{FI}(\tilde{\theta}, R) = V^{FI}(\tilde{\theta}) - R \geq 0$$
$$W^{HC}(\tilde{\theta}, R) = V^{HC}(\tilde{\theta}) + R \geq 0$$ (9.1-9.2)

where $\theta_t = \tilde{\theta}$.

Note that by [9.1-9.2] we are simply requiring that for both parties the expected value attached to the project is non-negative.

Second, note that

**Proposition 4** At $t = \tilde{t}$, given a certain tax rate, $s$, an agreement between FI and HC over $R$ always entails the instantaneous development of the following land surface:

$$\tilde{A} = \left\{ \frac{(1 - s)}{\sigma^2/\beta + \delta k} \tilde{\theta} \right\}^{1/\phi} \text{ (9.3)}$$

The interpretation of (9.3) is straightforward. By (6.1), the level of $\theta$ at $t = \tilde{t}$ is high enough for supporting some land development, $\tilde{A}$. Note in fact that for any $\theta > 0$, land should be developed up to the amount at which the control $\theta^*$ stops the conversion process, i.e. $\theta^* > \theta$. The extent of such amount of land depends, via (6.1), on, among other parameters, $s$. As shown by (9.3), the relationship between $\tilde{A}$ and $s$ is negative, i.e., $\partial \tilde{A}/\partial s < 0$. That is, the higher the corporate tax rate, the lower the surface that FI finds profitable to develop in the first place. Hence, in the technical parlance, viewing $\tilde{T}$ as a set of options to develop, HC is splitting it in a subset composed by $\tilde{A}$ options "in-the-money" and a subset composed by $\tilde{T} - \tilde{A}$ "out-of-the money". The first group of options must be exercised as soon as the contract is signed while the remainder may be exercised later using (6.1). Changing perspective, by $s$, HC is implicitly 1) illustrating his short-run goals for what concerns the development of the land surface $\tilde{T}$, and 2) setting the amount of land over which FI would exercise control. These considerations seem in line with what observe in the reality where HC are often willing to concede tax holidays to foreign investors.15

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15In this respect, note that, depending on $\tilde{\theta}$ and $\tilde{T}$, it may be feasible to set $s$ such that $\tilde{A} = \tilde{T}$.
5.1 Rental payment: cooperative and non-cooperative solutions

Meeting the goal of a fast and vast land development would, however, come at a cost in terms of tax revenues. As pointed out, this would in fact require a lower tax rate on profits accruing to FI. This loss may be balanced (or reduced) by setting a proper rental payment, $R$. Clearly, as stressed above, this is not a trivial issue since $R$ must be set such that FI's initiative is not deterred. This choice will be the object of this section where, given a certain taxation regime, we will study the definition of an optimal rental payment in two possible settings, namely a cooperative and non-co-operative one.

Cooperative solution - Assume that HC and FI are engaged in a cooperative cake splitting game where. Both parties are neutral to the risk of internal conflicts and we assign bargaining power, $\psi$ and $1-\psi$ with $\psi \in (0,1)$, to each of them, respectively.\(^{16}\) As well know, we may solve the underlying game by applying the Nash bargaining solution concept (Nash, 1950; Harsany, 1977).

A feasible Nash bargaining solution, $R^*_1 \geq 0$ solves the following maximization problem:\(^{17}\)

$$\max_{R_1 \geq 0} \Omega_1 = \psi \ln[W^{FI}(\bar{\theta}, R_1)] + (1-\psi) \ln[W^{HC}(\bar{\theta}, R_1)]$$ (10)

In the appendix we show that

**Proposition 5** At $t = \bar{t}$, when FI and HC jointly decide upon the optimal rental payment, $R^*_1$, in a Nash-bargaining frame, then the optimal payment is set as follows

$$R^*_1 = (1-\psi)V^{FI}(\bar{\theta}) - \psi V^{HC}(\bar{\theta})$$ (10.1)

The interpretation is straightforward. The optimal payment is set on the basis of the relative strength of the two parties. Note in fact that, as expected, $R^*_1$ is increasing in HC's bargaining power and decreasing in FI's strength. Note also that given a certain power allocation $(\psi, 1-\psi)$, a lower $R^*_1$ is paid as the expected value of tax revenues, $V^{HC}(\bar{\theta})$, increases. Consistently, a higher payment is due when a higher expected value is attached to FI's net revenues, $V^{FI}(\bar{\theta})$. Substituting (10.1) into (9.1-9.2) yields

$$W^{FI}(\bar{\theta}, R^*_1) = \psi V(\bar{\theta}), \quad W^{HC}(\bar{\theta}, R^*_1) = (1-\psi)V(\bar{\theta})$$ (10.2-10.3)

where $V(\bar{\theta}) = V^{FI}(\bar{\theta}) + V^{HC}(\bar{\theta})$.

That is, the two parties share the total value at stake, $V(\bar{\theta})$, in shares which are given by their respective bargaining powers. It is worth to highlight that, by bargaining, the two parties are basically setting an optimal risk-sharing contract. Note in fact that HC's revenues include a certain component represented by $R^*_1$ and a volatile one represented by tax revenues, $V^{HC}(\bar{\theta})$. In this respect, one may also view the tax rate $s$ as a share on FI's

\(^{16}\)Note that our frame may easily apply to the analysis of a Nash bargaining game where the two parties are characterized in terms of risk aversion. It would in fact suffice to set the Nash product equal to $(W^{FI})^p(W^{HC})^q$ where $0 < p \leq 1$ and $0 < q \leq 1$, measure the level of risk aversion for each of the parties involved.

\(^{17}\)The objective function (10) is defined on the net gains from bargaining. Disagreement pay-offs are null since without agreement the land development project is not activated.
volatile profits. In addition, as it can be easily shown, for \( s < 1/\beta \), the optimal rental payment, \( R^*_1 \), is decreasing in \( s \). That is, taxing FI’s profits, by lowering the rental payment, is equivalent to an implicit subsidy paid by HC to FI.

**Non-cooperative solution** - Assume that HC and FI are engaged in a two-stage game. HC sets in the first stage the rental payment maximizing local benefits, i.e., \( W^{HC}(\tilde{\theta}, R) \). If profitable, FI signs the leasing contract and contemplates land development in the second stage.

A feasible solution, \( R^*_2 \geq 0 \) solves the following maximization problem:

\[
\max_{R_2 \geq 0} \Omega_2 = W^{HC}(\tilde{\theta}, R_2), \quad \text{s.t.} \quad W^{FI}(\tilde{\theta}, R_2) \geq 0 \tag{11}
\]

It is easy to show that\(^{18}\)

**Proposition 6** At \( t = \tilde{t} \), when HC decide upon the optimal rental payment, \( R^*_2 \), in a non-cooperative setting, then the optimal payment is set as follows

\[
R^*_2 = V^{FI}(\tilde{\theta}) \tag{11.1}
\]

**Proof.** Straightforward from Proposition 5. ■

The interpretation is immediate. The optimal payment to be set by HC is equal to the expected value attached by FI to the development project. In other words, HC can implicitly fully expropriate the benefits accruing from the initiative that FI may undertake. Note in fact that

\[
W^{FI}(\tilde{\theta}, R^*_2) = 0, \quad W^{HC}(\tilde{\theta}, R^*_2) = V(\tilde{\theta}) \tag{11.2-11.3}
\]

It must, however, be said that a non-cooperative outcome is extremely unlikely. In the real word, HC must compete with other countries in order to attract FDI. Thus, competition for capitals, by increasing the bargaining power of foreign investors, leads to the development of negotiations where the two parties must play cooperatively.

### 5.2 Corporate taxation of profits

Let’s conclude this section by checking the impact that corporate taxation has on the payoffs accruing to both parties once an agreement has been reached. By taking the derivative of \( V(\tilde{\theta}) \) with respect to \( s \) we obtain:

\[
\frac{\partial V(\tilde{\theta})}{\partial s} = \frac{\partial V^{FI}(\tilde{\theta})}{\partial s} + \frac{\partial V^{HC}(\tilde{\theta})}{\partial s} = -\beta \frac{s}{(1-s)^2} k \int_A^L \left( \frac{\theta}{\theta^*(A)} \right)^{\beta} dA + \frac{\partial \tilde{A}}{\partial s} k < 0 \tag{12}
\]

which in turn implies

\[
\frac{\partial W^{FI}(\tilde{\theta}, R^*_1)}{\partial s} = \psi \frac{\partial V(\tilde{\theta})}{\partial s} < 0, \quad \frac{\partial W^{HC}(\tilde{\theta}, R^*_1)}{\partial s} = (1-\psi) \frac{\partial V(\tilde{\theta})}{\partial s} < 0 \tag{12.1-12.2}
\]

\(^{18}\)Note that the problem in (11) corresponds to the problem in (10) for the case where FI has no bargaining power, i.e., \( \psi \to 0 \).
That is, both parties would be better off if no taxes are imposed. By setting \( s = 0 \), land development timing would not be affected by tax considerations and the value of the land project would be maximized. Note in fact that for \( s = 0 \), \( V(e) \) is equal to \( V_{FI}(\theta) \) and

\[
\frac{\partial V_{FI}(\theta)}{\partial s} = -\frac{\beta}{\beta - 1} \frac{k}{1 - s} \int_{\tilde{A}} \left( \frac{\theta}{\theta^*(A)} \right)^{\beta} dA - \frac{\pi(\tilde{\theta}, \tilde{A})}{\delta - \mu} + \frac{\partial \tilde{A}}{\partial s} k < 0
\]  

(13)

In (13) the first term captures the negative effect induced by taxation on the timing of land development. The second term represents the impact of higher taxation on FI’s profits. The interpretation of the third term is more subtle. Let’s decompose the terms as follows

\[
\frac{\partial \tilde{A}}{\partial s} k = -\frac{k}{\beta - 1} \frac{\partial \tilde{A}}{\partial s} + \frac{\beta}{\beta - 1} \frac{k}{1 - s} \frac{\partial \tilde{A}}{\partial s}
\]

(13.1)

In (13.1) the first terms measures the positive effect of higher taxation on the set of options to develop held by FI. Basically, a higher taxation reduces the extent of land converted at \( t = \tilde{t} \). Thus, FI maintains flexibility by controlling a larger set of potential development options. This effect is counterbalanced by reduced profit in the short run due to having converted less land at \( t = \tilde{t} \). As shown above, this second effect is prevailing.

Let’s now analyze what a tax exemption would imply for HC. By setting \( s = 0 \) we would have

\[
W^{HC}(\tilde{\theta}, R^*_1) = R^*_1 = (1 - \psi)V_{FI}(\tilde{\theta})
\]

(14.1)

\[
W^{FI}(\tilde{\theta}, R^*_1) = \psi V_{FI}(\tilde{\theta})
\]

(14.2)

Each party receives a portion of \( V(\tilde{\theta}) (= V_{FI}(\tilde{\theta})) \) which is proportional to its own bargaining power. Note, however, that in this case the agreement will not entail any risk-sharing between the two parties since once HC has cashed the payment \( R^*_1 \) the whole uncertainty characterizing the project will only affects FI’s net benefits.

### 6 Empirical implementation

Reliable empirical information about past and ongoing large scale land deals in Africa is almost impossible to obtain. However, individual governments have, in few instances, given in to public requests, and now publish the contractual details of several recently signed large-scale land deals (Ethiopian Land Portal, 2012). In the empirical part of this article we demonstrate that our model closely reflects the conditions that are stated in some of such publicly available contracts. Based on estimates of medium-term price trends for the relevant agricultural crop (cotton in our case) and typical production cost data in the corresponding African regions we then establish plausible ranges for the parameters that enter our model exogenously. The model established in the previous sections is then calibrated to illustrate the impact that each parameter has on the expected net present value of a large-scale investment in agricultural land at the time of signing the contract. Due to the incertainty characterizing our exogenous parameters we refrain from the computation of exact numerical values and rather illustrate the relative magnitude of the expected effect that
Table 1: Exogenous model parameters for the Whitefield Cotton Farm in Ethiopia

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Value or Range</th>
<th>Assumptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$</td>
<td>Project size</td>
<td>10,000 ha</td>
<td>from Whitefield’s contract</td>
</tr>
<tr>
<td>$t$</td>
<td>Duration</td>
<td>25 years</td>
<td>from Whitefield’s contract</td>
</tr>
<tr>
<td>$k$</td>
<td>Cost of developing 1ha</td>
<td>13.48 TBirr, plowing 2.1 TBirr/ha and fixed cost to set up farm</td>
<td></td>
</tr>
<tr>
<td>$w$</td>
<td>Total Average Cost / ha</td>
<td>6.472 TBirr, for an assumed yield of 3000kg/ha</td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>cotton price volatility</td>
<td>$[0.05;0.5]$</td>
<td>average volatility in annual price; estimated as 0.24</td>
</tr>
<tr>
<td>$\mu$</td>
<td>cotton price drift</td>
<td>$[0.005;0.04]$</td>
<td>randomly drawn; original estimate 0.0078</td>
</tr>
<tr>
<td>$p_t$</td>
<td>starting price cotton</td>
<td>$[11,14]$</td>
<td>Average world cotton price (2010 in TBirr/ha)</td>
</tr>
<tr>
<td>$\iota$</td>
<td>Degree of DRTS</td>
<td>$[2;60]$</td>
<td>higher $l \rightarrow$ CRTS</td>
</tr>
<tr>
<td>$c$</td>
<td>Cobb-Douglas</td>
<td>0.25</td>
<td>Factor elasticity for non-land inputs</td>
</tr>
<tr>
<td>$\rho$</td>
<td>risk-free interest rate</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>$\lambda$</td>
<td>loss (probability)</td>
<td>$[0.04;0.08]$</td>
<td>Poisson process; lower bound: one event in 25 years</td>
</tr>
<tr>
<td>$\omega$</td>
<td>loss (share)</td>
<td>$[0.5;1]$</td>
<td>share of investment lost due to political event</td>
</tr>
<tr>
<td>$s$</td>
<td>Corporate income tax</td>
<td>$[0;0.5]$</td>
<td>Ethiopian tax office</td>
</tr>
</tbody>
</table>

As first step of the empirical implementation we calibrate the model established in the previous sections to a land lease contract that has been signed between the Government of Ethiopia and the Indian company "Whitefield Cotton". The contract covers 10000 hectares for cotton production.\(^{19}\) The agreement between "Whitefield Cotton" and the Ethiopian government has been signed on August 1, 2010 and the contract duration is of 25 years. The annual rent amounts according to the contract to 158 Birr/ha. Furthermore, the contract requires developing 25% of the land in year 1 and 100% in year 4. Both parties can terminate the contract within 6 months unless grand majeure forces (e.g. draught, civil conflict, etc.) are the reason. The total net present value for Ethiopia, after taking a negotiated 3-year grace period into account, accrues to 15426.8 thousand Birr (TBirr) for the whole farm, which is equivalent to 2.9 TBirr/ha (own computations based on Ethiopian Land Portal, 2012).

With this information we are able to calibrate the contractual part of the model. Furthermore, in order to determine the profitability of the cotton production process we use output- and input price data for cotton production around the time when the contract was signed. Table 1 presents all parameters that are exogenous to the model; some of them can, due to incertitude or lack of precise information available, only be considered within plausible ranges. Therefore, no attempt was made to present individual parameters at an overly ambitious level of accuracy, and instead the incertitude attached to these parameters has explicitly been incorporated into the model by allowing them to vary stochastically within the specified ranges.

The cotton price in Table 1 has been set according to the world market price around the time when the contract has been negotiated and is allowed to vary according to an uniform distribution. We initially estimate the drift and volatility of cotton prices according

\(^{19}\)The original contract is available at Ethiopian Land Portal (2012).
to a GARCH-1,1 model. However, drift and volatility estimates are sensitive to the time horizon that is considered. Despite long-term time series for cotton world market prices being available, we allow for slightly higher volatility than the one empirically estimated from the 1990s onwards. This decision was made because of the often cited claim that rising volatility has triggered the recent interest in large-scale land acquisitions. Therefore, investors may over-proportionally focus on recent short-term time windows with higher than long-term average price volatility, and price volatility may not only be measured according to real commodity prices but also based on price movements for financial derivatives that are based on agricultural raw materials (OECD, 2011).

Furthermore, both, a real world investor and our model require location-specific estimates of total- and variable cost of cotton production, as well as fixed cost for initial development of uncultivated agricultural land. For potential agricultural investors, the Ethiopian government provides illustrative standards gross margin calculations for the production of the most common cash crops. These standard gross margins already refer to typical large-scale investment projects and receive distribution through the diplomatic body of various foreign Ethiopian embassies (Ethiopian Embassy, 2013). Even though these gross margin computations may in some instances appear simplistic, we assume that they provide a fundamentally realistic approximation to the ex-ante cost estimations that investors will do about a prospective project in reality.

For the purpose of calibration, these cost figures have been re-expressed in terms of per hectare cost (Table 1). For instance, the cost of developing 1 ha has been estimated as cost for surveying, clearing and leveling of farm land and main canal drainage, plus building access through farm roads including hydraulic structure constructions. This total cost is estimated to range about Birr 4.2 million for a hypothetical 2000ha farm, which is 2.1Tbirr/ha (Ethiopian Embassy, 2013).

The specification of political risk of expropriation (or similar catastrophic events) that may occur seldom, but can nevertheless pose a major threat to the investment, is difficult. One option would be to consider country-specific risk indicators as for instance frequently issued by the World Bank. This can be combined with location-specific data about the past occurrence of extreme weather events such as droughts or floods. Table 1 shows that we have chosen both the probability of a catastrophic event and the share of the investment lost due to this event to vary in a relatively wide range in order to assess the sensitivity of the model with respect to such shocks, and in order to reflect the high degree of uncertainty that has to be attributed to these risk sources.

Furthermore, the model imposes for cotton production a Cobb-Douglas technology with decreasing returns to scale. Production inputs are land, and all other inputs required to farm this land, respectively.\footnote{We are aware of factor elasticity estimates for land only from developed countries and have chosen not to apply such estimates here.} The approximate factor elasticities have been obtained by computing the share of variable cost of farming 1 ha of cotton in total cost per hectare. The degree of decreasing returns to scale is captured by a separate parameter that needs to be specified exogenously in order to be able to derive the factor elasticity on land as composite to the factor elasticity for all other inputs. As the decreasing returns to scale parameter tends towards infinity, the technology approaches the behavior of constant returns to scale.
6.1 Simulation Experiment 1: Response Surface

We illustrate the effect of each of the exogenously chosen parameters on the project value to the Foreign Investor at the moment of signing the investment deal. For this purpose, 500 investment projects have been generated under parameter settings that were simultaneously and randomly chosen from the ranges specified in Table 1 ("Monte Carlo simulations"). Based on these data, an econometric response surface is estimated. Specifically, the econometric response surface is an Ordinary Least Squares (OLS) regression model that takes the following form:

\[ y_i = \beta_0 + \beta X_i + u_i \]  
(15)

where \( X_i \) is a vector containing the elements \( \sigma_i, \mu_i, p_i, t_i, \lambda_i, \omega_i, s_i \) for every investment project \( i = 1, ..., 500 \) and \( u_i \) is a term capturing random disturbances that are assumed to follow a standard normal distribution. The dependent variable in this regression model is expressed as the logarithm of the project value to FI.\(^{21}\) In order be able to present the responsiveness of the dependent variable with respect to simultaneous multivariate uncertainty in exogenous parameters in an even more parsimonious way, we have chosen to approximate the nonlinear functional relationships within the model by expressing the dependent and explanatory variables all as their logarithmic values. We furthermore found that the number of hectares that the investor is willing to develop instantaneously after signing the contract (\( \tilde{A} \) in equ. 9.3) is a very good predictor for the dependent variables; however, including this predictor as a regressor in the response surface is not possible because of its endogenous relationship with the dependent variable.

For the case of the Whitefield cotton contract, Figure 1 illustrates the occurrence of initial land development \( \tilde{A} \) for \( n=500 \) simulated projects, given the parameter settings specified in Table 1: In more than 300 out of 500 cases initial land development, \( \tilde{A} \), will cover between 9000 ha and the maximum of 10,000 ha. In about 150 out of the 500 cases, \( \tilde{A} \) falls into the category of up to 1000 ha. Interestingly, the results in Figure 1 show also that initial activation of land takes in less than 10% of these 500 simulated project cases values between 1000 and 9000 ha.

Results from Figure 1 indicate that initial land development according to our model takes overall the characteristic of an ‘everything or nothing’ strategy. According to this result may the government of Ethiopia reconsider the current practice to contractually oblige an initial land development during the first four years (Ethiopian Land Portal, 2012). Fixing the land development path in this artificial way may address the Ethiopian desire to avoid projects under which a foreign investors acquires land without actually getting any development started. However, our model suggests that the economic driving forces of land development on the side of the investor are likely very strong, making land development either profitable or not, and a contract that tries to regulate this may interfere severely with the investor’s perceived risk situation. In other words: If the investor finds the project overall convenient, land development will in most cases happen as soon as possible. However, there is also the

\(^{21}\) One may consider to capture non-linear functional relationships within the model through nonparametric regression techniques or, alternatively, according to parametric splines or other appropriate nonlinear transformations such as the Box Cox transformation.
Figure 1: Simulation results for land immediately developed after signing the contract

possibility that the investor initially does not find land development at large scale profitable. According to the model, such a situation indicates that the combination of uncertainties at the time of signing suggests to rather hold the option to develop the land later.

Table 2 shows econometric response surface estimates based on the Ordinary Least Squares Estimator and the experimental set up for the exogenous model parameters according to Table 1.

Table 2 contains results from two different response surface estimations, both using the same data and explanatory variables. The first set of columns in Table 2 refers, as described, to the log of the project value to FI at the moment of signing. Almost all regressors appear significant at the 5% level or better, and the coefficient of determination suggests a satisfactory fit to the data. One advantage of the log-log transformation is that estimated

|                          | 
|--------------------------|--------------------------|
| Table 2: Regression Results for Initial Project Value to the Foreign Investor  |  
|                          | ln[\[V^F I(\theta)\]] | ln[\[V^F I(\theta)/A\]] |
|                          | Estimate | Std. Error | Pr(>|t|) | Estimate | Std. Error | Pr(>|t|) |
| (Intercept)               | 0.3228   | 0.3008     | 0.2838   | -4.3111  | 1.2064     | 0.0004   |
| ln[\[\mu]\]              | 0.3673   | 0.0179     | <2e-16   | 0.3968   | 0.0721     | 5.89E-08 |
| ln[\[\sigma]\]           | -0.0269  | 0.0082     | 0.0012   | 0.0846   | 0.0331     | 0.0108   |
| ln[\[\tilde{p}\]]        | 4.218    | 0.1006     | <2e-16   | 2.5643   | 0.4035     | 4.74E-10 |
| ln[\[s\]]                | -1.4816  | 0.0489     | <2e-16   | -0.5844  | 0.1924     | 0.0025   |
| ln[\[\omega\]]           | -0.6388  | 0.0345     | <2e-16   | -0.4371  | 0.1384     | 0.0017   |
| ln[\[\lambda\]]          | -0.5691  | 0.0116     | <2e-16   | -0.4336  | 0.0464     | <2e-16   |
| ln[\[\xi\]]              | 0.5039   | 0.0359     | <2e-16   | 0.5588   | 0.1441     | 0.0001   |
| Adj.R^2                   | 0.92     |            |          | 0.29     |            |          |
regression coefficients can directly be interpreted as the corresponding partial elasticities, with a 1% change in the regressor inducing a corresponding percentage change in the dependent variable. The estimated coefficient for the intercept has to be interpreted as the log of the mean model response when all other regressors take zero values. A closer inspection of the estimated coefficients in Table 2 reveals that the partial elasticities of the market price of cotton, the drift of this market price and the DRTS technology parameter each have a positive effect on the project value from the viewpoint of FI. In line with expectations from the theoretical properties of the model, the estimated project value will *ceteris paribus* be higher if the natural conditions of the investment project allow for milder degrees of decreasing returns to scale rather than stronger decreasing returns to scale. In other words, as more closely as the production technology will in reality resemble constant returns to scale, as higher will be the expected project value.

Contrary, the estimated elasticities confirm that the share of the investment lost due to a potential adverse event and due to the corresponding probability of this event to occur, decrease the project value. The same holds for the introduction of corporate profit taxes. It is interesting to note that a 2% corporate tax can roughly offset the value gains from a 1% increase in the market price of cotton at the time signing the contract. The sign of the estimated coefficient on cotton price volatility (ln sigma) appears negative and significant. This contradicts reports in the literature (see introduction) that rising price volatility would, among other factors, actually attract global land deals. Due to the interplay of various factors in our model, however, the positive role of market price volatility on project value is dominated by the negative role that price volatility has on land development and initial land conversion (compare Figure 1). In the second set of regression results in Table 2 we therefore present a response surface regression with the dependent variable given by the project value to FI at the moment of signing (=dependent variable from first set of regression results) divided by $\hat{A}$. This dependent variable can be interpreted as the project value to FI per hectare that FI would immediately develop. In this regression, the estimated coefficient on cotton price volatility is positive and significant, which confirms the previous interpretation that volatility is driving project value in a positive way and initial land conversion in a negative way; apparently, the negative effect dominates the positive one for the case of the Whitefield cotton contract. All other estimated coefficients maintain their previously estimated sign. The substantially lower coefficient of determination in the second set of regressions is due to outliers that are generated by the distribution of $\tilde{A}$ (Figure 1): Most observations get divided by the maximum land available for development (10000ha), while $\tilde{A}$ varies widely below this value for only a small number of observations. However, given the overall robustness of the estimated coefficients when comparing the first against the second set of regression results in Table 2, we conclude from this scenario that the method of econometric response surface estimation with double-log specification can be a parsimonious way to assess the effect of individual exogenous parameters on the aggregated model response, both in absolute terms and relative to other exogenous parameters.
6.2 Simulation Experiment 2: Estimating Ethiopia’s Bargaining Power

The aim of the second response surface simulation scenario is to assess if the Ethiopian government may have exercised a bargaining power that can be considered in line with the public interest of the Ethiopian society; since bargaining power enters the model as a parameter in the range \([0,1]\), intuition may suggest that a bargaining share of 0.5 reflects a balanced negotiation power under which both parties meet ‘on eye level’. Major imbalances in this bargaining share instead may reflect either that one party has signed the contract without insisting to get a near to fair share in the expected total project value, or it may indicate that this party factors in additional benefits from this contract that are not directly observable. Such additional benefits may reflect the Ethiopian government’s hope that a project like "Whitefield" will generate further benefits through forward- and backward linkages within the local economy, and that the investor may provide e.g. infrastructure available for public use.

However, such unobserved benefits could potentially also reflect the attempt by some host country negotiators to acquire individual shares in this investment project (i.e., corruption) without necessarily passing them on to the public.

In order to determine ex post Ethiopia’s bargaining power in the case of the Whitefield cotton contract, it is therefore necessary to assess all benefits that the host country is definitely going to receive. In this respect, the Whitefield cotton contract states only the rental payment over the 25 year contract period, which amounts to a total NPV of 15426.8 TBirr for Ethiopia after taking a negotiated 3-year grace period into account.

A second potential source of revenue is the taxation of profits, once the farm has been established. However, since no income tax is mentioned in the Whitefield contract, we initially assume that no income taxes are levied. However, domestic businesses in Ethiopia certainly face corporate income taxes that progress according to firm profit. Expected profits from the Whitefield cotton project would usually fall into the highest tax category of 35%. However, the Ethiopian government frequently grants tax holidays of up to seven years, e.g. for startup firms. We therefore compare three different scenarios of 500 projects each. All three scenarios use exactly the same specifications for the exogenous parameters as in the previous response surface experiment. However, the first of this three bargaining share scenarios fixes the corporate income tax at zero, the second and third scenarios at 24% and 35%, respectively. The last two scenarios represent the most generous possible and the maximum possible taxation scenario, respectively, given that the official taxation rules for domestic firms would be applied also to the Whitefield cotton project.

Table 3 presents these three scenarios and reports the corresponding median and mean bargaining share that Ethiopia has exercised in the Whitefield cotton contract, given uncertainty in the parameters on price drift and volatility, degree of decreasing returns to scale, cotton price and risk of losing parts of the investment due to catastrophic events (Table 1).

When assuming zero corporate tax, as the Whitefield cotton contract suggest, Table 4 reveals that this would correspond to a rather low bargaining share of Ethiopia, with a mean around 3%. However, Figure 2 illustrates based on Kernel density estimates that this first

\(^{22}\) The rate has been computed accounting for a 7 year tax holiday.
Table 3: Ethiopia’s estimated bargaining share in the Whitefield cotton contract

<table>
<thead>
<tr>
<th>Corporate Tax</th>
<th>n</th>
<th>Median</th>
<th>Mean</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>500</td>
<td>0.027</td>
<td>0.033</td>
<td>Whitefield contract does not mention taxes</td>
</tr>
<tr>
<td>24%</td>
<td>500</td>
<td>0.270</td>
<td>0.257</td>
<td>35% adjusted for initial 7-year tax holiday</td>
</tr>
<tr>
<td>35%</td>
<td>500</td>
<td>0.383</td>
<td>0.430</td>
<td>Relevant Ethiopian corporate income tax</td>
</tr>
</tbody>
</table>

scenario can, despite its rather low mean, still in few instances reflect bargaining shares around 10% to 15%. For the second and third scenario, respectively, Figure 2 demonstrates that the distribution of simulated bargaining shares is much wider than for the first scenario, making a 'fair' (0.5) or near to fair bargaining share certainly realistic. However, Figure 2 also indicates that, in rare events of bargaining shares exceeding 50%, Ethiopia may actually have negotiated with Whitefield cotton an especially favorable deal.

Figure 2: Epanechnikov-Kernel density estimates of Ethiopia’s simulated bargaining share under different profit tax rates

In summary, results from the simulations indicate that for the case of the Whitefield cotton contract, Ethiopia may very well have negotiated reasonably as long as it is going to consider income taxation. However, in order to initially attract the investor, Ethiopia may have granted tax holidays up to seven years but our simulation results suggest that under assumed price drift and volatility combinations for cotton as in the year 2010 when the contract has been signed, such a tax holiday may have been unnecessary. Of course this does not automatically imply that the implementation of this land lease project may not have been unfair against other interest groups (even though for the case of Whitefield cotton we do not have such information); the model only states that the exercised bargaining power can broadly be justified as in line with the interest of the Ethiopian society as a whole.

20
7 Conclusions

We have developed a theoretical model that approximately reflects the typical bargaining situation between a foreign investor and a host country for many currently ongoing or recently signed large-scale land deals in Africa.

Our results from Proposition 1 show that the larger the surface under agriculture, the higher must be the agricultural profit to induce additional land conversion. This implies that the expected timing for the development of the next marginal unit of land increases as more land is developed. This in turn means that, even if global food prices remain at high level, the probability of further land conversion goes down. Proposition 2 shows the expected long-run growth rate of land development can be interpreted as a measure for the profitability of land conversion. Our land conversion rate suggests a straightforward relationship between price trends and volatility and project profitability that can be empirically tested. Proposition 3 shows that as future agricultural profits become more volatile, FI postpones land conversion. In contrast, a higher expected profit growth rate triggers a faster land conversion. As expected, political risk and consequent losses slow down development. A similar effect is associated to income taxation.

We have calibrated this model to one specific land contract in Ethiopia for which we were able to determine plausible ranges within which important exogenous model parameters likely are going to range. By using Monte Carlo simulations and letting these parameters simultaneously vary from corresponding random distributions, we were able to assess the model response surface econometrically and we have estimated the corresponding bargaining share that the Ethiopian government was likely able to raise in the case of this specific contract. Such response surface estimates, once established for a specific contract, may enable host country administrators to conduct rule of thumb predictions about the change of the project value if e.g. negotiations over potential tax exemptions take place.

Findings from the simulations indicate that the land development path is driven by the economic incentives and uncertainties, and the foreign investor will most likely seek to develop all land immediately, but in about one third of simulated cases he will instead postpone almost the entire land development. We therefore found that the host country’s attempt to fix a specific land development path in the negotiated contract is unnecessary, if not harmful, without yielding any definite payoff for the host country.

However, our simulations have also confirmed that the Ethiopian government, as long as it considers to tax corporate profits from this investment project according to the rules that apply to domestic firms, has on average exercised a near to fair bargaining share. Furthermore, it should be stressed that none of the additional benefits in terms of infrastructure or job creation that the host country may hope to receive, have been stated in case of the Whitefield cotton contract as definite deliverables. Additional benefits that may come in terms of infrastructure provision and job creation will be provided in the interest of the foreign investor anyway, if at all, as part of the profit seeking operation of cotton production. Therefore, Ethiopia does not need to ‘pay’ for the provision of these additional benefits from the investment.

The very strong "everything or nothing" land development dynamics indicate that specific regulations likely cannot revert the underlying economic incentives. Thus, if an investor will find a project overall profitable, Ethiopia can request a fair share of the total project value.
without having to be afraid to deter the investor. The role of cotton price volatility however is more ambiguous than often claimed in the literature: In our model we found the positive contribution to project value overcompensated by other factors.

Putting the findings from our analysis into the context of the scientific and grey literature on large-scale land projects in Africa shows that the frequently used term ‘land grabbing’ in this context suggests that something seems to be wrong at least with some of these investment projects. Indeed, reported violations of human- and local property rights seem to have occurred, and a recent World Bank report (Deininger et al. 2011) admits that negotiated benefits in terms of infrastructure have so far hardly been delivered up to the amount that was initially agreed upon.

This paper by no means intends to play down the seriousness of such incidences; however, our analytical framework intends to structure the phenomenon of large-scale land deals in developing- and least developed countries from the perspective of economic theory. In this context, there is little room for ‘good’ or ‘evil’ investors, as some authors want to see them (e.g Collier and Venebales, 2011). Furthermore, the misfunctioning of public institutions within weak states can neither be expected to improve very soon, nor should appeals to a ‘good investment code of conduct’ alone have much power, unless they are supported by very strong economic incentives (see von Braun and Meinzen-Dick, 2009).

In addition, despite attempts to empower local people during negotiations about large-scale land deals, it is perhaps more realistic to assume that people who could potentially get adversely affected by large scale land would initially have only very limited bargaining power. For this reason, there is no third party directly represented within our model. However, the model addresses major risk sources according to which foreign investors decide upon the optimal degree of development of the land under consideration. We have argued that political and social risk sources within the host country, such as riots, unrest, political instability and macroeconomic instability all add to the probability of losing a certain share of the investment. This implies that the socio-political-economic conditions within the host country and the administrative way how a land deal is implemented may crucially determine the success of the investment.

This however assigns an important implicit role to the local people and non-governmental institutions: According to our model, all these socio-political risk sources can potentially have an important impact on the total return to investment, and this impact may offset even very optimistic future price developments.

Furthermore, the assumed Nash bargaining leads to a solution which should approximately reflect the bargaining situation in large-scale land deals, and the model suggests in this context that a negotiation strategy of the local government, which might be targeted primarily towards the acquisition of a large share in the investment, could be short-sighted. Instead, if the total payoff to the host country is considered within a dynamic perspective, a smaller initial share in combination with the reduction of location-specific socio-political-economic risks increases economic incentives for investors to develop ceteris paribus more land into production stages of higher value added.

With respect to policy makers and non-governmental organizations, the findings from our analysis suggest that the role of location-specific risk sources such as socio political conditions, but also location-specific climatic conditions, should perhaps receive more attention when the phenomenon of large-scale land deals is assessed.
With respect to future research, we suggest that case studies as well as panel-data regressions will allow to put the comparative static results from our model to wider empirical test. Further research also needs to clarify to what extent a higher degree of land development also increases the forward- and backward linkages with the local economy for the practice of actually observed land deals. For now, the model suggest that, as long as a ruthless strategy against local people will create social tensions that raise the level of policy risk, the net expected present value of the land development project will decline. Thus, a core finding of the dynamic stochastic programming model presented here is that an economic rationale for long-term sustainable economic growth due to large scale foreign direct investment into land may very well exist, and is based on economic incentives to lower the uncertainty about social tensions that may lead to full or partial expropriation or destruction of the investment.
A Appendix

A.1 Profit function: a price-taker farmer

Suppose the farmer produces a certain agricultural product combining land, $A_t$, with a variable input factor (or several other input factors), $B_t$, e.g. labor, according to the following Cobb-Douglas production function

$$Q_t = (A_t^c B_t^{1-c})^{1-(1/c)}$$

(A.1.1)

where $c$ and $1-c$ are the cost shares for each specific input factor and $1 > c$ indicates the degree of decreasing return to scale.

In the following in order to simplify the notation we introduce $a = (1 - c)^{1/(1-c)}$ and $b = (1-c)/{(1-1/c)}$. Let $p_t$ and $w$ be respectively the output price and the unit price for $B_t$. The operating profit at each $t$ is then given by

$$\pi(p_t, B_t; A_t) = p_t Q_t - wB_t;$$

Since apart from land other inputs can be costlessly and instantaneously adjusted to maximize $\pi(p_t, B_t; A_t)$ over time then the instantaneous short-run profit is given by:

$$\pi(\theta_t, A_t) = \theta_t A_t^{1-\phi}/(1-\phi)$$

where $\phi = 1 - \gamma_a/(1 - \gamma_b)$ and $\theta_t = G p_t^{1/(1-\gamma_b)}$ with $G = \gamma_a (\gamma_b/w)^{\gamma_b/(1-\gamma_b)}$.

Now, assume that the price $p_t$ is stochastic and fluctuates according to the following geometric Brownian motion:

$$dp_t = \mu p_t dt + \sigma p_t dW_t$$

where $\mu$ and $\sigma_p$ are expected growth rate and volatility of $p_t$, and $dW_t$ is the increment of a Wiener process with $E[dZ_t] = 0$ and $E[dZ_t^2] = dt$.

It is immediate to show that $\theta_t$ follows the geometric Brownian motion

$$d\theta_t = [Gp_t^{\gamma_b/(1-\gamma_b)}/(1 - \gamma_b)](\mu p_t dt + \sigma p_t dW_t) + (\sigma_p^2/2)Gp_t^{1/(1-\gamma_b)}[p_t/(1 - \gamma_b)]^2 dt$$

$$= \{\mu p_t/(1 - \gamma_b) + (\gamma_b/2)(\sigma_p^2/(1 - \gamma_b))^2\}Gp_t^{1/(1-\gamma_b)} dt + \sigma p_t/(1 - \gamma_b)Gp_t^{1/(1-\gamma_b)} dW_t$$

$$= \mu \theta_t dt + \sigma \theta_t dW_t$$

(A.1.3)

where $\mu = \mu_p/(1 - \gamma_b) + \gamma_b\sigma_p/(1 - \gamma_b)^2/2$ and $\sigma = \sigma_p/(1 - \gamma_b)$.

A.2 Proof of Proposition 1

Assume $V^F(\theta, A)$ to be twice continuously differentiable. By using Ito’s lemma the Bellman equation in (5) can be rearranged as follows

$$\frac{1}{2}\sigma^2 \theta^2 V^F_{\theta\theta}(\theta, A) + \mu \theta V^F(\theta, A) - \delta V^F(\theta, A) = -(1-s)\pi(\theta, A), \text{ for } 0 \leq A \leq L$$

(A.2.1)

where $\delta = \rho + \omega \lambda$.

---

23We drop the time index for notational convenience.
Differentiating (A.2.1) with respect to $A$ we obtain
\[(1/2)\sigma^2\theta^2\psi_{\theta\theta}^F(\theta, A) + \mu \theta \psi_{\theta}^F(\theta, A) - \delta \psi^F(\theta, A) = -(1-s)\theta A^{-\phi} \tag{A.2.2}\]
where $\psi^F(\theta, A) = V^F_A(\theta, A)$ is the expected marginal net benefit from the conversion of a marginal unit of land.

The closed form solution for the differential equation in (A.2.2) is given by
\[
\psi^F(\theta, A) = f(A, \theta) + \sum_{i=1}^{2} h_i(A)\theta^{\beta_i} \tag{A.2.3}
\]
where $\beta_1 > 1$ and $\beta_2 < 0$ are the roots of the characteristic equation $\Phi(\beta) = (\sigma^2/2)\beta(\beta - 1) + \mu\beta - \delta = 0$, $h_1(A)$ and $h_2(A)$ are two constants to be determined and\(^{24}\)
\[
f(A, \theta) = (1-s)[\theta/(\delta - \mu)]A^{-\phi}
\]
The second term in (A.2.3) is solution to the homogenous part of (A.2.2) and represents the value of the option to develop an additional unit of land. However, since the value of such option should vanish as $\theta$ tends to 0 then we must set $h_2(A) = 0$.\(^{25}\)

The boundary conditions for (A.2.3) are:
\[
v^F(\theta^*, A) = k, \quad v^F_\theta(\theta^*, A) = 0, \quad h_1(\bar{A}) = 0 \tag{A.2.4-A.2.6}
\]
Conditions (A.2.4) and (A.2.5) are the value-matching and smooth-pasting conditions for the FI’s optimal development policy.\(^{26}\) Condition (A.2.6) simply imposes that $A \leq \bar{A}$.

Substituting (A.2.3) into [A.2.4-A.2.5] yields
\[
\begin{aligned}
&h_1(A)\theta^*(A)\beta + (1-s)[\theta^*(A)/(\delta - \mu)]A^{-\phi} = k \\
h_1(A)\beta\theta^*(A)^{\beta-1} + (1-s)[1/(\delta - \mu)]A^{-\phi} = 0.
\end{aligned}
\]
Solving the system for $\theta^*(A)$ and $h_1(A)$ we obtain:
\[
\begin{aligned}
\theta^*(A) &= \frac{\beta}{\beta - 1} \frac{k}{1 - s} (\delta - \mu) A^\phi, \tag{A.2.7a} \\
h_1(A) &= -\frac{1-s}{\beta} \frac{A^{-\phi}}{\delta - \mu} \theta^*(A)^{1-\beta} < 0. \tag{A.2.7b}
\end{aligned}
\]
Note that (A.2.7a) can be easily rearranged in terms of $\pi^*(A)$. That is
\[
\pi^*(A) = \frac{\beta}{\beta - 1} \frac{k}{1 - s} (\delta - \mu) \frac{A}{1 - \phi} \tag{A.2.8}
\]
\(^{24}\)This is determined by using the method of undetermined coefficients. Note that it corresponds to the net discounted value attached to the conversion of a marginal unit of land.
\(^{25}\)Note that $\beta = \beta_1$, hereafter.
\(^{26}\)Optimality requires that marginal benefit and marginal cost are tangent at the threshold $\theta^*$. See Dixit and Pindyck (1994, p. 364).
A.2.1 Comparative statics

In this section we analyze the impact of each parameter on the critical threshold for land development. It is immediate to show that

\[ \partial \pi^*(A)/\partial A = \pi^*(A)/A > 0, \partial \pi^*(A)/\partial s = \pi^*(A)/(1 - s) > 0 \text{ and } \partial \pi^*(A)/\partial k = \pi^*(A)/k > 0. \]

Rearranging (6.2) as follows

\[ (\sigma^2/2)k \leq \frac{A}{1 - \phi}, \]

it is easy to show that

\[ \partial \pi^*(A)/\partial \mu = (\sigma^2/2) \frac{d \beta}{d \mu} k \frac{A}{1 - \phi} = -\frac{(\sigma^2/2)\beta}{\sigma^2(\beta - 1/2) + \mu} \frac{A}{1 - \phi} < 0. \]

Taking the derivative with respect to \( \sigma^2 \) we obtain:

\[ \frac{\partial \pi^*(A)}{\partial \sigma^2} = \frac{k}{2} \left( \beta + \sigma^2 \frac{d \beta}{d \sigma^2} \right) \frac{A}{1 - \phi} = \frac{k}{2} \left\{ -\frac{(\sigma^2/2)\beta(\beta - 1)}{\sigma^2(\beta - 1/2) + \mu} \right\} \frac{A}{1 - \phi} > 0 \]

Finally, the derivative with respect to \( \delta \) is

\[ \frac{\partial \pi^*(A)}{\partial \delta} = [(\sigma^2/2) \frac{d \beta}{d \delta} + 1] k \frac{A}{1 - \phi} = \frac{\sigma^2 \beta + \mu}{\sigma^2(\beta - 1/2) + \mu} k \frac{A}{1 - \phi} > 0 \]

Note that since \( \delta = \rho + \omega \lambda \) then \( \partial \pi^*(A)/\partial \rho > 0 \), \( \partial \pi^*(A)/\partial \omega > 0 \) and \( \partial \pi^*(A)/\partial \lambda > 0 \).

A.3 Proof of Proposition 2

Plugging (A.2.7b) into (A.2.3) we obtain the expected marginal net value from the conversion of a marginal unit of land. That is

\[ v^{FI}(\theta^*(A), A) = f(A, \theta^*(A)) + h_1(A) \]

By integrating it over \( 0 \leq A \leq L \) we obtain the value function for FI\(^{27}\)

\[ V^{FI}(\theta, A) = \int_A^L v^{FI}(\theta^*(\xi), \xi) d\xi = (1 - s) \left[ \frac{\int_A^L \theta^*(\xi) \xi^{-\phi \left( \frac{\theta}{\sigma^2(\xi)} \right)}}{\beta} + \frac{\theta}{\delta - \mu} \frac{A^{1-\phi}}{1 - \phi} \right] \]

Let’s now compute the value function for HC. Once signed the contract, HC has implicitly transferred the option to develop the land surface \( L \) to the counterpart. This implies that the strategy for the exercise of such option is fixed by FI and taken as granted by HC which receives, as payment, a share on the profits accruing as land is developed over time. Given

\(^{27}\)Note that to guarantee the integral convergence we need to impose \( 1 - \phi \beta_1 > 0 \).
a certain land allocation \( A \leq L \), the value of the development project, \( V^{HC}(\theta, A) \), for HC is given by the solution of the following differential equation:

\[
(1/2)\sigma^2\theta^2V^{HC}_{\theta\theta}(\theta, A) + \mu\theta V^{HC}_\theta(\theta, A) - \delta V^{HC}(\theta, A) = -s\theta A^{1-\phi}/(1 - \phi) \tag{A.3.3}
\]

Differentiating (A.3.3) with respect to \( A \) we obtain

\[
(1/2)\sigma^2\theta^2v^{HC}(\theta, A) + \mu\theta v^{HC}_\theta(\theta, A) - \delta v^{HC}(\theta, A) = -sA^{-\phi} \tag{A.3.4}
\]

where \( v^{HC}(\theta, A) = V^{HC}_A(\theta, A) \) is the expected marginal net benefit from the conversion of a marginal unit of land.

The general solution to equation (A.3.4) is given by

\[
v^{HC}(\theta, A) = x(A)\theta^\beta + s\frac{\theta}{\delta - \mu}A^{-\phi} \tag{A.3.4a}
\]

where \( x(A) \) is a constant to be determined.

Since HC takes as given the land development strategy, we can determine \( v^{HC}(\theta, A) \) by simply imposing the a value-matching condition \( v^{HC}(\theta^*(A), A) = 0 \). This yields:

\[
x(A) = -s\frac{\theta^*(A)^{1-\beta}}{\delta - \mu}A^{-\phi} \tag{A.3.5}
\]

Finally, substituting (A.3.5) into (A.3.4a) and integrating \( v^{HC}(\theta, A) \) over \( 0 \leq A \leq L \) we have:

\[
V^{HC}(\theta, A) = s\left[ \int_A^L \frac{\theta^*(\xi)}{\delta - \mu} \xi^{-\phi}(\frac{\theta}{\theta^*(\xi)})^\beta d\xi + \frac{\theta}{\delta - \mu} A^{1-\phi} \right] = \frac{\beta}{\beta - 1 - s} \frac{s}{k} \int_A^L (\frac{\theta}{\theta^*(\xi)})^\beta d\xi + s\frac{\theta}{\delta - \mu} A^{1-\phi} \tag{A.3.6}
\]

A.4 Proof of Proposition 4

Relation (6.1) can be equivalently rearranged as follows:

\[
\eta_t = (1 - s)\theta A^{-\phi}, \quad \text{for } \eta \leq \tilde{\eta} = [(\sigma^2/2)\beta + \delta]k \tag{A.4.1}
\]

where the process \( \{\eta_t : t \geq 0\} \) and \( \tilde{\eta} \) represent the expected marginal profit and the marginal cost attached to the conversion of an additional hectare of land, respectively.\(^{28}\) Note that 1) \( \eta \) moves as \( \theta \) fluctuates and 2) \( A \) will increase to prevent the process from passing the barrier, \( \tilde{\eta} \), whenever \( \eta \) reaches the barrier \( \tilde{\eta} \).

Taking the logarithm of (A.4.1) we get:

\[
\ln \eta = \ln(1 - s) + \ln \theta - \phi \ln A \tag{A.4.2}
\]

\(^{28}\) Note that in the technical parlance \( \{\eta_t : t \geq 0\} \) is a regulated process having \( \tilde{\eta} \) as upper reflecting barrier. See Harrison (1985, chp. 2).
which, on the basis of a straightforward application of the Ito’s lemma, follows the same Brownian motion that \( \ln \theta \) does. That is

\[
d\ln \theta = (\mu - \sigma^2/2)dt + \sigma dZ
\]

Following Dixit (1993, pp. 58-68) the long-run density function for \( \ln \eta \) fluctuating between a lower reflecting barrier, \( l \to -\infty \), and an upper reflecting barrier, \( \ln \tilde{\eta} \), is given by the following truncated exponential distribution:

\[
f(\ln \eta) = \begin{cases} 
(2\mu/\sigma^2 - 1)e^{-(2\mu/\sigma^2 - 1)\ln(\tilde{\eta}/\eta)} & \mu > \sigma^2/2, \\
0 & \mu \leq \sigma^2/2.
\end{cases}
\]

for \(-\infty < \ln \eta < \ln \tilde{\eta}\) \hspace{1cm} (A.4.3)

It follows that the long-run average of \( \ln \eta \) is given by

\[
E[\ln \eta] = \int_{-\infty}^{\ln \tilde{\eta}} \ln \eta f(\ln \eta) d\ln \eta = \ln \tilde{\eta} - (2\mu/\sigma^2 - 1)
\] \hspace{1cm} (A.4.4)

Rearranging (A.4.2) and taking the expected value on both sides, we obtain

\[
E[\ln A] \approx [\ln (1 - s) + E[\ln \theta] - E[\ln \eta]]/\phi \\
= [\ln (1 - s) + \ln \theta_0 + (\mu - \sigma^2/2)t - E[\ln \eta]]/\phi
\] \hspace{1cm} (A.4.5)

Note that since \( E[\ln \eta] \) is independent on \( t \), differentiating (A.4.5) with respect to \( t \) gives the following expected long-run rate of land development:

\[
E[d\ln A]/dt = (\mu - \sigma^2/2)/\phi
\] \hspace{1cm} (A.4.6)

### A.5 Proof of Proposition 5

The first-order conditions for the maximization problem in (10) are\(^{29}\)

\[
\partial \Omega_1/\partial R|_{R=R_1^*} = -\frac{\psi}{W^{FI}(\tilde{\theta}, R_1^*)} + \frac{1 - \psi}{W^{HC}(\tilde{\theta}, R_1^*)} = 0
\] \hspace{1cm} (A.5.1)

From (A.5.1) we obtain

\[
R_1^* = (1 - \psi)V^{FI}(\tilde{\theta}) - \psi V^{HC}(\tilde{\theta})
\] \hspace{1cm} (A.5.2)

or, differently put, \( W^{HC}(\tilde{\theta}, R_1^*) = V^{HC}(\tilde{\theta}) + R_1^* = (1 - \psi)V(\tilde{\theta}) > 0 \).

\(^{29}\) As one can easily check, the second-order condition holds.
References


