Traceability and Reputation in Supply Chains

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Abstract

The paper studies the questions of why and when a supply chain should invest in a traceability system that allows the identification of which supplier is responsible for quality defects due to insufficient non-contractible effort under vertical complementarity in efforts to provide quality and repeated interaction. The downstream firm and consumers observe imperfect, lagged signals of intermediate and final quality. It is demonstrated that in deciding whether to maintain information regarding product origin, firms face a trade-off. On one hand, the downstream firm is tempted to condone limited upstream shirking when products are not traceable to their firm of origin. On the other hand, the downstream firm is tempted to vertically coordinate shirking in the provision of quality when products are traceable. Perfect traceability is not optimal in a supply chain with a small number of suppliers if (1) the ratio of the cost savings from upstream and downstream shirking is neither too high nor too low or (2) the downstream firm sufficiently infrequently detects input defects or (3) the consumer experience is a sufficiently noisy signal of quality. Sufficient conditions under which perfect traceability is not optimal in a supply chain with a large number of suppliers are also provided.

1. Introduction

In 2002, the European Union adopted legislation requiring all food to be traceable to its origins of production, and the United States and several other countries are considering the adoption of similar legislation (European Union 2002; Food and Drug Administration 2012). The rationale for mandatory traceability is that it enables timely identification of the source of contamination or defects in supply chains where inputs from different suppliers are commingled during the processing stage (Golan et al. 2004; Roth et al. 2008). Traceability can also be seen as a tool to maintain trust within a supply chain and build a reputation for quality when firms' behavior is not perfectly observed by consumers (Marucheck et al. 2011). In this paper, we study the effects of information about product movement through a supply chain on reputation building in the presence of moral hazard in the choices of multiple efforts to provide high quality.

We consider a two-stage supply chain with *anonymous*, *vertically complementary*, *experience* inputs in the sense that (a) the identity of the upstream supplier of a given product is unknown, (b) the final quality is low if quality at any stage of production is low, and (c) both intermediate and final product qualities are unverifiable and imperfectly observed after consumption (Buhr 2003; Gibbons 2005; Skilton and Robinson 2009).[1] One example is a meat supply chain. Meat processors source live animals from multiple producers and some quality attributes such as texture, taste, or contamination level are discovered after the identity of supplier has been separated from the meat cut. On-farm and off-farm practices that determine meat quality include genetic screening of animals, sanitation, veterinary care, as well as handling during transportation, slaughter, and storage (Dahl et al. 2004; Koohmaraie et al. 2005). Another example is a fresh produce supply chain where shipments from multiple growers are mixed during post-harvest processing. The quality of fruits and vegetables is controlled by growers through the choice of seed, planting, growing, and harvesting conditions, as well as continuous temperature control during shipping (Blackburn and Scudder 2008; Hardesty and Kusunose 2009).

In the environment studied in this paper, firms along the supply chain will shirk in the provision of quality to save costs unless they are sufficiently patient to build reputation of high quality. As in the case of reputation of a single firm, in our setting with vertically related firms customer trust is maintained in equilibrium because bad performances in the final good market are followed by a punishment phase during which the price is reduced or financial penalties are paid (Bar-Isaac and Tadelis 2008). The key feature of our model is that information about input origin has a *negative* as well as a *positive* effect on reputation building. On the one hand, traceability allows matching the levels of upstream and downstream efforts put into products. This makes it possible to *vertically coordinate* downstream and upstream shirk-

[1] An alternative plausible characterization of production technology is that inputs are "horizontal complements" in determining quality attributes such as food safety whereas low quality inputs from one supplier increase the likelihood that the quality of products that originate from other suppliers is also low as in Giraud-Heraud et al. (2012). It can be readily demonstrated that our analysis is also valid under such quality spillovers which tend to reduce the gains from shirking but also make monitoring of behavior of individual firms along the supply chain more difficult.

ing with a subset of suppliers. On the other hand, traceability allows targeted punishments for unilateral upstream shirking. It tends to be easier to maintain the credibility of the promise to punish upstream shirking when the downstream firm can identify individual upstream performances.

Whether the adoption of a traceability system is optimal in equilibrium is determined by comparing the strengths of the temptation to engage in vertically coordinated "top down" shirking and the temptation to condone occasional anonymous "bottom up" shirking. We find that perfect traceability is not optimal in a supply chain with a small number of suppliers if one or more of the following conditions hold: (1) the ratio of the cost savings from upstream and downstream shirking is neither too high nor too low, (2) the downstream firm not too often detects input defects, (3) the consumer experience is a sufficiently noisy signal of quality. We also provide sufficient conditions under which perfect traceability is not optimal in a supply chain with many suppliers.

In addition, we show that the returns to more precise inter-firm monitoring can increase or decrease following the adoption of a traceability system depending on information regarding quality that is derived from consumption experience. Since one can expect that downstream firms are more likely to detect upstream shirking in more vertically integrated supply chains, our model sheds new light on the empirical evidence that traceability tends to increase the degree of vertical integration in Banterle and Stranieri (2008).[2] Our model suggests that traceability leads to more (less) vertical integration in a supply chain if consumers are relatively likely (unlikely) to observe bad outcomes when the supply chain provides high quality products.

To evaluate the value of information about input origin, we merge the static model of quality leadership in a supply chain with complementary inputs of Hennessy, Roosen, and Miranowski (2001) and the dynamic model of firm reputation with imperfect monitoring of Cai and Obara (2009). Our results regarding conditions that determine whether or not traceability is adopted in equilibrium can be intuitively explained as follows. If the ratio of the upstream and downstream costs of provision of high quality is either very small or very large, consumers need to punish the downstream firm for bad performance in the final good market less harshly under traceability because the downstream firm has little to gain from vertically coordinated deviations relative to vertically uncoordinated deviations, and is able to punish shirkers individually. However, if the cost savings from shirking along the supply chain are similar, supplier anonymity raises profits because the equilibrium temptation to vertically coordinate shirking is stronger than the temptation to tolerate a few anonymous low quality inputs.

The intuition behind the effect of more precise consumer monitoring of upstream and downstream efforts on the value of traceability is also interesting. When the signals of final quality are impre-

[2] Factors influencing the adoption of traceability systems and their effects on vertical relationships in food supply chains are discussed in Galliano and Orozco (2011), Liao, Chang, and Chang (2011), and Heyder, Theuvsen, and Hollmann-Hespos (2012).

cise, the supply chain has little reputational capital at stake. As a result, supplier anonymity is optimal because no additional punishment in the final good market is necessary to make the downstream firm's promise to punish all suppliers for anonymous shirking credible. However, if the signal of final quality is precise, good reputation is very valuable. Then the promise to punish all suppliers for occasional anonymous shirking will not be credible without more severe punishments in the final good market than those that are necessary to assure that the downstream firm restrains itself from vertically coordinated shirking.

To our knowledge, this is the first paper to study an environment in which mandatory traceability can reduce profits when traceability per se is costless. The existing economic literature on traceability assumes an exogenous cost of traceability and considers essentially static settings that require ad hoc assumptions regarding contractual relationships between upstream and downstream firms, the allocation of liability, and the effects of traceability on demand (Starbird 2005; Pouliot and Sumner 2008, 2012; Souza-Monteiro and Caswell 2010; Resende-Filho and Hurley 2012).[3] In contrast, this paper studies a dynamic setting with vertically related firms that rely on self-enforcing (implicit) contracting and reputation of quality. The model's predictions regarding the value of traceability contribute to the empirical literature on traceability in supply chains (Resende-Filho and Buhr 2008; Pouliot 2011; Wang et al. 2010).

Investment in product quality under formal contractual arrangements between suppliers and manufacturers is studied in operations management literature. Baiman et al. (2000) and Lim (2001) consider contract design in static settings with one supplier and one manufacturer. Baiman et al. (2004) and Li et al. (2011) study quality investments in a supply chain where one manufacturer contracts with multiple suppliers and defective components cause the failure of the whole product (the "weakest-link" property). In their settings, the manufacturer makes a single product and does not observe whether individual components are defective without testing. Here we consider a multi-product environment with the "weakest-link" property and assume that downstream firm privately learns the qualities of intermediate products (whether individual components are defective) after processing has begun and the identity of the supplier has been separated from the product.

More generally, our paper is related to the literature on repeated games with imperfect monitoring concerned with the role of information structures. In particular, Kandori (1992) shows that in repeated games with imperfect public monitoring, the set of equilibrium payoffs expands when signals become more informative. In contrast, in our setting with private inter-firm monitoring of input quality, the set of equilibrium payoffs may expand when signals of product origin are garbled. Fong and Li (2010) show that intertemporal garbling of public signals can increase the efficiency of relational contracting by reduc-

[3] One exception is the study of traceability in a model of collective reputation with network monitoring in Saak (2012). While Saak focuses on the role of "horizontal" peer monitoring, we consider a supply chain in which the upstream firms do not observe one another's effort choices and study the effects of traceability on the vertical relationship between upstream and downstream firms.

ing temptations to renege. We consider garbling of private information about the sources of inputs within periods, which reduces temptations to shirk as well as renege.

Our paper also contributes to the literature that studies the role of external instruments such as explicit contracting in Baker et al. (1994), ownership structure in Rayo (2007), firm size in Cai and Obara (2009), and job design in Mukherjee and Vasconcelos (2011) in sustaining relational contracts and reputation mechanisms. In our paper, information about product flows is an external instrument that affects not only relational contracting between firms but also the reputation mechanism in the final good market. In this way, our paper contributes to the literature on optimal contracts in teams since in our model, the principal (downstream firm) cares about the workers' (upstream firms') efforts only because consumers do so (e.g., Che and Yoo 2001).

The rest of the paper is organized as follows. In Section 2, we present the model. In Section 3, we characterize equilibria in which firms achieve the highest expected joint profit in the regime with and without traceability. In Section 4, we establish conditions under which the joint profit is greater in the traceability regime than in the no-traceability regime and vice versa. Section 5 considers the model where the downstream firm detects input defects with a certain fixed probability. Section 6 concludes. Proofs are collected in the Appendix.

2. Model

We consider a supply chain with $1 \le n < \infty$ identical risk-neutral, long-lived upstream firms $U_1,...,U_n$, and one risk-neutral, long-lived downstream firm, *D*. Time is discrete and is indexed by t; let $\delta \in (0,1)$ denote the common discount factor. In each period, each supplier produces either one unit or none of a homogeneous intermediate good and sells it to the downstream firm. The downstream firm transforms the intermediate good into the final good on a one-to-one basis and sells at most *n* units in the final good market.

Upon delivery supplier j's input is randomly assigned a serial number (unique label) $i \in \mathrm{N}$, $j \in \mathrm{U}$, where $\mathrm{N} \equiv \{1,2,...,n\}$ and $\mathrm{U} \equiv \{U_1,...,U_n\}$. Let $o_{i,t} \in \mathrm{U}$ denote the upstream firm of origin of input $i \in \mathrm{N}$ and $m_{j,t} \in \mathrm{N}$ denote the unit originating from supplier $j \in \mathrm{U}$ in period t, where $o_{m_{j,t},t} = j$. By itself, serial number i is uninformative about the upstream firm of origin in the sense that $\Pr((o_{1,t},...,o_{n,t}) = (U_{\pi(1)},...,U_{\pi(n)})) = \frac{1}{n!}$ for any permutation $\{\pi(1),...,\pi(n)\} \in \Pi$, where Π is the set of all permutations of N. For concreteness, we assume that intermediate good i is used to produce unit i of the final good (recall that serial number i conveys no information about input origin $o_{i,t}$).

In each period, each supplier j can exert effort $e^u_{j,t} = 1$ or shirk $e^u_{j,t} = 0$ at cost $c^u e^u_{j,t}$, where

$c^u > 0$. The effort of supplier $o_{i,t}$ determines the quality of intermediate good i, $q_{i,t}^u = e_{o_{i,t},t}^u$. The downstream firm also can exert effort $e_{i,t}^d = 1$ or shirk $e_{i,t}^d = 0$ at cost $c^d e_{i,t}^d$ for each unit i, where $c^d > 0$. Let $x \equiv \frac{c^u}{c^d}$ denote the ratio of the costs of upstream and downstream efforts. The upstream and downstream efforts are complementary in determining the quality of unit i of the final good:

$$q_{i,t} = q_{i,t}^u e_{i,t}^d \in \{0,1\},\ i \in \mathrm{N}. \tag{1}$$

Our focus is on comparing equilibrium outcomes in the regimes with and without traceability that are defined as follows. In the traceability regime, the downstream firm observes signals $(o_{1,t}^T, ..., o_{n,t}^T)$ that are perfectly informative about input origins, where $\Pr((o_{1,t}^T, ..., o_{n,t}^T) = (o_{1,t}, ..., o_{n,t})) = 1$. In the no-traceability regime, the downstream firm observes signals $(o_{1,t}^N, ..., o_{n,t}^N)$ that are uninformative about input origins, where $\Pr((o_{1,t}^N, ..., o_{n,t}^N) = (o_{1,t}, ..., o_{n,t})) = \frac{1}{n!}$. Before the game starts, the firms publicly commit for the duration of the game to either the traceability (T) or no-traceability regime (N).[4]

On the demand side of the market, there is a large number of identical, anonymous risk-neutral consumers. It will be convenient to refer to the mass of consumers as a single player *C*. If the consumers do not buy, the consumers and firms get their outside payoffs normalized to zero. If the consumers buy product i in regime $k \in \{N, T\}$, the consumers' benefit is $v^k q_{i,t} + \underline{v}^k (1 - q_{i,t})$, where (a) $v^k - c^u - c^d > 0 > \max_{(e^u, e^d) \in \{(0,1),(1,0),(0,0)\}} \{\underline{v}^k - c^u e^u - c^d e^d\}$, that is, trade is efficient only if upstream and downstream firms put in efforts, and (b) $\underline{v}^T - \underline{v}^N \geq 0$ is the value generated by timely identification of input suppliers when quality is low. To focus on the potential effects of traceability on reputation, we assume that traceability does not affect costs or technology as well as consumption value of a high quality good, $v^N = v^T \equiv v$.[5]

Since an individual consumer's behavior is not observable by the downstream firm, consumers will maximize their current period payoffs. Following Cai and Obara (2009) and Cabral (2009), we assume that customers imperfectly observe efforts to provide quality because of random shocks to quality during production, distribution, and consumption. The signal of final quality, $y_{i,t} \in \{0,1\}$, is given by

$$\Pr(y_{i,t} = 1 \mid q_{i,t} = 1) = \alpha \text{ and } \Pr(y_{i,t} = 1 \mid q_{i,t} = 0) = \beta, \tag{2}$$

where $0 \leq \beta < \alpha < 1$ and $y_{i,t}$ are independent across products and across periods conditional on $q_{i,t}$.

[4] We also consider a regime with imperfect traceability where $\Pr((o_{1,t}^N, ..., o_{n,t}^N) = (o_{1,t}, ..., o_{n,t})) \in (\frac{1}{n!}, 1)$.

[5] The range of parameters such that traceability is optimal will contract if the direct costs of a traceability system are taken into account and expand for $v^N < v^T$. The cost of traceability systems that use technologies such as radio frequency identification is likely to fall over time (Kelepouris, Pramatari, and Doukidis 2007).

Equivalently, $y_{i,t}$ can represent a *realization* of final quality, and $v_h^k y_{i,t} + v_l^k (1-y_{i,t})$ can represent the stochastic consumption value, where $v_h^k = ((1-\beta)v - (1-\alpha)\underline{v}^k)/(\alpha-\beta)$ and $v_l^k = (\alpha\underline{v}^k - \beta v)$ $/(\alpha-\beta)$, so that $E[v_h^k y_{i,t} + v_l^k (1-y_{i,t}) \mid q_{i,t}=1] = v$, $k = N,T$. It will be convenient to let $f(y \mid q_1,..., q_n)$ $= \sum_{y_1,...,y_n \in \{0,1\}} 1_{\frac{1}{n}(y_1+...+y_n)=y} \Pr(y_1 \mid q_1) \times ... \times \Pr(y_n \mid q_n) = g(ny, \sum_{z=1}^n q_z)$ denote the probability that a given final quality profile $(q_{1,t},..., q_{n,t}) = (q_1,..., q_n)$ generates the signal of the average final quality $y = \frac{1}{n}\sum_{i=1}^n y_{i,t} \in \{0, \frac{1}{n},...,1\}$, where $g(i,m) = \sum_{h=\max[i-(n-m),0]}^{\min[m,i]} B_{m,h}^{\alpha} B_{n-m,i-h}^{\beta}$, $B_{n,m}^{\gamma} = \frac{n!}{m!(n-m)!}\gamma^m (1-\gamma)^{n-m}$.

The timing of events in each period t is as follows:

1. The downstream firm privately observes the lagged input quality profile, $(q_{1,t-1}^u,..., q_{n,t-1}^u)$, and the downstream firm and consumers jointly observe the final quality signal profile, $(y_{1,t-1},..., y_{n,t-1})$. These signals are non-verifiable. We show that the analysis generalizes to imperfect *inter-firm* monitoring of input quality in Section 5.
2. The downstream firm makes simultaneous secret take-it-or-leave-it contract offers to suppliers, where each offer specifies the input price $w_{j,t} \in \Re$. Then each supplier either accepts the contract ($d_{j,t}=1$) and privately chooses its level of effort $e_{j,t}^u$ or gets the outside option ($d_{j,t}=0$). The acceptance or rejection decisions are public information, but each input price $w_{j,t}$ is only observed by supplier j and the downstream firm.[6]
3. The downstream firm sets final good prices for the period, $(p_{1,t},..., p_{n,t}) \in \Re_+^n$, and consumers decide whether to purchase the final products, $(b_{1,t},..., b_{n,t}) \in \{0,1\}^n$. These prices are observed by the downstream firm and consumers but are not observed by suppliers.
4. The downstream firm privately observes the signals of origin of each input/product $(o_{1,t}^k,..., o_{n,t}^k)$ and then privately chooses the level of effort for each input/product $(e_{1,t}^d,..., e_{n,t}^d)$, $k \in \{N,T\}$.
5. Obligatory transfers are made and payoffs are realized.

The assumptions that the supply chain relies on reputational contracting rather than relational contingent contracting and that output prices cannot be negative simplify the exposition but are not essential for the analysis to follow. However, the assumptions about the information structure are essential fea-

[6] Since the supplier base is fixed and suppliers are capacity constrained, cutting suppliers off will reduce the *quantity* of output.

tures of our model and need justification. We assume that the suppliers do not observe prices and signals of quality in the final good market in order to ensure that the downstream firm cannot offer suppliers contracts contingent on (perhaps relative) performances in the final good market. In practice, such complicated contracts are rare, and suppliers in the exporting countries may lack information about demand and sales in the importing country as well as in the domestic market in some developing countries (e.g., Gulati et al. 2007).

We also assume that (a) the downstream firm makes contract offers to suppliers *in secret* and (b) chooses its own effort levels *before* the arrival of information about upstream effort/input quality (akin to a Stackelberg leader in the pricing game). A plausible alternative characterization of the contracting environment and technology is that contracts between the downstream firm and suppliers are observed publicly (by other suppliers and possibly consumers) or that the downstream firm chooses its own efforts *after* observing the upstream effort/input quality (akin to a Stackelberg follower in the pricing game). In either case, traceability does not expand the room for sophisticated deviations, and therefore, cannot have a negative effect on reputation building. The reason is that, when input prices are public, the downstream firm cannot *secretly* procure a mix of low and high quality inputs. On the other hand, if there is no lag in detecting input quality, the downstream firm *does not* need to know the supplier's identity to match low (high) upstream efforts with low (high) downstream efforts.

The existing literature typically assumes that some dimensions, though not related to product quality, of wholesale contracts are publicly disclosed (Piccolo and Miklos-Thal 2012). In our setting, one possible justification for the assumption that input prices are not publicly disclosed is that there is an equilibrium in which communication of input prices to third parties is not credible, if communication is modeled as cheap talk and low input prices convey low quality. A justification for the timing of downstream effort decisions and arrival of information about input quality is that provision of high quality may require planning or capital expenditures such as maintaining equipment or personnel training.

Equilibrium concept

In the unique static equilibrium of the stage game there is no trade since in one-time interaction the firms provide low quality. To state the equilibrium concept for the repeated game, we denote $\mathrm{H}_j^t = \{d_{j,z}, e_{j,z}^u, w_{j,z}\}_{z=1}^{t-1}$, $\mathrm{H}_D^t = \{\mathrm{d}_z, \mathrm{w}_z, \mathrm{q}_z^u, \mathrm{o}_z^k, \mathrm{e}_z^d, \mathrm{y}_z, \mathrm{p}_z\}_{z=1}^{t-1}$, and $\mathrm{H}_C^t = \{\mathrm{y}_z, \mathrm{p}_z, \mathrm{b}_z\}_{z=1}^{t-1}$ as the history of the game observed, respectively, by upstream firm $j \in \mathrm{U}$, downstream firm, and consumers, and $\mathrm{H}^t = \mathrm{H}_{U_1}^t \cup ... \cup \mathrm{H}_{U_n}^t \cup \mathrm{H}_C^t \cup \mathrm{H}_D^t = \{\mathrm{d}_z, \mathrm{w}_z, \mathrm{e}_z^u, \mathrm{q}_z^u, \mathrm{o}_z^k, \mathrm{e}_z^d, \mathrm{y}_z, \mathrm{p}_z, \mathrm{b}_z\}_{z=1}^{t-1}$ as the complete history of the game in the beginning of period t, where $k = N,T$, $\mathrm{w}_t = \{w_{U_1,t}, ..., w_{U_n,t}\}$, $\mathrm{o}_t^k = (o_{1,t}^k, ..., o_{n,t}^k)$ and so on. The customers purchasing decisions in period t are given by $(b_{1,t},...,b_{n,t}):\mathrm{H}_C^t \cup \mathrm{p}_t \to \{0,1\}^n$. The supplier j's acceptance and effort decisions in period t are given by, respectively, $d_{j,t}:\mathrm{H}_j^t \cup w_{j,t} \to \{0,1\}$ and $e_{j,t}^u:\mathrm{H}_j^t \cup w_{j,t} \cup d_{j,t} \to \{0,1\}$, $j \in U$. The downstream firm's input-pricing, product-pricing, and effort decisions in period t are given by, respectively, $\mathrm{w}_t:\mathrm{H}_D^t \to \Re^n$, $\mathrm{p}_t:\mathrm{H}_D^t \to \Re_+^n$, and $\mathrm{e}_t^d:\mathrm{H}_D^t \cup (\mathrm{p}_t,\mathrm{w}_t,\mathrm{d}_t,\mathrm{o}_t^k) \to \{0,1\}^n$.

The pure strategy of upstream firm j is given by $s_j = \{d_{j,t}, e_{j,t}^u\}_{t=1}^{\infty}$. The pure strategy of the downstream firm is given by $s_D = \{\mathrm{p}_t,\mathrm{w}_t,\mathrm{e}_t^d\}_{t=1}^{\infty}$. The pure strategy of consumers is $s_C = \{\mathrm{b}_t\}_{t=1}^{\infty}$. Let S_j denote the set of strategies for player j. For a given strategy profile $\{s_{U_1},...,s_{U_n},s_C,s_D\}$, the expected payoff of player j conditional on the complete history of the game H^t and a private history of offers and decisions within period t, h_j^t, starting from period t, is given by

$$\pi_j(\{s_{U_1},...,s_{U_n},s_C,s_D\},\mathrm{H}^t,\mathrm{h}_j^t) = E[\sum_{z=t}^{\infty}\delta^{z-t}d_{j,z}(w_{j,z}-c^u e_{j,z}^u) \mid \mathrm{H}^t,\mathrm{h}_j^t],\ j \in \mathrm{U},$$

$$\pi_j(\{s_{U_1},...,s_{U_n},s_C,s_D\},\mathrm{H}^t,\mathrm{h}_j^t) = E[\tfrac{1}{n}\sum_{z=t}^{\infty}\delta^{z-t}(\sum_{i=1}^{n}b_{i,z}p_{i,z} - d_{o_i,z}w_{o_i,z} - c^d e_{i,z}^d) \mid \mathrm{H}^t,\mathrm{h}_j^t],\ j = D,$$

$$\pi_j(\{s_{U_1},...,s_{U_n},s_C,s_D\},\mathrm{H}^t,\mathrm{h}_j^t) = E[\tfrac{1}{n}\sum_{z=t}^{\infty}\delta^{z-t}(\sum_{i=1}^{n}b_{i,z}(vq_{i,t}+\underline{v}(1-q_{i,t})-p_{i,z})) \mid \mathrm{H}^t,\mathrm{h}_j^t],\ j = C.$$

For convenience, we measure payoffs on a per-product basis. Since the players do not observe the complete history of the game, we define

$$\Pi_j(\{s_A,s_B,s_C,s_D\},\mathrm{H}_j^t,\mathrm{h}_j^t) = E_{\mu_j}[\pi_j(\{s_{U_1},...,s_{U_n},s_C,s_D\},\mathrm{H}^t,\mathrm{h}_j^t) \mid \mathrm{H}_j^t,\mathrm{h}_j^t]$$

as the expected payoff of player j following her private history H_j^t and choices h_j^t observed or made by player j during period t, where $\mathrm{h}_j^t = \mathrm{p}_t$ for $j = C$, $\mathrm{h}_j^t = \mathrm{p}_t, (\mathrm{p}_t,\mathrm{w}_t)$, or $(\mathrm{p}_t,\mathrm{w}_t,\mathrm{d}_t,\mathrm{o}_t^k)$ for $j = D$, and $\mathrm{h}_j^t = w_{j,t}$ or $(w_{j,t},d_{j,t})$ for $j \in \mathrm{U}$. Here the expectation is taken over all of the possible complete histories H^t according to player j's belief μ_j conditional on observing private history H_j^t and choices h_j^t that player j observes or makes during period t.

A perfect Bayesian equilibrium (PBE) in this model consists of the downstream firm's strategy, s_D^*, upstream firms' strategies $s_{U_1}^* = ... = s_{U_n}^*$, consumers' strategy s_C^*, the downstream firm's belief μ_D^*, the upstream firms' beliefs $\mu_{U_1}^* = ... = \mu_{U_n}^*$, and consumers' belief μ_C^* such that simultaneously

(i) $\Pi_j(\{s_{U_1}^*,...,s_{U_n}^*,s_C^*,s_D^*\},\mathrm{H}_j^t,\mathrm{h}_j^t) \ge \Pi_j(\{s_{U_1}^*,...,s_j,....,s_{U_n}^*,s_C^*,s_D^*\},\mathrm{H}_j^t,\mathrm{h}_j^t)$ for any $s_j \in \mathrm{S}_j$,

H^t_j, and h^t_j, $j \in U$,

$\Pi_D(\{s^*_{U_1},...,s^*_{U_n},s^*_C,s^*_D\},H^t_D,h^t_D) \geq \Pi_D(\{s^*_{U_1},...,s^*_{U_n},s^*_C,s_D\},H^t_D,h^t_D)$ for any $s_D \in S_D, H^t_D$, and h^t_D,

$b^*_{i,t} = 1$ if and only if $E_{\mu^*_C}[vq_{i,t} + \underline{v}(1-q_{i,t}) - p_{i,t} \mid \{s^*_{U_1},...,s^*_{U_n},s^*_B,s^*_D\},H^t_C,h^t_C] \geq 0$, $i \in N$, for any H^t_C and h^t_C, and

(ii) the beliefs are consistent with the strategy profile $\{s^*_{U_1},...,s^*_{U_n},s^*_C,s^*_D\}$ and are updated using the Bayes rule whenever possible.

Because there are many PBE in this game, to study the effects of traceability we will characterize equilibrium that yields the greatest joint payoff for the firms in the traceability and no-traceability regime. We will refer to such a PBE as a best equilibrium in a given regime. We then determine the regime that yields the greatest joint profit. The characterization of equilibrium involves solving for two optimal reputational (self-enforcing) contracts: one between an upstream firm and the downstream firm and one between the downstream firm and consumers. Without loss of generality, we can restrict our attention to a trigger strategy PBE where each upstream firm reverts to playing its static best response forever if the downstream firm offers a smaller-than-expected input price. Note that, as in the models of relational contracting with public monitoring (e.g. Levin 2003), the assumption that inter-firm monitoring is perfect (although private) implies that on the equilibrium path firms' and consumers' beliefs regarding the private histories of all other players will be degenerate since all players place probability one on the equilibrium behavior. We will specify beliefs when upstream firms' actions do not conform to the equilibrium play in what follows.

3. Equilibrium

The downstream firm can promise to punish upstream shirking (i) by excluding suppliers from all future transactions (that is, "cutting suppliers off"), or (ii) by procuring low quality inputs from one or more suppliers forever (that is, "spot-market governance" or "Nash reversion" in the supplier-downstream firm subgame) after the downstream firm observes that one or more suppliers shirk on efforts. In each case, there is no loss of generality in assuming that during the upstream punishment phase suppliers get their reservation utilities. An important difference between the punishments by reverting to "spot-market transactions" and "cutting suppliers off" is that consumers do not observe the former but do observe the latter. It will be shown that, without loss of generality, we can assume that in equilibrium the downstream firm promises to punish upstream shirking by infinite "Nash reversion" in upstream transactions with certain suppliers.

Consider the following strategy profile for some expected input payment w^k and output price function $p^k : \{0, \frac{1}{2}, 1\} \to [0, v]$ that will be specified later:

$$d^*_{j,t} = \begin{cases} 1, \text{if } w_{j,t} \geq 0 \\ 0, \text{if otherwise} \end{cases} \text{ and} \tag{3}$$

$$e^{u,*}_{j,t} = \begin{cases} 1, \text{if } (d_{j,z}, e^u_{j,z}) = (1,1) \text{ for all } z < t \text{ and } w_{j,z} \geq w^k \text{ for all } z \leq t \\ 0, \text{if otherwise} \end{cases}, \ j \in U, \tag{4}$$

$$(b^*_{1,t}, ..., b^*_{n,t}) = \begin{cases} (1,...,1), \text{if } p_{i,t} \leq p^k(y_{t-1}) \text{ for all } i \in \text{N}, \\ (0,...,0), \text{if otherwise} \end{cases}, \tag{5}$$

$$(p^*_{1,t}, ..., p^*_{n,t}) = (p^k(y_{t-1}), ..., p^k(y_{t-1})) \text{ for } k = N, T, \tag{6}$$

$$e^{d,*}_{i,t} = \begin{cases} 1, \text{if } (d_{j,z}, q^u_{m_{j,z},z}) = (1,1) \text{ for all } z \leq t, j = o_{i,t} \\ 0, \text{if otherwise} \end{cases}, \ i \in N, \text{ and} \tag{7}$$

$$w^*_{j,t} = \begin{cases} w^T, \text{if } (d_{j,z}, q^u_{m_{j,z},z}) = (1,1) \text{ for all } z < t, \\ 0, \text{if otherwise} \end{cases}, \ j \in U, \tag{8}$$

in the traceability regime, and

$$(e^{d,*}_{1,t}, ..., e^{d,*}_{n,t}) = \begin{cases} (1,...,1), \text{if } (d_{U_1,z}, ..., d_{U_n,z}) = (q^u_{1,z}, ..., q^u_{n,z}) = (1,...,1) \text{ for all } z \leq t-1 \\ (0,...,0), \text{if otherwise} \end{cases}, \tag{9}$$

$$(w^*_{U_1,t}, ..., w^*_{U_n,t}) = \begin{cases} (w^N, ..., w^N), \text{if } (d_{U_1,z}, ..., d_{U_n,z}) = (q^u_{1,z}, ..., q^u_{n,z}) = (1,...,1) \text{ for all } z \leq t-1 \\ (0,...,0), \text{if otherwise} \end{cases}. \tag{10}$$

in the no-traceability regime.[7] Next we will show that in both traceability regimes, provided certain parametric restrictions hold, there exist w^k and p^k, and an associated system of beliefs such that strategies (3) – (10) constitute a PBE where all firms exert efforts and trade in high quality products in every period. Since the proposed strategies are not path-dependent, the time subscript will be dropped.

[7] In a best equilibrium for any customer monitoring strategy that depends on unit-specific signals of quality $(y_{1,t-1}, ..., y_{n,t-1})$, there is an equivalent monitoring strategy that depends on the average signal y_{t-1}. Since each upstream firm is equally likely to supply the intermediate good that is transformed into unit i of the final good, y_{t-1} is a sufficient statistic for $(y_{1,t-1}, ..., y_{n,t-1})$ with respect to $(q_{1,t-1}, ..., q_{n,t-1})$.

Upstream incentive compatibility

If in equilibrium the downstream firm maintains a reputation for offering the expected input payment w^k and upstream firms maintain a reputation for supplying high quality inputs, the equilibrium payoff of upstream firm j satisfies the following value recursive equation

$$\pi_j^k = w^k - c^u + \delta\pi_j^k . \tag{11}$$

Equation (11) says that the upstream firm's value in the equilibrium is the sum of the current period profit, $w^k - c^u$, plus the discounted value from continuation, $\delta\pi_j^k$. The acceptance and effort strategies in (3) and (4) are optimal for an upstream firm as long as the upstream participation (UP) and incentive compatibility (UIC) conditions hold:

$$\pi_j^k \geq 0 \text{ and} \tag{UP}$$

$$\pi_j^k \geq w^k , \tag{UIC}$$

where the right-hand sides are the upstream firm's payoffs from rejecting a contract or accepting the contract and choosing a low level of effort in accordance with the downstream firm's input payment strategy in (8) in the traceability regime or (10) in the no-traceability regime.

Downstream incentive compatibility

Let

$$P^T(e_{U_1}^u,...,e_{U_n}^u, e_1^d,...,e_n^d) = \sum\nolimits_{y\in\{0,\frac{1}{n},\frac{2}{n}...,1\}} p^T(y) f(y \mid e_{U_1}^u e_1^d, e_{U_2}^u e_2^d,..., e_{U_n}^u e_n^d) \tag{12}$$

denote the expected price (before the signals of final quality are realized) when the downstream firm knows the level of effort for each supplier *and* the supplier of origin for each unit. Also, let

$$P^N(e_{U_1}^u,...,e_{U_n}^u, e_1^d,...,e_n^d) = \tfrac{1}{n!}\sum\nolimits_{y\in\{0,\frac{1}{n},...,1\}} \sum\nolimits_{(\pi(1),...,\pi(n))\in\Pi_n} p^N(y) f(y \mid e_{\pi(1)}^u e_1^d,..., e_{\pi(n)}^u e_n^d) , \tag{13}$$

denote the expected price when the downstream firm knows the level of effort for each supplier *but* does not know which unit originates from which supplier. In addition, let us define

$$P^N(\tfrac{1}{n},...,\tfrac{1}{n}, e_1^d,...,e_n^d) = \tfrac{1}{n}\sum\nolimits_{i=1}^{n} P^N(1_{1\neq i}, 1_{2\neq i} .., 1_{n\neq i}, e_1^d,..., e_n^d) . \tag{14}$$

In accordance with the proposed strategies in the no-traceability regime this is the expected price if in each of the preceding periods the downstream firm offered the expected payments $\{w_{U_1,z},..., w_{U_n,z}\}_{z=1}^{t} = \{w^N,..., w^N\}_{z=1}^{t-1}$ but observed $\{q_{1,z}^u,..., q_{n,z}^u\}_{z=1}^{t-1} \in \{(0,1,...,1),(1,0,1,..,1),...,(1,1,...,0),$ $(1,1,...,1)\}^{t-1}$ with at least one period in which $\{q_{1,z}^u,..., q_{2,z}^u\} \in \{(0,1,...,1),(1,0,1,..,1),...,(1,1,...,0)\}$ for some $z \leq t-1$. In accordance with the upstream effort strategy in (4), after such a history the downstream firm expects one of the upstream firms to shirk in every period but does not know which one does so.[8]

Suppose that in the previous period the levels of the upstream and downstream efforts are given by $\{e^u_{U_1},...,e^u_{U_n},e^d_1,...,e^d_n\}\in\{0,1\}^{2n}$. Then the expected equilibrium (per product) payoff for the downstream firm before the average signal of final quality is realized satisfies the following recursive value equation

$$\pi^k_D(e^u_{U_1},...,e^u_{U_n},e^d_1,...,e^d_n)=\max_{\tilde{w}_{U_1},...,\tilde{w}_{U_n}\geq 0,\tilde{e}^d_1,...,\tilde{e}^d_n\in\{0,1\}} P^k(e^u_{U_1},...,e^u_{U_n},e^d_1,...,e^d_n)-\tfrac{1}{n}\sum_{i=1}^{n}(\tilde{w}_{U_i}+c^d\tilde{e}^d_i)$$

$$+\delta\pi^k_D(1_{\tilde{w}_{U_1}\geq w^k}e^u_{U_1},...,1_{\tilde{w}_{U_n}\geq w^k}e^u_{U_n},\tilde{e}^d_1,...,\tilde{e}^d_n). \quad (15)$$

The right-hand side is the expected current period profit plus the firm's discounted future value provided that suppliers adhere to the acceptance and effort strategies (3) and (4). For the downstream firm to be willing to offer the expected input payments and put in efforts, the downstream firm's incentive compatibility (DIC) constraint requires that for all $y\in\{0,\frac{1}{n},...,1\}, w_{U_1},...,w_{U_n}\geq 0, e^d_1,...,e^d_n\in\{0,1\}$

$$p^k(y)-w^k-c^d+\delta\pi^k_D(1,...,1)\geq p^k(y)-\tfrac{1}{n}\sum_{i=1}^{n}(w_{U_i}+c^d e^d_i)+\delta\pi^k_D(1_{w_{U_1}\geq w^k}e^u_{U_1},...,1_{w_{U_n}\geq w^k}e^u_{U_n},e^d_1,...,e^d_n).$$

(DIC)

It remains to determine under what conditions the downstream firm's threat to punish upstream shirking in accordance with the input payment strategies in (8) and (10) is credible. In accordance with (4), the downstream firm believes that an upstream firm that shirked at any time in the past continues to shirk forever independently of the subsequent history of play. This is without loss because there is no upstream shirking on the equilibrium path. A novel feature of the moral hazard problem in a supply chain is that the punishment of upstream shirking by the downstream firm may not be incentive compatible since consumer monitoring of the final quality is imperfect. Because the incentive cost of maintaining credibility of inter-firm punishments depends on whether the downstream firm knows the identities of upstream shirkers, we will analyze inter-firm punishments separately in the traceability and no-traceability regime.

Customers' purchasing strategy

Since the equilibrium outcome is beyond the control of any single customer, the purchasing strategy in (5) characterizes optimal consumer behavior provided that the firms' announcements of the strategies in (3) –

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[8] For example, for $n=2$ from the definitions in (12) – (14), it immediately follows that $P^k(1,1,0,1)=P^k(1,1,1,0)$ $=P^k(0,1,1,1)=P^k(1,0,1,1)$, $P^k(0,1,0,1)=P^k(1,0,1,0)$, and $P^k(0,0,1,1)=P^k(1,1,0,0)=P^k(0,0,0,0)$ for $k=T,N$. In addition, we have $P^T(0,1,0,1)=P^T(0,1,1,1)$ because the downstream firm is able to vertically coordinate upstream and downstream shirking and working in the traceability regime; $P^N(1,0,1,0)=P^N(0,1,0,1)=P^N(0,1,1,0)=P^N(1,0,0,1)$ because each product is equally likely to originate from upstream firm U_1 or U_2; and $P^N(\frac{1}{2},\frac{1}{2},e^d_1,e^d_2)=P^N(0,1,e^d_1,e^d_2)=P^N(1,0,e^d_1,e^d_2)$ for all $e^d_1,e^d_2\in\{0,1\}$. Similar symmetry properties continue to hold for $n>2$.

(10) are credible, i.e. all of the upstream and downstream incentive compatibility conditions are satisfied. We assume that consumers believe that if the downstream firm offers a price that is inconsistent with the final good pricing policy in (6), it will never put in efforts or offer the expected input payments. This assures that the downstream firm never offers a price that is inconsistent with the pricing policy in (6) as long as its payoff from maintaining reputation for adhering to the output pricing policy exceeds its reservation payoff of zero.

3.1. Traceability regime

Inter-firm punishment credibility

It is easy to see that the downstream firm's promise to punish upstream shirking is credible since the downstream firm cannot achieve higher profits by continuing to offer the expected input payments to an identified upstream shirker. Suppose that the downstream firm detected that supplier U_1 had shirked for the first time in period $t-1$. In accordance with (8), the downstream firm must offer $w_{U_1,t} \leq 0$ forever. Such an "individual punishment" strategy is credible since the individual punishment incentive compatibility (IPIC) constraint is satisfied:

$$\max_{0 \leq w_{U_1} < w^T, w_{U_2},\ldots,w_{U_n} \geq 0, e_1^d,\ldots,e_n^d \in \{0,1\}} p^T(y) - \tfrac{1}{n}\sum_{i=1}^{n}(w_{U_i} + c^d e_i^d) + \delta\pi_D^k(0, 1_{w_{U_2} \geq w^k} e_{U_2}^u, \ldots, 1_{w_{U_n} \geq w^k} e_{U_n}^u, e_1^d, \ldots, e_n^d)$$

$$> \max_{w_A \geq w^T, w_{U_2},\ldots,w_{U_n} \geq 0, e_1^d,\ldots,e_n^d \in \{0,1\}} p^T(y) - \tfrac{1}{n}\sum_{i=1}^{n}(w_{U_i} + c^d e_i^d) + \delta\pi_D^k(0, 1_{w_{U_2} \geq w^k} e_{U_2}^u, \ldots, 1_{w_{U_n} \geq w^k} e_{U_n}^u, e_1^d, \ldots, e_n^d).$$

The left-hand side is the maximum out-of-equilibrium payoff for the downstream firm when U_1 supplies low quality inputs and the downstream firm offers a smaller-than-expected input payment to that supplier. It is easy to see that the IPIC constraint does not bind in equilibrium. The downstream firm and U_1 expect to play $(w_{U_1,z}, d_{U_1,z}, e_{U_1,z}^u) = (0,1,0)$ for all $z \geq t$. This constitutes equilibrium because the shirking supplier expects the payment that just covers the cost of production of low-quality inputs and the downstream firm expects that the shirking supplier will shirk in the future.

Best equilibrium for the supply chain

Our first result characterizes a best equilibrium in the traceability regime.

Proposition 1 *Suppose that* $v > \frac{c^d + c^u}{n(\alpha-\beta)(1-\alpha)^{n-1}}$. *There exists a threshold level of the discount factor* $\delta^T < 1$ *such that for all* $\delta \geq \delta^T$ *the strategies (3) – (8) constitute the best equilibrium in the traceability regime with* $w^T = \frac{1}{\delta} x c^d$ *and* $p^T(y) = v - \Delta_0^T 1_{y=0}$, *where* $\Delta_0^T \equiv \frac{(1+x/\delta)c^d}{\delta n(\alpha-\beta)(1-\alpha)^{n-1}}$. *The expected joint per product* *value of upstream and downstream firms in the best equilibrium is* $\pi^T \equiv \frac{1}{1-\delta}(v - c^u - c^d - (1-\alpha)^n \Delta_0^T)$.

The analytical expression for δ^T is provided in the proof of Proposition 1. The optimal product-pricing scheme $p^T(y)$ prescribes reducing the price by Δ_0^T following the worst possible signal of quality. The softest possible punishment leaves the downstream firm indifferent between staying on the equilibrium path and engaging in the most profitable deviation which involves vertically coordinating upstream and downstream shirking for a single unit. The equilibrium discount for bad performance in the final good market, Δ_0^T, equals the ratio of the downstream firm's per-product cost savings from coordinated shirking on efforts for one product, $\frac{1}{n}(\frac{1}{\delta}c^u + c^d)$, and the discounted incremental probability that consumers observe the worst possible performance, $\delta(\Pr(y_t = 0 \mid \sum_{i=1}^n q_{i,t} = n-1) - \Pr(y_t = 0 \mid \sum_{i=1}^n q_{i,t} = n)) = \delta((1-\beta)(1-\alpha)^{n-1} - (1-\alpha)^n) = \delta(\alpha-\beta)(1-\alpha)^{n-1}$. At optimum, the downstream firm is indifferent between giving up Δ_0^T in the next period with probability $(1-\alpha)^n$ and losing, on average,

$$\delta(1-\alpha)^n \Delta_0^T = \frac{1-\alpha}{\alpha-\beta}\frac{1}{n}(\frac{1}{\delta}c^u + c^d),$$

and giving up Δ_0^T with probability $(1-\alpha)^{n-1}(1-\beta)$ but saving $\frac{1}{n}(\frac{1}{\delta}c^u + c^d)$ per period:

$$\delta(1-\alpha)^{n-1}(1-\beta)\Delta_0^T - \frac{1}{n}(\frac{1}{\delta}c^u + c^d) = \frac{1-\alpha}{\alpha-\beta}\frac{1}{n}(\frac{1}{\delta}c^u + c^d).$$

The comparative statics about δ, α, β, and n are all intuitive and follow from the simple observation that the equilibrium expected loss of quality premium $(1-\alpha)^n \Delta_0^T = \frac{1-\alpha}{\delta n(\alpha-\beta)}(1 + x/\delta)c^d$ is decreasing in δ, α, and n, and increasing in β. Consumers punish the supply chain less for bad performance if the firms care more about the future (greater δ) or the public unit-specific signals of final quality are less noisy (greater α or smaller β) or the public signal of the average final quality is less noisy (greater n).

Also, note that the optimal input payment $w^T = \frac{1}{\delta}c^u$ is the smallest value that satisfies the UIC constraint. Our focus on the joint payoff is without loss of generality since the downstream firm can easily extract the entire surplus generated by the supply chain. For example, the downstream firm can propose to offer to each supplier j zero input payment, $w_{j,1} = 0$, in $t = 1$ and offer the expected payment, $w_{j,t} = w^T$, in $t = 2,3,\ldots$. Then all incentive compatibility conditions will still be satisfied, and in equilibrium we will have $\pi_D^T = \pi^T$ and $\pi_j^T = 0$, $j \in U$.[9]

[9] Note that the overall incentive costs will be the same in a model with relational contingent contracting where the downstream

We also need to show that the assumption that the downstream firm's promise to punish upstream shirking by supplier-specific spot-market (low-quality) transactions rather than by excluding that supplier from all future transactions is without loss of generality. Substituting the optimal price policy in (15), it is east to verify that in the best equilibrium with inter-firm punishment by supplier-specific reversion to low quality, the expected (per product) value of the downstream firm on an off equilibrium path, where one supplier (say supplier U_1) shirks and the other suppliers put in efforts is given by

$$\max_{e_1^d,\ldots,e_2^d} \pi_D^T(0,1,\ldots,1,e_1^d,\ldots,e_n^d) = v - \tfrac{n-1}{n}(w^T + c^d) - (1-\alpha)^{n-1}(1-\beta)\Delta_0^T > 0. \qquad (16)$$

Now suppose that in a best equilibrium the downstream firm promises to punish upstream shirking by exclusion. There are two possibilities. First, suppose that the value of the downstream firm on an off-equilibrium path where it transacts with only $n-1$ suppliers (and produces $n-1$ units of output for sale) is greater than (16). Then, as shown in Proposition 1, the punishment by supplier exclusion is not optimal since it is easier to satisfy the single binding DIC constraint when upstream shirking is punished by infinite supplier-specific reversion to low quality. Now suppose that the value of the downstream firm that transacts with $n-1$ suppliers is smaller than (16). Then the punishment of upstream shirking by exclusion is not credible since the downstream firm will prefer to continue to transact with n suppliers (and produce n units of output) and procure a mix of high and low quality inputs after it observes than one of the suppliers started shirking.

Before we analyze the equilibrium outcome in the no-traceability regime, note that the assumption that the downstream firm operates at *full* capacity and sells the products *jointly* (under one brand) is also without loss of generality. From Proposition 1 it follows that if the downstream firm transacts with $m < n$ suppliers, the greatest payoff for the downstream firm is given by

$$\tfrac{1}{1-\delta}(v - w^T - c^d - \tfrac{n}{m}(1-\alpha)^n \Delta_0^T) < \tfrac{1}{1-\delta}(v - w^T - c^d - (1-\alpha)^n \Delta_0^T) = \pi_D^T(1,\ldots,1) < n\pi_D^T(1,\ldots,1).$$

3.2. No-traceability regime

Because the downstream firm observes anonymous (unidentifiable by source) choices of upstream efforts, the credibility of the downstream firm's promise to punish upstream shirking can be harder to sustain. In the no-traceability regime, by Lemma 1 in Appendix, the downstream firm must punish all suppliers (that is, offer payments $(w_{U_1,t},\ldots,w_{U_n,t}) = (0,\ldots,0)$ forever or exit the market) once it detects that one (or more) of suppliers shirked in the previous period. We refer to this strategy as "group punishment". We first derive conditions such that group punishment is credible for $n = 2$.

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firm pays out a noncontractible bonus that equals the cost savings from upstream shirking, c^u, in the end of the period rather than $\frac{1}{\delta}c^u$ in the beginning of the period, because the savings from reneging on input payment will then accrue one period before upstream shirking begins.

3.2.1. Two suppliers

Inter-firm punishment credibility

Consider a history such that $\{q_{1,z}^u, q_{2,z}^u\}_{z=1}^{t-2} = \{1,1\}_{z=1}^{t-2}$, $\{w_{U_1,z}, w_{U_2,z}\}_{z=1}^{t-1} = (w^N, w^N)$, and the downstream firm observes $(q_{1,t-1}^u, q_{2,t-1}^u) = (0,1)$ for the first time in period t but does not know whether the input originated from U_1 or U_2. For concreteness, assume that $(o_{1,t-1}, o_{2,t-1}) = (U_1, U_2)$. First, suppose that the downstream offers $(w_{U_1,t}, w_{U_2,t}) \in \{(0, w^N), (w^N, 0)\}$. Then the downstream firm updates its beliefs regarding the upstream firms' effort strategies (the identity of the shirker) as follows:[10]

$$\Pr(\{e_{U_1,z}^u, e_{U_2,z}^u\}_{z=t}^{\infty} = \{0, 1_{w_{B,z} \geq w^N}\}_{z=t}^{\infty} \mid (q_{1,t}^u, q_{2,t}^u) \in \{(0,1),(1,0)\}, (w_{U_1,z}, w_{U_2,z}) = (0, w^N)) = 1, \quad (17)$$

$$\Pr(\{e_{U_1,z}^u, e_{U_2,z}^u\}_{z=t}^{\infty} = \{1_{w_{A,z} \geq w^N}, 0\}_{z=t}^{\infty} \mid (q_{1,t}^u, q_{2,t}^u) \in \{(0,1),(1,0)\}, (w_{U_1,t}, w_{U_2,t}) = (w^N, 0)) = 1, \quad (18)$$

$$\Pr(\{e_{U_1,z}^u, e_{U_2,z}^u\}_{z=t}^{\infty} = \{0,0\}_{z=t}^{\infty} \mid (q_{1,t}^u, q_{2,t}^u) = (0,0)) = 1. \quad (19)$$

For $(q_{1,t}^u, q_{2,t}^u) \in \{(0,1),(1,0)\}$, the downstream infers that it offered the input price of zero to the shirking supplier while it offered w^N to the supplier that did not shirk in the past. By offering $(w_{U_1,t}, w_{U_2,t}) = (0, w^N)$, the downstream firm punishes the actual shirker and earns $p_t - \frac{1}{2} w^N - \frac{1}{2}(e_1^d + e_2^d)c^d + \delta\pi_D^N(0,1,e_1^d, e_2^d)$. For $(q_{1,t}^u, q_{2,t}^u) = (0,0)$, the inference is that w^N was offered to the supplier that shirked in the past and 0 was offered to the supplier that did not shirk in the past. By offering $(w_{U_1,t}, w_{U_2,t}) = (w^N, 0)$, the downstream firm reneges on the expected payment to a non-shirking supplier and earns $p_t - \frac{1}{2} w^N - \frac{1}{2}(e_1^d + e_2^d)c^d + \delta\pi_D^N(0,0,e_1^d, e_2^d)$. Thus, the downstream firm punishes the actual shirker with probability $\frac{1}{2}$, and the maximum expected payoff under this strategy is given by

$$\max_{e_1^d, e_2^d \in \{0,1\}} p_t - \frac{1}{2} w^N - \frac{1}{2}(e_1^d + e_2^d)c^d + \delta(\tfrac{1}{2}\pi_D^N(0,0,e_1^d, e_2^d) + \tfrac{1}{2}\pi_D^N(0,1,e_1^d, e_2^d)). \quad (20)$$

If the downstream offers the expected payments to both the shirking and non-shirking supplier, it does not learn the identity of the shirker but knows that $(e_{U_1,t}^u, e_{U_2,t}^u) \in \{(0, 1_{w_{U_2,t} \geq w^N}), (1_{w_{U_1,t} \geq w^N}, 0)\}$. In this case, its maximum expected payoff is given by $\max_{e_1^d, e_2^d \in \{0,1\}} p_t - w^N - \frac{1}{2}(e_1^d + e_2^d)c^d + \delta\pi_D^N(\frac{1}{2}, \frac{1}{2}, e_1^d, e_2^d)$. Finally, if the downstream firm offers to both suppliers the input payments that just cover the cost of low quality inputs, it earns at most $\max_{e_1^d, e_2^d \in \{0,1\}} p_t - \frac{1}{2}(e_1^d + e_2^d)c^d + \delta\pi_D^N(0,0,e_1^d, e_2^d)$.

Therefore, the maximum expected payoff for the downstream firm after a history of the expected

[10] In accordance with (4) and (10), there is no need to specify the downstream firm's beliefs for $(q_{1,t}^u, q_{2,t}^u) = (1,1)$.

input payments $\{w_{U_1,z}, w_{U_2,z}\}_{z=1}^{t} = \{w^N, w^N\}_{z=1}^{t-1}$, high input qualities in all but the last period $\{q_{1,z}^u, q_{1,z}^u\}_{z=1}^{t-2} = \{1,1\}_{z=1}^{t-2}$, $(q_{1,t-1}^u, q_{1,t-1}^u) \in \{(0,1),(1,0)\}$, and downstream efforts $(e_{1,t}^d, e_{2,t}^d) = (\tilde{e}_1^d, \tilde{e}_2^d)$, satisfies the following recursive value equation

$$\pi_D^N(\tfrac{1}{2},\tfrac{1}{2},\tilde{e}_1^d,\tilde{e}_2^d) = \max[\max_{e_1^d,e_2^d\in\{0,1\}} P^N(\tfrac{1}{2},\tfrac{1}{2},\tilde{e}_1^d,\tilde{e}_2^d) - w^N - \tfrac{1}{2}(e_1^d + e_2^d)c^d + \delta\pi_D^N(\tfrac{1}{2},\tfrac{1}{2},e_1^d,e_2^d),$$

$$\max_{e_1^d,e_2^d\in\{0,1\}} P^N(\tfrac{1}{2},\tfrac{1}{2},\tilde{e}_1^d,\tilde{e}_2^d) - \tfrac{1}{2}w^N - \tfrac{1}{2}(e_1^d + e_2^d)c^d + \delta(\tfrac{1}{2}\pi_D^N(0,0,e_1^d,e_2^d,N) + \tfrac{1}{2}\pi_D^N(0,1,e_1^d,e_2^d)),$$

$$\max_{e_1^d,e_2^d\in\{0,1\}} P^N(\tfrac{1}{2},\tfrac{1}{2},\tilde{e}_1^d,\tilde{e}_2^d) - \tfrac{1}{2}(e_1^d + e_2^d)c^d + \delta\pi_D^N(0,0,e_1^d,e_2^d)]. \tag{21}$$

For the downstream firm to be willing to offer $\{w_{U_1,z}, w_{U_2,z}\}_{z=t}^{\infty} = \{0,0\}_{z=t+1}^{\infty}$ after observing $(q_{1,t-1}^u, q_{1,t-1}^u) \in \{(0,1),(1,0),(0,0)\}$ for the first time in period t, the group punishment incentive compatibility (GPIC) constraint requires

$$P^N(\tfrac{1}{2},\tfrac{1}{2},\tilde{e}_1^d,\tilde{e}_2^d) + \delta\pi_D^N(0,0,0,0) \ge \pi_D^N(\tfrac{1}{2},\tfrac{1}{2},\tilde{e}_1^d,\tilde{e}_2^d). \tag{GPIC}$$

It is easy to see that the threat to punish both upstream firms after the downstream firm observes $(q_{1,t-1}^u, q_{1,t-1}^u) = (0,0)$ for the first time is credible because, as in the case of the IPIC constraint, there is no uncertainty regarding the identities of upstream shirkers. The following result simplifies the analysis.

Lemma 2 *For $n = 2$, the GPIC constraint is satisfied if and only if*

$$\delta(\alpha-\beta)(\beta p^N(1) + (1-2\beta)p^N(\tfrac{1}{2}) - (1-\beta)p^N(0)) \le c^d Y(\tfrac{w^N}{c^d}), \tag{22}$$

where $Y(\tfrac{w^N}{c^d}) \equiv \min[\tfrac{w^N}{c^d} + 1, 2(1-\tfrac{1}{2}\delta)(\tfrac{1}{2}\tfrac{w^N}{c^d} + 1), \tfrac{2}{1+\delta}((1-\tfrac{1}{2}\delta)\tfrac{w^N}{c^d} + 1)]$.

The right-hand side of (22) is the discounted future incremental expected quality premium averaged out over time (multiplied by $1-\delta$) and the left-hand side is the current-period incremental costs averaged out over time if the downstream firm punishes none of the suppliers or punishes a random supplier after observing the first instance of unprompted upstream shirking. Consider a special case with a sufficiently small discount factor and no type II errors in consumer monitoring of quality ($\beta = 0$). Then the left-hand side becomes $\delta\alpha(p^N(\tfrac{1}{2}) - p^N(0))$ and the right-hand side becomes $w^N + c^d$. The GPIC constraint is satisfied if, having observed unprompted shirking, the downstream firm prefers (a) to save the expected payments and the cost of downstream efforts for both products, $w^N + c^d$, and earn $\delta p^N(0)$ in the next period than (b) to offer the expected payments to both suppliers and put its own efforts into both products and earn $\delta(\alpha p^N(\tfrac{1}{2}) + (1-\alpha)p^N(0))$ in the next period. In contrast, when the future is important, the downstream firm's option value of learning the first-time shirker's identity is high, and for $\delta \to 1$ the right-hand side of (22) converges to $\frac{1}{2}w^N + c^d$ because only the non-shirking supplier can receive the expected payment in the long run.

Best equilibrium for the supply chain

Let $l(x) \equiv \frac{c^d Y(\max[\frac{1}{\delta}x,1])}{\frac{1}{2}\max[\frac{1}{\delta}c^u, c^d]} = 2\frac{Y(\max[\frac{1}{\delta}x,1])}{\max[\frac{1}{\delta}x,1]}$ denote the ratio of the current-period incremental costs that the downstream firm incurs under the most profitable deviation from the group punishment strategy, $c^d Y(\max[\frac{1}{\delta}x,1])$, and the greater of the upstream and downstream per product cost savings from single-unit shirking, $\frac{1}{2}\max[\frac{1}{\delta}c^u, c^d]$. Also, let $\tilde{\Delta}_0^N \equiv \frac{1}{\delta(\alpha-\beta)}\max[\frac{1}{2(1-\alpha)}\max[\frac{1}{\delta}x,1], \frac{1}{2-\alpha-\beta}(\frac{1}{\delta}x+1)]c^d$, $\Delta_0^N \equiv \frac{1}{\delta(\alpha-\beta)^2}((\frac{1}{2}-\beta)\max[\frac{1}{\delta}x,1] \quad -(1-2\alpha)Y(\max[\frac{1}{\delta}x,1]))c^d$, and $\Delta_{0.5}^N \equiv \Delta_0^N + \frac{1}{\delta(\alpha-\beta)^2}(\frac{1}{2}\beta\max[\frac{1}{\delta}x,1] - \alpha Y(\max[\frac{1}{\delta}x, 1]))c^d$ denote the different values of the price discount (quality premium) for the final quality signals $y = 0$ and 0.5, respectively. Consider the following input payment and output-pricing schemes:

$$w^N = \begin{cases} w \in [\frac{1}{\delta}x, \max[\frac{1}{\delta}x, \min[\frac{1}{2}\frac{\alpha-\beta}{1-\alpha},1]]]c^d \text{ such that } l(\frac{w}{c^d}) \geq \frac{1-\beta}{1-\alpha}, \text{if } l(x) \geq \frac{1-\beta}{1-\alpha} \\ \max[\frac{1}{\delta}x,1]c^d, \text{if } l(x) < \frac{1-\beta}{1-\alpha} \end{cases} \tag{23}$$

and

$$p^N(y) = \begin{cases} v - \tilde{\Delta}_0^N 1_{y=0}, \text{if } l(x) \geq \frac{1-\beta}{1-\alpha} \\ v - \Delta_{0.5}^N 1_{y=0.5} - \Delta_0^N 1_{y=0}, \text{if } l(x) < \frac{1-\beta}{1-\alpha} \end{cases}. \tag{24}$$

Proposition 2 *Suppose that* $n = 2$ *and* $v > \frac{1}{\alpha-\beta}\max[\frac{1}{2(1-\alpha)}\max[x,1], \frac{1+x}{2-\alpha-\beta}, \max[x,1] - \frac{1-2\alpha}{\alpha-\beta}]c^d$. *Then there exists a threshold level of the discount factor* $\delta^N < 1$ *such that for all* $\delta \geq \delta^N$ *the strategies (3)-(6), (9)-(10) constitute the best equilibrium in the no-traceability regime with the expected input payment and output-pricing function given by (23) and (24), respectively. The expected per product value generated by the supply chain in the best equilibrium is*

$$\pi^N \equiv \begin{cases} \frac{1}{1-\delta}(v - c^u - c^d - \frac{(1-\alpha)^2}{2\delta(\alpha-\beta)}\max[\frac{\max[\frac{1}{\delta}x,1]}{1-\alpha}, \frac{\frac{1}{\delta}x+1}{1-\frac{1}{2}(\alpha+\beta)}]c^d), \text{if } l(x) \geq \frac{1-\beta}{1-\alpha} \\ \frac{1}{1-\delta}(v - c^u - c^d + \frac{1-\alpha}{\delta(\alpha-\beta)^2}((1-\alpha)Y(\max[\frac{1}{\delta}x,1]) - \frac{1+\alpha-2\beta}{2}\max[\frac{1}{\delta}x,1])c^d), \text{if } l(x) < \frac{1-\beta}{1-\alpha} \end{cases}.$$

The equilibrium properties of the output and input pricing policies hinge on whether the likelihood ratio $\frac{\Pr(y_{i,t}=0|q_{i,t}=0)}{\Pr(y_{i,t}=0|q_{i,t}=1)} = \frac{1-\beta}{1-\alpha}$ is smaller or greater than the ratio of the time-averaged costs of not immediately administering group punishment for shirking and the greatest per product savings from upstream or downstream single-unit shirking, $l(x)$. First, suppose that $l(x) \geq \frac{1-\beta}{1-\alpha}$. Then the optimal output-pricing rule that prescribes the loss of the quality premium $\widetilde{\Delta}_0^N$ following the worst possible signal of final quality is the softest punishment that ensures that the downstream firm is willing to offer the expected input payments and put in its own efforts, that is, the DIC constraint is satisfied. In this case, depending on the parameters of the model, the downstream firm's most profitable deviation is either (1) to shirk on downstream effort for one of the products and continue to put effort into the other product and offer the expected payments to both suppliers, thus earning $P^N(1,1,0,1) - w^N - \frac{1}{2}c^d$ per product per period; (2) to renege on the expected payment to one supplier and continue to offer the expected payment to the other supplier and put downstream efforts into both products, thus earning $P^N(0,1,1,1) - \frac{1}{2}w^N - c^d$ per product per period; or (3) to shirk and renege on the expected payments for all products, thus earning $P^N(0,0,0,0)$ per product per period.

In this case, the GPIC constraint is easy to satisfy in the sense that it requires a greater input payment without increasing the overall incentive costs (that is, the frequency or magnitude of the price discounts in the final good market). Specifically, for

$$x < \delta \text{ and } 2Y(\tfrac{1}{\delta}x) < \tfrac{1-\beta}{1-\alpha}, \tag{25}$$

the expected upstream payment is set above $\frac{1}{\delta}c^u$ (the smallest expected payment that satisfies the UIC constraint) in order to satisfy the GPIC constraint. However, if the likelihood ratio $\frac{1-\beta}{1-\alpha}$ is sufficiently small, the GPIC constraint is slack even at $w^N = \frac{1}{\delta}c^u$. Then the downstream firm's most profitable deviation is to shirk on downstream efforts and renege on the expected payments for all products, so that an expected payment above $\frac{1}{\delta}c^u$ will reduce the joint value because it will harden the DIC constraint.

Now we consider the case where $l(x) < \frac{1-\beta}{1-\alpha}$. Then the GPIC constraint requires that the downstream firm more frequently forgoes some quality premium. Although, as in the previous case, an expected payment above $\frac{1}{\delta}c^u$ for $x < \delta$ makes it easier to satisfy the GPIC constraint, now the product-pricing scheme needs to be adjusted to ensure that the downstream firm's threat to stop paying w^N to both suppliers if one of them shirks is credible. In this case, consumers reward the downstream firm more when the signal of the average quality is high than when the signal is mixed, $p^N(1) > p^N(\frac{1}{2}) > p^N(0)$. When the parameters of the model fall in this range, in the equilibrium the most profitable deviation for the downstream firm is to renege on the expected payment to one supplier but continue to offer the expected payment to the other supplier and put downstream efforts into both products. However, if one of the suppliers initiates (unprompted) shirking on his own, his identity is unknown to the downstream firm. What makes the threat of group punishment incentive compatible is that the downstream firm gives up a small incremental expected revenue, $(\alpha-\beta)((2\beta-1)\Delta_{0.5}^{N}+(1-\beta)\Delta_{0}^{N})$, relative to the large savings from *jointly* reneging on both expected payments and shirking on downstream efforts.

There are two instruments to satisfy the binding GPIC and DIC constraints: the expected payment, w^{N}, and the product-pricing policy, $p^{N}(y)$. Although the product-pricing policy is unique, there can be multiple expected input payments, w^{N}, that allow firms to achieve the highest expected joint profit in the no-traceability regime. Still, if condition (25) holds, that is, the ratio of upstream and downstream effort costs is sufficiently small and consumer information is sufficiently precise, the expected input payment is greater in the no-traceability regime, $w^{N}>w^{T}$, because the wholesale contracts are not publicly disclosed.

Also, as in the traceability regime, we can show that punishment of upstream shirking by exclusion is not optimal in a best equilibrium. Since the downstream firm does not observe the shirker's identity, by Lemma 1, we can restrict our attention to punishments by exclusion of all suppliers in which case the downstream firm's payoff equals zero. Because selling either uniformly low quality products or a mix of high and low quality products generates a positive payoff, the downstream firm cannot credibly promise to punish upstream shirking by excluding all suppliers from all future transactions.

3.2.2. Many suppliers and incomplete traceability

Writing down the GPIC constraint in a general case with $n>2$ suppliers is difficult because the set of experimentation strategies to identify upstream shirkers quickly becomes large. To circumvent this difficulty, we consider a particular information structure:

$$\Pr((o_{i,t}^{N},o_{k,t}^{N})=(U_{j-1},U_{j})\,|\,(o_{i,t},o_{k,t})\in\{U_{j-1},U_{j}\})$$
$$=\Pr((o_{i,t}^{N},o_{k,t}^{N})=(U_{j},U_{j-1})\,|\,(o_{i,t},o_{k,t})\in\{U_{j-1},U_{j}\}\}=\tfrac{1}{2}, \qquad (26)$$

for $i,k\in\mathrm{N}$, $j\in\{2,4,6,..,n\}$, where n is an even number. We refer to (26) as "$100\cdot(1-\frac{2}{n})\%$-traceability" since it partitions the set of suppliers U into $n/2$ pairs $\{\hat{U}_{j}\}_{j=1}^{n/2}$, where $\hat{U}_{j/2}=\{U_{j-1},$ $U_{j}\}$ for $j=2,4,...,n$, such that it is possible to identify that a given unit $i\in\mathrm{N}$ originated from a pair of suppliers $o_{i,t}\in\{U_{j-1},U_{j}\}$, but whether the unit was supplied by U_{j-1} or U_{j} is unknown.

Next we determine under what conditions the input and output pricing policies that are optimal in the complete traceability regime characterized in Proposition 1 constitute equilibrium in the "$100\cdot(1-\frac{2}{n})\%$-traceability" regime. Note that the downstream firm cannot engage in any new deviations under an incomplete traceability regime relative to the regime with complete traceability. Therefore, the DIC constraint is necessarily satisfied in the incomplete traceability regime under the pricing policy $p^N(y) = p^T(y)$ for $y = 0, \frac{1}{n}, ..., 1$, and $w^N = w^T$.

Under (26), the downstream firm's experimentation strategies to identify shirkers for $n > 2$ are essentially the same as in the case with two suppliers. The reason is that, as shown in the proof of Proposition 1, the DIC constraints for multi-unit deviations do not bind under the proposed pricing policies. Therefore, if an instance of shirking among the pair of suppliers $\{U_{j-1}, U_j\}$ is detected, the downstream firm is willing to continue to offer the expected input payments and supply high quality products from the remaining non-shirking suppliers $\mathrm{U} \setminus \{U_{j-1}, U_j\}$. From Lemma 2 it follows that the downstream firm's promise to punish $\{U_{j-1}, U_j\}$ is credible if

$$\delta(1-\alpha)^{n-2}(\alpha-\beta)(1-\beta)(p^T(1) - p^T(0)) \le \tfrac{2}{n} c^d Y(\tfrac{w^T}{c^d}), \text{ or } \tfrac{1-\beta}{1-\alpha} \le 2\frac{Y(x/\delta)}{1+x/\delta} \qquad (27)$$

The only difference compared with the GPIC constraint in (22) is the term $(1-\alpha)^{n-2}$ on the left-hand side and the term $\frac{2}{n}$ on the right-hand side of the first inequality. Under the pricing policy $p^T(y)$, the price premium can be lost only if the final qualities of the products from *all* non-shirking suppliers $\mathrm{U} \setminus \{U_{j-1}, U_j\}$, turn out to be low, which happens with probability $(1-\alpha)^{n-2}$. The relative weight of cost savings from reneging on the expected input payments and shirking on the downstream firm's efforts for the units generated by suppliers $\{U_{j-1}, U_j\}$ is now given by $\frac{2}{n}$ since the downstream firm continues to provide high quality products originating from the other suppliers. Note that condition (27) is independent of *n*, and it is satisfied whenever (a) firms are sufficiently patient, (b) consumer information is sufficiently noisy, and (c) the ratio of upstream and downstream costs of effort is sufficiently small.

4. Traceability versus no traceability

We are now in a position to compare the equilibrium profits in the traceability and no-traceability regimes. We begin with the case of two suppliers. It will be convenient to let $\alpha = 1-\beta \in (\frac{1}{2}, 1)$ and interpret α as the precision of consumer information.[11]

Proposition 3 *Suppose that* $n = 2$ *and* v *is sufficiently large.*[12] *In the best equilibrium, the expected joint*

[11] The value of traceability can also be positive or negative for $\alpha + \beta \neq 1$ and is decreasing in β. The case with $\alpha + \beta \neq 1$ is considered in Section 5.

[12] It can be shown that the threshold value of the discount factor δ^k such that an equilibrium with high quality provision exists, can be greater or smaller in the traceability regime than in the no-traceability regime, $\delta^T \geq (<) \delta^N$.

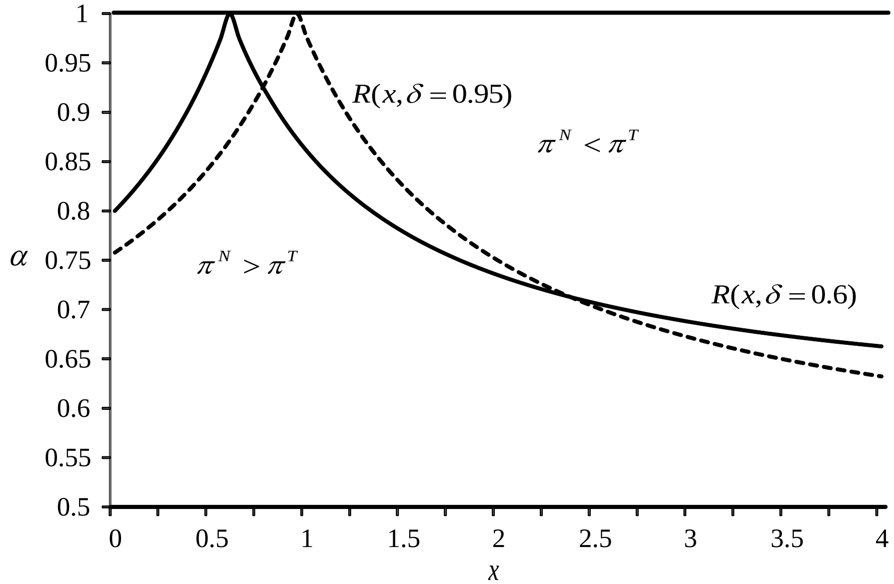
profit is greater (smaller) in the no-traceability regime than in the traceability regime, $\pi^T \le (>) \pi^N$*, depending on whether* $\alpha \le (>) R(x,\delta)$*, where function* $R(x,\delta) \equiv \left(1 + \frac{|\frac{1}{\delta}x - 1|}{2Y(\max[\frac{1}{\delta}x,1]) - \min[\frac{1}{\delta}x,1]}\right)^{-1}$ *is unimodal in* x*, reaches its maximal value of 1 at* $x = \delta$*, and*

$$\lim_{x\to 0} R(x,\delta) = \frac{2\min[2,3(1-\frac{1}{2}\delta),\frac{4-\delta}{1+\delta}]}{2\min[2,3(1-\frac{1}{2}\delta),\frac{4-\delta}{1+\delta}]+1} > \frac{2-\delta}{3-\delta} = \lim_{x\to\infty} R(x,\delta).$$

For $l(x) \ge \frac{\alpha}{1-\alpha}$, only the worst possible final quality triggers the loss of the quality premium in both regimes, $p^k(1) = p^k(\frac{1}{2}) = v > p^k(0)$, $k = T, N$, but the price discount is smaller in the no-traceability regime. The reason is that the DIC constraint is easier to satisfy when the downstream firm cannot vertically coordinate upstream and downstream shirking while the GPIC constraint imposes no additional incentive costs. Therefore, the no-traceability regime maximizes profits in this case. For $l(x) < \frac{\alpha}{1-\alpha}$, the GPIC constraint is hard to satisfy, which is reflected in the more frequent and significant price discounts for bad performance in the final good market. Comparing the expected revenues under the two regimes: $P^T(1,1,1,1) = v - (1-\alpha)^2 \Delta^T$ and $P^N(1,1,1,1) = v - 2\alpha(1-\alpha)\Delta^N_{0.5} - (1-\alpha)^2 \Delta^N_0$ shows that $P^T(1,1,1,1) \le (>) P^N(1,1,1,1)$ depending on whether $\alpha \le (>) R(x,\delta)$. If consumer monitoring of final quality is sufficiently noisy, information about input origin *decreases* profits because maintaining the credibility of the promise to abstain from vertically coordinating provision of low and high quality products imposes relatively large incentive costs. If consumers monitor final quality with little noise, information about input origin *increases* profits because it is difficult to make the promise to punish all suppliers for occasional shirking self-enforcing.

The general shape of the function $R(x,\delta)$ for different values of δ in Figure 1 illustrates comparative statics about the key parameters of the model: the ratio of the costs of upstream and downstream efforts, x, the precision of consumer information, α, and the discount factor, δ. The value of information about input origin, $\pi^T - \pi^N$, decreases when the ratio of upstream and downstream costs of efforts is closer to δ ($|x-\delta|$ is smaller) or when consumer information is less precise (α is smaller). Furthermore, the value of information about input origin is negative at $x = \delta$ for any value of α. Then only reneging on the expected payment or only shirking on the costs of the downstream effort leaves *half* of the potential cost savings from provision of a low quality product on the table without affecting the expected revenues. In this case, the commitment to abstain from vertically coordinated deviations is so valuable that the no-traceability regime dominates even when consumer information is nearly perfect. Note that the function $R(x,\delta)$ is asymmetric in x around δ because for $x < \delta$ the downstream firm may be able to slacken the GPIC constraint by increasing the expected input payment w^N, whereas this is not optimal for $x > \delta$.

Figure 1 The value of traceability for different combinations of the values of the ratio of upstream and downstream costs, x, and the precision of consumer information, α



As Figure 1 demonstrates, the effect of a larger discount factor, δ, on the value of traceability, $\pi^T - \pi^N$, is ambiguous because δ affects the importance of future payoffs for both the downstream firm and upstream firms. On the one hand, it is easier for a more patient downstream firm to satisfy the DIC constraint and more difficult to satisfy the GPIC constraint. This tends to increase the value of traceability. On the other hand, the minimum input payment, $\frac{1}{\delta}c^u$, decreases as δ increases. This increases the incremental payoff from vertically coordinated shirking compared with vertically uncoordinated shirking whenever the full cost of upstream effort, $\frac{1}{\delta}c^u$, is greater than the cost of the downstream effort, c^d. This tends to decrease the value of traceability.

Many suppliers

It is easy to see that in a supply chain with an arbitrary (even) number of suppliers complete traceability is not optimal if condition (27) is satisfied. The reason is that under parametric restriction (27) the downstream firm's promise to punish upstream shirking is credible in the "$100 \cdot (1 - \frac{2}{n})\%$ traceability" regime *even* under the input and output pricing policies that are optimal in the complete traceability regime. Since, as shown in the proof of Proposition 1, the DIC for a vertically coordinated deviation for a *single* unit binds in the best equilibrium in the traceability regime, it must be that the supply chain can generate greater profits in the no-traceability regime. Note that, under (26), the downstream firm can no longer vertically coordinate upstream and downstream shirking for a single unit (and *m*-unit deviations are easier for consumers to detect, $m \geq 2$). As a result, the severity of punishment for bad performance in the final good market can be reduced, while still maintaining the incentive compatibility of punishing a pair of suppliers if one member of the pair is caught shirking. The incentive cost of maintaining credibility of inter-firm punishment for shirking is relatively small when the downstream firm does not need to break off its supplier relations with all firms at once to punish shirking and (27) holds. Summarizing,

Proposition 4 *Suppose that v is sufficiently large and condition (27) holds. Then perfect traceability is not optimal for any* $n \geq 2$.

Supplier turnover

Here we discuss what happens in a more general model where the downstream firm can replace suppliers. The previous analysis corresponds to the case where the cost of searching for a new supplier is so large that it precludes any turnover among suppliers. A reduction in the cost of replacing suppliers makes it easier for the downstream firm to maintain credibility of the promise to punish upstream shirking in any traceability (including no-traceability) regime. This happens because the downstream firm no longer needs to forego all future profits when it excludes all current suppliers from future transactions. If the downstream firm can establish profitable relationships with new suppliers, it can cut off existing suppliers without the loss of sales in the final good market.

As a result, in equilibrium the incentive cost (due to offering lower output or higher input prices) will be proportional to the direct costs of replacing the supplier base (rather than the cost savings from shirking on quality as in the case with the fixed supplier base). Therefore, in a model where the downstream firm can cut off shirkers and (at a sufficiently small cost) start transacting with new suppliers, the downstream firm is more likely to achieve greater profits in the no-traceability regime. This happens because the downstream firm's ability to engage in vertically coordinated deviations is not affected by the turnover among suppliers, but the incentive cost necessary to sustain credibility of the promise to punish upstream shirking by supplier exclusion is now bounded by the cost of replacing suppliers.

5. Imperfect inter-firm monitoring of levels of upstream efforts

The previous analysis relied on a restrictive assumption that the signals of input quality, while lagged, are perfect. We now consider a model where the downstream firm's experience with inputs provides *imperfect* information about the levels of upstream efforts (in addition to possible noise regarding sources of inputs). For tractability, we again assume that the supplier base is fixed and there are two suppliers. Now the downstream firm observes lagged signals of input quality $y_{1,t}^u, y_{2,t}^u \in \{0,1\}$ that are correlated with the actual input quality $q_{1,t}^u, q_{2,t}^u$ as follows

$$\Pr(y^u_{i,t} = 1 \,|\, (q^u_{i,t}, q^u_{-i,t}) \in \{(1,1),(1,0)\}) = 1, \tag{28}$$

$$\Pr(y^u_{i,t} = 1 \,|\, (q^u_{i,t}, q^u_{-i,t}) = (0,1)) = 1 - \lambda > 0,$$

$$\Pr(y^u_{i,t} = 1 \,|\, (q^u_{1,t}, q^u_{2,t}) = (0,0)) = 0,$$

where $\lambda > 0$ is the precision of monitoring of upstream efforts. In accordance with (28), the downstream firm observes without noise if (a) a supplier puts in effort or (b) both suppliers shirk. The former assumption assures that there is no need for the downstream firm to actually punish suppliers on the equilibrium path. The latter assumption assures that in the no-traceability regime the downstream firm can learn the identity of an unprompted upstream shirker through one-time experimentation with input payment offers. Yet, the inter-firm monitoring of upstream efforts is now imperfect in the sense that the downstream firm detects unilateral upstream shirking with the fixed exogenous probability λ. We impose an additional assumption that $(y^u_{1,t}, y^u_{2,t}, e^d_{1,t}, e^d_{2,t})$ is a sufficient statistic for the pair $(y_{1,t}, y_{2,t})$ with respect to $q^u_{1,t}, q^u_{2,t}$, if the pairs of input qualities/upstream efforts and downstream efforts are viewed as random parameters:

$$\Pr((y_{1,t}, y_{2,t}) = (y_1, y_2), (y^u_{1,t}, y^u_{2,t}) = (y^u_1, y^u_2), (e^d_{1,t}, e^d_{2,t}) = (e^d_1, e^d_2), (q^u_{1,t}, q^u_{2,t}) = (q^u_1, q^u_2)) \tag{29}$$

$$= \Pr((y_{1,t}, y_{2,t}) = (y_1, y_2), (y^u_{1,t}, y^u_{2,t}) = (y^u_1, y^u_2), (e^d_{1,t}, e^d_{2,t}) = (e^d_1, e^d_2))$$

$$\times \Pr((y^u_{1,t}, y^u_{2,t}) = (y_1, y_2), (q^u_{1,t}, q^u_{2,t}) = (q^u_1, q^u_2)) \text{ for all } y_1, y_2, y^u_1, y^u_2, e^d_1, e^d_2, q^u_1, q^u_2 \in \{0,1\}.$$

This assures that the downstream firm cannot use the signals of final quality to make inferences about input quality/upstream efforts. Assumptions (2), (28), and (29) are satisfied only if $\beta = \alpha(1-\lambda)$.[13] All other elements of the model remain unchanged. Note that the case with perfect inter-firm monitoring of input quality with no type II errors in consumer monitoring ($\beta = 0$) is obtained for $\lambda = 1$.

Without loss, we assume that, in the traceability regime, upon observing $y^u_{i,t-1} = 0$ for the first time the downstream firm believes that upstream firm $o_{i,t}$ will never put in effort in the future. In the no-traceability regime, upon observing $(y^u_{1,t-1}, y^u_{2,t-1}) = (0,1)$ or $(1,0)$ for the first time, the downstream firm believes that one of the suppliers will never put in effort in the future. After observing $(y^u_{1,t-1}, y^u_{2,t-1})$ $= (0,0)$ the downstream firm believes that both suppliers will shirk in both regimes. Then the approach that was used to characterize the best equilibrium under perfect downstream monitoring of upstream efforts, $\lambda = 1$, remains valid for $\lambda < 1$. The conditions under which the strategy profile in (3) – (10) consti-

[13] A garbling process of inter-firm signals of upstream and downstream efforts that satisfies conditions (1), (2), (28) and (29) is given by $y_{i,t} = y^u_{i,t} e^d_{i,t} \varepsilon_{i,t} + (1 - e^d_{i,t} \max[e^u_{U_1,t}, e^u_{U_2,t}])\eta_{i,t}$, $i = 1,2$, where $\varepsilon_{i,t}, \eta_{i,t} \in \{0,1\}$ are noise terms that are independent across products and across periods, $\Pr(\varepsilon_{i,t} = 1) = \alpha \in (0,1)$ and $\Pr(\eta_{i,t} = 1) = \alpha(1-\lambda)$.

tutes a PBE are the same as before except the upstream no-shirking incentive compatibility constraint is now given by

$$\pi_j^k \geq w^k + (1-\lambda)\delta\pi_j^k,\ j \in \{U_1, U_2\},\ k \in \{N, T\}. \qquad \text{(UIC')}$$

The right-hand side is the upstream firm's payoff from accepting the contract and choosing a low level of effort in accordance with the downstream firm's input payment strategy in (8) in the traceability regime or (10) in the no-traceability regime, where with probability $1-\lambda$ the downstream firm does not detect unilateral upstream shirking. Upon simplification, the upstream incentive compatibility constraint becomes

$$w^k \geq \tfrac{1-(1-\lambda)\delta}{\delta\lambda} c^u = \tfrac{1}{\delta} x(\lambda) c^d,$$

where $x(\lambda) = \frac{1-(1-\lambda)\delta}{\lambda}\frac{c^u}{c^d} \geq \frac{c^u}{c^d}$ is decreasing in λ. Since none of the other participation and incentive compatibility constraints change, in order to characterize the best equilibrium under imperfect inter-firm monitoring of upstream efforts we just need to replace $x(= x(1))$ with $x(\lambda)$ in Propositions 1 and 2 and set $\beta = \alpha(1-\lambda)$. An increase in the precision of inter-firm monitoring, λ, raises the joint profits in both regimes because the signals of output quality become more informative since low quality products are relatively less likely to generate good signals, and the expected input payment (weakly) decreases since the upstream shirking is less likely to go undetected by the downstream firm.

Now we are in a position to compare the joint expected profits under imperfect inter-firm monitoring of upstream efforts in the regimes with and without traceability. Generalizing our finding in Proposition 3, firms achieve greater joint profits in the no-traceability regime than in the traceability regime depending on whether

$$\alpha \leq (>)(1 + \lambda \frac{|\frac{1}{\delta}x(\lambda) - 1|}{2Y(\max[\frac{1}{\delta}x(\lambda),1]) - \max[\frac{1}{\delta}x(\lambda),1]})^{-1}.$$

It is easy to check that the right-hand is decreasing in λ and it approaches one as $\lambda \to 0$. This implies that whenever the downstream firm detects upstream shirking sufficiently rarely, the supply chain achieves higher joint profits in the no-traceability regime. A smaller probability that the downstream firm detects upstream shirking and consumers observe bad signals when the actual product quality is low has two effects on the difference in the overall incentive costs with and without traceability. First, as λ decreases, the equilibrium input payment (weakly) increases. This increases or decreases the difference in the incentive costs of building reputation for high quality products in the traceability versus no-traceability regime depending on whether $x(\lambda) < (\geq) \frac{1}{\delta}\frac{c^u}{c^d}$, that is, whether the full cost of upstream and downstream effort become more similar or dissimilar as λ decreases. Second, a smaller λ also reduces the downstream firm's temptation to condone unilateral upstream shirking in the no-traceability regime since the signals of quality generated by products of uniformly low quality and those generated by products of mixed (high and low) quality are more likely to be similar. Given the posited information structures of inter-firm and consumer monitoring, as the precision of inter-firm monitoring decreases, the "temptation to condone anonymous upstream shirking" becomes weaker than the "temptation to vertically coordinate shirking" and, as a result, firms can achieve greater profits when information about input origin is not available.[14]

An interesting question is whether the precision of inter-firm monitoring and traceability are complements or substitutes.

Proposition 5 *Suppose that* v *and* δ *are sufficiently large. Then for sufficiently large (small)* α, $\pi^T - \pi^N$ *decreases (increases) in* λ.

Suppose that firms are sufficiently patient (δ is close to 1) and high quality products almost certainly generate high signals in the final good market (α is close to 1). Then the supply chain incurs very small overall incentive costs in the traceability regime while the group punishment incentive compatibility constraint is very difficult to satisfy. Therefore, an increase in the precision of inter-firm monitoring is more valuable to the firms in the no-traceability regime, and $\pi^T - \pi^N$ decreases in λ. Now suppose that even high quality products are unlikely to generate high signals in the final good market (α is close to 0). Then only the DIC constraint binds in equilibrium, and it is more difficult to satisfy in the traceability regime than in the no-traceability regime. As a result, the incremental returns to precision of inter-firm monitoring are greater in the traceability regime, i.e. $\pi^T - \pi^N$ increases in λ. If we interpret parameter λ as the degree of vertical integration in the supply chain, Proposition 5 demonstrates that mandatory traceability can increase or decrease the returns to vertical integration depending on information generated by the consumer experience with the product.

6. Conclusions

In this paper, we analyze a simple model of a supply chain in which the downstream firm imperfectly identifies product origins and input quality, and consumers imperfectly monitor effort choices by firms along the supply chain. We consider a setting with binary efforts where it is socially efficient for all firms to choose the high level of effort. We focus on the effects of traceability on the incentive costs necessary to build reputation for high quality and find that the firms may prefer not to maintain information about product flows even when the direct cost of doing so is zero.

[14] Note, however, that this result is obtained under the special information structure and may not hold more generally.

Our analysis can be extended in several dimensions. First, allowing for continuous efforts to provide quality may lead to a trade-off between the costs of building reputation and the levels of upstream and downstream efforts implementable in equilibrium in different traceability regimes. Second, in order to provide a clean characterization of the conditions under which information about product flow restricts the set of equilibrium payoffs, we made several simplifying assumptions about the structure of information regarding upstream and downstream efforts, and it is clearly of interest to extend the analysis to more general settings.

Finally, the model focused on experience quality attributes and assumed that neither the downstream firm nor consumers are able to observe the actual choices of efforts by firms along the supply chain. In practice, firms and consumers can rely on costly external instruments to monitor compliance with quality standards such as audits and inspections. On the one hand, external monitoring of efforts should decrease the value of traceability since the downstream firm will have more tools to punish upstream shirking. In addition to the exclusion of suspected shirkers or the reversion to the provision of low quality, the downstream firm can now identify and punish shirkers by increasing the intensity of audits. On the other hand, traceability can increase the effectiveness of audits and reduce the costs of providing inter-firm incentives to provide high quality. Investigating whether additional external means of identifying non-complying suppliers increase or decrease the benefits of adopting a traceability system is left for future research.

Appendix

The following notation will be used in the proofs to follow. Let $V^k > 0$ be the greatest per-product joint payoff that is achievable in equilibrium, $p \leq v$ be the equilibrium first-period final good price, and w be the equilibrium first-period transfer from the downstream firm to each upstream firm in regime $k = N, T$. Also, let u^k be the mapping that maps each public signal of average quality y to the equilibrium continuation per product joint payoff $u^k \in [0, V^k]$, let u_D^k be the mapping that maps signal y to the equilibrium continuation per product payoff for the downstream firm, and let u_U^k be the equilibrium value of each upstream firm $j \in \mathrm{U}$, where

$$u_D^k(y) + u_U^k(y) \leq u^k(y) \text{ for } y \in \{0, \tfrac{1}{n}, ..., 1\}. \tag{A1}$$

Note that, as long as the downstream firm makes the expected offers to consumers, the punishments of the downstream firm by consumers are on the equilibrium path since consumers imperfectly monitor effort choices and, by assumption, the relationship between the downstream firm and consumers is ongoing. However, inter-firm punishments are off the equilibrium path and are private to the downstream firm and the upstream firm in question. So to apply the self-generation principle (Abreu, Pearce, and Stacchetti 1990) and characterize the greatest equilibrium payoffs, the set of continuation payoffs needs to include both equilibrium and off-equilibrium payoffs. We will make use of the fact that on the equilibrium path the expected flow and continuation payoff of the downstream firm satisfy the following system of dynamic equations:

$$u_D^k(y) = r_D^k(y) - w - c^d + \delta \sum_{i=1}^{n} u_D^k(\tfrac{i}{n}) f(\tfrac{i}{n} \mid 1, ..., 1), \; y \in \{0, \tfrac{1}{n}, ..., 1\}, \tag{A2}$$

where $r_D^k(y)$ are the downstream firm's gross flow revenues when the signal of average quality in the previous period is y, $k = T, N$.

We look for equilibrium where deviations from the promised actions that are perfectly observed – an unexpected offer or upstream choice of low effort or rejection of a contract – imply reversion to the static equilibrium of the production subgame (i.e. the sequence of input payment offers and upstream acceptance and effort choices) or the entire game. In the traceability regime, there is no loss in assuming the worst punishment off the equilibrium path of play because the downstream firm and upstream firm perfectly observe the relevant actions (Abreu 1988). In the no-traceability regime, the assumption that firms can credibly threaten with the reversion to static equilibrium can appear to be restrictive since the downstream firm imperfectly monitors individual upstream efforts. However, by Lemma 1, the assumption that the downstream firm and all upstream firms revert to the static equilibrium following any upstream deviation or an unexpected input payment offer is without loss in the no-traceability regime as well.

Therefore, in a best equilibrium the following conditions must hold for all $y \in \{0, \tfrac{1}{n}, ..., 1\}$, $k = N, T$.

(i) The firms are willing to trade with each other and consumers:

$$u_D^k(y) \geq 0 \text{ and } u_U^k(y) \geq 0,\ j \in \mathrm{U}; \tag{A3}$$

(ii) Upstream firms are willing to choose high effort:

$$u_U^k(y) = w - c^u + \delta u_U^k(y) \geq w; \tag{A4}$$

(iii) The downstream firm is willing to offer the expected payments and exert efforts. Since the upstream firm responds by permanent shirking after being offered a smaller-than-expected payment, in the traceability regime this condition is satisfied if

$$p - w - c^d + \delta \sum_{y \in \{0, \frac{1}{n}, \ldots, 1\}} u_D^T(y) f(y \mid 1, \ldots, 1) \geq \max_{e_{U_1}^u, \ldots, e_{U_n}^u, e_1^d, \ldots, e_n^d} p - \tfrac{1}{n} \sum_{i=1}^{n} (e_{U_i}^u w + e_i^d c^d) \tag{A5}$$

$$+ \tfrac{\delta}{1-\delta} \Big(\sum_{y \in \{0, \frac{1}{n}, \ldots, 1\}} u_D^T(y) f(y \mid e_{U_1}^u e_1^d, \ldots, e_{U_n}^u e_n^d) - \delta u_D^T(y) f(y \mid 1, \ldots, 1) + w + c^d - \tfrac{1}{n} \sum_{i=1}^{n} (e_{U_i}^u w + e_i^d c^d) \Big),$$

and in the no-traceability regime this condition is satisfied if

$$p - w - c^d + \delta \sum_{y \in \{0, \frac{1}{n}, \ldots, 1\}} u_D^N(y) f(y \mid 1, \ldots, 1) \geq \max_{e_{U_1}^u, \ldots, e_{U_n}^u, e_1^d, \ldots, e_n^d} p - \tfrac{1}{n} \sum_{i=1}^{n} (e_{U_i}^u w + e_i^d c^d) \tag{A6}$$

$$+ \tfrac{\delta}{1-\delta} \Big(\tfrac{1}{n!} \sum_{y \in \{0, \frac{1}{n}, \ldots, 1\}} \sum_{\pi(1), \ldots, \pi(n) \in \Pi} u_D^N(y) f(y \mid e_{U_1}^u e_{\pi(1)}^d, \ldots, e_{U_n}^u e_{\pi(n)}^d) - \delta u_D^N(y) f(y \mid 1, \ldots, 1)$$

$$+ w + c^d - \tfrac{1}{n} \sum_{i=1}^{n} (e_{U_i}^u w + e_i^d c^d) \Big);$$

(iv) By Lemma 1 the downstream firm must be willing to punish all suppliers if one or more of them shirk in the no-traceability regime. By Lemma 2, for $n = 2$ this is incentive compatible if and only if[15]

$$\delta(\alpha - \beta)(\beta u_D^N(1) + (1 - 2\beta) u_D^N(\tfrac{1}{2}) - (1 - \beta) u_D^N(0)) \leq c^d Y(\tfrac{w}{c^d}). \tag{A7}$$

Note that to derive (A5), (A6), and (A7) we used (A2) to substitute for the downstream firm's expected gross flow revenue.

Lemma 1 *If there exists a PBE in which all firms exert efforts in every period in the no-traceability regime, then upstream shirking by one or more suppliers is punished by reversion to the static equilibria of the contracting subgame where either (i) the downstream offers a payment that just covers the cost of low quality inputs to all suppliers and suppliers produce low quality inputs forever or (ii) the downstream firm stops transacting with all suppliers and exits the market.*

Proof of Lemma 1 If suppliers are willing to put in efforts, it must be that their payoffs depend on input quality. Suppose that the downstream firm announces the following strategy of punishing unprompted upstream shirking: With probability $\varphi \in (0,1]$, punish a randomly chosen subset of suppliers (by offering them a contract with fixed payment $w = 0$ in the future) and continue to offer the expected payment w^N to the other suppliers; with probability $1 - \varphi$, punish all suppliers if one them shirks; and with probability

[15] For analytical tractability we consider the incentive compatibility of group punishment for $n > 2$ in a special case of imperfect traceability and for given input and output price policies.

one, punish all suppliers if all suppliers shirk. Then, provided that the no-upstream-shirking incentive compatibility (UIC) constraint is satisfied, the best response for a supplier that was offered the expected input payment in every period and shirked in period t for the first time is the following "stop shirking if not punished" strategy: Exert effort in every period $z > t$ if the downstream firm offers the expected payment; shirk in every period $z > t$ if the downstream firm offers a smaller-than-expected payment. Having done the same calculation, the downstream firm will prefer to continue to offer the expected payments to all suppliers in period $t+1$ after it observes low input quality $(q^u_{1,t},...,q^u_{n,t}) = (e^u_{o_{1,t},t},...,e^u_{o_{n,t},t}) \in \{(0,...,0),(0,1,...,1),(1,...,1,0)\}$ for the first time in period $t+1$. The reason is that the supplier that shirked in period t prefers to return to the equilibrium path in period $t+1$ if it is offered the expected payment w^N in period $t+1$, whereas suppliers that did not shirk in period t remain on the equilibrium path in period $t+1$ if they are offered the expected payment in period $t+1$. This demonstrates that in the no-traceability regime, the downstream firm's threat to punish a randomly chosen subset of suppliers, whether or not it includes the ones that actually shirked, is not credible for any $\varphi > 0$. If $\varphi = 0$, provided that the group punishment incentive compatibility constraint is satisfied, the downstream firm's threat of group punishment is self-enforcing because each supplier expects to be offered a zero payment forever (or being excluded from all future transactions) after shirking for the first time and has no reasons to put in efforts after destroying the group's reputation for input quality. ■

Proof of Lemma 2 Suppose the downstream firm has observed unprompted upstream shirking for the first time. Then from (21) it follows that the expected value of the downstream firm $\pi^N_D(\frac{1}{2},\frac{1}{2},e^d_{1,t},e^d_{2,t})$ in the explicit form is given by

$$\begin{aligned}
\pi^N_D(\tfrac{1}{2},\tfrac{1}{2},e^d_{1,t},e^d_{2,t}) = \max[\; & p_{t+1} + \tfrac{\delta}{1-\delta}P^N(0,0,0,0),\;\; p_{t+1} - w^N - c^d + \tfrac{\delta}{1-\delta}(P^N(\tfrac{1}{2},\tfrac{1}{2},11) - w^N - c^d), \\
& p_{t+1} - w^N - \tfrac{1}{2}c^d + \tfrac{\delta}{1-\delta}(P^N(\tfrac{1}{2},\tfrac{1}{2},01) - w^N - \tfrac{1}{2}c^d), \\
& p_{t+1} - \tfrac{1}{2}w^N - c^d + \tfrac{1}{2}\tfrac{\delta}{1-\delta}(P^N(0,1,1,1) - \tfrac{1}{2}w^N - c^d) + \tfrac{1}{2}\tfrac{\delta}{1-\delta}P^N(0,0,0,0), \\
& p_{t+1} - \tfrac{1}{2}w^N - \tfrac{1}{2}c^d + \tfrac{1}{2}\tfrac{\delta}{1-\delta}(P^N(0,1,0,1) - \tfrac{1}{2}w^N - \tfrac{1}{2}c^d) + \tfrac{1}{2}\tfrac{\delta}{1-\delta}P^N(0,0,0,0), \qquad \text{(A8)} \\
& p_{t+1} - \tfrac{1}{2}w^N - c^d + \tfrac{1}{2}\delta(p^N(0,1,1,1) - \tfrac{1}{2}w^N - \tfrac{1}{2}c^d + \tfrac{\delta}{1-\delta}(p^N(0,1,0,1) - \tfrac{1}{2}w^N - \tfrac{1}{2}c^d)) + \tfrac{1}{2}\tfrac{\delta}{1-\delta}P^N(0,0,0,0), \\
& p_{t+1} - \tfrac{1}{2}w^N - \tfrac{1}{2}c^d + \tfrac{1}{2}\delta(P^N(0,1,0,1) - \tfrac{1}{2}w^N - c^d + \tfrac{\delta}{1-\delta}(P^N(0,1,1,1) - \tfrac{1}{2}w^N - c^d)) + \tfrac{1}{2}\tfrac{\delta}{1-\delta}P^N(0,0,0,0)],
\end{aligned}$$

where $p_{t+1} = P^N(\frac{1}{2},\frac{1}{2},e^d_{1,t},e^d_{2,t})$. The right-hand side is the maximum expected payoff from all possibly optimal combinations of pairs of input payments and downstream efforts given that the upstream firms adhere to the acceptance and effort strategies in (3) and (4). To derive the value function in closed form in (A8), we used the properties of the expected product price conditional on the upstream and downstream efforts in (13) and (14), and the fact that if the downstream firm punishes none of the upstream firms, it does not learns the identity of the shirker, and if the downstream firm punishes a random upstream firm, it learns the identity of the unprompted shirker in the end of the period. Upon substituting (A8) into the GPIC constraint and simplification, the GPIC constraint reduces to the following system of inequalities

$$w^N + c^d \geq \delta(P^N(0,1,11) - P^N(0,0,0,0)),$$
$$w^N + \tfrac{1}{2}c^d \geq \delta(P^N(0,1,01) - P^N(0,0,0,0)),$$
$$(1-\tfrac{1}{2}\delta)(\tfrac{1}{2}w^N + c^d) \geq \tfrac{1}{2}\delta(P^N(0,1,1,1) - P^N(0,0,0,0)),$$
$$(1-\tfrac{1}{2}\delta)(\tfrac{1}{2}w^N + \tfrac{1}{2}c^d) \geq \tfrac{1}{2}\delta(P^N(0,1,0,1) - P^N(0,0,0,0),$$
$$(1-\tfrac{1}{2}\delta)\tfrac{1}{2}w^N + (1-\tfrac{3}{4}\delta)c^d \geq \tfrac{1}{2}\delta((1-\delta)P^N(0,1,1,1) + \delta P^N(0,1,0,1) - P^N(0,0,0,0)),$$
$$(1-\tfrac{1}{2}\delta)\tfrac{1}{2}w^N + \tfrac{1}{2}c^d \geq \tfrac{1}{2}\delta((1-\delta)P^N(0,1,0,1) + \delta P^N(0,1,1,1) - P^N(0,0,0,0)).$$

On substitution for the expected price conditional on upstream and downstream efforts in (13), this system of inequalities reduces to condition (22). ■

Proof of Proposition 1 To find an upper bound on the equilibrium joint payoff in equilibrium where all firms exert efforts in every period, we solve the following problem

$$\max_{w, u_U^T, \{u^T(y), u_D^T(y)\}_{y=\{0,\frac{1}{n},\dots,1\}}} \sum_{i=0}^{n} u^T(\tfrac{i}{n}) g(i,n) \qquad \text{(A9)}$$

subject to conditions (A1), (A2), (A3), (A4), and (A5). Clearly, at optimum the total value constraint (A1) binds and the participation constraints in (A3) are slack whenever the incentive compatibility constraints are satisfied. In the traceability regime, the DIC constraint in (A5) can be rewritten as follows:

$$p - w - c^d + \delta \sum_{i=0}^{n} u^T(\tfrac{i}{n}) g(i,n) \geq \max_{0 \leq m \leq n} \; p - \tfrac{m}{n}(w + c^d) + \tfrac{\delta}{1-\delta}\Big(\sum_{i=0}^{n} u^T(\tfrac{i}{n}) g(i,m) + \tfrac{n-m}{n}(w + c^d)\Big) \qquad \text{(A10)}$$
$$- \tfrac{\delta^2}{1-\delta} \sum_{i=1}^{n} u^T(\tfrac{i}{n}) g(i,n) \text{ for all } 0 \leq m \leq n-1.$$

The left-hand side is the equilibrium payoff when consumers are willing to pay up to p. The right-hand side is the maximum payoff that the downstream firm can achieve from stationary deviations. Given that the quality of the final product exhibits the "weakest-link" property, the downstream firm clearly prefers to vertically coordinate upstream and downstream shirking rather than (a) renege on the upstream payment to some suppliers and put in its own effort into the product originating from those suppliers, or (b) shirk on the downstream effort for some products and offer the expected payments for the inputs used in those products. So we only need to verify that the downstream firm does not prefer to engage in vertically coordinated deviations and provide $n - m \in \{1,2,\dots,n\}$ low quality units.

On simplification, constraints (A10) and (A4) can be rewritten as

$$\tfrac{n-m}{n}(w + c^d) \leq \delta \sum_{i=0}^{n} (g(i,n) - g(i,m)) u^T(\tfrac{i}{n}) \text{ for all } 0 \leq m \leq n-1 \qquad \text{(A5')}$$
$$\text{and } w \geq \tfrac{1}{\delta} c^u.$$

Consider the following relaxed version of problem (A9):

$$\max_{u^T(0), u^T(\frac{1}{n}), \dots, u^T(1) \in [0, V^T]} \sum_{i=0}^{n} u^T(\tfrac{i}{n}) g(i,n) \text{ subject to } \tfrac{1}{n}(w + c^d) \leq \delta \sum_{i=0}^{n} (g(i,n) - g(i,m)) u^T(\tfrac{i}{n}). \qquad \text{(A11)}$$

Clearly, the single constraint in (A11) must bind at optimum. Manipulating the binding constraint yields

$$u^T(0) = \tfrac{1}{(1-\alpha)^{n-1}(\alpha-\beta)}\left(-\tfrac{1}{\delta n}(w+c^d) + \sum_{i=1}^{n}(g(i,n)-g(i,m))u^T(\tfrac{i}{n})\right).$$

Substituting this into the objective function, the optimization problem (A11) reduces to

$$\max_{u^T(0),u^T(\frac{1}{n}),\dots,u^T(1)\in[0,V^T]} \frac{1}{\alpha-\beta}\left(\sum_{i=1}^{n}(B^\alpha_{n,i}(1-\beta)-(1-\alpha)\sum_{h=i-1}^{\min[n-1,i]}B^\alpha_{n-1,h}\beta^{i-h}(1-\beta)^{1-(i-h)})u^T(\tfrac{i}{n})\right.$$

$$\left.-(1-\alpha)(\tfrac{1}{\delta n}(w+c^d)\right) = \sum_{i=1}^{n} B^\alpha_{n-1,i-1}\; u^T(\tfrac{i}{n}) - \tfrac{1-\alpha}{\alpha-\beta}\tfrac{1}{\delta n}(w+c^d).$$

Because the coefficients on $u^T(\frac{i}{n})$ are positive for all $i=1,\dots,n$, the optimal solution is

$$u^T(y) = V^T - \tfrac{c^d+w}{\delta n(\alpha-\beta)(1-\alpha)^{n-1}}1_{y=0} \text{ for } y=0,\tfrac{1}{n}\dots,1. \qquad \text{(A12)}$$

Next note that evaluated at (A12) the DIC constraints in (A5) are slack for all $0\le m\le n-2$. To see why note that, upon substitution of (A12) into (A5'), conditions (A5') can be written as

$$\tfrac{n-m}{n}(w+c^d) \le \delta((1-\alpha)^n-(1-\alpha)^m(1-\beta)^{n-m})(u^T(1)-u^T(0))$$
$$=\delta(1-\alpha)^n(1-(\tfrac{1-\beta}{1-\alpha})^{n-m})(u^T(1)-u^T(0)) = \delta(1-\alpha)^n(1-(\tfrac{1-\beta}{1-\alpha})^{n-m})\tfrac{c^d+w}{\delta n(\alpha-\beta)(1-\alpha)^{n-1}},$$

or

$$\tfrac{1-\alpha}{\alpha-\beta}((\tfrac{1-\beta}{1-\alpha})^{n-m}-1)\ge n-m,$$

which hold with strict inequalities for all $0\le m\le n-2$. Therefore, the solutions to problems (A9) and (A11) coincide for a given value of w. That is, at optimum the downstream firm's most profitable deviation is to vertically coordinate upstream and downstream shirking for a single unit and put in efforts in all other products.

To find the maximum achievable joint payoff, V^T, note that the consumer price cannot exceed $p=v$, and at optimum, $u^T_D(y)=V^T-u^T_U$ for all $y=\frac{1}{n},\frac{2}{n},\dots,1$, satisfies the Bellman equation (A2):

$$u^T_D(y) = v-c^d-w+\delta\sum_{k=0}^{n}\frac{n!}{k!(n-k)!}\alpha^k(1-\alpha)^{n-k}u^T(\tfrac{k}{n}) \text{ for all } y=\tfrac{1}{n},\tfrac{2}{n},\dots,1. \qquad \text{(A13)}$$

Substituting (A12) in (A13) and solving for V^T yields:

$$V^T = \tfrac{1}{1-\delta}(v-c^u-c^d-\tfrac{1-\alpha}{n(\alpha-\beta)}(w+c^d)). \qquad \text{(A14)}$$

Using (A14), an upper bound on the expected (long-run) per-product joint payoff for a given value of $w\ge\frac{1}{\delta}c^u$ is given by

$$\tfrac{1}{1-\delta}(v-c^u-c^d-\tfrac{1-\alpha}{\delta n(\alpha-\beta)}(w+c^d)). \qquad \text{(A15)}$$

Since (A15) is decreasing in w, it follows that the upstream firm's incentive compatibility constraint must bind at optimum, $u^T_j = w = \frac{1}{\delta}c^u$, $j\in \mathrm{U}$. Finally, (A3) is satisfied if

$$u^T_D(0) = v-\tfrac{1}{\delta(\alpha-\beta)(1-\alpha)}\tfrac{1}{2}(\tfrac{1}{\delta}c^u+c^d)-\tfrac{1}{\delta}c^u-c^d+\tfrac{\delta}{1-\delta}(v-\tfrac{1-\alpha}{\delta(\alpha-\beta)}\tfrac{1}{n}(\tfrac{1}{\delta}c^u+c^d)-\tfrac{1}{\delta}c^u-c^d)\ge 0.$$

Next we observe that the strategy profile in (3) – (8) where the expected input payment and product-pricing scheme are given by $w^T=\frac{1}{\delta}c^u$ and $p^T(y)=r^T_D(y)=u^T_D(y)+\frac{1}{\delta}c^u+c^d$

$-\delta\sum_{y'\in\{0,\frac{1}{2},...,1\}} u_D^T(y') f(y'|1,...,1) = v - \Delta_0^T 1_{y=0} \ \geq 0$, respectively, constitutes a PBE because, as we showed, all of the upstream and downstream firms' participation and incentive compatibility constraints are satisfied, where $\Delta_0^T = \frac{c^d + c^u/\delta}{\delta n(\alpha-\beta)(1-\alpha)^{n-1}}$. The output prices are non-negative if

$$p^T(0) = v - \frac{1}{\delta(\alpha-\beta)(1-\alpha)^{n-1}} \frac{1}{n}(\frac{1}{\delta}x + 1)c^d \geq 0, \text{ or } n(\alpha-\beta)(1-\alpha)^{n-1} v\delta^2 - \delta c^d - c^u \geq 0.$$

Let $\delta^T \equiv \frac{c^d + \sqrt{(c^d)^2 + 8(\alpha-\beta)(1-\alpha)^{n-1} v c^u}}{4(\alpha-\beta)(1-\alpha)^{n-1} v} < 1$ denote the largest root of this quadratic equation assuming that condition $v > \frac{c^d + c^u}{n(\alpha-\beta)(1-\alpha)^{n-1}}$ holds. Note that from (A5‘) it follows that (A3) is satisfied with a slack for all $\delta \in [\delta^T, 1)$. Furthermore, this is the best equilibrium since all firms exert efforts in every period and the upper bound on the joint per product value of the supply chain (A15) is achieved. ■

Proof of Proposition 2 In the no-traceability regime, an upper bound on the expected per-product joint profit can be found as

$$\max_{w, u_A^N, \{u_D^N(y), u^N(y)\}_{y=0,0.5,1}} \alpha^2 u^N(1) + 2\alpha(1-\alpha)u^N(0.5) + (1-\alpha)^2 u^N(0),$$

subject to (A1), (A2), (A3), (A4), (A6), and (A7). As in the traceability regime, at optimum the feasibility constraint (A1) must bind, and the participation constraints in (A3) must hold with slack. Using (A2), the DIC constraint in (A6) can be written more explicitly as

$$\begin{aligned}
&p - w - c^d + \delta(\alpha^2 u_D^N(1) + 2\alpha(1-\alpha)u_D^N(0.5) + (1-\alpha)^2 u_D^N(0)) \\
\geq \max[\ &p - w - \tfrac{1}{2}c^d + \tfrac{\delta}{1-\delta}(\alpha\beta u_D^N(1) + (\alpha(1-\beta) + (1-\alpha)\beta)u_D^N(0.5) + (1-\alpha)(1-\beta)u_D^N(0) + \tfrac{1}{2}c^d), \\
&p - \tfrac{1}{2}w - c^d + \tfrac{\delta}{1-\delta}(\alpha\beta u_D^N(1) + (\alpha(1-\beta) + (1-\alpha)\beta)u_D^N(0.5) + (1-\alpha)(1-\beta)u_D^N(0) + \tfrac{1}{2}w), \qquad \text{(A16)} \\
&p - \tfrac{1}{2}w - \tfrac{1}{2}c^d + \tfrac{\delta}{1-\delta}\tfrac{1}{2}(\beta(\alpha+\beta)u_D^N(1) + ((\alpha+2\beta)(1-\beta) + (1-\alpha)\beta)u_D^N(0.5) + (1-\beta)(2-\alpha-\beta)u_D^N(0) + w + c^d), \\
&p + \tfrac{\delta}{1-\delta}(\beta^2 u_D^N(1) + 2\beta(1-\beta)u_D^N(0.5) + (1-\beta)^2 u_D^N(0) + w + c^d)] \\
&- \tfrac{\delta^2}{1-\delta}(\alpha^2 u(1) + 2\alpha(1-\alpha)u_D^N(0.5) + (1-\alpha)^2 u_D^N(0)).
\end{aligned}$$

Note that in the no-traceability regime, the downstream firm can no longer vertically coordinate upstream and downstream effort/no-effort choices. It is possible that the downstream firm's most profitable deviation is to renege on the payment for upstream effort for one of the firms (or shirk on the downstream effort for one of the products) but continue to put downstream efforts into both products (respectively, offer the expected payments to both upstream firms). Upon manipulation of terms in (A16) and (A4) the downstream and upstream firms' no-shirking and no-reneging constraints reduce to the following system of inequalities

$$\max[c^d, w] \leq 2\delta(\alpha-\beta)(\alpha u_D^N(1) + (1-2\alpha)u_D^N(0.5) - (1-\alpha)u_D^N(0)), \qquad \text{(A17a)}$$

$$w + c^d \leq 2\delta((\alpha(\alpha - \tfrac{1}{2}\beta) - \tfrac{1}{2}\beta^2)u_D^N(1) + ((1-\alpha)(2\alpha - \tfrac{1}{2}\beta) - \tfrac{1}{2}(\alpha+2\beta)(1-\beta))u_D^N(\tfrac{1}{2}) + ((1-\alpha)^2 - \tfrac{1}{2}(1-\beta)(2-\alpha-\beta))u_D^N(0)), \qquad \text{(A17b)}$$

$$w + c^d \leq \delta(\alpha-\beta)((\alpha+\beta)u_D^N(1) + 2(1-\alpha-\beta)u_D^N(0.5) - (2-\alpha-\beta)u_D^N(0)), \qquad \text{(A17c)}$$

$$w \geq \tfrac{1}{\delta}c^u. \qquad \text{(A17d)}$$

There are two cases to consider. Suppose that at optimum the GPIC constraint in (A7) does not bind. Then from conditions (A17a), (A17b), and (A17c) it follows that the optimal solution is

$$u_D^N(1) = u_D^N(0.5) = V^N - u_A^N,\ u_D^N(0) = V^N - u_A^N - \tfrac{1}{2\delta(\alpha-\beta)}\max[\tfrac{\max[w,c^d]}{1-\alpha}, \tfrac{w+c^d}{1-\frac{1}{2}(\alpha+\beta)}] \qquad \text{(A18)}$$

because the constraint (A15b) is necessarily slack (the downstream never engages in a vertically uncoordinated upstream and downstream shirking). From (A18) it follows that the inequality in (A7) is satisfied if $(1-\beta)\max[\tfrac{1}{2(1-\alpha)}\max[\tfrac{w}{c^d},1], \tfrac{1}{2-\alpha-\beta}(\tfrac{w}{c^d}+1)] \le Y(\tfrac{w}{c^d})$, or

$$\frac{2Y(\frac{w}{c^d})}{\max[\frac{w}{c^d},1]} \ge \frac{1-\beta}{1-\alpha}. \qquad \text{(A19)}$$

Using (A1), (A2), and (A16) to solve for V^N, we find that for a given value of $w \ge \tfrac{1}{\delta}c^u$, an upper bound on the achievable expected per-product joint profit is given by

$$\tfrac{1}{1-\delta}(v - c^u - c^d - \tfrac{(1-\alpha)^2}{2\delta(\alpha-\beta)}\max[\tfrac{\max[w,c^d]}{1-\alpha}, \tfrac{w+c^d}{1-\frac{1}{2}(\alpha+\beta)}]). \qquad \text{(A20)}$$

Because the expected joint value of the firms in the supply chain in (A20) is decreasing in or independent of w depending on whether $w \ge (<) \min[\tfrac{1}{2}\tfrac{\alpha-\beta}{1-\alpha},1]c^d$, an optimal expected payment is given by $w \in [\tfrac{1}{\delta}c^u, \max[\tfrac{1}{\delta}c^u, \min[\tfrac{1}{2}\tfrac{\alpha-\beta}{1-\alpha},1]c^d]]$ such that (A19) holds. The assumption that the GPIC constraint does not bind at optimum is satisfied if

$$\frac{2Y(\frac{c^u}{\delta c^d})}{\max[\frac{c^u}{\delta c^d},1]} \ge \frac{1-\beta}{1-\alpha}. \qquad \text{(A21)}$$

Therefore, the expected joint per product profit cannot exceed

$$\tfrac{1}{1-\delta}(v - c^u - c^d - \tfrac{(1-\alpha)^2}{2\delta(\alpha-\beta)}\max[\tfrac{1}{1-\alpha}\max[\tfrac{1}{\delta}c^u, c^d], \tfrac{1}{1-\frac{1}{2}(\alpha+\beta)}(\tfrac{1}{\delta}c^u + c^d)])$$

if condition (A21) holds.

Now suppose that

$$\frac{2Y(\frac{c^u}{\delta c^d})}{\max[\frac{c^u}{\delta c^d},1]} < \frac{1-\beta}{1-\alpha}, \qquad \text{(A22)}$$

and the GPIC constraint binds at optimum. Then we have

$$u_D^N(1) = V^N - u_A^N \text{ and } u_D^N(0) = \tfrac{1}{1-\beta}(\beta(V^N - u_A^N) + (1-2\beta)u_D^N(0.5) - \tfrac{1}{\delta(\alpha-\beta)}c^d Y(\tfrac{w}{c^d})). \qquad \text{(A23)}$$

On substitution of (A23) into the downstream firm's incentive compatibility constraints in (A17a), (A17b), and (A17c), it is easy to verify that the largest value of $u_D^N(0.5)$ that satisfies all the inequalities is given by $u_D^N(0.5) = V^N - u_A^N + \tfrac{1}{\delta(\alpha-\beta)^2}((1-\alpha)c^d Y(\tfrac{w}{c^d}) - \tfrac{1}{2}(1-\beta)\max[c^d, w])$, so that an upper bound on the expected joint profit for a given value of $w \ge \tfrac{1}{\delta}c^u$ is given by

$$\tfrac{1}{1-\delta}(v - c^u - c^d + \tfrac{1-\alpha}{\delta(\alpha-\beta)^2}((1-\alpha)Y(\tfrac{w}{c^d}) - \tfrac{1}{2}(1+\alpha-2\beta)\max[\tfrac{w}{c^d},1])c^d. \qquad \text{(A24)}$$

Suppose that $w > c^d$. Then the expected joint per product profit in (A24) is decreasing in w because $(1-\alpha)\ Y(\tfrac{w}{c^d}) - \tfrac{1}{2}(1+\alpha-2\beta)\tfrac{w}{c^d}$ is decreasing in w when condition (A22) holds (the function Y

is piecewise differentiable). Therefore, it must be that at optimum, either $w \le c^d$ if $\frac{1}{\delta} c^u < c^d$ or $w = \frac{1}{\delta} c^u$ if $\frac{1}{\delta} c^u \ge c^d$. If $\frac{c^u}{c^d} \ge \delta$, the upstream firm's no-shirking constraint binds at optimum, and the optimal expected payment to each upstream firm and firm's value are given by $u_A^N = u_B^N = w = \frac{1}{\delta} c^u$. When the ratio of the costs of upstream and downstream efforts is greater than the discount factor, increasing the expected payment to upstream firms cannot reduce the overall incentive costs for the downstream firm.

The remaining possibility that needs to be considered is that at optimum, $\frac{1}{\delta} c^u < w \le c^d$. Then the expected joint per product profit in (A24) is increasing in w because the function Y is increasing in w. Therefore, the optimal expected payment to upstream firms is $w = c^d$. Summarizing, the expected joint profit cannot exceed

$$\tfrac{1}{1-\delta}(v - c^u - c^d + \tfrac{1-\alpha}{\delta(\alpha-\beta)^2}((1-\alpha)Y(\max[\tfrac{c^u}{\delta c^d},1]) - \tfrac{1}{2}(1+\alpha-2\beta)\max[\tfrac{c^u}{\delta c^d},1])c^d \qquad \text{(A25)}$$

if condition (A22) holds. Also, note that the downstream firm's participation constraint in (A3) is satisfied if $u_D^N(0) \ge 0$.

The strategy profile in (3) – (6), (9), and (10), where the expected input payment and product-pricing scheme are given by $w^N = w$ and $p^N(y) = r_D^N(y) = u_D^N(y) + w + c^d - \delta \sum_{y' \in \{0,\frac{1}{2},1\}} u_D^N(y') f(y|1,1) \ge 0$, respectively, constitutes a PBE because, as we showed, all of the upstream and downstream firms' participation and incentive compatibility constraints are satisfied. The analytical expression for the output price is given by (24). The output price is non-negative for sufficiently large discount factors, $\delta \in [\delta^N, 1)$, if $v > \frac{1}{\alpha-\beta} \max[\frac{1}{2(1-\alpha)} \max[x,1], \frac{1+x}{2-\alpha-\beta}, \max[x,1] - \frac{1-2\alpha}{\alpha-\beta}]c^d$. As in the case of traceability regime, from (A16) it follows that (A3) holds with a slack for all $\delta \in [\delta^N, 1)$. Furthermore, this is the best equilibrium since all firms exert efforts in every period and the upper bound on the expected joint per product value of the supply chain in (A25) is achieved. ■

Proof of Proposition 3 From Propositions 1 and 2, it follows that the difference between the equilibrium expected joint profits in the traceability and no-traceability regime is given by

$$\pi^T - \pi^N = \begin{cases} -\frac{1}{1-\delta}\frac{1-\alpha}{2\delta(\alpha-\beta)} c^d \min[\min[\frac{1}{\delta}x,1], \frac{\alpha-\beta}{2-\alpha-\beta}(\frac{1}{\delta}x+1)], \text{if } l(x) \ge \frac{1-\beta}{1-\alpha} \\ -\frac{1}{1-\delta}\frac{1-\alpha}{2\delta(\alpha-\beta)^2} c^d (2(1-\alpha)Y(\max[\frac{1}{\delta}x,1]) + (\alpha-\beta)\min[\frac{1}{\delta}x,1] - (1-\beta)\max[\frac{1}{\delta}x,1]), \text{if } l(x) < \frac{1-\beta}{1-\alpha} \end{cases}.$$

We have $\pi^T - \pi^N \le (>) 0$ depending on whether

$$\alpha \le (>) \frac{2Y(\max[\frac{1}{\delta}x,1]) - \max[\frac{1}{\delta}x,1] + |\frac{1}{\delta}x - 1|\beta}{2Y(\max[\frac{1}{\delta}x,1]) - \min[\frac{1}{\delta}x,1]}.$$

For $\beta = 1-\alpha$, this condition becomes $\alpha \le (>) R(x,\delta)$. The function $R(x,\delta)$ is unimodal in $x > 0$ because it is increasing for $x < \delta$, decreasing for $x > \delta$, reaches its maximum value of 1 at $x = \delta$, and

$$\lim_{x\to 0} R(x,\delta) = \frac{2\min[2,3(1-\frac{1}{2}\delta),\frac{4-\delta}{1+\delta}]}{2\min[2,3(1-\frac{1}{2}\delta),\frac{4-\delta}{1+\delta}]+1} > \frac{2-\delta}{3-\delta} = \lim_{x\to\infty} R(x,\delta).$$ ■

Proof of Proposition 5 Upon substitution, the difference in the equilibrium joint payoffs is given by

$$\pi^T - \pi^N = \begin{cases} -\frac{1}{1-\delta}\frac{1-\alpha}{2\delta\alpha\lambda} c^d \min[\min[\frac{1}{\delta}x(\lambda),1], \frac{\alpha\lambda}{2(1-\alpha)+\alpha\lambda}(\frac{1}{\delta}x(\lambda)+1)], \text{if } l(x(\lambda)) \geq \frac{1-\alpha(1-\lambda)}{1-\alpha} \\ -\frac{1}{1-\delta}\frac{1-\alpha}{2\delta(\alpha\lambda)^2} c^d (2(1-\alpha)Y(\max[\frac{1}{\delta}x(\lambda),1]) + \alpha\lambda \min[\frac{1}{\delta}x(\lambda),1] \\ \qquad -(1-\alpha(1-\lambda))\max[\frac{1}{\delta}x(\lambda),1]), \text{if } l(x(\lambda)) < \frac{1-\alpha(1-\lambda)}{1-\alpha} \end{cases}.$$

Ignoring for the moment the participation constraints and taking the limit yields

$$\lim_{\alpha\to 1}\frac{\pi^T-\pi^N}{1-\alpha} = \frac{1}{1-\delta}\frac{1}{2\delta}\frac{1}{\lambda}c^d \,|\frac{1-\delta}{\delta\lambda}\frac{c^u}{c^d} + \frac{c^u}{c^d} - 1| \text{ and} \tag{A26}$$

$$\lim_{\alpha\to 0}\pi^T - \pi^N = -\frac{1}{1-\delta}\frac{1}{4\delta}c^d(\frac{1-(1-\lambda)\delta}{\delta\lambda}\frac{c^u}{c^d}+1). \tag{A27}$$

For sufficiently large δ, the right-hand side of (A26) is decreasing in λ and the right-hand side of (A27) is increasing in λ. Proposition 5 follows by continuity. ■

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