

האוניברסיטה העברית בירושלים
The Hebrew University of Jerusalem



המרכז למחקר בכלכלה חקלאית
The Center for Agricultural
Economic Research

המחלקה לכלכלה חקלאית ומנהל
The Department of Agricultural
Economics and Management

Discussion Paper No. 4.02

Growth, Scarcity and R&D

by

Yacov Tsur and Amos Zemel

Papers by members of the Department
can be found in their home sites:

מאמרים של חברי המחלקה נמצאים
גם באתרי הבית שלהם:

<http://departments.agri.huji.ac.il/economics/indexe.html>

P.O. Box 12, Rehovot 76100

ת.ד. 12, רחובות 76100

Growth, Scarcity and R&D

Yacov Tsur¹ and Amos Zemel²

June 2002

Abstract: The limits imposed on economic growth by resource scarcity can be alleviated only by the development of backstop substitutes, hence the intricate relationship between growth, scarcity and R&D in backstop technologies. This work attempts to unfold this intricacy by combining growth based on natural and backstop resources with R&D activities that gradually reduce the backstop cost. We classify prototypical economies in terms of their production technology, learning ability and time preferences. Depending on the economy's type and capital endowment, we find a wide variety of optimal R&D and capital formation processes, ranging from cases where R&D diverts an economy that otherwise would consume its entire capital onto a path of sustained growth, to cases where R&D is unwarranted.

JEL Classification: C61, D92, O13, O32, Q32

Keywords: Growth, Nonrenewable Resources, Backstop Technologies, R&D, Scarcity

¹ Corresponding author: Department of Agricultural Economics and Management, The Hebrew University of Jerusalem, P.O. Box 12, Rehovot, 76100, Israel, tsur@agri.huji.ac.il (Tel +972-8-9489372, Fax +972-8-9466267).

² The Jacob Blaustein Institute for Desert Research and Department of Industrial Engineering and Management, Ben Gurion University of the Negev, Sede Boker Campus, 84990, Israel (amos@bgumail.bgu.ac.il). Zemel gratefully acknowledges financial support from the Paul Ivanier Center of Robotics and Production Management at Ben-Gurion University of the Negev.

1. Introduction

The limits to economic growth imposed by the finite Earth resources have been a matter of concern since Malthus' day. While at the pre-industrial era the chocking forces were thought to emanate from shortage of land and food, post-industrial concerns revolve around depletion of natural resources and degradation of environmental quality. We all hope that what got us off the hook of Malthusian starvation, namely, technological progress, will allow a sustained, long-run growth in spite of increased scarcity and pollution. Translating such hopes into practical policies has attracted considerable research efforts and gave rise to a large body of literature that deals with economic growth, resource scarcity and R&D. The present study follows this line of research.

Our analysis derives from three main strands of literature. The first is concerned with economic growth, pioneered by Ramsey (1928) and by Solow (1956, 1957) who emphasized the role of technical change, and extended to include learning by doing (Arrow [1962]), independent (albeit exogenous) R&D sector (Shell [1966]), and endogenous learning and R&D (Romer [1986, 1990], Grossman and Helpman [1993] and others).

Second, we build on the literature on intertemporal exploitation of natural resources, initiated by the seminal work of Hotelling (1931), calling attention to effects of resource scarcity (see the recent survey by Krautkraemer [1998]). Of particular relevance in the present context are the works of Burt and Cummings (1970) and Rausser (1974) that stressed the role of natural resources as input of production and incorporated technical change in the resource extraction industry, either as an exogenous process (Burt and Cummings) or in the form of learning by doing (Rausser).

Finally, we derive from the vast literature on R&D investments in backstop technologies that will substitute a finite stock of primary resource when the latter runs out or becomes too expensive to use. Examples include the works of Dasgupta and Heal (1974), Solow (1974), Stiglitz (1974), Dasgupta and Stiglitz (1981) and Hung and Quyen (1993). Of particular relevance are the works of Dasgupta, Heal and Majumdar (1977) and of Kamien and Schwartz (1978) who studied policies that combine R&D in backstop with economic growth.

A characteristic feature of these backstop-R&D models is that the backstop technology arrival is a discrete event whose time of occurrence (which may be uncertain) is affected by the R&D policy. Tsur and Zemel (2000, 2002) depart from this characteristic aspect by considering a continuous improvement of an existing backstop technology, realized through R&D efforts that accumulate in the form of knowledge to gradually reduce the cost of the backstop technology. While a continuous mode of technical change underlies most of the growth literature, it is rarely assumed in the resource literature. Yet, empirical analyses provide ample evidence for the gradual nature of the development of many substitutes for natural resources (see, e.g., Chakravorty et al. [1997] for the gradual reduction in the cost of renewable substitutes for fossil energy).

In this work we incorporate Tsur and Zemel's R&D-cum-scarcity structure within a growth model, thereby linking together economic growth, resource scarcity and R&D in backstop technologies. The resulting model contains the same basic components as that of Kamien and Schwartz (1978). However, while Kamien and Schwartz (1978) assume that technical change (backstop development) appears in the form of (uncertain) discrete breakthroughs, we maintain a continuous backstop development process. Our approach, thus, follows that of the endogenous growth

framework, but its prime concern is the development of substitutes for finite resources.

The recent work of Tahvonen and Salo (2001) also includes growth, primary-backstop resources competition and technical change processes. Their technical change process differs from ours in that they model it as the result of learning by doing that depends on the use of the primary resource and on the capital stock. In our framework, technical change is induced by R&D activities devoted specifically for that purpose and the temporal allocation of these activities is our main concern. In comparison to the growth literature, Tahvonen and Salo's model is on the line of Arrow's (1962) learning by doing model, whereas our model has the structure of Shell's (1966) model but with endogenous R&D sector. Other models incorporating both growth and environmental R&D (e.g. Bovenberg and Smulders, 1996) are somewhat less related to the present study because they are set in the context of environmental pollution rather than of resource scarcity.

The structure of the paper is as follows. Section 2 sets up the basic model and provides necessary conditions for optimum. In Section 3 we revisit the standard growth model, neglecting resource scarcity and R&D. The results are used as a reference for the more complex scenarios. Sections 4 and 5 incorporate, in turn, R&D and scarcity. Various types of economies are identified and classified in terms of their response to R&D, production technology and time preferences. For each type, the long run equilibrium and the dynamic path leading to it are characterized. We find that R&D may turn a decaying economy (one in which capital diminishes in the long run) into a converging (capital approaches a positive and finite steady state) or even a diverging (sustained growth) economy. The effects of resource scarcity also

differ markedly between the various types of economies. Some concluding comments are offered in Section 6.

2. Model Formulation

The economy produces a single composite good, using capital (K) and resource (energy, say, denoted by x) as inputs (to focus attention on resource scarcity and R&D we assume that labor is constant and suppress it as an argument). The production function $F(K,x)$ assumes standard properties: $F(0,x) = F(K,0) = 0$, and for positive K and x , $F_1 \equiv \partial F/\partial K > 0$, $F_2 \equiv \partial F/\partial x > 0$, $F_{11} \equiv \partial^2 F/\partial K^2 < 0$, $F_{22} \equiv \partial^2 F/\partial x^2 < 0$, $F_{12} \equiv \partial^2 F/\partial K \partial x > 0$ and $J \equiv F_{11}F_{22} - F_{12}^2 > 0$.

The state of the economy at each point of time depends on a capital stock, K , a natural (primary) resource stock, Q , and a stock of knowledge, N , affecting the unit cost of backstop technology. The resource input can be derived from two sources: a nonrenewable (primary) resource stock, or a (practically unlimited) backstop technology. Thus, $x = q+b$, where q and b represent the primary and backstop input rates, respectively. The costs of supplying these inputs are $M(q)$ and $B(N)b$, where $B(N)$ is the unit cost of backstop supply. The cost of supplying the input mix $x = q+b$ with knowledge N is thus $M(q)+B(N)b$. We assume that $M(q)$ is increasing and strictly convex and the unit backstop cost function $B(N)$ satisfies $B'(N) < 0$ and $B''(N) > 0$ for $0 \leq N < \bar{N}$, $B'(\bar{N}) = 0$ and $B(N) = B(\bar{N})$ for $N \geq \bar{N}$. Thus, there exists some minimal backstop unit cost that cannot be further reduced even by increasing knowledge beyond the level \bar{N} .

The composite good produced at any instant of time is used for consumption, c , supply of the primary resource input, $M(q)$, supply of the backstop input, $B(N)b$, and

R&D expenditures, R . The residual is investment (or disinvestment) in physical capital:

$$\dot{K}_t \equiv dK_t / dt = F(K_t, x_t) - c_t - M(q_t) - B(N_t)b_t - R_t. \quad (2.1)$$

The stocks of knowledge and of the natural resource evolve in time according to

$$\dot{N}_t = R_t \quad (2.2)$$

and

$$\dot{Q}_t = -q_t. \quad (2.3)$$

Utility is derived from consuming according to an increasing and strictly concave instantaneous utility function $u(c)$ with $\lim_{c \rightarrow 0} \{u'(c)\} = \infty$. The optimal policy consists of consumption (c), resource inputs (q, b) and R&D (R) trajectories according to

$$V(K_0, N_0, Q_0) = \text{Max}_{\{c, q, b, R\}} \left\{ \int_0^{\infty} u(c_t) e^{-rt} dt \right\} \quad (2.4)$$

subject to (2.1), (2.2), (2.3), K_0 and Q_0 given, $N_0 = 0$, $K_t \geq 0$, $N_t \geq 0$, $Q_t \geq 0$, $c_t \geq 0$, $q_t \geq 0$, $b_t \geq 0$ and $0 \leq R_t \leq \bar{R}$, where r is the time rate of discount and \bar{R} is an exogenous upper bound on the rate of R&D expenditures.

Requiring R to be nonnegative implies irreversible R&D in the sense that it is impossible to transform knowledge back into capital and use it for consumption or any other purpose. The assumptions on the unit backstop cost $B(N)$ imply that it can never be optimal to extend the state of knowledge beyond \bar{N} , hence attention will be confined to the $[0, \bar{N}]$ interval.

With λ , γ and μ representing the current-value costate variables of K , N and Q , respectively, the current-value Hamiltonian associated with (2.4) is

$$H_t = u(c_t) + \lambda_t [F(K_t, x_t) - c_t - M(q_t) - B(N_t)b_t - R_t] + \gamma_t R_t - \mu_t q_t \quad (2.5)$$

and the necessary conditions for interior optimum c^* , q^* and b^* are, respectively,

$$u'(c_t^*) = \lambda_t, \quad (2.6)$$

$$F_2(K_t, x_t^*) = M'(q_t^*) + \mu_t / \lambda_t, \quad (2.7)$$

and

$$F_2(K_t, x_t^*) = B(N_t). \quad (2.8)$$

To identify corner solutions, let $q^\#$ denote the q -level at which $F_2(K, \cdot)$ and $M'(\cdot) + \mu/\lambda$ intersect (i.e. $F_2(K, q^\#) = M'(q^\#) + \mu/\lambda$). The primary and backstop resources are used simultaneously when $M'(0) + \mu/\lambda < B(N) < M'(q^\#) + \mu/\lambda$. Otherwise, $q^* = 0$ when $B(N) \leq M'(0) + \mu/\lambda$ and $b^* = 0$ when $B(N) \geq M'(q^\#) + \mu/\lambda$ (see Figure 2.1).

Figure 2.1

Maximizing the Hamiltonian with respect to R gives

$$R_t^* = \begin{cases} \bar{R} & \text{if } \lambda_t < \gamma_t \\ 0 & \text{if } \lambda_t > \gamma_t \\ \text{singular} & \text{if } \lambda_t = \gamma_t \end{cases} \quad (2.9)$$

(the singular solution is characterized below), while λ , γ and μ evolve according to

$$\dot{\lambda}_t - r\lambda_t = -\lambda_t F_1(K_t, x_t), \quad (2.10)$$

$$\dot{\gamma}_t - r\gamma_t = -\lambda_t [-B'(N_t)b_t] \quad (2.11)$$

and

$$\dot{\mu}_t - r\mu_t = 0. \quad (2.12)$$

The transversality conditions associated with the nonnegativity of K , N and Q are, respectively,

$$\lim_{t \rightarrow \infty} \{K_t \lambda_t e^{-rt}\} = 0, \quad (2.13a)$$

$$\lim_{t \rightarrow \infty} \{N_t \gamma_t e^{-rt}\} = 0 \quad (2.13b)$$

and

$$\lim_{t \rightarrow \infty} \{Q_t \mu_t e^{-rt}\} = 0. \quad (2.13c)$$

We turn now to characterize the optimal policy, starting with the reference case of growth without scarcity and R&D (Section 3) and later on incorporating R&D (Section 4) and scarcity (Section 5).

3. Abundant primary resources and no R&D

Without R&D and with abundant primary resource, the model specializes to a standard growth model, to be used as a reference. It differs from the neoclassical growth model only in that resource input substitutes labor input, hence we briefly summarize its properties. Equations (2.7)-(2.8) reduce to $F_2(K, x(K)) = M'(q(K))$ and $F_2(K, x(K)) = B$, where B is the fixed unit cost of backstop supply. So long as $B < M'(q^\#)$ the demand for the primary resource is fixed at the level (see Figure 2.1)

$$q^B = M'^{-1}(B). \quad (3.1)$$

As the stock of capital increases, the curve depicting $F_2(K, \cdot)$ in Figure 2.1 shifts upward. The particular K level under which $F_2(K, \cdot)$ and $M'(\cdot)$ intersect at the cost level B is denoted K^B , i.e.,

$$F_2(K^B, q^B) = B. \quad (3.2)$$

K^B is the minimal capital level above which the backstop resource will be used: $b_t > 0$ and $q_t = q^B$ whenever $K_t > K^B$ (see Figure 3.1).

Figure 3.1

Denoting

$$\rho(K) = F_1(K, x(K)). \quad (3.3)$$

we find

$$\rho'(K) = \begin{cases} (J - F_{11}M'')/(F_{22} - M'') & \text{if } K < K^B \\ J / F_{22} & \text{if } K > K^B \end{cases} \quad (3.4)$$

where it is recalled that F_1 is the derivative of F with respect to its first argument and J is the determinant of the Hessian matrix of F . The assumptions regarding F and M ($J > 0$ and $M'' > 0$) ensure that $\rho(K)$ decreases and the equation $\rho(K) = r$ has at most one root:

$$\hat{K} = \begin{cases} 0 & \text{if } r \geq \rho(0) \\ \rho^{-1}(r) & \text{if } \rho(0) > r \geq \rho(\infty) \\ \infty & \text{if } r < \rho(\infty) \end{cases} \quad (3.5)$$

We obtain the following characterization for optimal capital formation:

- (i) If $\hat{K} < \infty$ then \hat{K} is an attractive steady state to which the optimal capital process K_t^* converges monotonically from any positive initial state.
- (ii) If $\hat{K} = \infty$ the economy grows permanently.

Notice from (3.5) that growth can be sustained in the long run ($\hat{K} = \infty$) only if $\rho(\infty) > r$ (i.e., the value of marginal product of capital always exceeds the discount rate). Notice also that the economy will decay to $\hat{K} = 0$ (capital diminishes in the long run) when $r > \rho(0)$ (i.e., the discount rate exceeds the maximal value of marginal product of capital). The latter situation does not occur when the Innada condition ($F_1 \rightarrow \infty$ as $K \rightarrow 0$) hold. These results are in accordance with the well-known properties of the neoclassical growth model.

We verify the characterization formulated in (i) and (ii) above using the $h\delta$ -method of Tsur and Zemel (2001). According to this method one examines whether any given state K can qualify as an optimal steady state by comparing the value $W(K)$ associated with the steady state policy

$c(K) = F(K, x(K)) - M(q(K)) - Bb(K)$ with the value $V^{h\delta}(K)$ obtained with the

small-variation policy $c_t^{h\delta} = c(K) - \delta$ for $t \leq h$ and $c_t^{h\delta} = c(K_h)$ for $t > h$, where $h > 0$

and δ are arbitrary small numbers. Applying the method, we obtain

$V^{h\delta}(K) - W(K) = L(K)h\delta / r + o(h\delta)$, where the function $L(K)$ is given by

$L(K) = u'(c(K))(\rho(K) - r)$ and the higher order term $o(h\delta)$ satisfies $o(\varepsilon)/\varepsilon \rightarrow 0$ as

$\varepsilon \rightarrow 0$. For any K at which $L(K)$ does not vanish, we can assign its sign to δ , so that

$V^{h\delta}(K) - W(K) > 0$, implying that the small-variation policy yields a higher value

than the steady state policy and K does not qualify as an optimal steady state. It

follows that any optimal steady state must be a root of $L(K)$. The only possible

exception is the corner state $K = 0$ which can be a steady state when $L(0) \neq 0$, since

only $\delta = 0$ is feasible at this state. Now, $u'(c) > 0$ hence the root of $L(K)$ defines the

state \hat{K} of (3.5). When $L(0) > 0$ the corner state $K = 0$ cannot be a steady state

(unless $K_0 = 0$ which leaves no other feasible policy), because $\rho(K) > r$ near the corner

state entails $\dot{\lambda} < 0$ (see 2.10) while (2.6) and $u''(c) < 0$ imply that the consumption

rate c increases permanently, which is inconsistent with a decaying economy. Similar

considerations forbid the capital process to grow indefinitely when the root \hat{K} is

finite and the characterization follows from the observation that the optimal state

process of this autonomous problem must evolve monotonically in time.

Having characterized the optimal capital process, the control variables c , b and

q are uniquely determined. When $K < \hat{K}$ capital increases. With $\rho'(K) < 0$ we find

that $\rho(K) > r$ hence the consumption rate c increases together with K . Furthermore,

when $K < K^B$ we have $b = 0$ and $F_2(K, q) = M'(q)$. Thus, $\dot{q} = [F_{21} / (M'' - F_{22})] \dot{K}$, hence

q and K evolve in the same direction over time. Similarly, when $K > K^B$, the primary

resource input use is constant at q^B and $F_2(K, q^B + b) = B$. Thus, $\dot{b} = -(F_{21} / F_{22}) \dot{K}$ and

b evolves over time in the same direction as K . Thus, all the economic indicators (capital stock, consumption and resource use) increase or decrease together towards their long run levels.

4. R&D

In this section we maintain the assumption of abundant primary resource stock and study the effects of R&D. We characterize the optimal learning and capital accumulation policies, relegating proofs and technical derivations to the appendix.

The R&D efforts contribute to the accumulation of a knowledge base that, in turn, reduces the cost of the backstop technology. With unlimited stock of the primary resource, μ vanishes identically at all times and Equations (2.7)-(2.8) specialize to $F_2(K,x(K,N)) = M'(q(K,N))$ and $F_2(K,x(K,N)) = B(N)$ to determine $q(K,N)$ and $b(K,N)$, as illustrated in Figure 4.1. It is seen that increasing the knowledge stock N decreases the primary resource input q and increases both the backstop input b and the total resource input x .

Figure 4.1

Let

$$\rho(K,N) = F_1(K,x(K,N)) \quad (4.1)$$

and rewrite Equation (2.10) as

$$\dot{\lambda}_t = \lambda_t [r - \rho(K_t, N_t)]. \quad (4.2)$$

The characterization of the optimal policy is most conveniently carried out in terms of two lines defined in the state ($N-K$) space. First, note that implementing the singular solution introduced in Equation (2.9) (with $0 < R < \bar{R}$) during a finite period of time implies that $\lambda = \gamma$ and $\dot{\lambda} = \dot{\gamma}$ during that period. Thus, Equations (2.10)-(2.11) reduce to

$$\rho(K_t, N_t) = -B'(N_t)b(K_t, N_t) \quad (4.3)$$

along the singular path. Equation (4.3) admits at most one positive value of K for every N and defines a line in the N - K plane, which we call the *singular* line and denote by $K^S(N)$. Since $\rho(K, \bar{N}) > 0$, $\partial b/\partial K \geq 0$ and $B'(\bar{N}) = 0$, the singular line diverges at \bar{N} , i.e. $K^S(\bar{N}) = \infty$. Thus, the singular line must increase over some N intervals. Although far from necessary, it simplifies the analysis to maintain that the singular line is nondecreasing and we therefore assume that $K^S(N) \geq 0$ for all $N < \bar{N}$.

An additional line in the N - K plane is defined by $\dot{\lambda} = 0$, or in view of Equation (4.2), by

$$\rho(K, N) = r. \quad (4.4)$$

This line gives the unique optimal steady state of K for any given (fixed) knowledge level N (see Section 3); it is denoted $\hat{K}(N)$ and is referred to as the K -line. Using Equations (4.1) and (2.7) or (2.8) we find that $J\hat{K}'(N) = -B'(N)F_{12}$ or $\hat{K}'(N) = 0$ when $b(\hat{K}(N), N) > 0$ or $b(\hat{K}(N), N) = 0$, respectively. It follows that the K -line is nondecreasing over $[0, \bar{N}]$. The identification of $\hat{K}(N)$ as the steady state for fixed N carries over to the present context in that any optimal steady state must fall on the K -line (see appendix).

Condition (2.9) identifies three possible R&D regimes: no R&D ($R = 0$), denoted *o*-regime; maximal R&D ($R = \bar{R}$), denoted *m*-regime; and singular R&D ($0 \leq R \leq \bar{R}$), denoted *s*-regime. The optimal policy consists of selecting between these three regimes at different phases of the planning horizon. We begin by formulating the optimal policy corresponding to each regime, which, given the

restriction on R&D, is obtained as outcome of a single-state problem. Then show how to form the optimal policy from these three regimes.

The s-regime: In this regime the (K,N) process is restricted to lie on the singular line $K = K^S(N)$, defined by Equation (4.3). This regime, it turns out, is trapping in that it cannot be optimal to leave it once entered. On the singular line $\dot{K} = K'^S(N)\dot{N}$ and the optimal s -regime policy initiated at some point $(N, K^S(N))$ on the singular line is the outcome of

$$V^S(N) = \text{Max}_{\{q,b,R\}} \left\{ \int_0^{\infty} u[F(K^S(N_t), x_t) - M(q_t) - B(N_t)b_t - R_t(1 + K'^S(N_t))]e^{-rt} dt \right\} \quad (4.5s)$$

subject to $\dot{N}_t = R_t$; $0 \leq R_t \leq \bar{R}$; $q_t \geq 0, b_t \geq 0$ and $N_0 = N$, where Equation (2.1) has been used to express the consumption rate in terms of production, input costs and R&D expenditures.

The o-regime: In this regime no R&D is implemented, hence knowledge remains constant. An o -regime may last indefinitely or be replaced by an s -regime at some time T , but it is never optimal to switch from the o -regime to an m -regime. The optimal o -regime policy that departs from some point (N,K) and switches to an s -regime at some time T is the outcome of

$$V^0(K; N) = \text{Max}_{\{c,q,b,T\}} \left\{ \int_0^T u(c_t)e^{-rt} dt + e^{-rT}V^S(N) \right\} \quad (4.5o)$$

subject to $\dot{K}_t = F(K_t, x_t) - c_t - M(q_t) - B(N)b$, $q_t \geq 0, b_t \geq 0, c_t \geq 0, K_0 = K, K_T = K^S(N)$.

If the optimal policy for the o -regime leads to a steady state without switching to the s -regime, T in Equation (4.5o) is set at $T = \infty$, and the problem reduces to the no-R&D problem of Section 3. In this case, the steady state must be $\hat{K}(N)$.

m-regime: Here R&D is implemented at a maximal rate ($R = \bar{R}$). An m -regime cannot be terminal (since it is not optimal to increase knowledge beyond

\bar{N}), hence it must lead to either an s -regime or to an o -regime. In the former case, the optimal policy is the outcome of

$$V^m(K; N) = \text{Max}_{\{c, q, b, T\}} \left\{ \int_0^T u(c_t) e^{-rt} dt + e^{-rT} V^S(N + \bar{R}T) \right\} \quad (4.5m)$$

subject to $\dot{K}_t = F(K_t, x_t) - c_t - M(q_t) - B(N + \bar{R}t)b - \bar{R}$, $q_t \geq 0$, $b_t \geq 0$, $c_t \geq 0$, $K_0 = K$ and $K_T = K^S(N + \bar{R}T)$. If the m -regime switches to an o -regime, then V^S in Equation (4.5m) is replaced by $V^0(K_T; N + \bar{R}T)$ and the condition on K_T is relaxed.

The characterization of the optimal growth/R&D policy entails selecting among the three regimes, reducing the two-state problem (2.4) into a series of single-state problems that define these regimes. This task, it turns out, depends on the relative positions of the singular and K -lines. We analyze in detail four prototypical situations that span a wide variety of economies:

Type 1: The singular line crosses the K -line once from below (Figure 4.2);

Type 2: The singular line crosses the K -line once from above (Figure 4.4a-b);

Type 3: The singular line lies below the K -line for all $N \leq \bar{N}$ (Figure 4.6).

Type 4: The singular line lies above the K -line for all $N \leq \bar{N}$ (Figure 4.7).

The analysis can be extended to situations where the two lines intersect more than once. Multiple crossing introduces some ambiguity regarding the identification of the optimal steady state, but otherwise yields no further insight and is therefore ignored (see Tsur and Zemel, 2001, for a discussion of multiple candidate equilibria in a related context). When the K - and singular lines intersect (Types 1 and 2), the intersection point is denoted (\hat{N}, \hat{K}) .

Type 1: As is the case for all economy types, the optimal policy depends on the capital endowment K_0 . For Type 1, K_0 is compared with a critical capital stock

$K^1 > K^S(0)$ defined by the property that the m -regime policy $V^m(K^1; 0)$ brings the (N, K) process to the intersection point (\hat{N}, \hat{K}) (Figure 4.2). When $K_0 \leq K^1$, the optimal policy is to initially implement the o -regime policy if $K_0 < K^S(0)$ or the m -regime policy if $K_0 > K^S(0)$, increasing capital in the former case and decreasing it in the latter case until the singular line is reached. Upon reaching the singular line, the s -regime policy is implemented to drive the (N, K) process along the singular line to a steady state at the intersection point (\hat{N}, \hat{K}) . If $K_0 = K^S(0)$, the singular policy is immediately implemented and the (N, K) process evolves along the singular line towards the steady state (\hat{N}, \hat{K}) .

If $K_0 > K^1$, the m -regime policy is first implemented, followed by an o -regime policy initiated at some knowledge level $N > \hat{N}$ and capital stock $K > K^S(N)$ above the singular line. The latter regime involves decreasing capital and the system approaches the steady state $\hat{K}(N)$ on the K -line segment below the singular line.

Four state-space trajectories initiated with capital endowments K_0 below $K^S(0)$, between $K^S(0)$ and K^1 , equal to K^1 and above K^1 , respectively, are depicted as arrows in Figure 4.2. The arrows indicate the direction in which the processes evolve. The corresponding optimal time trajectories of the R , N and K processes initiated with $K_0 < K^S(0)$ and $K^S(0) < K_0 < K^1$ are shown in Figures 4.3a and 4.3b, respectively.

Figure 4.2

Figures 4.3a & 4.3b

We conclude that the intersection point (\hat{N}, \hat{K}) serves as an attractive steady state for all Type-1 economies endowed with capital below or at K^1 , while the K -line segment $(\hat{K}(\hat{N}), \hat{K}(\bar{N}))$ serves as a basin of attraction when initial capital exceeds K^1 .

We further note that this type of economies encourages R&D, in that knowledge is increased to \hat{N} or above. When capital endowment lies below $K^S(0)$, the R&D program is delayed to allow for capital accumulation, while higher capital endowments call for vigorous initial R&D activities ($R = \bar{R}$).

Type 2: For these economies steady states are restricted to lie on the K -line segment $[\hat{K}(0), \hat{K}(\hat{N})]$. While the policy of reaching the intersection point (\hat{N}, \hat{K}) along the singular line and settling there as a steady state (as in Type 1) cannot be optimal, it may be optimal to implement the s -regime policy along the singular line all the way to \bar{N} , with capital increasing indefinitely. The behavior under arbitrary endowment depends on the optimal policy when $K_0 = K^S(0)$. There are two possibilities in this case: either to implement the o -regime policy i.e., to decrease capital to $\hat{K}(0)$ without any R&D (Figure 4.4a), or to employ the singular policy with knowledge increasing towards \bar{N} and capital increasing indefinitely (Figure 4.4b). A Type-2 economy for which the first response is optimal is denoted Type 2a, while economies for which the second possibility is preferable are classified as Type-2b.

For a Type-2a economy, (in which avoiding R&D is preferable when $K_0 = K^S(0)$), there exists a critical capital level $K^2 > K^S(0)$ such that no R&D and convergence to $\hat{K}(0)$ are optimal whenever $K_0 \leq K^2$. However, when $K_0 > K^2$ R&D is initially warranted and the m -regime (with $R = \bar{R}$) is implemented first to be followed either by an o -regime or by an s -regime. In the former case, the switch to the o -regime (zero R&D) takes place above the singular line and to the left of the intersection point and the (N, K) process converges to a steady state $\hat{K}(\hat{N})$ on the K -line segment between $\hat{K}(0)$ and \hat{K} . With larger initial capital stocks, the initial m -regime policy leads the (N, K) process to the singular line where it is replaced by the

s -regime with a sustained growth along the singular line. The various possibilities are displayed in Figure 4.4a.

For a Type-2b economy (in which $K_0 = K^S(0)$ initiates the singular policy), $K_0 > K^S(0)$ implies the m -regime until the singular line is reached, followed by a singular policy, i.e., sustained growth along the singular line towards (\bar{N}, ∞) . To characterize optimal processes initiated below the singular line, we introduce a third threshold capital level $\hat{K}(0) \leq K^3 < K^S(0)$. If $K^S(0) > K_0 > K^3$, the o -regime is first implemented, while capital increases without R&D until the singular line is reached at $K^S(0)$. Then, a switch to the singular policy takes place and the economy grows permanently along the singular line (Figure 4.4b). If $K_0 < K^3$, R&D is unwarranted; the o -regime policy is implemented to eventually enter a steady state at $\hat{K}(0)$. (When $K^3 = \hat{K}(0)$, it is optimal to reach the singular line from any initial state.) Figure 4.5 depicts time trajectories of R , N and K for Type-2b economies endowed with $K^S(0) > K_0 > K^3$. Following some delay, the economy will engage in R&D and enjoy sustained growth.

Figure 4.4a and Figure 4.4b

Figure 4.5

In a world without R&D, the capital process of an economy for which $\hat{K}(0) = 0$ will eventually vanish regardless of the initial stock (see Section 3). Allowing a Type-2 economy to engage in R&D can change its fate in a fundamental way, depending on its initial capital stock: the economy may converge to a steady state with positive capital and knowledge stocks, or even sustain growth in the long run.

Type 3: Here the K -line is always above the singular line (Figure 4.6)—a feature that forbids the existence of any steady state (see appendix). A Type-3 economy, therefore, must grow permanently along the singular line. When $K_0 < K^S(0)$, R&D is delayed under the o -regime while capital is increased until $K^S(0)$ is reached, following which the singular policy is implemented and the economy keeps growing along the singular line. When $K_0 > K^S(0)$, a vigorous R&D program is initiated under the m -regime until the singular line is reached, at which time the singular policy is adopted and the economy grows indefinitely along the singular line. This type of economy is most conducive to R&D in that R&D drives such economies to a sustained growth path, no matter how small the economy is to begin with, (provided it has a positive amount of capital). Possible trajectories in the state space are presented in Figure 4.6.

Figure 4.6

Type 4: Here the K -line is always below the singular line (Figure 4.7) and any point along the K -line is a legitimate steady state. Increasing capital indefinitely along the singular line cannot be optimal for this type (see appendix). The optimal R&D regime is determined in terms of some threshold capital level $K^4 > K^S(0)$ such that when $K_0 \leq K^4$, no R&D is ever warranted, implying an o -regime policy that leads to a steady state at $(0, \hat{K}(0))$. If $K_0 > K^4$, an m -regime policy ($R = \bar{R}$) is initially implemented, followed by an o -regime policy with no R&D and decreasing capital, leading to an eventual steady state on the K -line. The termination of R&D (the switch from an m -regime to an o -regime) occurs at a state above the singular line. Optimal state-space trajectories for a Type-4 economy endowed with different capital stock levels are presented in Figure 4.7.

Figure 4.7

5. Scarcity

When the primary resource stock is finite, the effect of scarcity comes in through the costate variable μ (see Equation (2.7)). Scarcity is particularly effective when $M'(0) < B(\bar{N})$, in which case the primary stock is depleted. From Equation (2.12), then, we see that μ increases exponentially in time, and the market share of the primary resource declines correspondingly. This is the main feature that distinguishes the finite resource problem from its abundant resource counterpart.

A question arises regarding the way in which the primary and backstop resources are to be used along the path to depletion. It turns out that when the marginal cost of primary resource supply slopes upward (i.e., $M'(q) > 0$) both resources are used simultaneously and the transition from the primary to the backstop resource is smooth. The smooth transition is obtained by adjusting the R&D process and the resource scarcity rent so that at the depletion time, the unit backstop price $B(N)$ just equals $M'(0) + \mu/\lambda$, hence Equations (2.7) and (2.8) imply that q must tend to zero as the depletion time is approached. The proof of this continuity property is similar to its counterpart in Tsur and Zemel (2002) and is therefore omitted.

We consider now the effect of scarcity on the long run equilibrium. Letting the superscript d denote post depletion quantities, we have $q^d = 0$ and $F_2(K, b^d(K, N)) = B(N)$. Comparing with the case of infinite stock, we find that $b^d(K, N) = x(K, N) > b(K, N)$. Thus, recalling that $F_1(K, x(K, N)) = r$ defines the K -line with an infinite primary stock, the corresponding line for the post depletion period is defined by $F_1(K, b^d(K, N)) = r$. Since $b^d(K, N) = x(K, N)$, the K -line is not affected by the depletion event.

If R&D is not allowed, so that knowledge is fixed at N , the economy under scarcity settles at the same steady state $\hat{K}(N)$ obtained with abundant resource, although the corresponding rate of consumption, c , is decreased by the cost difference $B(N)b^d - [B(N)b + M(q)] = B(N)q - M(q)$. Allowing for R&D requires considering the modification of the singular line, defined by Equation (4.3). Since $b^d(K,N) > b(K,N)$ and $\partial b/\partial K > 0$, we find that depletion shifts the singular line towards smaller values of K , i.e., the post-depletion singular line lies below its pre-depletion counterpart (Figure 5.1).

How does this shift affect the long run equilibrium? For Type 1 economies the intersection point shifts towards larger values of K and N along the K -line (Figure 5.1). For this type, the intersection point serves as the steady state of all processes (unless the initial capital is very large) and primary resource scarcity increases the rate of backstop supply and justifies additional R&D to obtain a larger knowledge stock.

Figure 5.1

Applying these considerations to Type 2 economies, it is seen that the intersection point shifts towards smaller K and N levels along the K -line. The intersection point, however, is not a steady state for Type 2 economies and the long run behavior of the economy—either a steady state on the K -line or a sustained growth along the singular line—depends on its capital and primary resource endowments. When the sustained growth is the preferred option, lowering the singular line represents a higher knowledge level for every capital stock, indicating again that scarcity increases the benefit derived from knowledge hence the production of the latter is enhanced.

Since the post-depletion singular line lies below its pre-depletion counterpart, a Type 3 economy remains of the same type also after depletion, exhibiting sustained

growth along a lower singular line. For this type, the effect of scarcity is similar to that of a Type-2 economy that grows permanently.

Of the four economy types, Type 4 is the least favorable for R&D (for R&D to be exercised at all in a Type-4 economy, the economy must have inherited a very large initial capital stock). Yet, a downward shift of the singular line may change the type of the economy. If such a change indeed occurs, then the effect of scarcity is quite dramatic when the economy turns (after-depletion) into Type 3, sustaining both capital and knowledge formation in the long run. A significant change in the eventual state of the economy can also take place if it is transformed to Type 1 or Type 2.

6. Concluding comments

The view that technological progress induced by R&D programs is instrumental to ensure economic growth in a world of limited resources is widely accepted. Yet, the temporal allocation of R&D efforts in the development of substitutes for scarce natural resources is a matter of much debate, reflecting diverse views about the extent of scarcity as well as the nature, organization and finance of R&D programs. Changing economic and environmental conditions, competition with rival firms, as well as uncertainty regarding the amount of R&D efforts required to innovate and the potential benefits associated with an innovation are just a few of the factors that determine optimal R&D programs.

Evidently, no tractable model can account for all these aspects, just as in each practical application some considerations are likely to dominate others. Here we study R&D policies in the context of a growth model with scarce resources.

Analyzing the dynamic processes describing the optimal accumulation of capital and knowledge stocks, some simple conclusions can be derived: Since R&D spending comes at the expense of consumption and capital accumulation, it can be justified

only if future saving in backstop costs outweighs present loss. In our model, this distinction shows up in terms of the difference in the slopes of the singular and K -lines, giving rise to a wide range of R&D behavior: for some economy types, R&D is advantageous and the economy will either settle at a state with larger capital stock (compared with the reference case of no R&D) or even sustain growth in the long run. Other types do not support R&D efforts and the economy behaves as if this option does not exist.

Second, capital endowment is important in determining both long run equilibria and dynamic behavior. For all economy types, a small or moderate initial capital calls for delaying R&D programs, while a large capital endowment implies vigorous R&D activities at the outset. We identify economies that will shrink to a vanishing capital level without ever engaging in R&D if endowed with small initial capital, while under a larger capital endowment will carry out R&D and increase the knowledge base to a level that supports sustained growth.

Finally, resource scarcity, in general, encourages R&D by increasing the user cost of the primary resource, thereby increasing the benefit derived from reducing the backstop cost. The depletion of a limited primary resource stock may shift the long run equilibrium towards higher capital and knowledge levels, as compared with the steady state obtained with unlimited primary resource, or even to a path of sustained growth and learning.

Appendix

We derive here the optimal policy for the four economy types of Section 4. The analysis is carried out in the state (N, K) space, in which the K - and singular lines lie. The optimal (N, K) process departs from $(0, K_0)$ and its evolution depends on its position vis-à-vis the K - and singular lines. We shall frequently use terms like “above the singular line,” meaning “when the (N, K) process lies above the singular line.”

The discussion in Section 2 reveals that it cannot be optimal to increase the knowledge level above \bar{N} . Moreover, Equation (2.2) and the requirement that R&D cannot be negative imply that the knowledge process N is monotonic and bounded, hence must approach some steady state on $[0, \bar{N}]$.

The K -line $\hat{K}(N)$ is defined in Equation (4.4) by $\rho(\hat{K}(N), N) = r$. From Equations (3.4) and (4.1) we know that $\partial\rho/\partial K < 0$, hence $\rho(K, N) < r$ above the K -line. Using Equation (4.2) we find that $\dot{\lambda} > 0$ above the K -line, hence Equation (2.6) and $u''(c) < 0$ imply $\dot{c} < 0$. The reverse relations hold below the K -line, yielding

Claim 1: The optimal consumption process decreases in time above the K -line and increases in time below it. ■

Since a steady state involves a constant rate of consumption, Claim 1 implies

Claim 2: An optimal steady state (N^*, K^*) falls on the K -line, i.e. $K^* = \hat{K}(N^*)$. ■

Indeed, Claim 2 is consistent with the identification of $\hat{K}(N)$ as the steady state of K for fixed N .

Consider the function $\Lambda(K, N) = \rho(K, N) + B'(N)b(K, N)$. The singular line $K^S(N)$ is defined in Equation (4.3) by $\Lambda(K^S(N), N) = 0$. We see that $K^S(0) > 0$ and recall from

Section 4 that $K^S(N)$ increases to infinity as $N \rightarrow \bar{N}$. Using $\partial\rho/\partial K < 0$, $\partial b/\partial K \geq 0$ and $B' \leq 0$, we obtain

Claim 3: $\Lambda(K,N) > 0$ below the singular line and $\Lambda(K,N) < 0$ above it. ■

According to Equation (2.9), the optimal rate of R&D investment is determined by $\zeta = \gamma - \lambda$: $R_t = \bar{R}$ when $\zeta > 0$ (the *m*-regime); $R_t = 0$ when $\zeta < 0$ (the *o*-regime), and R assumes a singular solution (*s*-regime) when $\zeta = 0$ (which must occur on the singular line). Subtracting Equation (2.10) from Equation (2.11) one finds

$$\dot{\zeta} = \lambda\Lambda(K, N) + r\zeta. \quad (\text{A.1})$$

Since λ is always positive (see Equation 2.6) we conclude:

Claim 4: (a) When the *m*-regime holds below the singular line, ζ increases at a rate faster than e^{rt} . (b) When the *o*-regime holds above the singular line, ζ decreases at a rate faster than $-e^{rt}$. ■

Observe that allowing the faster-than-exponential divergence of Claim 4 to proceed permanently is inconsistent with the transversality conditions (2.13) associated with the nonnegative long run values of K and N . Since a steady state involves an *o*-regime policy, Claim 4 implies

Claim 5: A steady state cannot fall above the singular line. ■

Claim 4 entails restrictions also on the dynamic processes under the various regimes. For example, if the capital-decreasing *m*-regime policy is adopted at or below the singular line, the sub-optimal behavior of Claim 4a will be followed permanently. Hence,

Claim 6: An *m*-regime policy can hold only above the singular line. ■

In fact, an m -regime policy can hold only during a finite period, after which it must be replaced by either an o -regime policy (above the singular line) or an s -regime policy (on the singular line).

As long as an o -regime policy holds, the capital process is monotonic in time because the problem is essentially one-dimensional (knowledge remains constant under the o -regime). If an o -regime policy holds above the singular line, it must involve decreasing capital until the singular line is reached, for otherwise the sub-optimal behavior of Claim 4b will be followed permanently. Now, ζ must be negative when the singular line is reached from above by an o -regime policy (with $R = 0$). Since no other regime can hold below the singular line (Claim 6), this K -decreasing o -regime policy must converge to a steady state on the K -line segment below the singular line.

Initiated below the singular line, the (N,K) process under an o -regime policy cannot cross it. Neither can it switch to another regime below the singular line (an s -regime can hold only on the singular line and Claim 6 precludes the m -regime below the singular line). The only two possibilities left are to converge to a steady state below the singular line or to reach the singular line (with $\zeta = 0$) and switch to the s -regime. We summarize these considerations in

Claim 7: (a) When initiated above the singular line, an o -regime policy continues permanently and the (N,K) process (with N remaining constant) converges to a steady state on the K -line segment below the singular line. (b) When initiated below the singular line, an o -regime policy either converges to a steady state below this line or reaches the singular line where it switches to the s -regime. ■

Turning to the s -regime, we recall that the singular policy can proceed only along the singular line. Moreover, using Equation (A.1) we find that once a singular

policy has been initiated (with $\dot{\zeta} = \zeta = 0$), the (N, K) process cannot leave the singular line without violating Claim 6 or 7 (this is why the s -regime is trapping). In view of Claim 2, the following characterization holds:

Claim 8: An s -regime policy either converges to a steady state on the intersection point (\hat{N}, \hat{K}) of the K - and singular lines or follows a sustained growth path along the singular line towards (\bar{N}, ∞) . ■

To decide between the two options offered in Claim 8, consider an s -regime policy that is carried out permanently along a singular line segment above the K -line (e.g. Figures 4.2 and 4.7). According to Claim 1, this policy involves a decreasing consumption process. However, the policy of staying permanently at the initial state (diverting to consumption the funds allocated by the singular policy to increase the capital and knowledge stocks), is feasible and yields a higher utility. Therefore, the singular policy cannot be optimal. Of course, a singular policy that drives the (N, K) process along a segment above the K -line during a finite period, and then moves on to a singular segment below the K -line (as in Figure 4.4b), cannot be ruled out. These considerations imply

Claim 9: An s -regime policy cannot be confined to a segment of the singular line above the K -line. ■

We apply these results to characterize the optimal processes corresponding to the four economy types introduced in Section 4. It turns out that the steady states themselves, as well as whether the economy converges to a steady state, depend on the initial capital level.

Type 1: A Type-1 economy is characterized (Figure 4.2) by the property that the K -line crosses the singular line from above, implying that $K^S(\hat{N}) > \hat{K}'(\hat{N})$. It follows from Claims 2 and 5 that an optimal steady state must lie on the K -line segment between \hat{N} and \bar{N} .

Suppose $0 < K_0 < K^S(0)$. Claim 6 forbids the m -regime while the s -regime can be adopted only on the singular line, hence it must be optimal to initially delay R&D and apply the o -regime policy. Since $\hat{K}(0) > K^S(0)$, Claim 7b implies that it is optimal to delay R&D (keeping $R=N=0$) until K_t reaches $K^S(0)$, and proceed thereafter along the singular line towards the intersection point (\hat{N}, \hat{K}) .

According to Claim 9, it cannot be optimal to continue the singular policy past the intersection point (where the singular line lies above the K -line). The only steady state allowed on the singular line by Claim 2 is the intersection point (\hat{N}, \hat{K}) . Thus, we deduce from Claim 8 that the optimal (N, K) process must converge to the steady state (\hat{N}, \hat{K}) .

With higher initial capital stock $K_0 > K^S(0)$, delaying R&D is no longer advantageous (Claim 7a) and the optimal policy is to initially set $R = \bar{R}$, increasing knowledge and decreasing capital until the (N, K) process reaches the singular line at some time. From that time on, R is reduced to the singular value, and the process continues along the singular line to the steady state (\hat{N}, \hat{K}) .

Evidently, the higher the initial stock K_0 , the higher is the point at which the singular line is reached. In fact, there exists some threshold initial stock $K^1 > \hat{K}(0)$ such that the (N, K) process initiated from $(0, K^1)$ under the m -regime policy meets the singular line exactly at (\hat{N}, \hat{K}) . To see this, we solve Equations (2.1) and (2.10)

backwards in time, by setting $\tau = \hat{N} / \bar{R} - t$; $N_\tau = \hat{N} - \bar{R} \tau$ and the initial values

$$K(\tau = 0) = \hat{K}; \quad \lambda(\tau = 0) = u'(\hat{c}) \quad \text{where } \hat{c} \text{ is the consumption rate at the steady state.}$$

The threshold stock is determined from the solution by $K^1 = K(\tau = \hat{N} / \bar{R})$. Using Claim 3 and the time-reversed version of Equation (A.1) with $\zeta_0 = 0$, it is verified that

$$\zeta_\tau = -\int_0^\tau \Lambda(K_s, N_s) \lambda_s e^{r(s-\tau)} ds > 0 \quad \text{along the solution and the } m\text{-regime policy is}$$

indeed optimal. When $K_0 > K^1$, the m -regime policy brings the process to \hat{N} above the singular line. In such cases, the m -regime policy continues to higher knowledge levels, but at some point above the singular line R&D activities abruptly cease, switching to an o -regime policy that leads the process to a steady state on the K -line segment below the singular line. Thus, (\hat{N}, \hat{K}) is the optimal steady state whenever $K_0 \leq K^1$, while higher initial capital stocks imply higher asymptotic knowledge and capital levels.

Type 2: Here the K -line crosses the singular line from below, with

$$K^{S'}(\hat{N}) < \hat{K}'(\hat{N}) \quad (\text{Figures 4.4a-b}). \quad \text{Claims 2 and 5 restrict optimal steady states to lie}$$

on the K -line segment $[\hat{K}(0), \hat{K}(\hat{N})]$. In contrast to Type 1 economies, Claim 9

forbids the optimal process to converge to the intersection point (\hat{N}, \hat{K}) along the

singular line. However, following the singular line all the way to \bar{N} cannot be ruled

out. The dynamic behavior, then, depends on the optimal policy for an economy

endowed with $K_0 = K^S(0)$: either to decrease K (o -regime) and end up at $(0, \hat{K}(0))$

(Figure 4.4a), or to follow a sustained growth path along the singular line towards

(\bar{N}, ∞) (Figure 4.4b). We analyze each of these possibilities separately.

In the former case, the optimality of an o -regime policy leading to $(0, \hat{K}(0))$ from $(0, K^S(0))$ implies that the same holds for any initial capital below $K^S(0)$ (Claims 6 and 7b). Moreover, since ζ is negative at $(0, K^S(0))$ (otherwise an o -regime policy is not optimal), $\zeta < 0$ also for some capital stocks above $K^S(0)$, implying the same policy from these states as well (Claim 7a). For larger capital stocks, however, ζ must turn positive. To see this, we solve again the time-reversed version of Equation (A.1) with $N_\tau = 0$ (the reversed-time τ is normalized such that $\tau = 0$ indicates the time when the singular line is crossed and $\zeta_0 < 0$ is the corresponding value of ζ at that time) and find $\zeta_\tau = [\zeta_0 - \int_0^\tau \Lambda(K_s, 0) \lambda_s e^{rs} ds] e^{-r\tau}$. For sufficiently large τ , this result entails $\zeta_\tau > 0$ which is inconsistent with the o -regime. Thus, there exists a threshold level $K^2 > K^S(0)$ such that the o -regime policy leading to $(0, \hat{K}(0))$ is adopted whenever the initial capital does not exceed it (Figure 4.4a).

If, however, $K_0 > K^2$, then initially R&D will be implemented under the m -regime. The (N, K) process under this policy cannot cross the singular line (Claim 6) and the policy must be replaced by an o -regime policy leading to a steady state below the singular line (Claim 7a) or by an s -regime policy leading to (\bar{N}, ∞) . The switch to the o -regime must take place above the singular line and to the left of the intersection point (where the K -line is below the singular line). The switch to the s -regime must take place on the singular line and will be optimal at larger initial capital levels (see Figure 4.4a).

An alternative policy at $(0, K^S(0))$ is to follow the singular policy to (\bar{N}, ∞) (Figure 4.4b). In this case, $K_0 > K^S(0)$ implies an m -regime policy leading to the singular line. (The o -regime policy would cross the singular line, as Claim 7a ensures, which is not optimal in this case.) The optimal (N, K) process then follows

sustained growth along the singular line (with an s -regime policy) to (\bar{N}, ∞) .

$K_0 < K^S(0)$ calls for an o -regime policy, under which capital either converges to $\hat{K}(0)$ or increases to $K^S(0)$ (Claim 7b). If at $K_0 = \hat{K}(0)$ it is optimal to increase capital to the level $K^S(0)$ and to proceed under the s -regime thereafter, then sustained growth along the singular line is obtained in the long run from any initial capital. Otherwise, a threshold state K^3 that lies between $\hat{K}(0)$ and $K^S(0)$ serves to determine the desirable policy: $K_0 < K^3$ entails convergence to $\hat{K}(0)$, while $K^S(0) > K_0 \geq K^3$ implies an o -regime until the (N, K) process reaches the singular line and switches to the sustained growth path along it thereafter (Figure 4.4b).

Type 3: Here the K -line is always above the singular line. Claims 2 and 5 forbid the existence of any steady state, hence the economy must grow permanently along the singular line. When $K_0 < K^S(0)$ the o -regime is invoked, increasing capital until $K^S(0)$ is reached (Claim 7b), following which the process evolves along the singular line. In contrast, when $K_0 > K^S(0)$ the o -regime policy is not allowed (Claim 7a) and the m -regime policy is followed until the singular line is reached and the s -regime takes over (see Figure (4.6))

Type 4: In Type 4 economies the K -line always lies below the singular line. In this case, no point along the K -line can be ruled out as a steady state. According to Claim 9, a sustained growth path along the singular line cannot be optimal. Therefore when $K_0 \leq K^S(0)$, Claim 7b implies an o -regime policy leading to the steady state $(0, \hat{K}(0))$. Moreover, $\zeta < 0$ at the state $(0, K^S(0))$, hence the same policy applies also for some initial states above the singular line. Higher initial states, however, call for a different policy: Following the arguments for the threshold state K^2 in Type 2 economies, we introduce the threshold capital stock $K^4 > K^S(0)$ such that when

$K_0 > K^4$ it is desirable to initially activate the R&D program at full capacity (m -regime). The singular policy is not favored by Type 4 economies (Claim 9), hence the m -regime cannot extend to the singular line and must be replaced by an o -regime policy at some (N, K) levels above the singular line, leaving the optimal capital process to decrease towards a steady state on the K -line, as Claim 7a implies (see Figure 4.7).

References

- Arrow, K. J. (1962) The economic implications of learning by doing, *Review of Economic Studies* 29, 155-173.
- Bovenberg, L. A. and S. A. Smulders (1996) Transitional impacts of environmental policy in an endogenous growth model, *International Economic Review*, 37, 861-893.
- Burt, O. and R. Cummings (1970) Production and investment in natural resource industries, *American Economic Review*, 60 (June), 576-590.
- Chakravorty, U., J. Roumasset and K. Tse (1997) Endogenous substitution among energy resources and global warming, *Journal of Political Economy*, 105, 1201-1234.
- Dasgupta, P.S. and G.M. Heal (1974) The optimal depletion of exhaustible resources, *Review of Economic Studies*, 41, 3-28.
- Dasgupta, P.S. M. Heal and M. Majumdar (1977) Resource depletion and research and development, In: *Frontiers of Quantitative Economics* (M. Intriligator, Ed.), Vol. 3, pp. 483-506, North Holland.
- Dasgupta, P.S. and J. Stiglitz (1981) Resource depletion under technological uncertainty, *Econometrica*, 49, 85-104.
- Grossman, M. G. and E. Helpman (1993) *Innovation and growth in the global economy*, The MIT Press: Cambridge.
- Hotelling, H. (1931) The Economics of Exhaustible Resources, *Journal of Political Economy*, 39, 137-175.
- Hung, N.M. and N.V. Quyen (1993) On R&D timing under uncertainty: The case of exhaustible resource substitution, *Journal of Economic Dynamic and Control*, 17, 971-991.
- Kamien, M. and N.L. Schwartz (1978) Optimal exhaustible resource depletion with endogenous technical change, *Review of Economic Studies*, 45, 179-196.
- Krautkraemer, J. A. (1998) Nonrenewable resource scarcity, *Journal of Economic Literature*, XXXVI (December) 2065-2107.
- Ramsey, F. P. (1928) A mathematical theory of saving, *Economic Journal* (December), 543-559.
- Rausser, G. C. (1974) Technological change, production and investment in natural resource industries, *American Economic Review*, 64 (December), 1049-1059.

- Romer, P. M. (1986) Increasing returns and long run growth, *Journal of Political Economy*, 94, 1002-1037.
- Romer, P. M. (1990) Endogenous technical change, *Journal of Political Economy*, 98 (October), S71-S102.
- Shell, K. (1966) Towards a theory of inventive activity and capital accumulation, *American Economic Review*, (May) 62-68.
- Solow, M. R. (1974) Intergenerational equity and exhaustible resources, *Review of Economic Studies*, 41, 29-45.
- Solow, M. R. (1957) Technical change and the aggregate production function, *Review of Economics and Statistics*, 39, 312-320.
- Solow, M. R. (1956) A contribution to the theory of economic growth, *The Quarterly Journal of Economics*, 70, 65-94.
- Stiglitz, J. (1974) Growth with exhaustible natural resources: efficient and optimal growth paths, *Review of Economic Studies*, 41, 123-137.
- Tahovenen, O. and S. Salo (2001) Economic growth and transition between renewable and nonrenewable energy resources, *European Economic Review*, 45, 1379-1398.
- Tsur, Y. and A. Zemel (2002) Optimal transition to backstop substitutes for nonrenewable resources, *Journal of Economic Dynamics and Control* (in press).
- Tsur, Y. and A. Zemel (2001) The infinite horizon dynamic optimization problem revisited: A simple method to determine equilibrium states, *European Journal of Operational Research*, 131, 482-490.
- Tsur, Y. and A. Zemel (2000) R&D Policies for desalination technologies, *Agricultural Economics*, 24, 73-85.

Figure 2.1: The determination of q^* and b^* , given K , N and μ/λ .

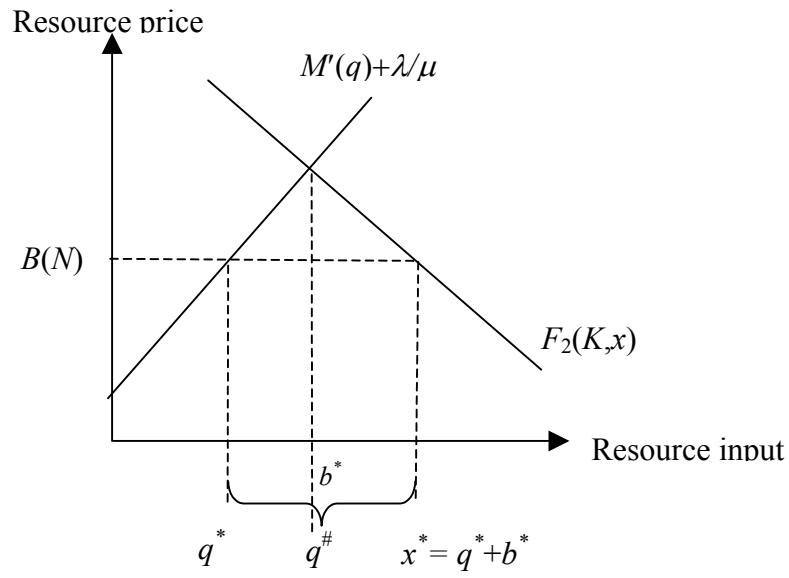


Figure 3.1: Resource input demands with capital levels $K^2 < K^B < K^1$.

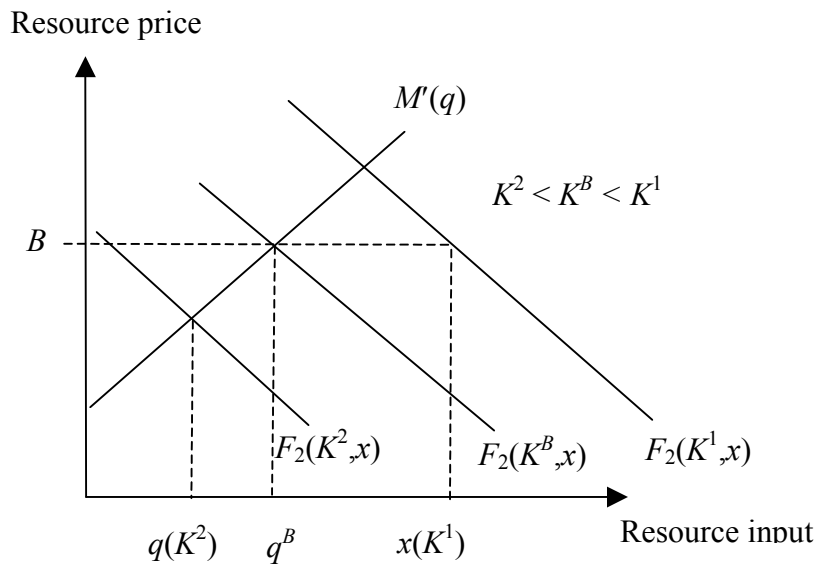


Figure 4.1: Demand for resource input at different knowledge levels.

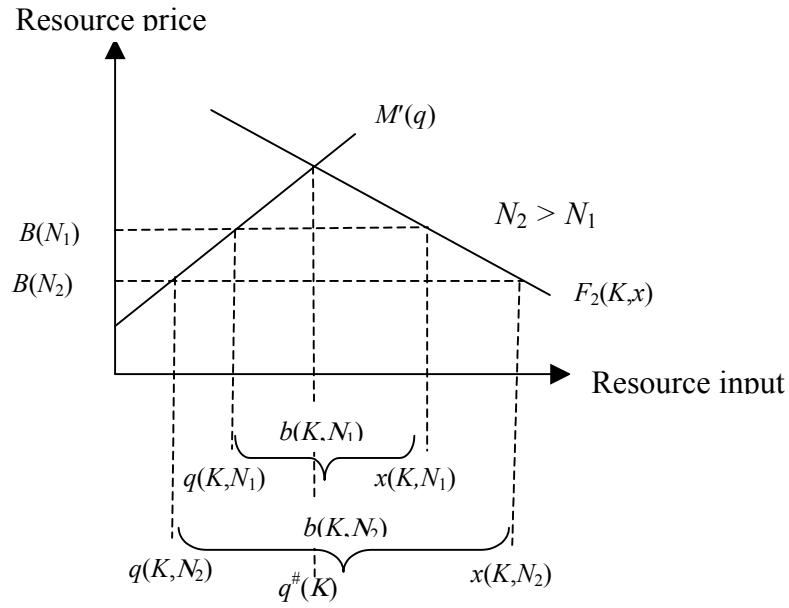


Figure 4.2: Possible trajectories for Type-1 economies endowed with various initial capital levels.

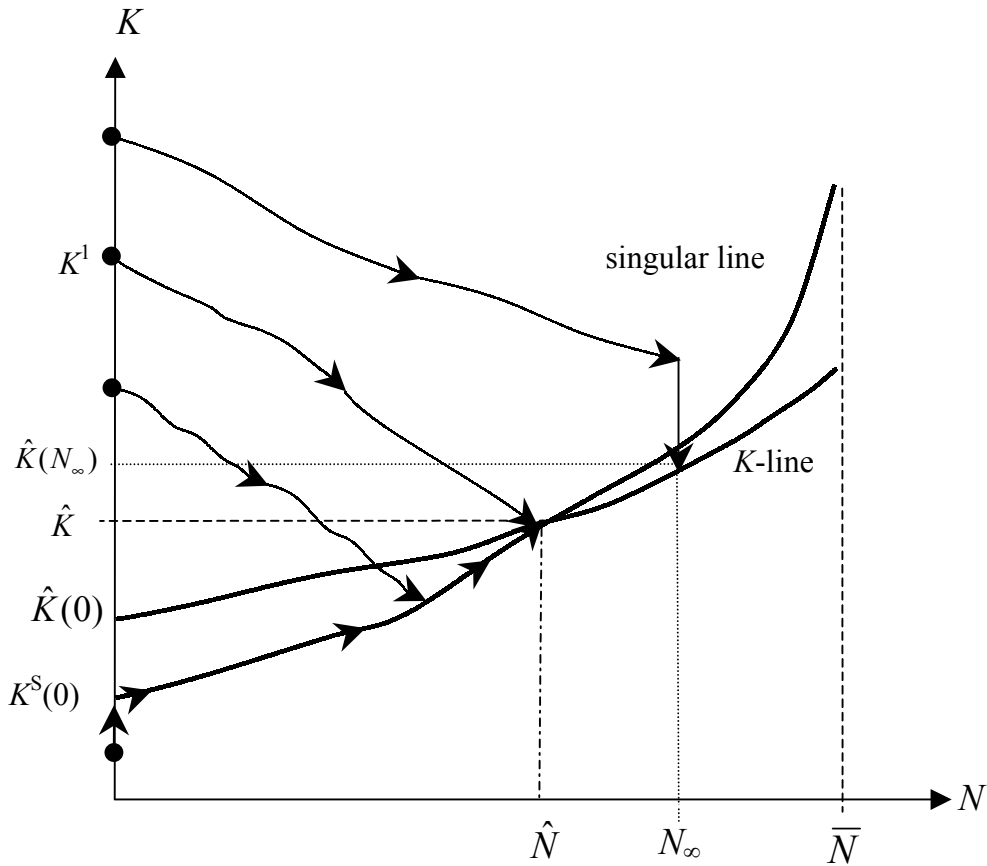


Figure 4.3a: Time trajectories for Type 1 economies when $K_0 < K^S(0)$. R&D is delayed to allow capital to grow more rapidly until it reaches $K^S(0)$, at which time the singular policy is implemented and R&D is so tuned as to drive knowledge and capital processes to their long run equilibrium (\hat{N}, \hat{K}) .

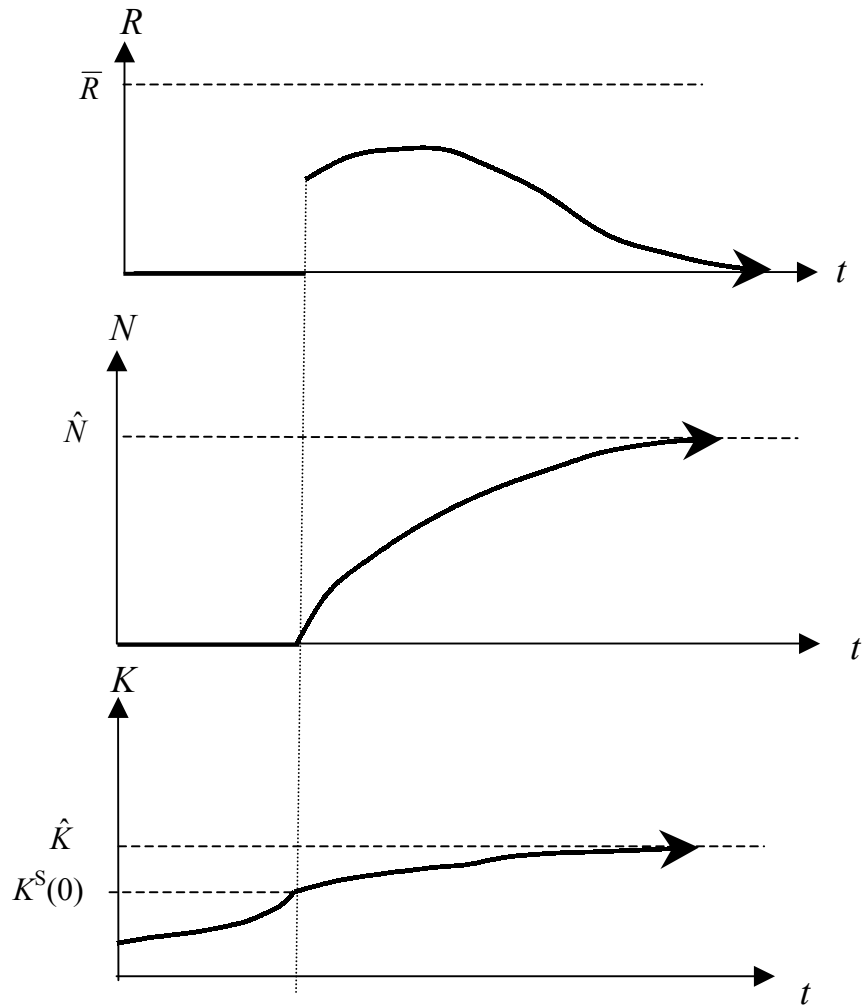


Figure 4.3b: Time trajectories for Type 1 economies when $K^S(0) < K_0 < K^1$. R&D is initiated at a full capacity \bar{R} while capital decreases until the (N,K) process reaches the singular line at time τ , at which time the singular policy is implemented and the R&D-cum-capital accumulation evolves to its long run equilibrium at (\hat{N}, \hat{K}) .

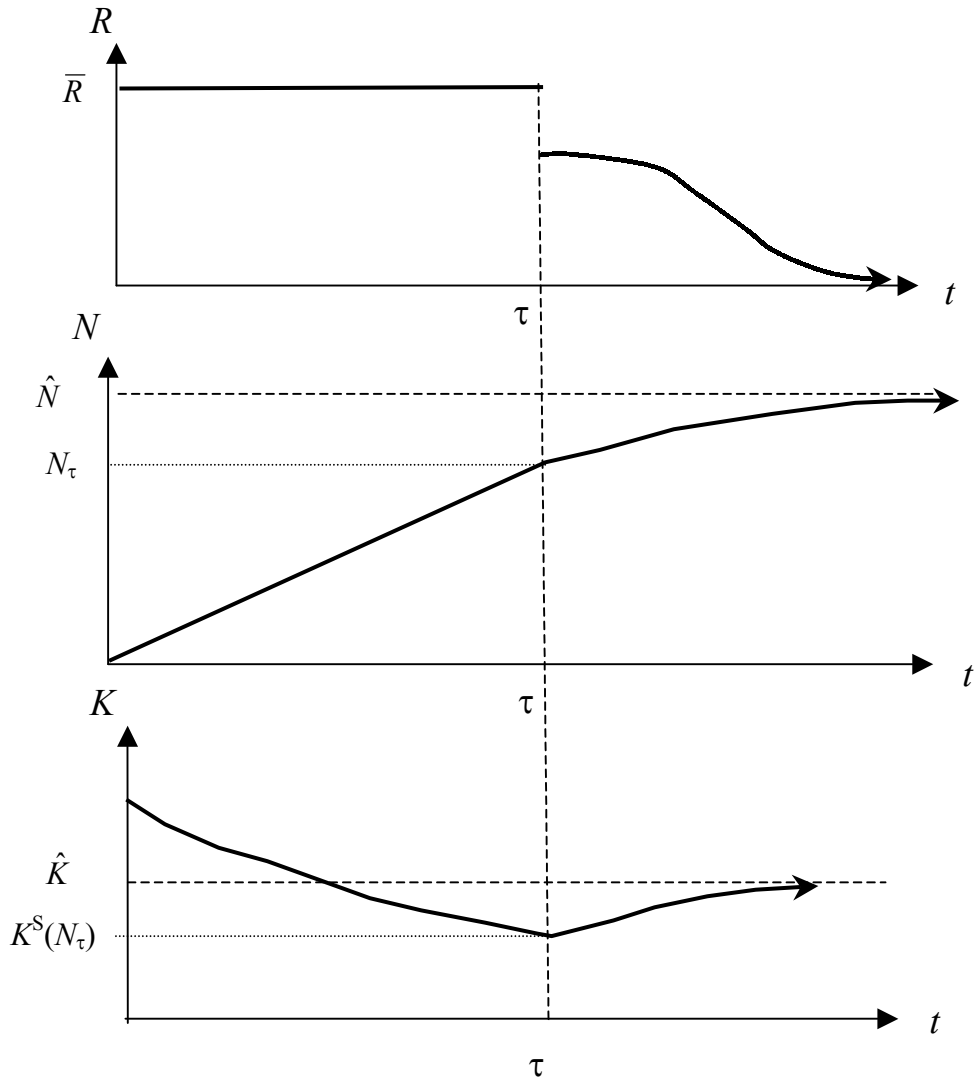


Figure 4.4a: Possible trajectories for Type-2a economies endowed with various initial capital levels K_0 when R&D is unwarranted under $K_0 = K^S(0)$.

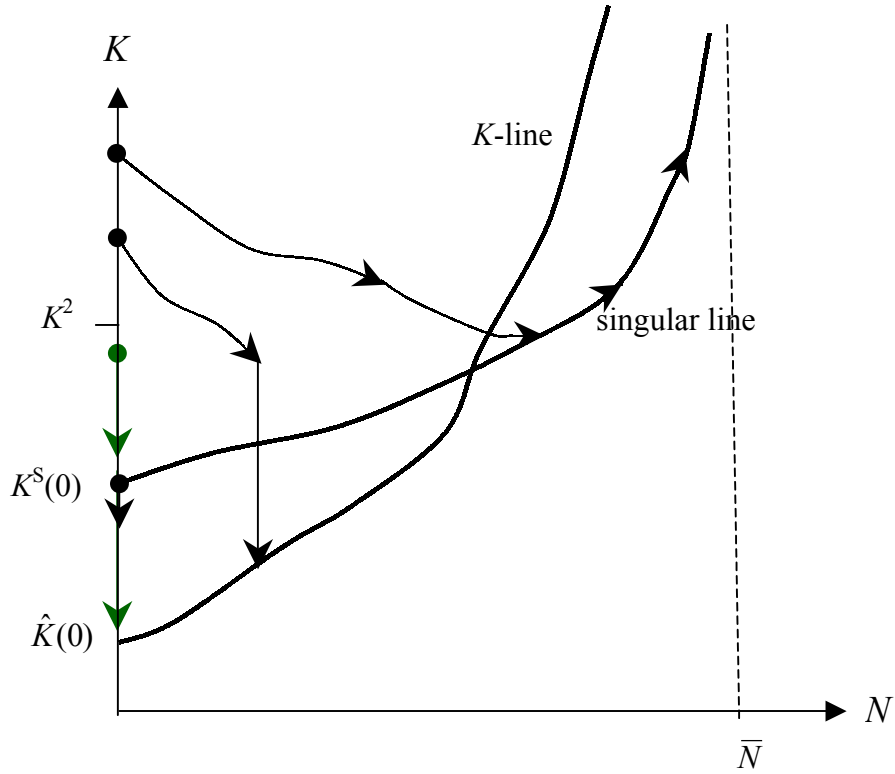


Figure 4.4b: Possible trajectories for Type-2b economies for various initial capital levels K_0 when R&D is warranted under $K_0 = K^S(0)$.

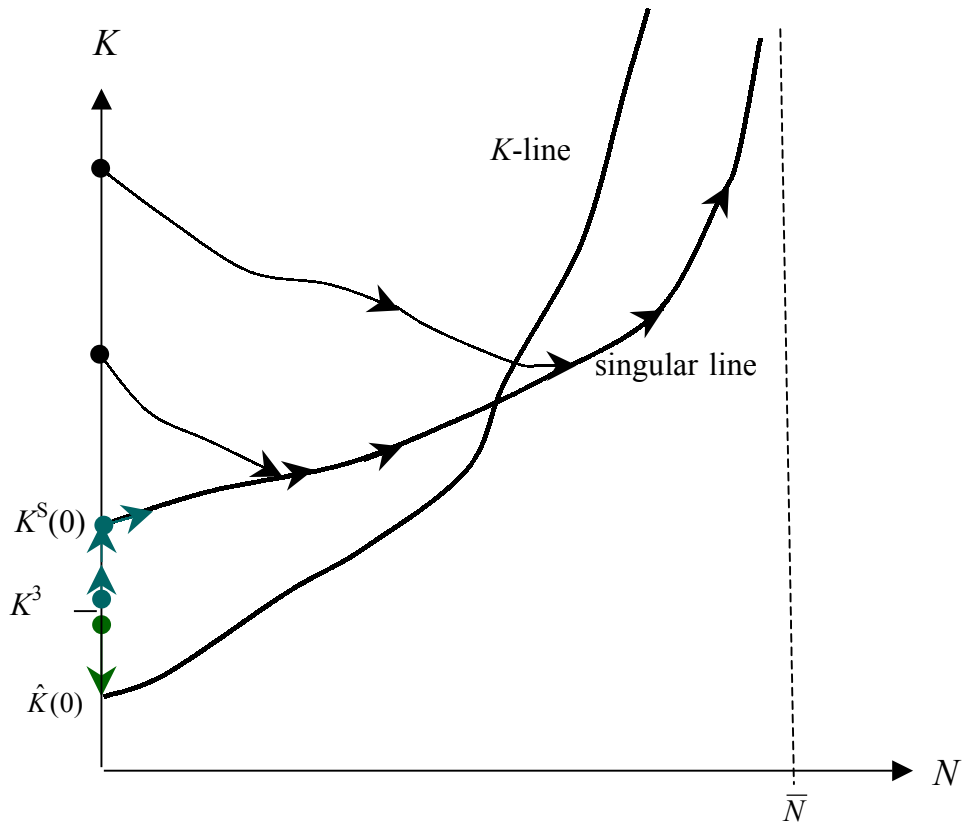


Figure 4.5: Optimal time trajectories for a Type 2b economy when $K^3 < K_0 < K^S(0)$. R&D is initially delayed while the economy accumulates capital. At the time capital reaches $K^S(0)$, the singular policy is implemented and the economy evolves along the singular path that sustains growth in the long run.

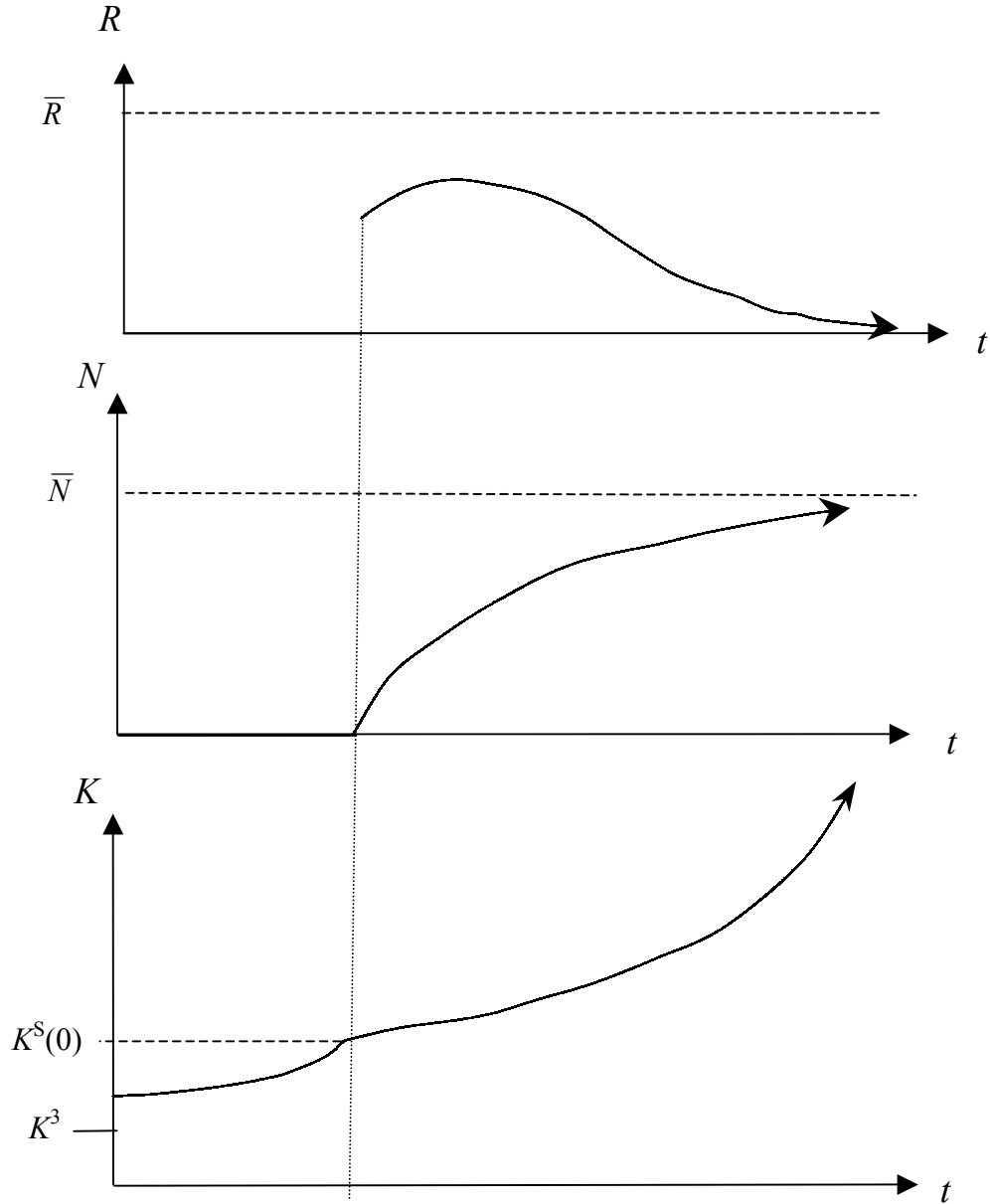


Figure 4.6: Optimal state-space trajectories for a Type 3 economy endowed with various capital stock levels.

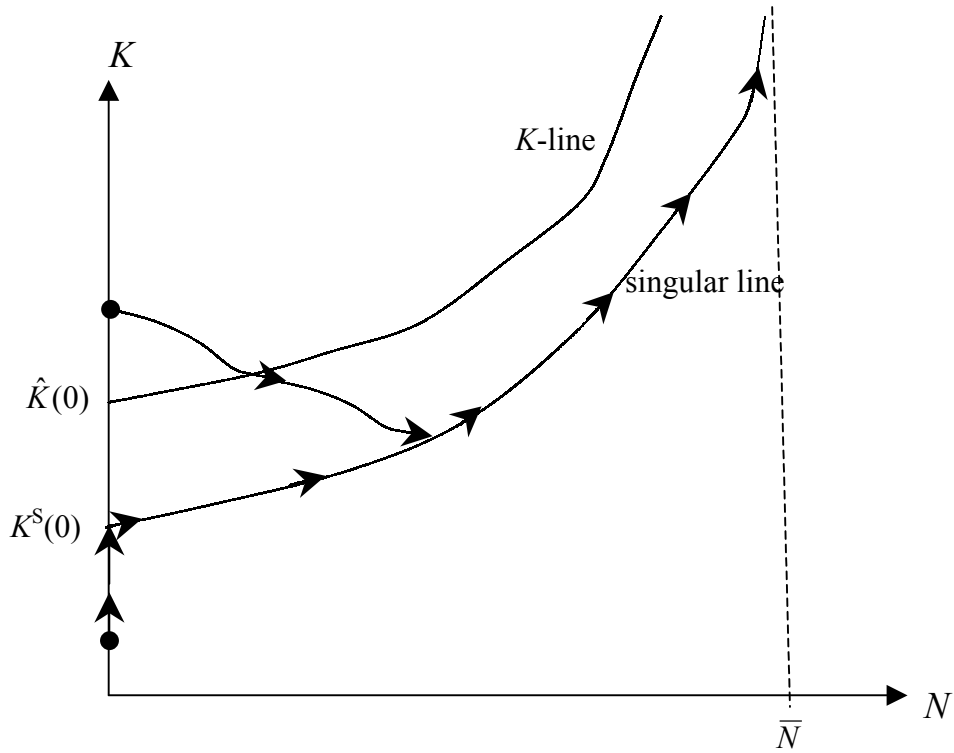


Figure 4.7: Optimal state-space trajectories for a Type 4 economy endowed with various capital stock levels.

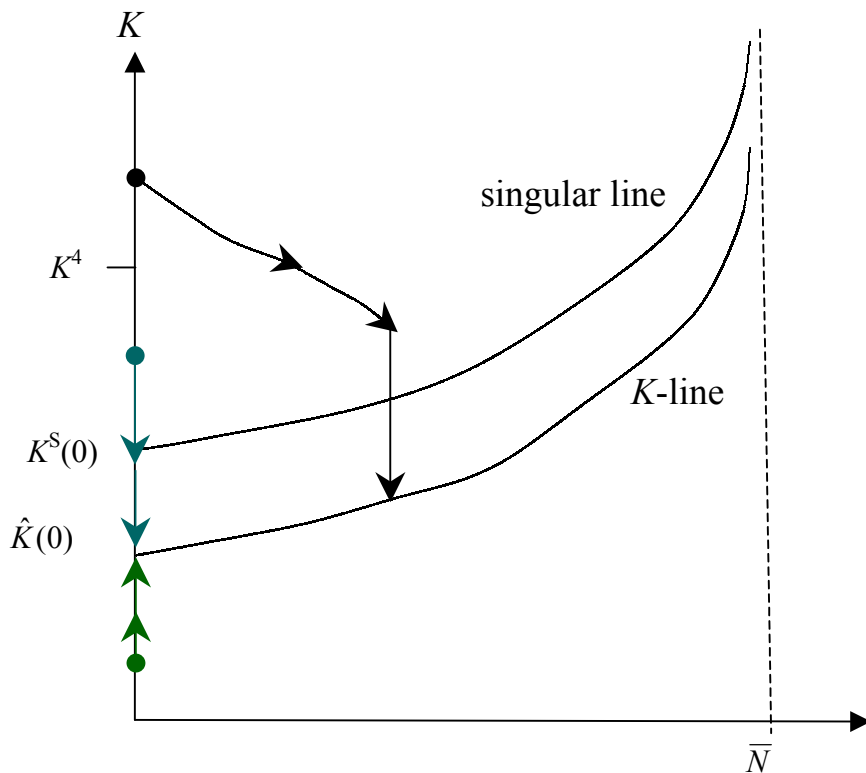
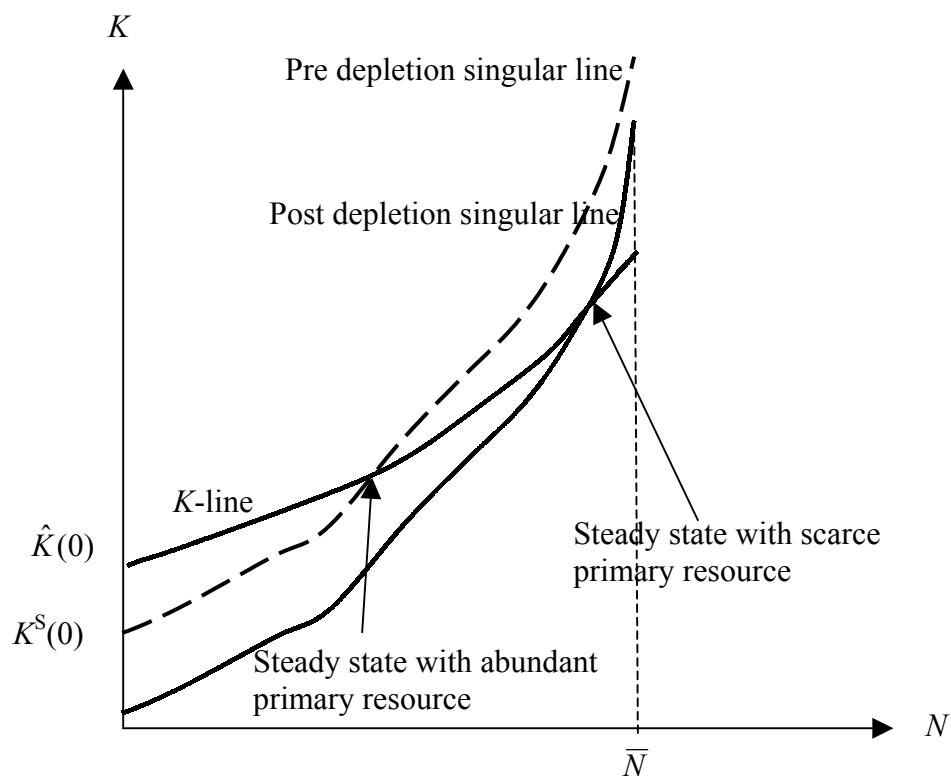


Figure 5.1: Effect of scarcity on the steady states of Type 1 economies. The K -line is not affected by depletion. The singular line shifts downward. The post depletion steady state has larger capital and larger knowledge than its abundant-resource counterpart.



PREVIOUS DISCUSSION PAPERS

- 1.01 Yoav Kislev - Water Markets (Hebrew).
- 2.01 Or Goldfarb and Yoav Kislev - Incorporating Uncertainty in Water Management (Hebrew).
- 3.01 Zvi Lerman, Yoav Kislev, Alon Kriss and David Biton - Agricultural Output and Productivity in the Former Soviet Republics.
- 4.01 Jonathan Lipow & Yakir Plessner - The Identification of Enemy Intentions through Observation of Long Lead-Time Military Preparations.
- 5.01 Csaba Csaki & Zvi Lerman - Land Reform and Farm Restructuring in Moldova: A Real Breakthrough?
- 6.01 Zvi Lerman - Perspectives on Future Research in Central and Eastern European Transition Agriculture.
- 7.01 Zvi Lerman - A Decade of Land Reform and Farm Restructuring: What Russia Can Learn from the World Experience.
- 8.01 Zvi Lerman - Institutions and Technologies for Subsistence Agriculture: How to Increase Commercialization.
- 9.01 Yoav Kislev & Evgeniya Vaksin - The Water Economy of Israel--An Illustrated Review. (Hebrew).
- 10.01 Csaba Csaki & Zvi Lerman - Land and Farm Structure in Poland.
- 11.01 Yoav Kislev - The Water Economy of Israel.
- 12.01 Or Goldfarb and Yoav Kislev - Water Management in Israel: Rules vs. Discretion.
- 1.02 Or Goldfarb and Yoav Kislev - Sustainable Salt Regime in the Coastal Aquifer (Hebrew).
- 2.02 Aliza Fleischer and Yacov Tsur - Measuring the Recreational Value of Open Spaces.
- 3.02 Yair Mundlak, Donald F. Larson and Rita Butzer - Determinants of Agricultural Growth in Thailand, Indonesia and the Philippines