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# Endangered aquifers: Groundwater management under threats of catastrophic events

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# Endangered aquifers:
# Groundwater management under threats of catastrophic events

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**Abstract**: We study optimal management of groundwater resources under risk of occurrence of undesirable events. The analysis is carried out within a unified framework, accommodating various types of events that differ in the source of uncertainty regarding their occurrence conditions and in the damage they inflict. Characterizing the optimal policy for each type, we find that the presence of event uncertainty has profound effects. In some cases the isolated steady states, characterizing the optimal exploitation policies of many renewable resource problems, become equilibrium intervals. Other situations support isolated equilibria, but the degree of prudence they imply is sensitive to the nature of the event risk.

Keywords: event uncertainty; groundwater; renewable resource; management; environmental catastrophes.

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## 1. Introduction

Overexploitation of groundwater resources—when pumping exceeds recharge—is pervasive worldwide (Postel, 1999). Such a situation involves shrinking groundwater stocks, which may lead to one of the following outcomes: i) The extraction cost increases to a point where it is no longer beneficial to pump above recharge and the aquifer settles at a steady state. ii) The groundwater stock is depleted, and from that time onward only the recharge can be pumped. iii) An event that adversely affects future exploitation benefits is triggered, e.g., seawater intrusion or the penetration of polluted water from nearby sources. The theory of groundwater management under the first two scenarios is well developed (see Burt, 1964; Gisser and Sanchez, 1980; Feinerman and Knapp, 1983; and Tsur and Graham-Tomasi, 1991, among many others), but the effects of the third type of outcome are not fully explored. This paper undertakes to characterize optimal groundwater management under the threat of occurrence of adverse environmental events.

An example in mind is the exploitation of a coastal aquifer. Excessive extraction, over and above natural recharge, leads to a decline in the groundwater head, which, in turn, may result in seawater intrusion. If seawater intrusion is a gradual process that can be monitored and controlled by adjusting extraction rates, the associated damage can be avoided. Often, however, seawater intrusion occurs abruptly as soon as the fresh water head declines below some threshold level, inflicting a severe damage or rendering the aquifer useless for a long time. In such cases, seawater intrusion can be treated as a discrete event. When the threshold level that triggers the event is known with certainty, it is easy to avoid the damage by ensuring that the threshold level is never reached. In most cases, however, the threshold is only partially known, due, e.g., to lacking information regarding subsurface flows. Moreover, the occurrence conditions may be affected also by stochastic environmental conditions that are not within the managers' control. Accounting for this kind of events, we enter the realm of *event uncertainty*.

Impacts of event uncertainty on optimal exploitation policies have been studied in a variety of resource management problems, including pollution-induced events (Cropper, 1976, Clarke and Reed, 1994, Tsur and Zemel, 1996, 1998b), forest fires (Reed, 1984, Yin and Newman, 1996), species extinction (Reed, 1989, Tsur and Zemel, 1994), seawater intrusion into coastal aquifers (Tsur and Zemel, 1995), and political crises (Long, 1975, Tsur and Zemel, 1998a). Typically, occurrence risk implies prudence, and the exploitation policies are more conservative than those obtained under certainty. In some cases, however, event uncertainty encourages more vigorous extraction policies in order to derive maximal benefit prior to occurrence. Tsur and Zemel (1998b) trace these apparently conflicting results to differences in the occurrence conditions and the damage inflicted by the events and consequetnly classify events as reversible or irreversible, and endogenous or exogenous.

In the context of groundwater, irreversible events are those that, once occurred, render the aquifer obsolete. Reversible events, on the other hand, entail a heavy penalty (e.g. the cleaning cost of a polluted aquifer) but otherwise do not prevent further exploitation of the resource. The adjective 'endogenous' signifies events whose occurrence is determined solely by the exploitation policy, although some essential information (e.g., the exact threshold level for seawater intrusion) is not a-priori known. In contrast, exogenous events are triggered also by stochastic environmental conditions (the expansion of a nearby source of pollution), which are outside the managers' control.

It turns out that the distinction among the different types of event uncertainty bears profound consequences for optimal management policies and often alters properties that are considered standard. For example, in a renewable resource context, the optimal stock process typically approaches an isolated equilibrium (steady) state. This feature, it turns out, no longer holds under endogenous event uncertainty: The unique equilibrium state expands into an equilibrium interval and the eventual steady state depends on the initial stock.

In this paper we present the problem of the optimal management of groundwater resources under event uncertainty in a unified framework that accommodates all the above-mentioned types of events. We begin, in the next section, by characterizing the optimal extraction policy under certainty. First we analyze the standard reference case of the *nonevent* problem, in which no event can ever interrupt the extraction plan, and then add *certain* events that occur when the groundwater stock shrinks to a known critical level. Since we show that under these conditions it is never optimal to trigger the event, it follows that the optimal policy is insensitive to the nature of the event (reversible or irreversible) or to the amount of damage it inflicts. This insensitivity, however, disappears when we deal (Section 3) with uncertain situations. We show that under endogenous uncertainty the optimal policy is to drive the stock process to the nearest edge of an equilibrium interval. The size of this equilibrium interval (which measures the degree of prudence implied by the events) turns out to depend on the expected damage from immediate occurrence. Under exogenous uncertainty, on the other hand, no extraction policy is perfectly safe and the equilibria are confined to isolated states. The effect of exogenous uncertainty is measured by the shift of these equilibrium states (relative to the nonevent counterpart) and is sensitive to the hazard and penalty associated with the events.

## 2. Groundwater management under certainty

We consider first the management of a confined groundwater basin (aquifer) under full certainty. Let $S_t$ denote the groundwater stock level at time $t$ and $R(S_t)$ the natural recharge rate (net water inflow excluding extraction), assumed decreasing and concave with $R(\bar{S}) = 0$ where $\bar{S}$ is the aquifer's capacity. Thus, recharge attains a maximal rate at an empty aquifer, diminishes with $S$ at an increasing rate and vanishes when the aquifer is at a full capacity $\bar{S}$. With $x_t$ representing groundwater extraction, the aquifer's stock evolves with time according to

$$dS_t / dt \equiv \dot{S}_t = R(S_t) - x_t . \tag{2.1}$$

The benefit derived from consuming water at the rate $x$ is $Y(x)$, where $Y$ is increasing and strictly concave with $Y(0) = 0$. The cost of extracting at the rate $x$ while the stock level is $S$ is $C(S)x$, where the unit cost $C(S)$ is nonincreasing and convex. The instantaneous net benefit is then given by $Y(x) - C(S)x$. It is assumed that $Y'(0) > C(\bar{S})$, so that some extraction is worthwhile under the most favorable conditions.

*2.1. Nonevent*: When no event can interrupt groundwater extraction, the optimal plan is obtained by solving

$$V^{ne}(S_0) = \max_{\{x_t\}} \int_0^\infty [Y(x_t) - C(S_t)x_t]e^{-rt} dt \tag{2.2}$$

subject to (2.1), $x_t \geq 0$; $S_t \geq 0$ and $S_0$ given. The optimal processes associated with the nonevent problem (2.2) will be indicated with an *ne* superscript. This standard problem has been treated by a variety of optimization methods (see, e.g., Tsur and Graham-Tomasi, 1991; Tsur and Zemel, 1994, 1995) and we summarize the main findings below.

We note first that because problem (2.2) is autonomous, (time enters explicitly only through the discount factor), the optimal stock process $S_t^{ne}$ evolves monotonically in time (Tsur and Zemel, 1994). Since $S_t^{ne}$ is bounded in $[0,\bar{S}]$ it must approach a steady state in this interval. Using the variational method of Tsur and Zemel (2001), possible steady states are located by means of a simple function $L(S)$ of the state variable, denoted the evolution function (see Appendix). In particular, an internal state $S \in (0,\bar{S})$ can qualify as an optimal steady state only if it is a root of $L$, i.e $L(S) = 0$, while the corners 0 or $\bar{S}$ can be optimal steady states only if $L(0) \le 0$ or $L(\bar{S}) \ge 0$, respectively.

For the case at hand, the evolution function corresponding to (2.2) is given by (see (A.3)):

$$L(S) = (r - R'(S))\left\{\frac{-C'(S)R(S)}{r - R'(S)} - [Y'(R(S)) - C(S)]\right\}. \tag{2.3}$$

The properties of the functions $Y$, $R$ and $C$ imply that the term inside the curly brackets is decreasing while $r - R'(S) > 0$. Moreover, the assumption that some exploitation is profitable at a full aquifer, i.e., $Y'(0) > C(\bar{S})$, implies that $L(\bar{S}) = (r - R'(\bar{S}))(C(\bar{S}) - Y'(0)) < 0$. Thus, either $L(0) \ge 0$, in which case $L(S)$ has a unique root in $[0,\bar{S}]$ or $L(0) < 0$. Let $\hat{S}$ represent the root of $L(S)$ if $L(0) \ge 0$ and $\hat{S} = 0$ otherwise . We have, therefore established:

**Property 1:** $\hat{S}$ is the unique steady state to which the optimal stock process $S_t^{ne}$ converges monotonically from any initial state.

The vanishing of the evolution function at an internal steady state represents the tradeoffs associated with groundwater exploitation. A steady state is optimal if any diversion from it inflicts a loss. Consider a variation on the steady state policy $x = R(\hat{S})$ in which extraction is increased during a short (infinitesimal) time period $dt$ by a small (infinitesimal) rate $dx$ above $R(\hat{S})$ and retains the recharge rate thereafter. This policy yields the additional benefit $(Y'(R(\hat{S})) - C(\hat{S}))dxdt$. But it also decreases the groundwater stock by $dS = -dxdt$, which, in turn, increases the unit extraction cost by $C'(\hat{S})dS$ and the extraction cost by $R(\hat{S})C'(\hat{S})dS$. The present value of this permanent flow of added cost is given by $R(\hat{S})C'(\hat{S})dS/(r - R'(\hat{S}))$.[1] At the root of $L$, these marginal benefit and cost just balance, yielding an optimal equilibrium state. The state of depletion $S = 0$ can be a steady state also when $L(0) < 0$, or equivalently when $Y'(R(0)) - C(0) > -C'(0)R(0)/[r - R'(0)]$. This is the case when the marginal benefit exceeds the added extraction cost even when the latter is at its maximum.

While Property 1 implies that the stock process must approach $\hat{S}$, the time to enter the steady state remains a free choice variable. Using the conditions for an optimal entry time, we establish in the Appendix that the optimal extraction rate $x_t^{ne}$ smoothly approaches the steady state recharge rate $R(\hat{S})$ and the approach of $S_t^{ne}$ towards the steady state $\hat{S}$ is asymptotic, i.e.,

**Property 2:** Initiated away from $\hat{S}$, the optimal stock process $S_t^{ne}$ will not reach $\hat{S}$ at a finite time.

Since problem (2.2) is autonomous, the optimal extraction can be expressed in terms of the state $S$ alone. Let $x^{ne}(S)$ denote optimal extraction when the stock is $S$. The necessary conditions for optimum give rise to the following first order, nonlinear differential equation for $x^{ne}(S)$ (see Appendix)

$$x^{ne}{}'(S) = \frac{[r - R'(S)][Y'(x^{ne}(S)) - Y'(R(S))] - L(S)}{Y''(x^{ne}(S))[R(S) - x^{ne}(S)]}. \tag{2.4}$$

with the boundary condition $x^{ne}(\hat{S}) = R(\hat{S})$, implied by the smooth transition to the steady state. To allow the use of (2.4) as the basis for a numerical solution, one can remove the singularity at $\hat{S}$ and obtain (see Appendix)

$$x^{ne}{}'(\hat{S}) = R'(\hat{S}) + \sqrt{\frac{r^2}{4} + \frac{L'(\hat{S})}{Y''(R(\hat{S}))}} - \frac{r}{2}, \tag{2.5}$$

which serves as the starting step for the integration scheme.

Given $x^{ne}(S)$, the optimal stock process $S_t^{ne}$ is determined by integrating (2.1)

$$t = \int_{S_0}^{S_t^{ne}} \frac{dS}{R(S) - x^{ne}(S)}, \tag{2.6}$$

and the value function is obtained from the Dynamic Programming equation (see, e.g., Kamien and Schwartz, 1981, p.242)

$$rV^{ne}(S) = Y(x^{ne}(S)) - C(S)x^{ne}(S) + [R(S) - x^{ne}(S)][Y'(x^{ne}(S)) - C(S)] \tag{2.7}$$

Moreover, it is shown in the Appendix that $x_t^{ne} = x^{ne}(S_t^{ne})$ is also monotonous in time:

**Property 3:** $x_t^{ne}$ decreases with time while $S_t^{ne} > \hat{S}$ and increases with time when $S_t^{ne} < \hat{S}$.

Observe that the decrease in the extraction rate when the groundwater stock is above the steady state takes place even though the natural rate of recharge increases as the stock declines. Thus, the two flow processes that drive the stock dynamics (extraction and recharge) work together to slow down the rate of approach to the eventual steady state.

*2.2. Irreversible Events*: Suppose now that driving the stock to some critical level $S_c$ triggers the occurrence of some catastrophic event, e.g., the intrusion of saline water into the reservoir, rendering the groundwater useless thereafter and ceasing extraction activities. We refer to such occurrence as an *irreversible* event. Obviously, if $S_c < \hat{S}$ the event risk has no bearing on the optimal policy, because extraction falls short of the recharge rate for all $S < \hat{S}$ even without the event risk (Property 1), hence the critical level will never be approached. We consider, therefore, the case $S_0 > S_c > \hat{S}$.

Let $T$ denote the event occurrence time ($T = \infty$ if the stock never shrinks to $S_c$ to trigger the event). The *certainty* problem with *irreversible* event risk is formulated as

$$V^{ci}(S_0) = \max_{\{T, x_t\}} \int_0^T [Y(x_t) - C(S_t)x_t]e^{-rt}dt, \tag{2.8}$$

subject to (2.1), $x_t \geq 0$; $S_t \geq 0$; $S_T = S_c$ and $S_0 > S_c$ given. Problem (2.8) differs from the nonevent problem (2.2) by the additional decision variable $T$ and the additional constraint $S_T = S_c$. Optimal processes corresponding to (2.8) are indicated with a *ci* superscript (*c* for certainty, *i* for irreversible).

The event occurrence is evidently undesirable, since just above $S_c$ it is preferable to extract at the recharge rate and enjoy the benefit flow associated with it rather than extract above recharge, trigger the event and lose all future benefits. Thus, the event should be avoided:

**Property 4:** When the critical level $S_c$ is known, $S_t^{ci} > S_c$ for all $t$ and $T = \infty$.

The certainty-irreversible event problem, thus, obtains the same form as the non-event problem (2.2), but with the additional constraint $S_t > S_c$. The evolution function (2.3), therefore, applies to this problem as well, but only roots in the range $[S_c, \bar{S}]$ (rather than $[0, \bar{S}]$) can be feasible steady states. Being monotonous and bounded, the optimal stock process $S_t^{ci}$ must approach a steady state. However, with $S_c > \hat{S}$ the function $L(S)$ is negative in the feasible interval $[S_c, \overline{S}]$, hence no internal steady state can be optimal. The only remaining possibility is the critical level $S_c$, because the negative value of $L(S_c)$ does not exclude this corner state. We have, therefore, established

**Property 5:** When the critical level $S_c$ corresponding to an irreversible event is known and lies above $\hat{S}$, the optimal stock process $S_t^{ci}$ converges monotonically to a steady state at $S_c$.

According to this property, in the long run $S_t^{ci}$ must lie above its nonevent counterpart $S_t^{ne}$. It turns out that this relation holds for the complete duration of the process, as stated in

**Property 6:** $S_t^{ci} > S_t^{ne}$ for all $t > 0$.

Both processes depart from the same initial stock $S_0$ at $t = 0$. According to Property 6, $x_0^{ne} > x_0^{ci}$ and the policy under event risk is always more conservative, in the sense of leaving more water in the aquifer. To see why, suppose that $S_\tau^{ne} = S_\tau^{ci}$ at some time $\tau > 0$. Then, $S_t^{ne} = S_t^{ci}$ must hold during the entire time interval $[0, \tau]$ hence the extraction rates must also coincide during this period. This, in turn, implies, (see (2.4)) that the two stock processes must evolve together also from $\tau$ onwards, violating Properties 1 and 5.

In fact, the extra caution due to the event risk is manifest also by the optimal extraction rates:

**Property 7:** $x^{ci}(S) < x^{ne}(S)$ for any $S > S_c$.

The property follows directly from Property 6, when we consider the two optimization problems initiated at $S_0 = S$.

*2.3. Reversible events*: Assume now that the damage inflicted by the event can be cured at some cost. For example, in some cases it may be possible to drive back the saline water by introducing large quantities of freshwater from other sources into the reservoir. Under such circumstances we refer to the event as *reversible* and specify the post-event value as $\phi(S_c) = W(S_c) - \psi$, where

$$W(S) = [Y(R(S)) - C(S)R(S)]/r \tag{2.9}$$

is the steady state value derived from keeping the extraction rate at the natural recharge rate $R(S)$, and the penalty $\psi > 0$ is the (once and for all) curing cost. The post-event value $\phi$, thus, accounts both for the fact that the stock cannot be further decreased (to avoid a second occurrence) and for the curing cost. The aquifer management problem under reversible events is modified to

$$V^{cr}(S_0) = \max_{\{T,x_t\}} \int_0^T [Y(x_t) - C(S_t)x_t]e^{-rt}dt + e^{-rT}\phi(S_T) \tag{2.10}$$

subject to (2.1), $x_t \geq 0$; $S_t \geq 0$; $S_T = S_c$ and $S_0 > S_c$ given. Optimal processes associated with (2.10) are indicated with a *cr* superscript.

Observe that (2.8) and (2.10) differ only in the post-event value. It follows that an irreversible event is a special case of reversible events with a penalty that equals the steady state value $W(S_c)$. Not surprisingly, the optimal policies for the two types of event turn out to be the same. To see this, note first that just as it is not desirable to trigger an irreversible event, it is also not desirable to do so with a reversible event because the post-event value is smaller than the steady state value that can be secured by avoiding the event occurrence:

**Property 8:** When the critical level corresponding to a reversible event with any positive penalty is known, the optimal stock process $S_t^{cr} > S_c$ and $T = \infty$.

Note that the reversible event may not be as harmful as the irreversible event (since the penalty may be smaller than $W(S_c)$). Nonetheless, for both types of events, the penalty is never realized (Properties 4 and 8) and its exact value (so long as it is positive) is irrelevant. It follows that the certainty policies do not depend on the nature of the event nor on the penalty it inflicts:

**Property 9:** When the critical stock level at which the event is triggered is known, the optimal policies under reversible and irreversible events are the same.

The lack of sensitivity of the optimal policy to the details of the catastrophic event is evidently due to the ability to avoid the event occurrence altogether. This may not be feasible (or optimal) when the critical stock level is not a-priory known. The optimal policy may, in this case, lead to unintentional occurrence, whose exact consequences must be accounted for in advance. We turn, in the following section, to analyze the effect of uncertain catastrophic events on groundwater management policies.

**3. Uncertain Events**

Often the conditions that lead to the event occurrence are imperfectly known, or are subject to environmental uncertainty outside the planner's control. In some cases the critical level is a priori unknown, to be revealed only by the event occurrence. Alternatively, the event may be triggered at any time by external effects (such as subsurface flows of fresh and saline water) with a probability that depends on the current aquifer state. We refer to the former type of uncertainty—that due to the planner's ignorance regarding the conditions that trigger the event—as endogenous uncertainty (signifying that the event occurrence is solely due to the exploitation decisions) and to the latter as exogenous uncertainty. It turns out that the optimal policies under the two types of uncertainty are quite different. These policies are characterized below.

*3.1 Endogenous events*: We consider events that occur as soon as the groundwater stock reaches some critical level $S_c$, which is imperfectly known. The uncertainty regarding the occurrence conditions, thus, is entirely due to the planner's ignorance concerning the critical level rather than to the influence of exogenous environmental effects. Let $F(S) = \Pr\{S_c \le S\}$ and $f(S) = dF/dS$ be the probability distribution and the probability density associated with the critical level $S_c$. The hazard function, measuring the conditional density of occurrence due to a small stock decrease given that the event has not occurred by the time the state $S$ was reached, is defined by

$$h(S) = f(S)/F(S). \tag{3.1}$$

We assume that $h(S)$ does not vanish in the relevant range, hence no state below the initial stock can be considered a-priori safe.

The event occurrence time $T$ is also uncertain, with a distribution that is induced by the distribution of $S_c$ and depends on the extraction plan. Upon occurrence, the penalty $\psi$ is inflicted and a further decrease in stock is forbidden, leaving the post-event value $\phi(S) = W(S) - \psi$. For irreversible events, the post-event value $\phi$ vanishes. Given that the event has not occurred by the initial time, i.e., that $T > 0$, we seek the extraction plan that maximizes the expected benefit

$$V^{en}(S_0) = Max_{\{x_t\}} E_T\left\{\int_0^T [Y(x_t) - C(S_t)x_t]e^{-rt}dt + e^{-rT}\phi(S_T)\Big|T > 0\right\} \tag{3.2}$$

subject to (2.1), $x_t \geq 0$; $S_t \geq 0$ and $S_0$ given. $E_T$ in (3.2) represents expectation with respect to the distribution of $T$. Optimal processes corresponding to the endogenous uncertainty problem (3.2) are denoted by the superscript *en*.

As the stock process evolves in time, the managers' assessment of the distributions of $S_c$ and $T$ can be modified since at time $t$ they know that $S_c$ must lie below $Min_{0\leq\tau\leq t}\{S_\tau\}$, for otherwise the event would have occurred at some time prior to $t$. Thus, the expected benefit in the objective of (3.2) involves $Min_{0\leq\tau\leq t}\{S_\tau\}$, i.e., the entire history up to time $t$, complicating the optimization task. The evaluation of the expectation in (3.2) is simplified when the stock process evolves monotonically in time, since then $Min_{0\leq\tau\leq t}\{S_\tau\} = S_0$ if the process is nondecreasing (and no information relevant to the distribution of $S_c$ is revealed) and $Min_{0\leq\tau\leq t}\{S_\tau\} = S_t$ if the process is nonincreasing (and all the relevant information is given by the current stock $S_t$). It turns out that

**Property 10:** The optimal stock process $S_t^{en}$ evolves monotonically with time.[2]

Property 10 allows to confine attention to monotonic processes. Roughly speaking, the property is based on the idea that if the process reaches the same state at two different times, and no new information on the critical level is revealed during that period, then the planner faces the same optimization problem at both times. This rules out the possibility of a local maximum for the process, because $Min_{0\leq\tau\leq t}\{S_\tau\}$ remains constant around the maximum, yet the conflicting decisions to increase the stock (before the maximum) and decrease it (after the maximum) are taken at the same stock levels. A local minimum can also be ruled out even though the decreasing process modifies $Min_{0\leq\tau\leq t}\{S_\tau\}$ and adds information on $S_c$. However, it cannot be optimal to decrease the stock under risk (before the minimum) and then increase it (with safety, after the minimum) from the same state. In fact, at any state along the optimal process, non-occurrence of the event cannot modify earlier decisions. Therefore, prior to occurrence no need ever arises to update the original plan, and the open- and closed-loop solutions are the same (see Tsur and Zemel, 1994, for a complete proof).

For nondecreasing stock processes it is known with certainty that the event will never occur and the uncertainty problem (3.2) reduces to the nonevent problem (2.2). When the stock process decreases, the distribution of $T$ is obtained from the distribution of $S_c$ as follows:

$$1 - F_T(t) \equiv \Pr\{T > t | T > 0\} = \Pr\{S_c < S_t | S_c < S_0\} = F(S_t)/F(S_0). \tag{3.3}$$

The corresponding density and hazard-rate functions are also expressed in terms of the critical stock distribution:

$$\begin{aligned} &\text{(a)} \quad f_T(t) = dF_T(t)/dt = f(S_t)[x_t - R(S_t)]/F(S_0), \\ &\text{(b)} \quad h_T(t) = \frac{f_T(t)}{1 - F_T(t)} = h(S_t)[x_t - R(S_t)]. \end{aligned} \tag{3.4}$$

Let $I(\cdot)$ denote the indicator function that obtains the value one when its argument is true and zero otherwise, and observe that $E_T\{I(T > t) | T > 0\} = 1 - F_T(t) = F(S_t)/F(S_0)$. Writing the objective of (3.2) as

$E_T\left\{\int_0^\infty [Y(x_t) - C(S_t)x_t]I(T > t)e^{-rt}dt + e^{-rT}\phi(S_T) \middle| T > 0\right\}$, the expectation for decreasing processes is readily evaluated, yielding

$$V^{aux}(S_0) = \max_{\{x_t\}} \left\{ \int_0^\infty \{Y(x_t) - C(S_t)x_t + h(S_t)[x_t - R(S_t)]\phi(S_t)\} \frac{F(S_t)}{F(S_0)} e^{-rt} dt \right\} \tag{3.5}$$

subject to (2.1), $x_t \geq 0$; $S_t \geq \hat{S}$ and $S_0$ given. The allocation problem for which (3.5) is the objective is referred to as the *auxiliary* problem, and optimal processes corresponding to this problem are denoted by the superscript *aux*. It turns out that the auxiliary problem is relevant only for stock levels above $\hat{S}$, hence $\hat{S}$ replaces the depletion level ($S$=0) as the lowest feasible stock for this problem. In similarity with the previously defined problems, the optimal stock process associated with the auxiliary problem evolves monotonically with time. Notice that at this stage it is not clear whether the uncertainty problem (3.2) reduces to the nonevent problem or to the auxiliary problem, since it is not a priori known whether $S_t^{en}$ decreases with time. We shall return to this question soon after the optimal auxiliary processes are characterized.

Using (A.3), we obtain the evolution function corresponding to the auxiliary problem (3.5)

$$L^{aux}(S) = [L(S) + h(S)r\psi]F(S)/F(S_0). \tag{3.6}$$

In (3.6), $L(S)$ is the evolution function for the nonevent problem, defined in (2.3), and $h(S)$ is the hazard function, defined in (3.1). Occurrence of the event inflicts an instantaneous penalty $\psi$ (or equivalently, a permanent loss flow at the rate $r\psi$) that could have been avoided by keeping the stock at the level $S$. The second term in the square brackets of (3.6) gives the expected loss due to an infinitesimal decrease in stock. Moreover, this term is positive at the lower bound $\hat{S}$, while $L(\hat{S}) = 0$, hence $L^{aux}(\hat{S}) > 0$, implying that $\hat{S}$ cannot be an optimal equilibrium for the auxiliary problem. Whether or not the auxiliary evolution function has a root in $(\hat{S}, \bar{S})$ (where $L(S) < 0$) depends on the size of the expected loss: for moderate losses, $L^{aux}$ vanishes at some stock level $\hat{S}^{aux}$ in the interval $(\hat{S}, \bar{S})$, which is the optimal steady state for the auxiliary problem. We assume that the root $\hat{S}^{aux}$ is unique[3]. Higher expected losses ensure that $L^{aux} > 0$ throughout, and the auxiliary process converges to a steady state at the upper bound $\hat{S}^{aux} = \bar{S}$. It follows that

**Property 11**: $\hat{S}^{aux}$ is the unique steady state to which the optimal stock process $S_t^{aux}$ converges monotonically from any initial state in $[\hat{S}, \bar{S}]$.

Events for which $\phi(S) = W(S) - \psi = 0$ are denoted irreversible. Noting (2.9) and $R(\bar{S}) = 0$, we see that $W(\bar{S}) = 0$, while $L(\bar{S}) < 0$. Thus, for irreversible events $L^{aux}(\bar{S}) < 0$. It follows that the auxiliary evolution function must have a root in the interval $(\hat{S}, \bar{S})$, and the auxiliary equilibrium level for irreversible events must be an internal state.

We apply these results to characterize the optimal extraction plan for the endogenous uncertainty problem (3.2). A detailed analysis is presented in Tsur and Zemel (1995). Here we outline the main considerations:

(i) When $S_0 < \hat{S}$, the optimal nonevent stock process $S_t^{ne}$ increases in time. With event risk, it is possible to secure the nonevent value by applying the nonevent policy, since an endogenous event can occur only when the stock decreases. The introduction of occurrence risk cannot increase the value function, hence $S_t^{en}$ must increase. This implies that the uncertainty and nonevent processes coincide, $S_t^{en} = S_t^{ne}$ for all $t$, and increase monotonically towards the steady state $\hat{S}$.

(ii) When $S_0 > \hat{S}^{aux} > \hat{S}$, both $S_t^{ne}$ and $S_t^{aux}$ decrease in time. If $S_t^{en}$ is increasing, it must coincide with the nonevent process $S_t^{ne}$, contradicting the decreasing trend of the latter. A similar argument rules out a steady state policy.

Thus, $S_t^{en}$ must decrease, coinciding with the auxiliary process $S_t^{aux}$ and converging with it to the auxiliary steady state $\hat{S}^{aux}$.

(iii) When $\hat{S}^{aux} \geq S_0 \geq \hat{S}$, the nonevent stock process $S_t^{ne}$ decreases (or remains constant if $S_0 = \hat{S}$ ) and the auxiliary stock process $S_t^{aux}$ increases (or remains constant if $S_0 = \hat{S}^{aux}$ ). If $S_t^{en}$ increases, it must coincide with $S_t^{ne}$, and if it decreases it must coincide with $S_t^{aux}$, leading to a contradiction in both cases. The only remaining possibility is the steady state policy $S_t^{en} = S_0$ at all *t*.

We summarize these considerations in

**Property 12:** (a) $S_t^{en}$ increases at stock levels below $\hat{S}$.

(b) $S_t^{en}$ decreases at stock levels above $\hat{S}^{aux}$.

(c) All stock levels in $[\hat{S}, \hat{S}^{aux}]$ are equilibrium states of $S_t^{en}$.

The various possibilities are illustrated in Figure 1. The equilibrium interval of Property 12(c) is unique to optimal stock processes under endogenous uncertainty. Its boundary points attract any process initiated outside the interval, (as indicated by the direction of the arrows in the Figure), while processes initiated within it must remain constant. This feature is evidently related to the splitting of the endogenous uncertainty problem into two distinct optimization problems depending on the initial trend of the optimal stock process. At $\hat{S}^{aux}$, the expected loss due to occurrence (represented by the second term of (3.6)) is so large that entering the interval by reducing the stock cannot be optimal even if under certainty extracting above the recharge rate would yield a higher benefit. Within the equilibrium interval, the planner can take advantage of the possibility to eliminate the occurrence risk altogether by not reducing the stock below its current level. As we shall see below, this possibility is not available under exogenous uncertainty, hence the corresponding management problem does not give rise to equilibrium intervals.

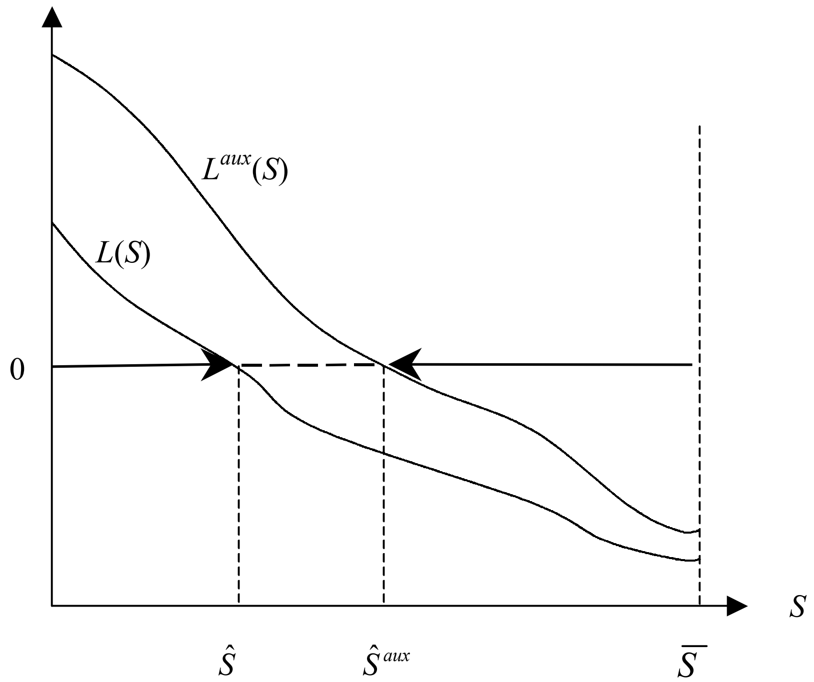


**Figure 1:** The evolution functions $L(S)$ (Eq. 2.3) and $L^{aux}(S)$ (Eq. 3.6) corresponding to the nonevent and auxiliary problems, respectively. The arrows indicate the direction in which the optimal process $S_t^{en}$ evolves. $[\hat{S}, \hat{S}^{aux}]$ is the equilibrium interval for this process.

Endogenous uncertainty, then, implies more conservative extraction than the nonevent policy for any initial stock above $\hat{S}$. Observe that the steady state $\hat{S}^{aux}$ of Property 12(b) is a *planned* equilibrium level. In actual realizations, the process may be interrupted by the event at a higher stock level and the *actual* equilibrium level in such cases will be the occurrence state $S_c$. The situation is depicted in Figure 2, in which the optimal stock processes corresponding to the nonevent and the endogenous uncertainty problems are compared.

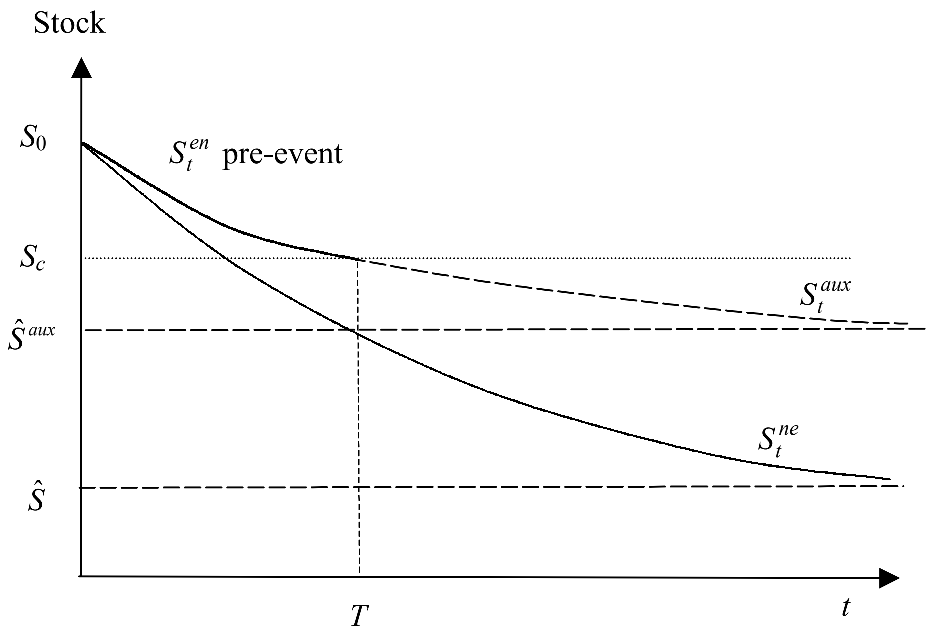


**Figure 2:** A comparison between the optimal processes $S_t^{ne}$ (corresponding to the nonevent problem) and $S_t^{en}$ (corresponding to endogenous uncertainty). The latter process is interrupted by the event at time $T$ and the rest of the process (depicted by the dashed-line process $S_t^{aux}$) is never realized.

It is also noted that under endogenous uncertainty, reversible and irreversible events differ only in the precise location of the root of the auxiliary evolution function. A large expected loss with $h(\bar{S})r\psi > -L(\bar{S})$ excludes the possibility of a root in $[\hat{S}, \bar{S}]$, hence the uncertainty process will never decrease; the expected loss in this case is too high to justify taking the risk of triggering the event at any stock level. For irreversible events such a situation cannot occur: since $W(\bar{S}) = 0,$ the expected loss near $\bar{S}$ is small and the auxiliary root must lie below this state.

A feature similar to both the certainty and endogenous uncertainty processes is the (planned) smooth transition to the steady states. When the initial stock is outside the equilibrium interval, the condition for an optimal entry time to the steady state implies that extraction converges smoothly to the recharge rate and the planned steady state will not be entered at a finite time. Thus, Property 8 extends also to endogenous uncertainty. It follows that when the critical level actually lies below $\hat{S}^{aux}$, uncertainty will never be resolved and the planner will never know that the adopted policy of approaching $\hat{S}^{aux}$ is indeed safe. Of course, in the less fortunate case in which the critical level lies above the steady state, the event will occur, resolving uncertainty at a finite time (see Figure 2).

*3.2 Exogenous events*: Random catastrophic events can be triggered by *exogenous* environmental conditions that are not within the resource managers' control. The current groundwater stock level can affect the *hazard* of immediate occurrence, but whether the latter will actually take place is determined by stochastic exogenous conditions. This type of event uncertainty was introduced by Cropper (1976) and analyzed by Clarke and Reed (1994) and by Tsur and Zemel (1998b) in the context of environmental pollution control. Here we consider the implications of this kind of uncertainty on groundwater resource management. Under exogenous uncertainty, the knowledge that a certain stock level has been reached in the past without triggering the event is not a safeguard from occurrence at the same stock level sometime in the future, lest the exogenous conditions turn out to be less favorable. Therefore, the mechanism that gives rise to the safe equilibrium intervals under endogenous uncertainty does not work here, and we shall show below that such intervals do not characterize the optimal processes under exogenous uncertainty.

As above, the post-event value is denoted by $\phi(S)$, which vanishes identically for all $S$ under irreversible events. The expected value from an extraction plan that can be interrupted by an event at time $T$ is again given by the objective of (3.2), but for exogenous events the probability distribution of $T$, $F(t) = \Pr\{T \le t\}$, is defined in terms of a stock-dependent hazard rate $h(S_t) = f(t)/[1-F(t)]$ as

$$F(t) = 1 - \exp\{-\int_0^t h(S_\tau)d\tau\}. \tag{3.7}$$

We assume that no stock level is completely safe, hence $h$ does not vanish and the integral in (3.7) diverges for any feasible process as $t \to \infty$. We further assume that $h(S)$ is decreasing, i.e., filling the aquifer reduces the occurrence hazard.

Using (3.7) to evaluate the expected value derived from any feasible process we obtain the exogenous uncertainty problem:

$$V^{ex}(S_0) = \underset{\{x_t\}}{Max} \int_0^\infty h(S_T)\exp[\int_0^T h(S_t)dt]\{\int_0^T [Y(x_t) - C(S_t)x_t]e^{-rt}dt + e^{-rT}\phi(S_T)\}dT \tag{3.8}$$

subject to (2.1), $x_t \ge 0$; $S_t \ge 0$ and $S_0$ given. Unlike the auxiliary problem (3.5) for endogenous events, (3.8) provides the correct formulation for the exogenous uncertainty problem regardless of whether the stock process decreases or increases.

To characterize the steady state, we need to specify the value $W^{ex}(S)$ associated with the steady state policy $x^{ex} = R(S)$. Exogenous events may interrupt this policy, hence $W^{ex}(S)$ differs from the value function $W(S)$ of (2.5) obtained from the steady state policy under certainty or endogenous uncertainty. An occurrence inflicts the penalty, but does not affect the hazard of future events. The post-event policy, then, is to remain at the steady state and receive the post-event value $W^{ex}(S) - \psi$. Under the steady state policy, (3.7) reduces to the exponential distribution $F(t) = 1 - exp(-h(S)t)$, yielding the expected value $W^{ex}(S) = W(S) - [W(S) - W^{ex}(S) + \psi]h(S)/[r+h(S)]$. Solving for $W^{ex}(S)$, we find

$$W^{ex}(S) = W(S) - \psi h(S)/r, \tag{3.9}$$

where the second term represents the expected loss over an infinite time horizon. The explicit time dependence of the distribution $F(t)$ of (3.7) does not allow to present the optimization problem (3.8) in an autonomous form. Nevertheless, the argument for the monotonicity of the optimal stock process $S_t^{ex}$ holds, and the associated evolution function can be derived (Tsur and Zemel, 1998b), yielding

$$L^{ex}(S) = L(S) - d[\psi(S)h(S)]/dS. \tag{3.10}$$

For reversible events with a fixed penalty and decreasing hazard one finds that $L^{ex}(S) > L(S)$. Since $L(S)$ is positive below $\hat{S}$, so must $L^{ex}(S)$ be, precluding any steady state below $\hat{S}$. Thus, the root $\hat{S}^{ex}$ of $L^{ex}(S)$ must lie above the nonevent equilibrium, implying

**Property 13:** The optimal stock process under exogenous uncertainty converges monotonically to the root $\hat{S}^{ex}$. When the hazard-rate function $h(S)$ is decreasing, $\hat{S}^{ex} > \hat{S}$ and the extraction policy is more conservative than its nonevent counterpart.

Property 13 is due to the second term of (3.10) which measures the marginal expected loss due to a decrease in stock. The latter implies a higher occurrence risk, which in turn calls for a more prudent extraction policy. Indeed, if the hazard is state-independent, the second term of (3.10) vanishes, implying that the evolution functions of the nonevent and exogenous uncertain event problems are the same and so are their steady states. In this case, extraction activities have no effect on the expected loss hence the tradeoffs that determine the optimal equilibrium need not account for the penalty, no matter how large it may be. For a decreasing hazard, however, the degree of prudence (measured by the shift $\hat{S}^{ex} - \hat{S}$ in the equilibrium state) increases with $\psi$ and the sensitivity to the penalty size is regained.

For irreversible events, $\psi = W^{ex}(S)$ and (3.9) implies that $\psi = rW(S)/(r+h(S))$, hence the second term of (3.10) becomes $-[h(S)rW(S)/(r+h(S))]'$ which is usually of indefinite sign because $W(S)$ can increase with $S$ at low stock levels. The case of a constant hazard, ($h(S)=h>0$) is of particular interest. In this case, we use (2.3) and (2.9) to reduce (3.10) to

$$L^{ex}(S) = L(S) - W'(S)hr/(r+h) = \{L(S) - h[Y'(R(S)) - C(S)]\}r/(r+h). \tag{3.11}$$

It follows that the steady state cannot lie at or above $\hat{S}$: In this range, $L(S) \leq 0$, implying that $Y'(R(S)) - C(S) \geq -C'(S)R(S)/(r - R'(S)) > 0$. Thus, both terms in the curly brackets of (3.11) are negative and $L^{ex}(S) < 0$, excluding a steady state. Therefore, $\hat{S}^{ex} < \hat{S}$ and the uncertainty policy is *less* conservative than its nonevent counterpart.

**Property 14:** When the hazard of irreversible exogenous events is constant, the optimal steady state $\hat{S}^{ex}$ lies below $\hat{S}$, and uncertainty induces higher extraction rates.

The intuition behind Property 14 is clear: with a stock-independent hazard rate, the extraction policy does not affect the occurrence probability. However, since the post-event value vanishes, the planners wish to accumulate as much benefit as possible prior to occurrence, speeding up the extraction activities and reducing the equilibrium stock. In terms of (3.10), we see that the penalty $\psi = W^{ex}(S)$ *increases* with the stock, hence reducing the latter is equivalent to reducing the expected loss, encouraging vigorous extraction. Similar results have been derived by Clarke and Reed (1994) for catastrophic environmental pollution.

The results presented in this section highlight the sensitivity of the optimal uncertainty processes to the details of an interrupting event. The type of uncertainty determines the equilibrium structure: endogenous uncertainty gives rise to equilibrium intervals while exogenous uncertainty implies isolated equilibrium states. In most cases, the expected loss due to occurrence encourages prudent extraction policies, but the opposite behavior is optimal under constant hazard of irreversible exogenous events.

**4. Concluding comments**

While it is widely recognized that uncertainty may have profound effects on groundwater management, the precise manner in which the optimal extraction rules should be modified is often ambiguous. In this work we concentrate on a particular type of uncertainty, namely event uncertainty, under which the occurrence date of some catastrophe cannot be predicted in advance. The occurrence of the catastrophic event, which significantly reduces the value of the resource, might be advanced by the extraction activities. Event uncertainty, therefore, renders intertemporal considerations particularly relevant to the design of optimal extraction rules: Unlike other sources of uncertainty (time-varying costs and demand, stochastic recharge processes, etc.) under which the extraction policy can be updated along the process to respond to changing conditions, event uncertainty is resolved only by occurrence, when policy changes can no longer be useful. Thus, the expected loss due to the catastrophic threats must be fully accounted for prior to occurrence, and the resulting policy rules are significantly modified.

In this work we study optimal groundwater extraction under the threat of events that differ in the damage they inflict and the conditions that trigger occurrence. We demonstrate the sensitivity of the optimal management policy to the details of the hazard and damage specifications. The analysis is presented here in the context of groundwater resources but has wide application in a variety of resource situations involving event uncertainty.

## Appendix

### A1: The evolution function

Tsur and Zemel (2001) consider possible equilibrium states for general infinite horizon optimization problems of the form

$$V(S_0) = \max_{\{x_t\}} \int_0^\infty B(S_t, x_t) e^{-rt} dt \tag{A.1}$$

subject to $\dot{S} = g(S_t, x_t)$, $\underline{S} \le S_t \le \overline{S}$, $\underline{x} \le x_t \le \overline{x}$, $S_0$ given, assuming that the steady state policy is feasible, i.e. there exists a "recharge" function $\underline{x} \le R(S) \le \overline{x}$ such that setting $x = R(S)$ in $g$ yields $g(S,R(S)) = 0$. $B$ in (A.1) is the benefit flow, and $g$ determines the state dynamics, while some of the inequality constraints can be relaxed by assigning infinite values to the corresponding bounds. The steady state policy $x = R(S)$, then, yields the value

$$W(S) = B(S,R(S))/r. \tag{A.2}$$

For the nonevent problem (2.2), (A.2) reduces to (2.9). The optimality of the steady state policy for a given state $S$ is tested by comparing $W(S)$ with the value obtained from a slight variation on this policy. It is established that an interior state $S$ can be an optimal steady state only if it is a root (zero) of the *evolution function*, defined as

$$L(S) = r\left( \frac{\partial B(S,R(S))/\partial x}{\partial g(S,R(S))/\partial x} + W'(S) \right). \tag{A.3}$$

If $L(S)$ does not vanish, a feasible variation on the steady state policy yielding a value larger than $W(S)$ can be found, hence $S$ is not an optimal steady state. Corner states make an exception by the possibility to qualify as optimal steady states without being roots of $L(S)$, depending on the sign obtained by the evolution function at these states.

In particular, the lower bound $\underline{S}$ can be an optimal steady state if $L(\underline{S}) < 0,$ while the upper bound can be an optimal steady state if $L(\overline{S}) > 0.$

Specializing (A.3) to the nonevent and auxiliary problems (2.2) and (3.5), we obtain the corresponding evolution functions (2.3) and (3.6).

**A2: The dynamics of the nonevent processes**

We assume below that the nonevent steady state $\hat{S}$ is internal and suppress, for brevity, the superscript *ne* from the associated optimal processes. Let $T$ denote the time at which the optimal nonevent stock process $S_t$ enters the steady state $\hat{S}$. The nonevent problem (2.2) is recast in the form

$$V^{ne}(S_0) = \max_{\{T,x_t\}} \int_0^T [Y(x_t) - C(S_t)x_t]e^{-rt}dt + e^{-rT}W(S_T) \qquad \text{(A.4)}$$

subject to (2.1), $x_t \geq 0$; $S_t \geq 0$; $S_T = \hat{S}$ and $S_0$ given. Denoting the current-value costate variable by $\lambda_t$, we obtain the current-value Hamiltonian

$$H(S,x,\lambda) = Y(x) - C(S)x + \lambda[R(S) - x]. \qquad \text{(A.5)}$$

Necessary conditions for optimum include

$$Y'(x) - C(S) - \lambda = 0, \qquad \text{(A.6)}$$

and

$$\dot{\lambda} - r\lambda = -\partial H / \partial S = C'(S)x - \lambda R'(S). \qquad \text{(A.7)}$$

The transversality condition associated with the free choice of the entry time $T$ is

$H(\hat{S}, x_T, \lambda_T) - rW(\hat{S}) = 0,$ or, noting (A.6) and (2.9)

$Y(x_T) - C(\hat{S})x_T + [Y'(x_T) - C(\hat{S})][R(\hat{S}) - x_T] = Y(R(\hat{S})) - C(\hat{S})R(\hat{S}),$ giving

$$Y(x_T) - Y(R(\hat{S})) = Y'(x_T)[x_T - R(\hat{S})]. \qquad \text{(A.8)}$$

Recalling the concavity of $Y$, (A.8) implies

$$x_T = R(\hat{S}), \tag{A.9}$$

hence the transition to the steady state extraction rate must be smooth.

Taking the time derivative of (A.6) we obtain $\dot{\lambda} = Y''(x)\dot{x} - C'(S)[R(S) - x]$. Comparing with (A.7), we can eliminate the co-state variable and its time derivative

$$\begin{aligned} Y''(x)\dot{x} = [r - R'(S)][Y'(x) - C(S)] + C'(S)R(S) = \\ [r - R'(S)][Y'(x) - Y'(R(S))] - L(S). \end{aligned} \tag{A.10}$$

For an autonomous problem, the optimal extraction is a function of the state $S$ alone, $x_t = x(S_t)$ hence $\dot{x} = x'(S)[R(S) - x]$. Therefore, (A.10) reduces to a first order differential equation for $x(S)$:

$$x'(S) = \frac{[r - R'(S)][Y'(x(S)) - Y'(R(S))] - L(S)}{Y''(x(S))[R(S) - x(S)]}. \tag{A.11}$$

with the boundary condition $x(\hat{S}) = R(\hat{S})$, representing the smooth transition to the steady state established by (A.9). When $\hat{S}$ is internal, both numerator and denominator of (A.11) vanish at this state. Nevertheless, the equation can be reduced, using l'Hopital's rule, to a quadratic equation in the difference $x'(\hat{S}) - R'(\hat{S})$, yielding (2.5) (see Tsur and Zemel, 1994).

Once the solution $x(S)$ of (A.11) is given, (2.1) is readily integrated, yielding (2.6). Since (2.5) ensures that the difference $x'(\hat{S}) - R'(\hat{S})$ is finite, the singularity of (2.6) at the steady state implies that the integral diverges when its upper limit is set at $\hat{S}$, giving $T = \infty$ and establishing Property 2. The derivation of Properties 4 and 8, corresponding to known critical stocks, is simpler: the transversality condition associated with the free choice of the entry time $T$, $H(S_c, x_T, \lambda_T) - r\phi(S_c) = 0$, is written in the form (c.f. the derivation of (A.8))

$$Y(x_T) - Y(R(S_c)) - Y'(x_T)[x_T - R(S_c)] = -r\psi < 0, \tag{A.12}$$

and the concavity of $Y$ implies that (A.12) cannot be solved with $x_T \geq R(S_c)$. Thus, the transversality condition cannot be satisfied in finite time and the event is never triggered.

Consider now the decreasing function $J(S) = -C'(S)R(S)/[r-R'(S)]$. From (A.7) we deduce that $J(\hat{S}) = \lambda_\infty$. For finite times, we use (2.1) and write (A.7) as $\dot{\lambda}_t = [r - R'(S_t)]\lambda_t + C'(S_t)x_t = [r - R'(S_t)][\lambda_t - J(S_t)] - C'(S_t)\dot{S}_t$, or

$$d[\lambda_t + C(S_t)]/dt = [r - R'(S_t)][\lambda_t - J(S_t)]. \tag{A.13}$$

When the stock process $S_t$ lies above $\hat{S}$, it decreases, hence both $C(S_t)$ and $J(S_t)$ must increase. Suppose that $J(S_t) > \lambda_t$. According to (A.13), the process $\lambda_t + C(S_t)$, hence $\lambda_t$ itself, must decrease with time. It follows that the difference $J(S_t) - \lambda_t$ increases in time, violating the end condition $J(\hat{S}) = \lambda_\infty$. Thus,

$$J(S_t) < \lambda_t \quad \text{for all } S_t > \hat{S}. \tag{A.14}$$

According to (A.6), $\lambda_t + C(S_t) = Y'(x_t)$ hence (A.13) and (A.14) imply that $Y'(x_t)$ increases and $x_t$ decreases with time whenever $S_t > \hat{S}$. The same considerations show that $x_t$ increases with time when the stock process lies below $\hat{S}$, establishing Property 3. In fact, the extraction and stock processes always show the same trend, hence the function $x(S)$ of (A.11) must increase.

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**References**

Burt, O. R. (1964) Optimal resource use over time with an application to groundwater, *Management Science* 11, 80-93.

Clarke, H.R. and W.J. Reed (1994) Consumption/pollution tradeoffs in an environment vulnerable to pollution-related catastrophic collapse, *Journal of Economic Dynamics and Control* 18, 991-1010.

Cropper, M.L. (1976) Regulating activities with catastrophic environmental effects, *Journal of Environmental Economics and Management* 3, 1-15.

Feinerman, E. and K.C. Knapp (1983) Benefits from groundwater management: magnitude, sensitivity and distribution, *American Journal of Agricultural Economics* 65, 703-710.

Gisser, M. and D.A. Sanchez (1980) Competition versus optimal control in groundwater pumping, *Water Resources Research* 16, 638-642.

Kamien, M.I. and N.L. Schwartz (1981) *Dynamic Optimization: The calculus of Variations and Optimal Control in Economics and Management.* North-Holland, New York.

Long, N.V. (1975) Resource extraction under the uncertainty about possible nationalization, *Journal of Economic Theory* 10, 42-53.

Pindyck, R.S. (1984) Uncertainty in the theory of renewable resource markets, *Review of Economic studies* 51, 289-303.

Postel (1999) *Pillar of sand: Can the irrigation miracle last?* W.W. Norton Company, New York.

Reed, W.J. (1984) The effect of the risk of fire on the optimal rotation of a forest, *Journal of Environmental Economics and Management* 11, 180-190.

Reed, W.J. (1989) Optimal investment in the protection of a vulnerable biological resource, *Natural Resource Modeling* 3, 463-480.

Tsur, Y. and T. Graham Tomasi (1994) The buffer value of groundwater with stochastic surface water supplies, *Journal of Environmental Economics and Management* 21, 201-224.

Tsur, Y. and A. Zemel (1994) Endangered species and natural resource exploitation: extinction vs. coexistence, *Natural Resource Modeling* 8, 389-413.

Tsur, Y. and A. Zemel (1995) Uncertainty and irreversibility in groundwater resource management, *Journal of Environmental Economics and Management* 29, 149-161.

Tsur, Y. and A. Zemel (1996) Accounting for global warming risks: resource management under event uncertainty, *Journal of Economic Dynamics and Control* 20, 1289-1305.

Tsur, Y. and A. Zemel (1998a) Trans-boundary water projects and political uncertainty, in: *Conflict and Cooperation on Trans-boundary Water Resources,* eds. R. Just and S. Netanyahu - Kluwer Academic Pub. 279-295.

Tsur, Y. and A. Zemel (1998b) Pollution control in an uncertain Environment, *Journal of Economic Dynamics and Control* 22, 967-975.

Tsur, Y. and A. Zemel (2001), The infinite horizon dynamic optimization problem revisited: A simple method to determine equilibrium states, *European Journal of Operational Research* 131 482-490.

Yin, R. and D.H. Newman (1996) The effect of catastrophic risk on forest investment decisions, *Journal of Environmental Economics and Management* 31, 186-197.

**Endnotes:**

[1] The effective discount rate equals the market rate $r$ minus the marginal recharge rate $R'$ because by reducing the stock by a marginal unit and depositing the proceeds at the bank the resource owner gains the market interest rate $r$ plus the additional recharge rate $-R'$ (see Pindyck 1984).

[2] For degenerate problems that allow multiple optima, the property ensures that at least one optimal plan is monotonic.

[3] The case of multiple roots is discussed in Tsur and Zemel (2001). The possibility of more than one root entails some ambiguity on the identification of the steady state but contributes no further insight.