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# Patents as Options: Path-Dependency and Patent Value

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## Abstract

Enabled by the Bayh-Dole Act (1980), universities license access to innovations protected by US patents. Despite the growing importance of license revenue to cash-strapped land-grant universities that generate a large share of agricultural innovations, there has been no formal attempt to determine an optimal pricing strategy for patent licenses. We recognize that patents are options on the stream of future revenues, and apply option-valuation techniques to determine optimal pricing strategies for university technology officers. We find that path-dependency in license revenue streams creates significant differences in the optimal pricing strategy relative to more standard risk-neutral pricing models, but that path-dependent pricing more nearly approximates observed patent prices. While non-path dependent prices yield conventional sensitivities to volatility, mean-reversion and returns-growth, path-dependent prices show highly non-linear comparative statics. These results are important both for patent licensees, and for licensors seeking to maximize license revenue.

keywords: Bayh-Dole Act, innovation, licenses, option valuation, patents, path-dependency.  
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# 1 Introduction

The importance of universities' role in generating commercially-relevant research has risen sharply in recent years. In fact, Lach and Schankerman (2008) report that universities conduct 53% of all basic research and that "...the number of U.S. patents awarded to university inventors annually increased from 500 in 1982 to 3,255 in 2006. During the period 1991–2006, the annual number of licenses granted more than tripled and license revenues increased from \$186 million to about \$1.4 billion (Association of University Technology Managers, 2006).” Despite the economic importance of licensing patents to university revenues, prices for these licenses tend to be determined in *ad hoc* ways through institutional mechanisms that are unlikely to arrive at efficient, or economically justifiable prices. If the market for innovations were deeper, if participants were well-informed and trading institutions were clear and transparent, there would likely be no need for university administrators to have a formal model to help them license work conducted by their faculty. However, none of these conditions currently exist, so the development of a mutually-agreeable pricing system is critically important for the growth and development of the market for patents in general, and agricultural patents in particular. In this study, we develop a general approach for pricing licenses on patented innovations, and apply two specific pricing models to a case study on licenses to patents for new apple varieties.<sup>1</sup>

We argue that a patent is an option on a stream of profit generated by an innovation, so should be priced as such. Empirical models of patent valuation have long-recognized the isomorphic nature of patents and options on real investments (Pakes and Schankerman 1984; Pakes 1986; Lanjouw 1998; Bloom and van Reenen 2002), but few reflect the attributes of patents that are relevant for their efficient pricing. We agree that patents entail a fixed and irreversible investment, the associated returns-stream is typically highly uncertain, and

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<sup>1</sup>The value of a patent and the value of a license to that patent are regarded as equivalent throughout this analysis. That is, we assume the market for licenses should be regarded as competitive by participants, and that the license itself grants exclusive planting rights to the buyer.

the patent licensor has at least a temporary monopoly right to exploit the market value of the innovation, so patents are appropriately valued using real option valuation techniques. However, the appropriate pricing model is non-standard. First, at the core of any option valuation model is the assumed data generating process for the returns upon which the option is drawn. Bloom and van Reenen (2002) model the underlying returns process to patenting in terms of a geometric Brownian motion (GBM), which results in a standard Black-Scholes (1973) type of valuation model. Agricultural innovations in particular, however, entail a number of complications that likely require the application of a far more flexible and robust valuation method. Our pricing model accomodates the complex nature of the returns process that underlie patent values. Second, patent prices are likely to be path-dependent because exercise is at the discretion of the holder, not fixed by contract. We extend an approach developed by Longstaff and Schwartz (2001) for valuing path-dependent, American-style options and apply this model to price patents on agricultural innovations. Third, we recognize that university technology-transfer offices can choose the timing of their sale of the license. Therefore, we calculate option prices over a number of expiry dates (years between the license sale and the patent's expiry) to uncover any non-linearities in the relationship between license prices and the timing of a license-auction. In doing so, we offer a means by which university-technology transfer offices may be able to value discoveries by university-based researchers in a more transparent, efficient way.

We find that prices for licenses to agricultural innovations appear to be priced as if producers recognize the path-dependency inherent in patent values. Moreover, because of this path-dependency, we find some evidence that the sensitivity of license prices to key model parameters (volatility, mean reversion and growth rates) tends to be highly non-linear, unlike prices determined using a non-path dependent model. Building a market for licenses in which innovations are priced efficiently, and the incentives to innovate are aligned with those to commercialize innovations, requires participants on both sides of the market

to understand how license values are determined in an economically-justifiable way.

Our study contributes to the literature on licensing patents on agricultural innovations in a number of ways. First, we introduce a simple and flexible, yet realistic model for patent-license pricing to the agricultural sector. Second, our findings provide a critical tool for university technology managers responsible for pricing innovations from university faculty. Third, our patent-license pricing model addresses a key weakness in existing markets for agricultural innovations – the lack of a clear, transparent price that both parties to the transaction can agree is economically justifiable. Just as the creation of the Black-Scholes model some 40 years ago provided the platform for explosive growth of the financial options and derivatives markets, our tool may provide a catalyst for greater liquidity in the market for food innovations. Although our specific example concerns new apple varieties, our findings are sufficiently general to be of interest to a wide variety of university technology managers charged with generating as much revenue as possible from their research program.

The rest of the paper is organized as follows. In the next section, we provide some background on the legal and institutional environment surrounding patent licensing by universities. In a third section, we describe two patent valuation models and an empirical model of the data generating process for the returns to owning an agricultural patent. We then describe the data used in our empirical application and the assumptions governing the application of each model in a fourth section. In section five we present the results obtained from each valuation method, and discuss some implications for both sellers and buyers of patent licenses. We conclude in the sixth section and suggest some avenues for future research.

## **2 Background on Licensing University-Based Innovations**

There are many potential explanations why universities are licensing more and more innovations to commercial enterprises. First, changes in public policy that allow universities

to benefit financially from the output of federally-funded faculty labs has opened the door for virtually all universities to create administrative positions, technically and generically referred to as technology transfer offices (TTOs), to serve as liaisons between researchers and private-sector firms. In 1980, only 25 universities reported operating TTOs, while there were over 200 by 1990 (Mowery et al., 2001). Previous to the 1980 Supreme Court decision in *Diamond versus Chakrabarty*, university researchers could not patent broad discoveries of specific molecules and organisms. With this decision, the commercial potential of the entire field of biotechnology was revealed to university researchers. Later that year, passage of the Bayh-Dole Act (The Patent and Trademark Amendment Act of 1980) ceded intellectual property rights to university research from the federal government to universities. In 1984, PL 98-620 further expanded the rights of universities regarding the type of research that could be patented and licensed and provided more flexibility as to who universities could assign these rights to (Henderson, Jaffe and Trajtenberg 1998). In 2000, the Technology Transfer and Commercialization Act further updated regulations regarding licensing federally-funded research to reflect new technologies in entirely new industries. Whether the successive relaxation of federal control over publicly-funded research is responsible for the growth in patenting, however, is still an open empirical issue (Mowery et al. 2001).

Second, greater opportunity for financial benefit was coincident with greater need for funding. Sharp reductions in public funding for post-secondary education in the United States has forced university administrators to look toward patent licensing as an alternative source of operating and endowment funds. Efforts to profit from licensing research, however, are not always – or even typically – successful. Trune and Goslin (1998) find that fully 60% of research universities lose money on their TTOs. Using more recent data, Bulut and Moschini (2006) show that between 1998 and 2002 the top 20 universities were responsible for some 83% of license revenue. Moreover, Henderson, Jaffe, and Trajtenberg (1998) and Mowery and Ziedonis (2002) document a remarkable decline in the “quality” of the patents

licensed in the surge following passage of Bayh-Dole.

Third, starting in the early 1980s, firms began to recognize the advantages of building on both basic and applied research coming out of universities. Underwritten by major research universities such as Stanford, Columbia, MIT or U. C. Berkeley, academic research comes with an implicit warranty, or at least an assurance of quality inputs.

Despite opponents of Bayh-Dole who argue that publicly-funded research should be in the public domain by its very nature, there are strong economic arguments favoring licensing university research (Rubenstein 2003). First, once the patent is filed, the nature of the innovation is fully and completely disclosed to other researchers. Second, because the innovation met the criteria required to be awarded a patent, the science has been deemed to be “...new, useful and nonobvious.” Third, by awarding exclusivity to the buyer, thereby reducing fears that others will free-ride on the research, the overall returns to the innovation may be greater than if rights were granted in open-source form. Fourth, the risk to license buyers is lower if the innovation is backed by either a government agency or major research university as both have an incentive to uphold the integrity of the research-and-discovery system. Fifth, federal agencies and universities may be able to select licensees that would be more successful in commercializing the innovation as they are not bound by either previous business relationships or vertical-ownership restrictions. Consequently, we would expect the social returns to licensed university research to be significantly positive.

Historically, publically-funded universities and research centers developed new varieties for many agricultural commodities and made them freely available to end users. Patented cultivars have become very common for various annual crops, and in recent years we are seeing a dramatic rise in patented fruit varieties, most notably with apples (see Brown and Maloney, 2009). Many of the patented apple varieties that exist today were developed and are promoted by European organizations such as Better3Fruits, Consorzio Italiano Vivaisti, International Fruit Obtention, Inova, Kiku Ltd., and Varicom, among others. In addition,

there have also been some patented apple varieties introduced by organizations in New Zealand (e.g., ENZA) and by university breeding programs (such as Cornell University, the University of Minnesota, and Washington State University) in the United States.

The transition to patented apple varieties, relative to patented seeds for annual crops, presents a more complex set of economic issues for plant breeders and growers. The most important difference relates to lag between the time decisions are made about new variety selections and the time when a fruit can be marketed. Over this time period there are a number of factors that could change the economic conditions for the new variety. For example, new varieties will continue to be introduced and subsequent introductions may easily provide substantial improvements (in terms of production, storage capacity, or market acceptance). Growers invest a significant amount of money each time they plant a new orchard and it represents a long-term financial commitment. In the case of a patented fruit variety, the grower also needs to consider the additional cost of the patent as well as the terms of the license as part of the decision process. The TTO's also need to be cognizant of these terms of the license in their efforts to maximize revenues for the innovations created by universities. The value of the payment for the patent, however, is calculated in a relatively *ad hoc* way with no formal valuation model to guide the process. Clearly, pricing new plant technology to industry is an option-valuation problem that has not been solved by the principals involved.

## **3 Model of Patent Valuation**

### **3.1 Overview**

Our real-option model of patent valuation compares two alternative valuation methods, under different assumptions regarding the important sources of uncertainty governing patent values. We first consider a relatively standard risk-neutral valuation model (Cox, Ingersoll and Ross 1985) in which the option can only be exercised on the date of expiry (European option



assumption) and the duration of cash flows does not depend on the duration of investment. We then extend our valuation model to incorporate more realistic assumptions that may be important in determining optimal patent values: (1) the option to exercise before expiry (American option assumption), (2) the co-dependency of cash-flow duration and post-patent investment duration, and (3) the value of the embedded option to remove patented trees and replace them with alternatives. Each of these additional assumptions implies that the standard risk-neutral valuation method must be extended to include path-dependency in a manner similar to Longstaff and Schwartz (2001). We then describe the Longstaff and Schwartz (2001) least-squares Monte Carlo (LMS) technique as it applies to the valuation of agricultural patents as complex-options.

## **3.2 Modeling the Returns Process**

If patents are nothing more than real options, then patent prices are based on the value of an underlying returns index. There are five essential elements that contribute to the value of a patent: (1) the cash flows to the patented innovation, (2) the length of time to patent expiration, (3) the post-patent investment required to generate cash flows, (4) the volatility of the underlying cash flows, and (5) the risk-free interest rate. How these elements interaction to influence patent value, and the model used to price the patent, however, depends on the nature and timing of the cash flows and post-patent investment. If investment, and hence option exercise, is assumed to be a one-time event with a specific date, then pricing models for European options are appropriate. If, however, exercise can occur at any date chosen by the patent purchaser, then the value of the option is path-dependent and valuation models for American-type options are more appropriate.

Regardless of the valuation method, however, the core of each approach involves assumptions regarding the path of returns to the innovation (the underlying security in options terminology). Others assume returns to the innovation evolve according to a Geometric

Brownian Motion (GBM) process that is standard in the options-valuation literature, but often not descriptive of the actual process followed by returns. For each model, we assume instead that returns to a demand-side agricultural innovation evolve according to a mean-reverting GBM with a poisson jump process (Merton 1976; Jorion 1989, Naik and Lee 1990; Schwartz 2004). Returns are likely to be mean-reverting as GBM assumes returns can vary away from the mean without bound – an assumption that is untenable in a relatively competitive commodity market in which new innovations are constantly coming to the market and driving prices back toward the average cost of production. Returns are also likely to experience Poisson jumps for both biological and economic reasons. Biologically, new varieties often experience problems only after they have been in the ground for a certain amount of time. Pests, post-harvest degradation or unsuitability to specific soil types are all examples of problems in the past that have reversed the fortunes of once-promising new specialty crop varieties. Economically, the most important reason why returns may experience a negative jump is the development of a new variety that proves to be superior to the previous innovation. Almost by definition, hybrid crops are developed from the best traits of all existing varieties, so success is only achieved if old varieties are rendered obsolete in the consumer’s mind.

Consequently, the most general form of the returns equation is written as:

$$dR_t/R_t = (\kappa(R_t^m - R_t) - \lambda\phi)dt + \sigma dz + \phi dq, \quad (1)$$

where  $\kappa$  is the rate of mean reversion per unit of time,  $dt$ ,  $\sigma$  is the standard deviation of the diffusion process,  $dz$  is an increment of a standard Weiner process with zero mean and variance equal to  $dt$ ,  $R_t$  is the cash flow from the new product with mean  $R_t^m$ , jumps occur according to a Poisson process  $q$  with average arrival rate  $\lambda$  and a random percentage shock,  $\phi$ . The random shock, in turn, is assumed to be log-normally distributed with mean  $\phi - 0.5\delta^2$  and variance,  $\delta^2$ . The Poisson process  $q$  describes a random variable that assumes a value

of 0 with probability  $1 - \lambda$  and 1 with probability  $\lambda$ .

Estimates of (1) are obtained by maximum likelihood estimation over the entire sample data set, using the likelihood function:

$$L(R_t|\theta) = -T\lambda - \frac{T}{2} \ln(2\pi) + \sum_{t=1}^T \ln \left[ \sum_{n=0}^N \frac{\lambda^n}{n!} \frac{1}{\sqrt{\sigma_t + \delta^2 n}} \exp \left( \frac{-((dR_t/dt)/R_t - \kappa(R_t^m - R_t) - n\phi + \sigma/2 + n\delta^2/2)^2}{2(\sigma + \delta^2 n)} \right) \right], \quad (2)$$

where  $n$  is a number of jumps that spans the number of observed shocks in the data (Jorion 1989). In (2) we approximate the change of  $dR_t/dt$  with a discrete change:  $dR_t = R(t) - R(t - 1)$ . In the next section, we show how parameter estimates from (2) are then used to forecast returns to the new variety and, hence, determine equilibrium prices for patents on the innovation. Given this stochastic returns process, the real option implied by this process is then valued using well-understood Monte Carlo option valuation techniques as explained next.

### 3.3 Patent Pricing Under Risk-Neutral Valuation

Proper pricing of patents is critical for their successful trade. If the prices at which patents are licensed is somehow wrong from the perspective of the buyer or seller, then the likelihood of an active market for agricultural innovations developing in the future is very low. If the uncertainty inherent in licensing a new variety represents a hedgeable risk, or one that growers can transfer by trading an underlying futures contract, then it would be possible to price patents using a traditional, no-arbitrage, Black-Scholes pricing model. However, innovations are not tradable assets. Without an effective hedge, it is necessary to consider the market price of risk and devise a way of estimating its impact on patent prices.

We account for the market price of risk using the risk-neutral valuation model of Cox, Ingersoll and Ross (1985). Applying this model involves a three-stage algorithm. First, the

returns process must be reduced to a martingale,  $Q$ , (essentially, a zero-drift stochastic process) by estimating the distribution governing the diffusion of returns and removing all systematic components from the observed process. This step – "risk neutralizing" the process – means that the best guess of returns at time  $t_1$  is its value at  $t_0$ , or:  $E[R_{t_1}] = R_{t_0}$ . By removing the predictable components of each part of the returns process, we change the Weiner process  $dz$  to  $dv$ , where  $v$  is a  $Q$ -Weiner process (Alaton, et al. 2004). The second step consists of forming an expectation of the intrinsic value of the patent under the  $Q$  measure defined by the risk-neutralized process. In the third step we discount the expected payoff value back to the current date at the risk-free rate. This discounted expected payoff is the market equilibrium price of the patent.

More formally, given a constant market price of risk, the martingale that defines total (deterministic and random) time-variation in the underlying returns index becomes:

$$dR_t/R_t = dR_t^m/R_t^m + (\kappa(R_t^m - R_t) - \lambda\phi - \psi_t\sigma)dt + \sigma dv + \phi dq, \quad (3)$$

where  $dv$  is now a  $Q$ -Wiener process and  $\psi_t$  is the market price of risk, expressed on a per unit basis. With this function, we then use the parameters estimated above to find the expected returns value at an "expiry" date  $T$ , given a value for the market price of risk. Finding the market price of risk, however, represents a significant empirical problem.

Typically, researchers attempt to calibrate the market price of risk using price series from similar instruments that are traded on organized exchanges. For new apple varieties, however, no such exchange exists, nor do we anticipate that the market will develop sufficient to support such an exchange. Nonetheless, we are able to simplify the problem somewhat. It is a basic tenet of asset pricing that a portfolio of two derivatives written on  $R_t$  can be constructed such that their combined return is equal to the risk-free rate. Thus, if we define the rate of drift in (3) as  $\mu = dR_t^m/R_t^m + (\kappa(R_t^m - R_t) - \lambda\phi)$ , the return to the risk-neutralized process must be equal to the risk-free rate:  $\mu - \psi_t\sigma_t = r$ . Using any asset pricing model – the

discrete-time capital asset pricing model (CAPM) for example – it must also be the case that the return to any particular asset must be equal to the risk-free rate plus a security-specific market-risk premium:  $\mu = r + \beta(r_m - r)$ , where  $r_m$  is the return to the market portfolio, and  $\beta$  measures the systematic risk of the security. In the CAPM, however, we know that  $r_m - r = \psi_t$  so the risk premium to any asset is determined by the market price of risk and the security-specific measure of systematic risk. Systematic risk, in turn, depends on the covariance of asset and market returns and the variance of market returns:  $\beta = \sigma_{BM}/\sigma_m^2$ , so any security with returns that are statistically independent of the market must have a zero market price of risk. Because this is indeed likely to be case for the returns to new varieties of apple, we set  $\psi_t = 0$  in (3) and calculate the equilibrium price by discounting the expected terminal value of the patent at the risk free discount rate. This terminal value, however, depends critically upon the assumed expiry date and, in fact, if one exists.

### 3.4 Empirical Patent Pricing Model: European-Option

The theoretical framework described in the previous section is used to price a complete chain of patent prices for a hypothetical new apple variety, where the chain is defined over a number of discrete expiry dates. Given the underlying returns index and a time to expiration, the other elements needed to price patents on new apple varieties are the designated "strike" returns level and the risk-free rate. For any real option, the strike returns level is defined as the amount of the investment required to exercise the option, or to plant trees and generate positive returns. In this section, we assume this decision is made at one point in time. In the next section, we consider a more general model in which the option to invest, or to abandon the license, can be made at anytime. For illustrative purposes, we use investment amounts estimated for a new variety of apple in Washington State (Gallardo and Galinato 2012) and estimate cash flows using net returns for Cripps Pink apples in Washington State (WGCH 2012) . We assume a risk-free rate of interest,  $r$ , of 3%, which is reflective of short-term

interest rates in the fall of 2012. However, it is important to note that the choice of the risk-free interest rate is not one of the more important variables influencing the value of the option.

In the absence of a closed-form solution to the option pricing problem, Monte Carlo simulation procedures are used to estimate the fair value of call options at each strike level for various times to expiry. Monte Carlo simulation has been used extensively in the literature in valuing options as it is an effective and easily generalizable way to value an option where the underlying index follows a complex process. The steps in the Monte Carlo simulation are as follows. First, the temporal  $Q$ -Wiener process in equation (3),  $dv$ , is specified as  $\epsilon_t\sqrt{t}$  where  $\epsilon_t \sim N(0, 1)$  and  $t$  is the time to expiration of the option expressed in days. Second, the jump diffusion process described in equation (3) is also modeled within the same Monte Carlo algorithm, where the two stochastic elements of the jump diffusion process are the arrival rate and the distribution of the random shock. Hence, for a given time to expiration  $t$ , a Monte Carlo simulation is run using 10,000 draws from the distribution of  $\epsilon_t$ , the distribution governing the arrival rate of the jumps in the jump diffusion process, and the distribution of the random shock. The Monte Carlo simulation produces a distribution of option payoff values as expressed in equation (3). The mean of the payoff distribution is then discounted back to the present by the time to maturity  $t$  using rate  $r$  yielding the option value. Therefore, in general, the value of the call option at a given expiry date  $t$  and strike level  $x$ ,  $C(t)$ , can be expressed as:

$$C(t) = e^{-rt} \int_x^{\infty} f(R_t)(R_t - x) dR_t, \quad (4)$$

where the integral is approximated using the Monte Carlo algorithm. We calculate the value of the option for a range of parameter values: For the expiry date, the mean rate of returns growth, the rate of mean reversion and the volatility of the returns series. In this way, we determine the "Greeks" for our numerical option procedure and compare to the values

obtained under a continuous-exercise alternative.

### 3.5 Empirical Patent Pricing Model: American-Option

If the option can be exercised at any date chosen by the patent holder, then the valuation problem becomes an optimal stopping problem. In other words, at each potential exercise date the holder is assumed to compare the immediate returns to exercise with the discounted value of cash flows under continuation (non-exercise). As soon as the returns to exercise exceed the expected present value of returns from continuation, the holder will optimally exercise the option, or use his or her right to invest in the new apple variety. Because the exercise date depends upon current versus expected discounted future returns, the option is path-dependent so the standard approach for European options described above cannot be used. Rather, we use the Least Squares Monte Carlo Simulation (LMS) approach of Schwartz and Moon (2001), Longstaff and Schwartz (2001), Schwartz (2004) and Miltersen and Schwartz (2004) to generate approximate values for the patent under a number of different parametric assumptions.<sup>2</sup> We compare the value of the patent calculated this way to the value calculated under the European expiry assumption above in order to obtain an estimate of the value of the "early exercise" option inherent in a patent for any long-lived investment in a new plant variety.

LMS valuation is by now well understood as an accepted approach to path-dependent option valuation so we only describe the intuition that underlies the algorithm here.<sup>3</sup> The fundamental problem addressed by the LMS algorithm is that when exercise is at the discretion of the holder, the value of the option depends on the conditional expectation of cash flows under continuation. Simulating the path of net returns using Monte Carlo methods

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<sup>2</sup>Others developed similar approximation models for pricing path-dependent options (Carriere 1996; Broadie and Glasserman 1997a, b, c; Broadie, Glasserman and Jain 1998; Broadie et al. 1998) but the LMS algorithm represents the simplest and fastest algorithm to date.

<sup>3</sup>Interested readers are referred to Longstaff and Schwartz (2001) for formal proofs that the option values that emerge approximate arbitrage-free prices consistent with the logic underlying any option value model (Black and Scholes 1973; Merton 1973).

and using least squares to estimate the cash flows under continuation produces best, linear, unbiased estimates of the conditional expectation of returns across the entire distribution of possible returns paths. Once the incremental investment required to continue the project is greater than the present value of the present value of expected returns, the option to abandon is exercised the value of the project is zero from there forward. Determining the value of the option under each path using backwards induction solves the optimal stopping problem and, when these values are discounted back to the present at the risk-free rate of interest and averaged across all returns paths, produces an approximation to the current value of the option. When the number of possible exercise dates is large, the algorithm is relatively complex and the values likely to diverge significantly from those expected under a fixed-exercise (European) assumption. In our application, however, we assume a relatively small number of potential exercise dates (monthly), both for realism and tractability.

Specifically, growers are assumed to be able to exercise their option under the patent on the first of each month under patent-expiry assumptions of one year, three years, five years, ten years and fifteen years. Although these expiry choices are admittedly arbitrary, they reflect the range of dates at which a decision must be made whether to continue to invest in developing the new variety, or abandoning the patent in favor of either an existing or another new variety. In our stylized model, therefore, we have exercise dates of  $t = 1, 2, 3 \dots T$ , after which time the new trees become fully bearing and cash flows reflect the full profitability of the new variety.

Our assumed process for cost-to-completion is based on Schwartz (2004). Because actual costs are not observable, the process cannot be estimated as it is for returns. Therefore, we base our cost process on reasonable assumptions regarding volatility and single-period observations of establishment and production costs for a similar type of apple (Gallardo and Galinato 2012). Specifically, the cost-to-completion process is assumed to be stochastic and is written:



$$dK = -I dt + \sigma(IK)^{1/2} dt, \quad (5)$$

where  $I$  is the periodic investment (control) and  $K$  is the cost to completion, or the total amount of capital required (Schwartz 2004). In this expression, cost-to-completion falls at a rate  $I$ , but varies according to the degree of "technical uncertainty" (Pindyck 1993) that is resolved through further investment, learning and experience with the new variety. While investing in a new apple variety involves less technical difficulty than developing a new drug or electronic device, variation in soil type, moisture, pests and post-production problems all represent sources of uncertainty that are unique to agricultural products. Uncertainty of the agronomic form is nonetheless resolved in the same way as other types of technical uncertainty: Through learning and experience.

Based on this cost process, the LMS algorithm proceeds as follows. In the first step, we forecast  $N$  paths of  $T$  returns using the process described in (3).<sup>4</sup> Next, we generate  $N$  paths of  $T$  values for the cost-to-completion using (5). In the third step, for all paths in which investment has not been completed, nor has the investment already been abandoned, we estimate the conditional expected return at each possible exercise date by regressing the value of the project (discounted) on a series of basis functions of the state variables of the problem (current returns). While Longstaff and Schwartz (2001) describe a range of basis functions that can serve this purpose, we follow Schwartz (2004) and use a high-degree polynomial function (six terms). We then compare the fitted values from these regressions to the incremental value of investment required to maintain the project at each possible exercise date. If the amount of investment required is greater than the conditional expected value, the option to abandon is exercised immediately and the value of the project is zero from that point forward. We follow this process recursively, from the last exercise date to the first, to determine the optimal stopping point for the investment. Fourth, we determine the

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<sup>4</sup>Our Monte Carlo simulation uses 10,000 paths.

value of the option along each path, which can take on three types of values: (1) a multiple of the cash flows expected at the expiry date of the patent if the project is never abandoned, (2) the value of the project at the optimal stopping point, before the decision to abandon is taken, or (3) zero if the project is abandoned immediately. For the fifth and final step, we discount the value determined in step 4 to the current period at the risk-free rate, average the present values across all paths and interpret the result as the optimal patent price.

As in the case of the fixed-exercise model, we calculate these "American" patent price over a range of values for the returns growth rate, the rate of mean reversion, and the volatility of the returns series. Although there are a number of other parameters that may be of interest, these three (in addition to the expiry date) represented a minimal set that describe the most important differences between European and American-option assumptions for patent prices.

## 4 Data Description

There are few fruit patented fruit varieties with a sufficient shipment history to allow estimation of a representative price process. Pink Lady (also known as Cripps Pink) apples, however, provide a unique opportunity to conduct a case-study of how proprietary rights to an apple variety should be valued in the market. Our price data are from the Washington Growers Clearing House (WGCH) which collects detailed, monthly data on FOB prices and shipments from the Yakima Valley in Washington State. The WGCH price and shipment data describe a sample period of 134 months between 2000 and 2011 – a period sufficiently long to allow the Cripps Pink apple to establish a reputation in the consumer market, earn super-normal profits for growers licensing trees, and thereby to attract other new, competitive apple varieties. In particular, our data spans the period during which the Honeycrisp apple was introduced (beginning in September 2009). Honeycrisp is another proprietary variety licensed by the University of Minnesota to a number of grower cooperatives throughout the

U.S. Honeycrisp apples immediately established a reputation among consumers for a crisp texture, sweet taste and large size that are apparently preferred. FOB prices for Honeycrisp apples have averaged nearly three-times the levels achieved by Cripps Pink apples over a similar time period, and have taken market share from all apples, whether proprietary or not. In terms of the price process for Cripps Pink apples described above, the introduction of Honeycrisp constitutes a discrete event that we model through the Poisson-jump term.

Table 1 provides some summary evidence regarding the price-path and shipment levels for our new apple variety grown in Washington state, while figures 1 and 2 show the price and quantity paths, respectively, for both Cripps Pink and Honeycrisp apples. The data in this table and figures are relatively typical for a new apple variety: Both prices and shipments begin from a low level and grow to somewhat of a steady-state level over time, but exhibit considerable volatility both within and between years. Figure 1 suggests that the introduction of Honeycrisp in 2009 may represent a significant competitive event to growers of Cripps Pink apples as Cripps Pink were no longer the newest apple in the store, but this simple graphic may reflect other factors as well. We test this more formally below.

[table 1 in here]

[figures 1 and 2 in here]

We augment the price and shipment data with establishment and production cost data for comparable apples (Gallardo and Galinato 2012). Although our focus is on Cripps Pink and we parameterize the stochastic returns process with Cripps Pink data, no establishment cost or production cost data were available for this variety. Therefore, we assume the cost per tree values for Cripps Pink are the same as for Honeycrisp, and scale the per acre cost values to observed densities for Cripps Pink. Conversations with extension economists suggested that this was a reasonable way to proceed in the absence of data specific to Cripps Pink. We assume that the cost-to-completion is entirely invested over a five-year period, and that the patent expiry date is either 1, 3, 5, 10 or 15 years in the future. Since 1994, patents extend

for a period of 20 years after initial filing, but licenses are often not granted until well after the initial filing period. We demonstrate the sensitivity of patent prices to variations in the expiry date in the results section that follows.

## 5 Results and Discussion

In this section, we present the results obtained from estimating the stochastic process for Cripps Pink apple returns, and test the importance of the introduction of Honeycrisp apples. After establishing the time-series properties for our focus variety (Cripps Pink) we then present the results from the risk-neutral patent valuation algorithm and compare the results with those obtained under alternative assumptions that require path-dependent valuation methods (Longworthy and Schwartz 2001). We complete the section with a sensitivity analysis to key process parameters, most importantly the timing and importance of the development of a competing apple variety (Honeycrisp).

Table 2 presents the parameter estimates for the stochastic price-process in (2). We first convert prices to per-tree net returns by subtracting the per-pound costs of production and multiplying by a fixed per-tree yield value. We then remove the month-and-annual fixed effects that are evident from figure 1 and test for the effect of Honeycrisp introduction on Cripps Pink apple returns. Whether accounting for the Honeycrisp introduction through either a binary variable that assumes a value of 1 after Honeycrisp was first introduced, or by including Honeycrisp volume directly, the result was the same. Namely, after controlling for these other temporal factors, the introduction of Honeycrisp did not have a significant effect on Cripps Pink returns. We interpret this result as implying not that Honeycrisp was unimportant, but rather that the effect is picked up by the month- and annual-fixed effects.

[table 2 in here]

Consequently, the parameters in table 2 do not include a specific variable measuring the introduction of a competitive apple, but rather a generic shock that captures either positive or

negative shocks to the price process. We begin by estimating the most parsimonious version of (2) and test against successively more comprehensive versions. Comparing a Brownian motion to a mean-reverting Brownian motion process with a likelihood-ratio (LR) test yields a chi-square statistic of 22.89 (critical value with 1 degree of freedom is 3.841) so we easily reject the simpler specification. Next, we compare a mean-reverting process to one that includes a Poisson jump term, again using a LR test. This comparison yields a chi-square statistic of 87.516 (critical value with 3 degrees of freedom is 7.815) so we again reject the less-comprehensive specification. Therefore, the last two columns in table 2 represent the parameters used for our option-pricing algorithm. These parameters imply that Cripps Pink returns diminish by approximately \$0.10 per tree per month, variations away from the mean returns diminish only very slowly over time, and that there is a roughly 36% probability of returns falling by \$0.34 per tree during any given month. In the analysis below, we conduct sensitivity analysis with respect to these key parameters.

The results in table 3 show patent prices obtained under our base scenario, and their sensitivity to variation in returns growth.<sup>5</sup> We show both American and European patent prices for comparison purposes. The base scenario is defined as the combination of expiry dates and growth rates that most nearly approximates that observed in practice. By comparing our estimated patent price with actual license contracts, we can get a sense of how growers are actually bidding for patents, that is, whether they are bidding as if there is a fixed exercise date, or whether the option is instead path-dependent. Although there is no public data base of license prices, recent experience suggests that licenses are valued at approximately \$2.15 per tree, which consists of a \$1.00 fixed fee and 5% of revenues. Because licenses tend to be sold several years after the patent is issued, the most realistic base scenario assumes a growth rate of  $\mu = -0.10$  and a time-to-expiry of  $T = 10$ . Based on these assumptions, the patent

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<sup>5</sup>The growth rate in the base scenario is assumed to be -0.06, which is close to the estimated rate of -0.05. These values were chosen as they present the best illustration of the option strings obtained over a large number of growth-rate values.

price under a fixed exercise date assumption (European) is \$0.711, while it is \$2.198 under a continuous-exercise assumption. Clearly, the American-option assumption more nearly approximates that observed in practice. Growers, however, appear to impute a faster rate of returns-diminition than that estimated with the Cripps Pink price process. Price estimates for other growth rates and expiry dates show a similar, large difference between the values obtained under European and American exercise assumptions. We interpret this difference as measuring the value of the flexibility of being able to abandon the investment at any time prior to completing the investment.

[table 3 in here]

Examining the pattern of option prices under alternative growth-and-expiry assumptions reveals an interesting comparison between the European and the American models. While the fixed-exercise assumption yields a very conventional pattern of prices as time to expiry rises – patent prices fall uniformly – under continuous exercise we see something quite different. For low growth rates – 10% or below – patent prices similarly fall with the time to expiry as expected. However, when returns fall more slowly, the value of the patent can actually rise as the time to expiry increases. After the investment is fully paid off (5 years), all returns to growing the apple are pure profit (over investment costs), so when returns stay positive for a longer period of time, it is possible that the present value will in fact rise with the expiry date. Said differently, the option to abandon becomes more valuable as the opportunity cost of doing so rises. Second, note how the patent price is concave in the rate of growth for the 10 and 15-year expiry dates, but diminishes linearly for 1, 3 and 5 year expiry dates, and all of the European-option scenarios. Linearly-declining patent prices with the rate of growth is the conventional outcome, so what is different about the long-horizon American option prices? When buying a patent, there are essentially two assets of value: a right to the future stream of income to the variety, and the right to abandon the project should it become unprofitable. For a fixed expiry date, and for dates prior to the pay-off

date for continuous exercise, the value of the patent is dominated by the former: A smaller stream of income will lower the value of the patent. After the cost-to-completion is paid off, however, the value of being able to abandon the project rises as the growth rate of returns falls. The larger the avoided-loss, the larger the value of the option to do so.

Other sensitivities also highlight the sharp contrast between path-dependent and non-path-dependent patent prices. We allow the rate of mean reversion ( $\tau$ ) to vary in table 4, and observe a similar pattern to that shown in table 3. Namely, with a fixed expiry date, prices fall uniformly as the rate of mean reversion rises across all expiry dates. This is to be expected as negative reversion to the mean implies that returns can grow without bound in response to a positive shock. Because the optionality of a patent allows growers to avoid the opposite occurrence – returns that fall – the option price must be higher for negative rates of mean reversion. If we allow for continuous exercise, however, we again see that patent prices fall and then rise in the rate of mean reversion for expiry dates of 10 and 15 years. As in the previous case, these expiries are beyond the point where the cost-to-completion is paid off, so option prices can be higher with faster mean-reversion because this implies greater stability. Option values are generally lower with less volatility.

[table 4 in here]

We examine how general this volatility effect is by varying the standard deviation of the returns process over a range of plausible values. These results are shown in table 5. Again, the non-path-dependent prices show very conventional sensitivities in most cases. Greater volatility leads to higher license prices at all expiry dates, but prices rise with time to expiry for the highest volatility rates in the table (0.20 and 0.25). Beyond a certain point, the higher option premium associated with truncating the distribution of returns at the negative end outweighs the effect of discounting. Path-dependent prices, on the other hand, show highly non-linear sensitivities with respect to volatility. At each expiry date, prices are concave in volatility. That is, prices rise to a certain point and then fall if volatility becomes too great.

Two effects are at work here. First, note that all prices are higher than their corresponding non-path-dependent values because of the value of the ability to exercise the option any time the conditional expected returns fall below the incremental investment required. This value rises in volatility because the probability of the conditional expected return falling below the incremental investment value higher for greater levels of volatility. Beyond a threshold point, however, the likelihood of abandoning a potentially-viable project early rises. Recall that the value of an abandoned project is zero. Therefore, the higher the level of volatility, the more abandoned projects are averaged into the Monte Carlo pricing algorithm and the value of the option falls accordingly. Prices are also concave with respect to expiry. With zero volatility, the conventional result obtains: Longer times to expiry cause the option price to fall due to the discounting process. At higher levels of volatility, and low rates of mean-reversion, however, there is a greater probability that returns will grow quickly in the future. Higher conditional expected returns increase the value of continuation, or of not abandoning the project, and hence, the higher threshold value before the option is exercised.

[table 5 in here]

Our findings hold many implications for both innovators and licensors. Because most markets for licenses to agricultural innovations are thin, there is no "market" price as there is no liquidity without competitive bidding. Consequently, innovators need a model to determine what an economically-justifiable price would be. Our model provides an estimate of what a justifiable price would be. Moreover, we show that simple application of conventional, European-option pricing models like the Black-Scholes, would dramatically undervalue the innovation if growers are indeed able to exercise their option under the patent in a continuous way. Under our "most likely" parametric assumptions, innovators would earn roughly three-times the revenue under an American-option pricing model relative to an European-option alternative. On the buying side, growers should enter negotiations similarly well-informed of the true value of what they are bidding on. Because our model provides a transparent,



mutually-agreeable accounting for the economic value of licenses, our findings may allow for greater liquidity in the market for university technology licenses. Our comparative static results also inform participants on both sides of the market the precise consequences of faulty assumptions regarding the market for the innovation. Bidders cannot assume simple linear (or even monotonically increasing or decreasing) relationships between prices and such variables as volatility, drift, mean-reversion and time to expiry. Rather, bidding on licenses requires that bidders obtain accurate information and adjust their bids accordingly.

## 6 Conclusion

This study investigates the optimal pricing of licenses on university-created agricultural innovations. Our pricing model relies on the assumption that patents provide the holder the right but not the obligation to either undertake the investment required to bring the innovation to market, or to abandon it before fully committing the necessary capital. As such, patents are derivative securities, or options, on the underlying innovation. Unlike options with a fixed exercise date, however, the option implied by holding a patent can be exercised any time at the discretion of the holder. Consequently, the value of licenses on patents to an agricultural innovation are path-dependent, or depend on the entire history of returns and not just those that prevail on one specific day.

We develop risk-neutral option pricing models under both a fixed-exercise (European) and a path-dependent (American) assumption, and compare their values both to each other and to those observed in real-world bidding. We estimate the stochastic process that underlies each model using data from a proprietary apple variety that was developed a number of years ago (Cripps Pink) and has been sold commercially for over 10 years. Because the process that best fits the Cripps Pink data is a mean-reverting Brownian motion with drift and a Poisson jump term, standard Black-Scholes option pricing models are not available. Therefore, for the fixed-exercise model, our pricing model uses a Monte Carlo solution algorithm, while

we use a least squares Monte Carlo (LMS, Longstaff and Schwartz 2001) model for the path-dependent option.

We find that the path-dependent model most nearly approximates the license prices observed in reality. If this is indeed the case, then neither buyers nor sellers in the license market can rely on conventional option "Greeks" or comparative statics to key model parameters. While conventional sensitivities tend to be monotonic in either volatility (except in the case of volatility smiles), mean-reversion or growth-rates, we find that the sensitivities for path-dependent option prices are highly non-linear and are indeed convex (concave) over ranges that likely span values that can arise in the real-world. Consequently, developing a more liquid market for licenses to agricultural innovations requires broad dissemination and agreement on a model similar to ours.

Future research in this area should consider other agricultural products beyond the apple variety considered here. Many different types of innovation are being licensed by universities, and the license prices will be unique to each. Further, our model makes many parametric assumptions in the absence of specific data on cost-to-completion and production cost. More accurate data on the fundamental profitability of each variety and the investment required to bring each to production would be necessary for wide-spread application of this type of pricing model. Finally, more institutional data on the nature of the licensing process would be helpful. While we understand that licenses are sold many years after the patent is applied for, we have no specific information on the actual lag between filing the patent and selling the license. This information will be critical in determining license prices sufficiently accurate to allow a trade to develop.

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Table 1: Summary of Pricing Data and Parameter Assumptions

	Units	Mean	Std. Dev.	Min.	Max	N
Volume	40 lb. cartons	83.851	82.192	0.000	301.00	134
Price	\$ / carton	27.507	10.514	11.31	65.53	134
Production Cost	\$ / carton	8.28				
Establishment Cost	\$ '000 over 5 years	85.82				
Yield	lbs. / tree	35.03				
Density	trees / acre	723				
Risk Free Rate	% / year	3.00				
Cost-to-Completion Volatility	% / year	0.200				

Source: Gallardo and Galinato (2012) and Washington Growers Clearing House (2012).

Table 2: Returns Process Estimates: Cripps-Pink Apples, WA State

	Brownian Motion		BM / Mean Reversion		BM / MR / Jump	
	Estimate	t-ratio	Estimate	t-ratio	Estimate	t-ratio
Growth Rate	0.3642*	24.6800	0.0056	0.1188	-0.1030*	-2.2556
Volatility	0.0056	0.1060	0.3066*	8.3881	0.0249*	4.5934
Mean Reversion	N.A.	N.A.	-0.3127*	-5.0841	0.0030	1.4736
Poisson Probability	N.A.	N.A.	N.A.	N.A.	0.3648*	8.6636
Variance of Jump	N.A.	N.A.	N.A.	N.A.	0.9429*	6.5949
Magnitude of Jump	N.A.	N.A.	N.A.	N.A.	-0.3448	-1.8197
LLF	-121.5420		-110.0970		-66.3390	

Note: A single asterisk indicates significance at a 5% level.

Table 3: Patent Price Estimates and Growth Sensitivity

American Exercise Option Pricing Model					
Growth Rate	Time to Expiry				
	1 Year	3 Years	5 Years	10 Years	15 Years
-0.14	5.473	4.553	2.772	2.286	2.228
-0.12	5.705	5.224	3.548	1.627	1.505
-0.10	5.812	5.808	5.613	2.198	1.232
-0.08	5.998	6.217	7.065	6.424	4.354
-0.06	6.124	6.483	7.912	9.493	9.580
-0.04	6.136	6.637	8.251	10.040	10.044

Fixed-Expiry Date Option Pricing Model					
Growth Rate	Time to Expiry				
	1 Year	3 Years	5 Years	10 Years	15 Years
-0.14	1.237	0.999	0.805	0.464	0.263
-0.12	1.261	1.063	0.893	0.575	0.367
-0.10	1.285	1.129	0.991	0.711	0.508
-0.08	1.310	1.201	1.098	0.877	0.701
-0.06	1.335	1.275	1.216	1.081	0.962
-0.04	1.361	1.354	1.347	1.331	1.321



Table 4: Patent Price Estimates and Mean Reversion Sensitivity

American Exercise Option Pricing Model					
Rate of Mean Reversion	Time to Expiry				
	1 Year	3 Years	5 Years	10 Years	15 Years
0.15	5.801	5.424	3.823	1.658	1.522
0.1	5.805	5.584	4.431	1.454	1.283
0.05	5.806	5.706	5.034	1.538	1.152
0	5.813	5.816	5.649	2.256	1.247
-0.05	5.819	5.898	6.109	3.542	1.936
-0.1	5.829	5.967	6.439	5.288	3.989
-0.15	5.836	6.041	6.653	6.401	5.973

Fixed-Expiry Date Option Pricing Model					
Rate of Mean Reversion	Time to Expiry				
	1 Year	3 Years	5 Years	10 Years	15 Years
0.15	1.283	1.098	0.896	0.421	0.131
0.1	1.284	1.109	0.933	0.532	0.246
0.05	1.284	1.121	0.965	0.632	0.385
0	1.285	1.129	0.992	0.715	0.515
-0.05	1.286	1.139	1.015	0.779	0.616
-0.1	1.287	1.147	1.034	0.828	0.685
-0.15	1.288	1.154	1.051	0.863	0.731

Table 5: Patent Price Estimates and Volatility Sensitivity

American Expiry Option Pricing Model					
Volatility	Time to Expiry				
	1 Year	3 Years	5 Years	10 Years	15 Years
0	5.445	5.161	4.038	1.531	1.216
0.05	5.892	5.921	6.343	4.473	3.087
0.1	6.027	6.138	6.985	6.599	6.227
0.15	6.119	6.211	6.432	8.109	7.013
0.2	5.990	6.006	6.585	6.890	6.809
0.25	5.879	5.832	5.411	6.723	6.600

Fixed-Expiry Date Option Pricing Model					
Volatility	Time to Expiry				
	1 Year	3 Years	5 Years	10 Years	15 Years
0	1.269	1.086	0.928	0.623	0.416
0.05	1.301	1.175	1.058	0.811	0.621
0.1	1.334	1.272	1.204	1.051	0.923
0.15	1.368	2.377	1.369	1.359	1.337
0.2	1.402	1.492	1.554	1.759	2.066
0.25	1.437	1.616	1.762	2.283	3.224

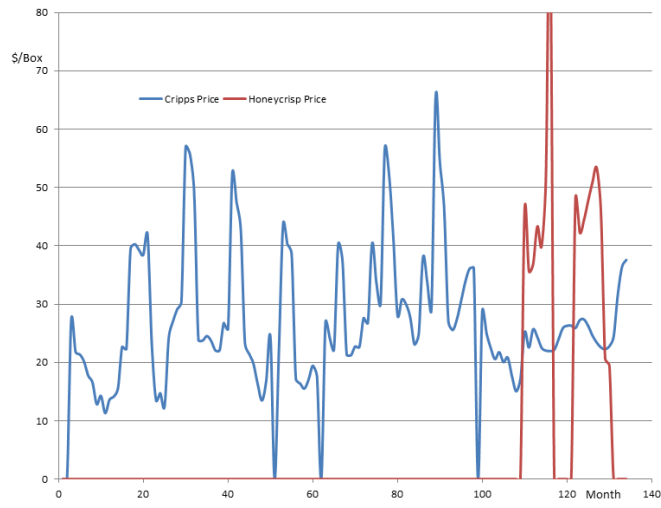


Figure 1: Price Paths for Cripps Pink and Honeycrisp Apples: 2001 - 2012

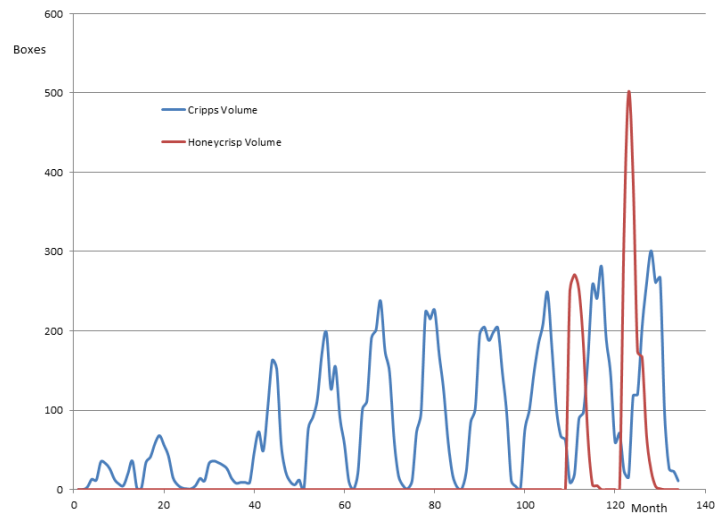


Figure 2: Shipments for Cripps Pink and Honeycrisp Apples, 2001 - 2011