JANUARY 96

ALLOCATION OF WATER AMONG AGRICULTURAL AND URBAN USERS IN ISRAEL

by

U. Chakravorty, E. Brill and E. Hochman*

*For presentation at the AARES 40th Annual Conference, University of Melbourne, 11–16 February, 1996. The authors are Associate Professor, University of Hawaii; Graduate Student and Professor, Hebrew University of Jerusalem, respectively.

ALLOCATION OF WATER AMONG AGRICULTURAL AND URBAN USERS IN ISRAEL

Introduction

Israel is probably one of the most water scarce countries in the world. Although relative to other countries, it manages its existing water resources very efficiently, supply of new water is extremely limited. The country is longitudinal in shape, with water being carried from the north to the south via a national aquaduct. Desalination and recycling options are under consideration in the built-up areas in the south.

In this paper we develop a stylized, spatial model of water allocation in the urban and agricultural sectors in Israel. The model is in line with previous work on spatial water modelling done by the authors (see for e.g., Chakravorty, Hochman and Zilberman, 1995) although it focusses on pumping costs and alternative sources of water generation, while abstracting from explicitly considering conveyance losses. This is because in Israel the conveyance facilities are already in place and conveyance costs are fixed and may be considered "small" compared to the cost of pumping water over relatively large distances.

The following section develops a simple spatial optimal control model of water allocation in Israel. The next section provides an outline of how bargaining theory may be applied to determine water allocation and pricing for the agricultural and urban sectors. The concluding section discusses welfare effects and policy implications that may be derived from the proposed model.

The Model

The schematic is shown in Fig.1. A fresh water source is located at point A. Water is pumped from A via the aquaduct to an urban center located at B. Water is withdrawn along the canal for use on agricultural land along AB. Residual water from A is supplemented by water from desalination at B. This water is used by the city and then recycled. A fraction of the water is lost during recycling and the remaining is pumped back in the direction BA for use in agriculture. Thus farmers located closer to A are likely to use freshwater while those located closer to B may use recycled water for irrigation. The exact boundary between the two groups will be determined by the model.

The model notation is as follows. Let the amount of water generated at A be denoted by z_0 which is endogenously determined. Agricultural firms are located along a continuum from A to B and withdraw $q(x)$ units of water at each location x where x is the distance measured from A along AB. The production function for agriculture is given by f(q) where f has the usual neoclassical properties: $f'>0$, $f''<0$. The width of the farmed area is assumed to be a constant α. The cost function for generating z_0 units of water is given by $C(z_0)$, and it has the usual properties, i.e., $C'(z_0)>0$, $C''(z_0)>0$. Let the residual amount of water flowing at any location be denoted by $z(x)$. Then the pumping cost of water at each location is $mz(x)$ where m is the constant unit cost of pumping.

At B which is the urban center, S units of water are desalinized at a unit cost of a, where S is a choice variable to be determined. The residual water from farming $z(X)$ and the desalinized water S are used for urban and municipal uses, given by an inverse demand function $D_u^{-1}(S+z(X))$. The water from the urban center is recycled at a unit cost of b, and the residual portion of the water is pumped back into the canal in the direction of A. Let v denote distance from B along BA. Then $v = X\text{-}x$. Firms located along v withdraw q(v) units of water from the canal. Let X^ be the location which divides the agricultural region (see fig.1). That is firms to the left of A use fresh water and firms to the right of A use recycled water. Let Y be the aggregate agricultural output from the entire project. Then the inverse demand function for agricultural output is given by $D_a^{-1}(Y)$. The optimization problem can be written as

$$\max_{q(x),z(X),z_0,S,\hat{X}} \int_0^{\hat{X}} [D_a^{-1}(Y) f(q)\alpha - mz(x)]dx - C(z_0) + \int_0^{X-\hat{X}} [D_a^{-1}(Y) f(q(v))\alpha - mz(v)]dv$$
$$+ \int_0^{S+z(X)} D_u^{-1}(\tau)d\tau - aS - b\beta(S + z(X)) - mz(\hat{X})(X-\hat{X}) \qquad (1)$$

where the first set of terms under the integral sign in the objective function represents the area under the demand curve for agricultural products less the cost of pumping water in the region that uses fresh water. Similarly the second set of terms under the integral sign denotes corresponding terms for the region between X and X^ that uses recycled water. The terms under the last integral sign represents benefit to urban consumers. The remaining terms denote the cost of water generation, the cost of desalination and recycling, and finally the cost of pumping the residual fresh water from X^ to X to be used by the city.

Using the change of variables v = X – x, so dv = – dx and v=0 implies x=X, v=X–X^ implies x = X–X+X^ = X^, the above equation becomes

$$\underset{q(x),z(X),z0,S,\hat{X}}{Max} \int_0^{\hat{X}} [D_a^{-1}(Y)f(q)\alpha - mz(x)]dx - C(z0) + \int_{\hat{X}}^{X} [D_a^{-1}(Y)f(q(X-x)\alpha - mz(X-x)]dx$$

$$+ \int_0^{S+z(X)} D_u^{-1}(\tau)d\tau - aS - b\beta(S+z(X)) - mz(\hat{X})(X-\hat{X}) \qquad (2)$$

subject to the following constraints, where $W = \beta(S+z(X))$, the amount of recycled water available:

$$\int_X^{\hat{X}} q(X-x)\alpha \; dx \leq W \qquad (3)$$

$$\dot{z}(x) = - q(x)\alpha, \; 0 \leq x \leq \hat{X} \qquad (4)$$

$$\dot{w}(v) = - q(v)\alpha, \; 0 \leq v \leq X - \hat{X} \qquad (5)$$

which gives

$$\dot{w}(X-x) = - q(X-x)\alpha, \quad X \leq x \leq \hat{X}. \qquad (5')$$

Then the Hamiltonian can be written as

$$H = D_a^{-1}(Y)f(q)\alpha - mz(x) + D_a^{-1}f(q(X-x))\alpha - mw(X-x) - \lambda(x)q(x)\alpha - \theta(X-x)q(X-x)\alpha \qquad (6)$$

where w and λ are the usual co–state variables attached to the equations of motion (4) and (5').

The first order conditions can be obtained as follows:

$$D_a^{-1} f'(q) = \lambda(x), \quad 0 \leq x \leq \hat{X} \tag{7}$$

$$\dot{\lambda}(x) = m, \quad 0 \leq x \leq \hat{X} \tag{8}$$

$$D_a^{-1}(Y) f'(q(X-x)) = \theta(X-x), \quad X \leq x \leq \hat{X} \tag{9}$$

$$\dot{\theta}(X-x) = m, \quad X \leq x \leq \hat{X} \tag{10}$$

$$D_u^{-1}(S + z(X)) = a + b\beta \tag{11}$$

Solving equations (8) and (10) give

$$\lambda(x) = \lambda(0) + mx \tag{12}$$

$$\theta(X-x) = mx + \theta(X) \tag{13}$$

Conditions (12) and (13) suggest that both shadow prices increase linearly with distance as shown in Fig.2. Thus it follows that freshwater allocated to agriculture will decrease away from the source, and recycled water allocation will also decrease away from the city. The latter condition implies that when

$$x=0,\ \theta(X) = \theta(X),\ \theta(\hat{X}) = \theta(X - (X-\hat{X})) = m(X - \hat{X}) + \theta(X).$$

Finally, the transversality conditions are obtained by differentiating the objective function (2), denoted NB(•) with respect to the following variables:

$$\lambda(0) = \partial(NB)/\partial(z_0) = C'(z_0) \tag{14}$$

$$\theta(X) = -\ \partial(NB)/\partial W = b. \tag{15}$$

$$\lambda(X) = \partial(NB)/\partial z(X) = D_u^{-1}(S + z(X)) - b\beta \text{ [redacted]} \tag{16}$$

Using (11), (16) becomes

$$\lambda(X) = a + b\beta - b\beta \text{ [redacted]} = a \text{ [redacted]} \tag{16'}$$

Now applying (14) and (16') to (12), we get

$$\lambda(X) = C'(z_0) + mX = a \text{ [redacted]} \Rightarrow C'(z_0) = a - \text{[redacted]} mX \text{ [redacted]}$$

therefore

$$\lambda(\hat{X}) = C'(z_0) + m\hat{X} = a - \text{[redacted]} m(X - \hat{X}).$$

Similarly,

$$\theta(\hat{X}) = m(X - \hat{X}) + \theta(X) = m(X - \hat{X}) + b.$$

The boundary between firms using freshwater and those using recycled water is given by X^, which is determined by differentiating (2):

$$\partial(NB)/\partial\hat{X} = D_a^{-1}(Y)f(q(\hat{X}))\alpha - mz(\hat{X}) - [D_a^{-1}(Y)f(q(X-\hat{X})) - mw(X - \hat{X})] + mz(\hat{X}) = 0 \tag{17}$$

The first two terms in (17) represent rents to freshwater at x=X^. The second two terms denote rents to recycled water. Equation (17) suggests that at the boundary X^, there is a wedge between the two rents, which is the marginal cost of pumping the residual amount of water z(X^) one more unit of length (see Fig.3). At X^, the rent from recycled water denoted R_c is higher than the rent from fresh water R_f. Thus the boundary is closer to the city than at the location where the two rents are equal. This makes intuitive sense, since the cost of transporting the residual freshwater to the city must be accounted for, thus farmers will continue to be allocated fresh water until X^, even though the rents from freshwater are lower than from recycled water.

Outline of a Bargaining Solution

An intuitive model of bargaining between the city and the farmers can be presented as in Fig.4. The x and y-axes denote the total suplus accruing to the agricultural and the urban sectors, respectively. The utility possibility frontier is shown. Point 'a' is the disagreement point. If the outcome is 'a' then the farmers will use only fresh water and there will be no use of recycled water in the agriculture sector. Thus there will be no recycling of urban water. This is the present situation in Israel. If the solution is at point 'b', farmers will be able to obtain the urban water at zero cost and will pay the cost of recyling and pumping at each location. If the solution is at point 'c', the city will maximize its profits.

If we assume equal bargaining power for both the city and the farmers, the solution is at point 'd', which can be found by maximizing the area akdj given the constraint on the total amount of water available.

Concluding Remarks

Some implications of the effect of parameters on the optimal solution can be explored. For example, if desalination is expensive, pumping costs are high (increased energy prices), more fresh water may be used for agriculture as well as in the city. In the extreme case, there may be no desalination and X^ will approach X. Alternatively, if fresh water is scarce or costly, the major source of irrigation may be recycled water. Similarly, high recycling costs or lower efficiency will result in more fresh water agriculture. An increase in demand for urban water will imply more desalination, more farmland under recycled water irrigation and less generation of fresh water.

Further work needs to be done in developing the bargaining solution in a more precise fashion. In particular, alternative assumptions regarding ownership of the property rights to the water can be imposed: e.g., the (i) agricultural sector has rights to the fresh water and the urban sector owns the recycled water (ii) the government has rights to both sources of water. Simulations of the alternative solutions will be run using Israeli water supply and demand parameters.

References

U. Chakravorty, E. Hochman and D. Zilberman, "A Spatial Model of Optimal Water Conveyance," *J. of Environmental Economics and Management*, August 1995.

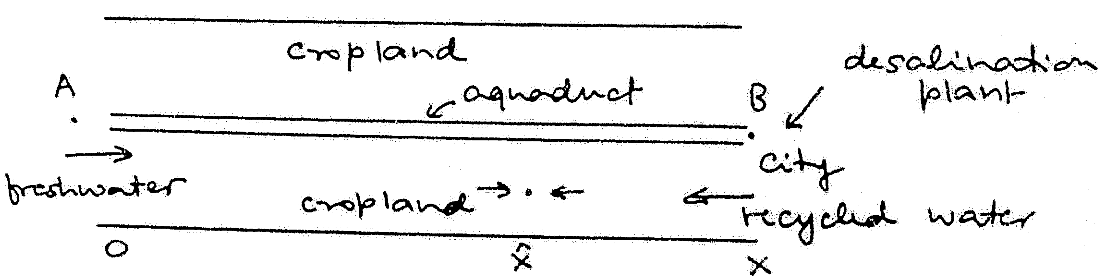

Fig. 1. Schematic

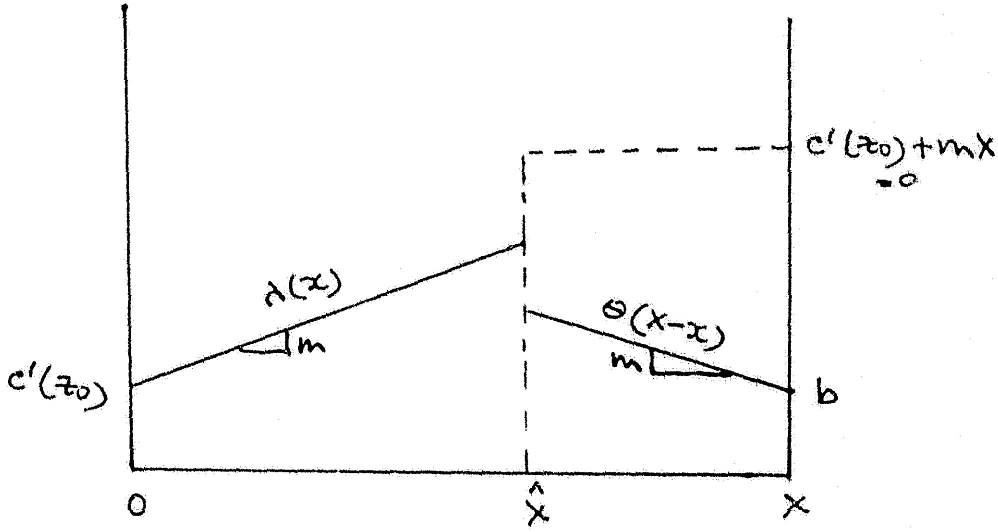

Fig. 2. Shadow prices

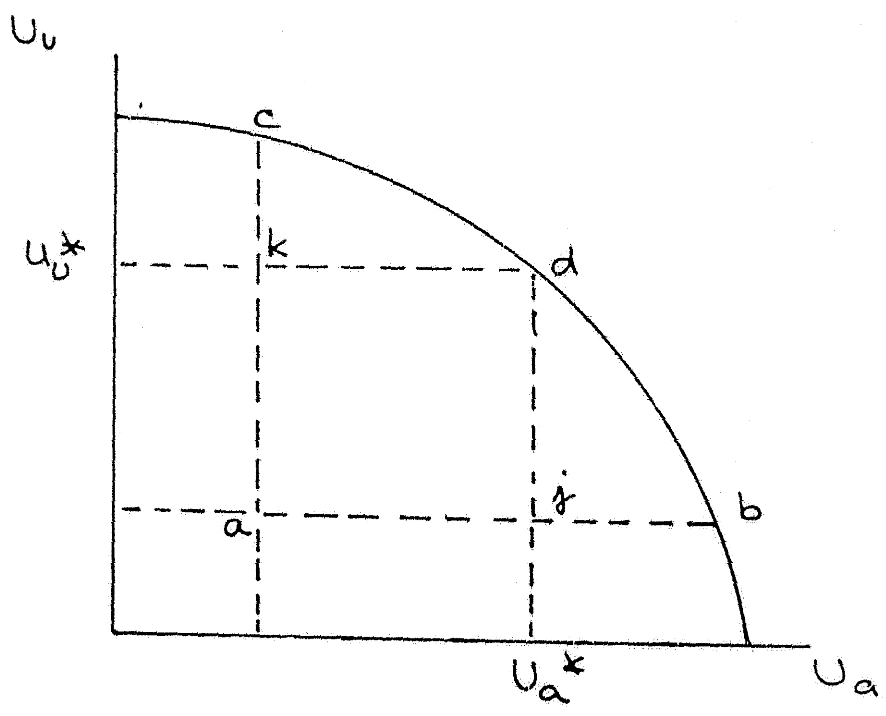

Fig. 4. Possible bargaining solutions

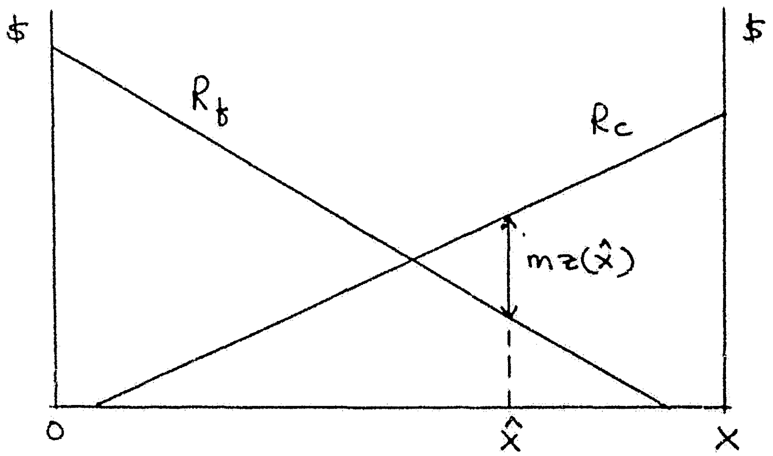

Fig. 3 Rents