On Demand Analysis and Dynamics: A Benefit Function Approach

by

Jean-Paul Chavas

Professor of Agricultural and Applied Economics

University of Wisconsin, Madison, WI 53706

Email: jchavas@wisc.edu.

Selected Paper prepared for presentation at the Agricultural & Applied Economics Association's

2013 AAEA & CAES Joint Annual Meeting, Washington, DC, August 4-6, 2013

On Demand Analysis and Dynamics: A Benefit Function Approach

by

Jean-Paul Chavas

University of Wisconsin Madison

Abstract: This paper develops an economic and econometric analysis of demand dynamics, with an application to US aggregate data over the period 1948-2010. The model builds on duality and the benefit function, which provide strong linkages with the theory. The research involves the specification and estimation of dynamic price-dependent demands as representations of marginal benefits. The analysis uncovers strong statistical evidence of demand dynamics, especially for food. We find that the marginal benefit of food declines with food consumption and that this effect becomes much stronger in the long run. We also find that, while food and service are always complements, the strength of this complementarity relationship increases sharply in the longer run.

Keywords: demand, duality, benefit, dynamics

JEL: D11, D12, C51

On Demand Analysis and Dynamics: A Benefit Function Approach

1. Introduction

Much research has been done on the economics of demand and its empirical investigation. The Almost Ideal Demand System (AIDS) model proposed by Deaton and Muellbauer (1980a, 1980b) provides a way to conduct demand analysis is a way that is consistent with economic theory while being econometrically tractable. And the Quadratic AIDS (QAIDS) model proposed by Banks et al.'s (1997) gives a useful generalization allowing for more flexible income effects. But the analysis has typically focused on a static approach to the investigation of consumer behavior. Yet, dynamics play an importnat role in consumer demand. Examples include habit formation (e.g., Pollak 1970), linkages between demand and health (e.g., Grossman, 1972, 2000) and situations of addiction (e.g., Becker and Murphy 1988; Gruber and Koszegi 2001). At this point, there is a need for a better integration of economics dynamics with the analysis of consumer behavior, both conceptually and empirically. This provides the main motivation for this paper.

The AIDS/QAIDS models have relied on duality properties related to the expenditure function and quantity-dependent demands. This paper takes a different approach: it relies on duality properties associated with the benefit function and price-dependent demands. As argued by Anderson (1980), Barten and Bettendorf (1989) and others, price-dependent demands are inverse demands that treat prices as a function of quantities. They can be motivated in the presence of production quotas or when supply adjustments are slow. In such situations, it is reasonable to conduct demand analysis using price-dependent demands where quantities are treated as predetermined. This paper explores the use of price-dependent demands reflecting marginal benefits for consumers. The approach relies on the benefit function first proposed by Allais (1943) and analyzed by Luenberger (1992). Defining the benefit function as a measure of the consumer's willingness-to-pay to obtain a bundle of goods, Luenberger (1992) and Chavas and Baggio (2010) established two keys results: 1/ the duality between the benefit function and the expenditure function; and 2/ the interpretation of marginal benefits as price-dependent demands (expressing marginal willingness-to-pay as a function of the quantities consumed). This paper uses this framework to specify and estimate price-dependent demand functions. And it builds on this approach to investigate the role of dynamics in consumption behavior.

This raises several issues. First, what is the conceptual basis for analyzing consumption dynamics? Second, how can we specify price-dependent demands that are flexible but remain closely linked with consumer theory? Third, how can we introduce dynamics in demand analysis in a way that is empirically tractable while maintaining strong linkages with the theory? The contributions of this paper are to provide a positive and constructive answer to each of these three questions. Another contribution is the econometric application to aggregate US demand. Besides illustrating the usefulness of the approach, the application documents the importance of dynamics and provides new and useful information on the nature and magnitude of temporal adjustments in consumption behavior.

This paper is organized as follows. First, a conceptual model of consumer dynamics is developed. This is presented in section 2, where the dynamics is captured by state variables that represent physical capital, human capital, health, habit and/or addiction. Section 3 reviews duality theory and its usefulness in demand analysis. It includes the linkages established by Luenberger (1992) and Chavas and Baggio (2010) between consumer preferences, the benefit function and the evaluation of marginal benefits as price-dependent demands. The specification and estimation of dynamic price-dependent demands are discussed in section 4. The proposed specification is consistent with consumer theory, flexible and empirically tractable. Section 5 presents an econometric application to the dynamics of demand, using aggregate US data over the period 1948-2010. It finds strong statistical evidence of demand dynamics, especially for food. Implications of the results for dynamics and economics are evaluated in section 5. We find that the marginal benefit of food declines with food consumption and that these effects become much stronger as dynamic adjustments take place. This is consistent with the adverse health effects of both undernutrition and overnutrition. Our analysis also provides useful information on the dynamics of substitution/complementarity relationships existing among goods.[1] While aggregate goods are found to be complements in the short run, we uncover evidence that substitution can arise in the longer run. We also find that, while food and service are always complements, their complementarity relationship becomes much stronger in the long run. Finally, section 6 concludes.

2. Dynamics of Household Consumption: Theory

We first review the linkages between economic theory and consumption dynamics. Consider a household facing a T-period planning horizon. At time t, the household purchases a consumption bundle $x_t = (x_{1t}, \ldots, x_{nt})' \in X \equiv R^n_+$, $t = 1, \ldots, T$.[2] At time t, the household faces a vector of state variables $y_t = (y_{1t}, \ldots, y_{mt})' \in R^m$. Given initial conditions y_1, the states evolve according to the state equation[3]

$$y_{t+1} = f_{t+1}(x_t, y_t), \tag{1}$$

$t = 1, 2, \ldots, T\text{-}1$. Equation (1) represents the dynamics of variables characterizing the stock of tangible or intangible goods affecting household welfare. Then, $[f_{i,t+1}(x_t, y_t, e_{t+1}) - y_{it}]/y_{it}$ is the growth rate (or the rate of decline if negative) of y_{it} from time t to t+1. The vector y_t can include durable goods, physical capital as well as human capital. It can also include health (Grossman, 1972, 2000), habit (Pollak 1970) and/or addiction (Becker and Murphy 1988; Gruber and Koszegi 2001). In these examples, as stated in (1), consumption x_t can affect the future evolution of the state vector y_{t+1}. For instance, food intake affects nutrition and health, education influences knowledge, medical care affects health, smoking is habit-forming and addictive, etc.

Let $p_t = (p_{1t}, \ldots, p_{nt})' \in R^n_{++}$ be the price vector for x_t. At time t, the household budget constraint is

$$p_t' x_t \leq h_t(y_t), \tag{2}$$

where $p_t' x_t = \sum_{i=1}^{n} p_{it} x_{it}$ is consumption expenditure and $h_t(y_t) > 0$ denotes household income, t = 1, …, T. Household income $h_t(y_t)$ includes exogenous income as well as the monetary payoff generated by y_t (e.g., payoff from capital).

Following Koopmans (1960) and Koopmans et al. (1964), consider that preferences at time t are represented by the recursive utility function[4]

$$u_t(x_t, y_t, u_{t+1}(x_{t+1}, y_{t+1}, u_{t+2}(\ldots))), \tag{3}$$

t = 1, 2, …, T, with $u_{T+1}(\cdot) = v_{T+1}$ being the utility obtained at time T+1. We assume throughout that the utility function $u_t(x_t, y_{t+1}, u_{t+1})$ is strictly increasing in x_t and u_{t+1}. Equation (3) covers as special cases a broad class of dynamic utility specifications. This includes models of time-additive utility where $u_t(\cdot) = \sum_{\tau=t}^{T} r(\tau, t)\, v_\tau(x_\tau, y_\tau)$, $r(\tau, t)$ being a discount factor. When the discount factor satisfies $r(\tau, t) = \beta^{\tau-t}$ where β is constant, this implies exponential discounting of future utility, as commonly found in the analysis of consumer behavior (e.g., Deaton 1992; Deaton and Muellbauer 1980a; Samuelson 1928). Other special cases include hyperbolic discounting (where $r(\tau, t) = [1 + \alpha\,(\tau - t)]^{-\gamma/\alpha}$ with $\alpha > 0$ and $\gamma > 0$ (Loewenstein and Prelec

1992)) and quasi hyperbolic discounting (where $r(\tau, t) = \delta\, \beta^{\tau\text{-}t}$, with $\delta \in (0, 1]$ and $\beta \in [0, 1)$ (Laibson 1997)). Finally, as argued by Koopmans (1960), the recursive utility specification in (3) provides a general representation of time discounting. In general, the discounting of next-period utility in (3) is given by the discount factor $\frac{\partial u_t}{\partial u_{t+1}}(x_t, y_t, u_{t+1})$. At time t, defining the rate of impatience as $R_t \equiv [1/(\partial u_t/\partial u_{t+1})] - 1$, follows that $R_t \to 0$ as $(\partial u_t/\partial u_{t+1}) \to 1$, reflecting that next-period utility is given as much weight as current utility when the individual is "very patient". Alternatively, $R_t \to \infty$ as $(\partial u_t/\partial u_{t+1}) \to 0$, implying that next-period utility receives little weight when the individual is "very impatient". When the discount factor $\frac{\partial u_t}{\partial u_{t+1}}(x_t, y_t, u_{t+1})$ in (3) depends on x_t and y_t, time discounting is then endogenous (as it depends on household decisions). This is useful in the analysis of linkages between discounting and dynamic behavior (e.g., Becker and Mulligan 1997).

Given initial conditions y_1 and under recursive preferences (3), optimal household decisions are given by the maximization problem

$$\text{Max } \{u_1(x_1, y_1, u_2(x_2, y_2, u_3(\ldots))): \text{equations (1) and (2)}, x_t \in X, t = 1, \ldots, T\}. \qquad (4)$$

Equation (2) involves consumer prices over the planning horizon $\{p_t: t = 1, \ldots, T\}$. We do not impose a priori restrictions about prices p_t and allow them to evolve over time in an arbitrary manner. But we make the following assumptions:

<u>Assumption A1</u>: The prices $\{p_t: t = 1, \ldots, T\}$ are taken as given by the household.

<u>Assumption A2</u>: The prices $\{p_t: t = 1, \ldots, T\}$ are known to the household.

Assumption A1 is commonly made in consumer theory, as price determination is assumed to take place outside the realm of the household. While assumption A2 is more restrictive, it will help simplify our analysis. By taking the path of prices $\{p_t: t = 1, \ldots, T\}$ as known and given, Assumptions A1-A2 avoid any issue related to how the household anticipates future prices. In this context, using backward induction, equation (4) can be alternatively written as Bellman's equation

$$V_t(y_t) = \text{Max}_{x_t} \{u_t(x_t, y_t, V_{t+1}(f_{t+1}(x_t, y_t)): p_t' x_t \leq h_t(y_t), x_t \in X\}, \tag{5}$$

$t = T, T\text{-}1, \ldots, 1$, where $V_{T+1}(\cdot) = v_{T+1}$. Note that the value function $V_t(y_t)$ has a "t" subscript and thus can change over time. This time-varying property can come from several sources. It can arise if the function $u_t(\cdot)$ or $f_t(\cdot)$ changes over time t. Or it can arise if T is "not high enough" to allow reaching a steady state within the planning horizon. In the context of (5), it will be convenient to define

$$v_t(x_t, y_t) \equiv u_t(x_t, y_t, V_{t+1}(f_{t+1}(x_t, y_t)). \tag{6}$$

Below, we rely extensively on the utility function $v_t(x_t, y_t)$. Equation (6) shows that (x_t, y_t) have two effects in the utility function $v_t(x_t, y_t)$: a direct effect through on u_t, and an indirect effect through the next-period utility V_{t+1} and the next-period state equation f_{t+1}.[5] We make the following assumptions:

Assumption A3: The utility function $v_t(x_t, y_t)$ is continuous in x_t on X.

Assumption A4: The utility function $v_t(x_t, y_t)$ is quasi-concave in x_t on X.

Assumption A5: The utility function $v_t(x_t, y_t)$ is strictly increasing in x_t on X.

Using equation (6), the maximization problem in (5) reduces to

$$W_t(p_t, h_t(y_t), y_t) = \text{Max}_{x_t} \{v_t(x_t, y_t): p_t' x_t \leq h_t(y_t), x_t \in X\}, \tag{7}$$

where $W_t(p_t, h_t(y_t), y_t) = V_t(y_t)$.[6] Equation (7) is a standard utility maximization problem. Note that it allows the state variables y_t to affect both utility $v_t(x_t, y_t)$ and household income $h_t(y_t)$. The optimal solution to (7) is the Marshallian demand for x_t, denoted by $x_t^*(p_t, h_t(y_t), y_t)$. In this context, the dynamics of demand is captured by the dynamics of the state variables y_t given in equation (1). This provides the conceptual basis for our analysis of consumption dynamics.

3. Duality

This section reviews how duality theory provides alternative characterizations of the economics associated with the optimization problem (7). We start with well-known duality relationships related to the expenditure function

$$E_t(p_t, y_t, U_t) \equiv \mathrm{Min}_{x_t \in X} \{p_t' x_t: v_t(x_t, y_t) \geq U_t\}, \tag{8}$$

which has for solution the Hicksian demand for x_t, denoted by $x^c(p_t, y_t, U_t)$. In general, the expenditure function $E_t(p_t, y_t, U_t)$ in linear homogenous and concave in p_t, and non-decreasing in U_t (Deaton and Muellbauer 1980a). And $x_t^c(p_t, y_t, U_t) \in \nabla_p E_t(p_t, y_t, U_t)$, where $\nabla_p E_t(p_t, y_t, U_t)$ is the subdifferential of $E_t(p_t, y_t, U_t)$ with respect to p_t. Under differentiability, this reduces to Shephard's lemma:

$$x_t^c(p_t, y_t, U_t) = \partial E_t(p_t, y_t, U_t)/\partial p_t. \tag{9a}$$

In addition, under assumptions A3, A4 and A5, the following duality results hold (e.g., Deaton and Muellbauer 1980a; Mas-Colell et al. 1995; Jehle and Reny 2001)

$$E_t(p_t, y_t, W_t(p_t, h_t, y_t)) = h_t, \tag{9b}$$

and

$$x_t^*(p_t, h_t, y_t) = x_t^c(p_t, y_t, W_t(p_t, h_t, y_t)). \tag{9c}$$

Equation (9b) establishes the duality relationships between the expenditure function $E_t(\cdot)$ in (8) and the indirect utility function $W_t(\cdot)$ in (7). And equation (9c) states the duality between the Marshallian demands $x_t^*(\cdot)$ and the Hicksian demands $x_t^c(\cdot)$. These relationships are useful in the empirical analysis of consumer behavior. They support the following approach: 1/ specify a parametric form for $E_t(\cdot)$ in (8); 2/ use (9a) to obtain the Hicksian demands $x_t^c(\cdot)$; 3/ solve (9b) for $W_t(\cdot)$; and 4/ use (9c) to obtain the associated Marshallian demand $x_t^*(\cdot)$. This approach has been successfully applied to the Almost Ideal Demand System (AIDS; Deaton and Muellbauer, 1980b) and the Quadratic AIDS (QAIDS; Banks et al.).[7]

Next, we explore duality properties related to the benefit function, as analyzed by Luenberger (1992). As shown below, these properties provide an important first step toward our empirical analysis of consumption dynamics. Consider a reference bundle $g = (g_1, \ldots, g_n)' \in R_+^n$ satisfying $g \neq 0$. Following Luenberger (1992), the benefit function is defined as

$$B_t(x_t, y_t, U_t) = \text{Max}_\beta \{\beta\text{: } v_t(x_t - \beta\, g, y_t) \geq U_t,\ (x_t - \beta\, g) \in X\} \tag{10}$$

$$\text{if there is a } \beta \text{ satisfying } v_t(x_t - \beta\, g, y_t, z_t) \geq U_t \text{ and } (x_t - \beta\, g) \in X,$$

$$= -\infty \text{ otherwise.}$$

Given y_t, the benefit function $B_t(x_t, y_t, U_t)$ in (10) measures the number of units of the reference bundle g reflecting the distance between point x_t and consumption levels generating utility level U_t. When the reference bundle g is chosen to have a unit price, then $B_t(x_t, y_t, U_t)$ in (10) becomes a monetary measure of willingness to pay for the household to obtain the goods x_t starting from utility level U_t.

The properties of the benefit functions have been investigated by Luenberger (1992, 1995) and Chambers et al. (1996). Under assumptions A3 and A4, $B_t(x_t, y_t, U_t)$ is concave in x_t and non-increasing in U_t. And it satisfies the translation property: $B_t(x_t + \alpha\, g, y_t, U_t) = \alpha + B_t(x_t, y_t, U_t)$. When $B_t(x_t, y_t, U_t)$ is differentiable in x_t, this implies $\frac{\partial B_t}{\partial x_t}(x_t, y_t, U_t)\, g = 1$, where $\frac{\partial B_t}{\partial x_t} = \left(\frac{\partial B_t}{\partial x_{1t}}, \ldots, \frac{\partial B_t}{\partial x_{nt}}\right)$ is a $(1 \times n)$ vector.[8] And if $B_t(x_t, y_t, U_t)$ is twice continuously differentiable in x_t

on X, then $\frac{\partial^2 B_t}{\partial x_t^2}$ g = 0. We call the (n×n) matrix $\frac{\partial^2 B_t}{\partial x_t^2}$ the "Luenberger matrix." Under assumptions A3 and A4, it follows that the Luenberger matrix $\frac{\partial^2 B_t}{\partial x_t^2}$ is symmetric, negative semidefinite and singular.

Luenberger (1992) established the close relationships existing between the benefit function $B_t(\cdot)$ in (10) and the expenditure function $E_t(\cdot)$ in (8). First, the following duality relationship holds:

$$E_t(p_t, y_t, U_t) = \text{Inf}_{x_t \in X} \{p_t' x_t - B_t(x_t, y_t, U_t)\}, \tag{11a}$$

Second, for $x_t \in$ int(X) and under assumptions (A3) and (A4),

$$B_t(x_t, y_t, U_t) = \text{Inf}_{p_t \geq 0} \{p_t' x_t - E_t(p_t, y_t, U_t): p_t' g = 1\}, \tag{11b}$$

which has for solution $p_t^b(x_t, y_t, U_t)$. Below, we call $p_t^b(x_t, y_t, U_t)$ the price-dependent (inverse) Luenberger demands expressing prices p_t as a function of quantities x_t and y_t, holding utility U_t constant. Note that the prices p_t has been normalized in (11b) to satisfy p_t' g = 1. The properties of the inverse Luenberger demand $p_t^b(x_t, y_t, U_t)$ have been investigated by Luenberger (1996), Courtault et al. (2004), Briec and Gardères (2004), Färe et al. (2008), Baggio and Chavas (2009), McLaren and Wong (2009) and Chavas and Baggio (2010). In general, $p_t^b(x_t, y_t, U_t) \in \nabla_x B_t(x_t, y_t, U_t)$, where $\nabla_x B_t(x_t, y_t, U_t)$ is the subdifferential of $B_t(x_t, y_t, U_t)$ with respect to x_t. Under differentiability, this reduces to

$$p_t^b(x_t, y_t, U_t) = \partial B_t(x_t, y_t, U_t)/\partial x_t. \tag{12a}$$

In addition, when $x \in$ int(X) and under assumption A5, Luenberger (1992, 1995) proved that the benefit function $B_t(x_t, y_t, U_t)$ satisfies

$$B_t(x_t, y_t, v_t(x_t, y_t)) = 0. \tag{12b}$$

Equation (12b) establishes the duality between the benefit function $B_t(x_t, y_t, U_t)$ in (10) and the utility function $v_t(x_t, y_t)$. In addition, under assumption A3, A4 and A5, Chavas and Baggio (2010) showed that

$$p_t^*(x_t, y_t) = p_t^b(x_t, y_t, v_t(x_t, y_t)), \tag{12c}$$

where $p_t^*(x_t, y_t)$ are the <u>price-dependent Marshallian demands</u> where prices have been normalized to satisfy $p_t' g = 1$. Equation (12c) states the duality between the price-dependent Marshallian demands $p_t^*(\cdot)$ and the price-dependent Luenberger demands $p_t^b(\cdot)$. Equation (12c) is crucial for any empirical analysis: it relates unobservable Luenberger demands $p_t^b(x_t, y_t, U_t)$ (which depend on unobservable utility U_t) to observable Marshallian demands $p_t^*(x_t, y_t)$. Below, we exploit this relationship and follow the following approach: 1/ specify a parametric form for $B_t(\cdot)$ in (10); 2/ use (12a) to obtain the price-dependent Luenberger demands $p_t^b(\cdot)$; 3/ solve (12b) for $v_t(\cdot)$; and 4/ use (12c) to obtain the associated price-dependent Marshallian demands $p_t^*(\cdot)$.

It is of interest to relate the above results to previous literature. First, as noted above, the solution $p_t^b(x_t, y_t, U_t)$ in (11b) is obtained subject to the price normalization rule $p_t' g = 1$. Other solutions can be obtained under different normalization rules. This includes the normalization rule $p_t' x_t = 1$, yielding the price-dependent (inverse) Hicksian demand defined as $p_t^c(x_t, y_t, U_t) \in \text{argmin}_{p_t \geq 0} \{p_t' x_t: E_t(p_t, y_t, U_t) \geq 1\}$ (Deaton 1979). In general, $p_t^c(x_t, y_t, U_t) = k\, p_t^b(x_t, y_t, U_t)$ for some $k > 0$ as the two price-dependent demands differ only in the way prices are normalized (Chavas and Bagio 2010).

Second, while we focused our attention on the benefit function $B_t(x_t, y_t, U_t)$ in (10), an alternative representation has been based on the distance function (e.g., Deaton 1979; Anderson 1980; Barten and Bettendorf 1989). Following Shephard (1953), the distance function is defined as $D_t(x_t, y_t, U_t) = \max_\alpha \{\alpha: v_t(x_t/\alpha, y_t) \geq U_t\}$ for $x_t \in X$.[9] Under assumptions A3 and A4, the distance function is dual to the expenditure function $E_t(p_t, y_t, U_t)$ and satisfies $D_t(x_t, y_t, U_t) = \mathrm{Inf}_{p_t \geq 0}\{p_t' x_t: E_t(p_t, y_t, U_t) = 1\}$, with $p_t^c(x_t, y_t, U_t) \in \nabla_x D_t(x_t, y_t, U_t)$, $\nabla_x D_t(x_t, y_t, U_t)$ being the subdifferential of $D_t(x_t, y_t, U_t)$ with respect to x_t (Deaton 1979; Anderson 1980). While the benefit function $B_t(x_t, y_t, U_t)$ in (10) differ from the distance functions $D_t(x_t, y_t, U_t)$, they are closely related. Indeed, when $g = x_t$, Chambers et al. (1996) showed that they satisfy the following relationship: $D_t(x_t, y_t, U_t) = 1/[1 - B_t(x_t, y_t, U_t)]$.[10] Finally, as noted by Luenberger (1992, 1995, 1996), the benefit function has one advantage over the distance function: it has better aggregation properties across heterogeneous consumers.[11]

4. Parametric Specification

Building on the duality relationships (12a), (12b) and (12c), this section presents a specification of the benefit function that is both flexible and empirically tractable. Our analysis proceeds in three steps. In a first step, consider the case where the benefit function $B_t(x_t, y_t, U_t)$ in (10) satisfies

$$B_t(x_t, y_t, U_t) = \alpha_t(x_t, y_t) + \beta(x_t)/[U_t + \gamma(x_t)], \tag{13}$$

where $\beta(x_t) > 0$, $[U_t + \gamma(x_t)] \neq 0$, and $\alpha_t(x_t, y_t)$, $\beta(x_t)$ and $\gamma(x_t)$ are differentiable functions. Note that $\alpha_t(x_t, y_t)$ is allowed to be time-varying; and it captures the effects of the state variables y_t. In this context, equation (12a) gives the price-dependent Luenberger demands

$$p_t^b(x_t, y_t, U_t) = \partial B_t(x_t, y_t, U_t)/\partial x_t$$

$$= \frac{\partial \alpha_t(x_t, y_t)}{\partial x_t} + \frac{\partial \beta(x_t)}{\partial x_t}/[U_t + \gamma(x_t)] - \frac{\partial \gamma(x_t)}{\partial x_t}\, \beta(x_t)/[(U_t + \gamma(x_t)^2]. \tag{14}$$

When $\alpha_t(x_t, y_t)$ takes a flexible form (e.g., quadratic in x_t), this term provides a convenient representation of the effects of quantities x_t on Luenberger prices and consumer welfare. And when nonzero, the terms $\frac{\partial\beta(x_t)}{\partial x_t}$ and $\frac{\partial\gamma(x_t)}{\partial x_t}$ in (14) provide a flexible representation of the effects of utility U_t on Luenberger prices p_t^b. Using the duality relationships (12b)-(12c), note that $U_t = v_t(x_t, y_t)$ and (13) imply that $1/[U_t + \gamma(x_t)] = -\alpha_t(x_t)/\beta(x_t)$. Using (12c) and (14), the associated Marshallian price-dependent demands are

$$p_t^*(x_t, y_t) = p_t^b(x_t, y_t, v_t(x_t, y_t))$$

$$= \frac{\partial\alpha_t(x_t, y_t)}{\partial x_t} - \frac{\partial \ln(\beta(x_t))}{\partial x_t}\ \alpha_t(x_t) - \frac{\partial\gamma(x_t)}{\partial x_t}\ \alpha_t(x_t, y_t)^2/\beta(x_t). \tag{15}$$

In a second step, we propose a way to make the function $\alpha_t(x_t, y_t)$ empirically tractable. As discussed in section 2, the state variables y_t include habit formation, health, addiction and other stock dynamics that are often not directly observable. This indicates a need to express $\alpha_t(x_t, y_t)$ in a form that can be estimated. Note that the dynamics of y_t in (1) can be alternatively expressed as

$$y_t = f_t(x_{t-1}, y_{t-1}) = f_t(x_{t-1}, f_{t-1}(x_{t-2}, y_{t-2}))$$

$$= \ldots$$

$$= f_t(x_{t-1}, f_{t-1}(x_{t-2}, f_{t-2}(x_{t-3}, \ldots, f_2(x_1, y_1))))$$

$$= g_t(x_{t-1}, x_{t-2}, x_{t-3}, \ldots, x_1),$$

where the initial states y_1 are treated as given. It follows that $\alpha_t(x_t, y_t)$ can be written as

$$\alpha_t(x_t, y_t) = \alpha_t(x_t, g_t(x_{t-1}, \ldots, x_1)). \tag{16}$$

Given (16), the third step involves choosing a parametric specification for $\alpha_t(x_t, g_t(x_{t-1}, \ldots, x_1))$, $\beta(x_t)$ and $\gamma(x_t)$ in (13) and (16). We propose the following specification[12]

$$\alpha_t(x_t, g_t(x_{t-1}, \ldots, x_1)) = x_t' \,[a_0 + a_T\, t + 0.5\, a_x\, x_t + a_L\, x_{t-1} + a_p\, (\partial B_{t-1}/\partial x_{t-1})'],$$

$$= x_t' \,[a_0 + a_T\, t + 0.5\, a_x\, x_t + a_L\, x_{t-1} + a_p\, p_{t-1}^*], \text{ using (12a) and (12c),} \tag{17a}$$

$$\beta(x_t) = \exp(x_t' \, b) > 0, \tag{17b}$$

and

$$\gamma(x_t) = x_t' \, c, \tag{17c}$$

where a_x is a (n×n) symmetric matrix, and the a's, b and c are conformable vectors/matrices of parameters. Equation (17a) provides a flexible specification allowing for quadratic current quantity effects (captured by the parameters in a_x), time-varying demand (reflected by the time trend t and the parameters a_T), dynamics in quantities (depicted by the lagged quantities x_{t-1} and the parameters a_L) as well as dynamics in prices (captured by the lagged prices p_{t-1}^* and the parameters a_p). Using (17a)-(17c), it follows that (15) can be written as

$$p_t^* = a_0 + a_T \, t + a_x \, x_t + a_L \, x_{t-1} + a_p \, p_{t-1}^* - b \, \alpha_t(\cdot) - c \, \alpha_t(\cdot)^2/\exp(x_t' \, b) + e_t, \tag{18}$$

where $e_t = (e_{1t}, \ldots, e_{nt})' \in R^n$ has been added, e_t being an error term assumed to be serially uncorrelated and normally distributed with mean zero and variance $E(e_t \, e_t') = \Omega$, Ω being a (n×n) symmetric and positive semidefinite matrix. Equation (18) is a system of n nonlinear equations that can be estimated econometrically. It satisfies the price normalization rule $p_t^{*\prime} \, g = 1$. To be globally valid, this normalization rule implies the following restrictions on the parameters[13]

$$a_0' \, g = 1, \tag{19a}$$

$$a_T' \, g = 0, \tag{19b}$$

$$a_x' \, g = 0, \tag{19c}$$

$$a_L' \, g = 0, \tag{19d}$$

$$a_p' \, g = 0, \tag{19e}$$

$$b' \, g = 0, \tag*{(19e)}$$

$$c' \, g = 0, \tag{19f}$$

and

$e_t' \, g = 0.$ (19g)

In addition to (19a)-(19g), the symmetry of the (n×n) matrix a_x implies the following symmetry restrictions

$a_x = a_x'$, (19h)

which are integrability conditions that must be satisfied to make the price equations (18) consistent with the benefit function (13).[14]

Note that the restriction (19g) implies that the variance of e_t, Ω, is necessarily singular. Following Barten (1969), the system of equations (18) can then be estimated after dropping one equation. In the context of maximum likelihood estimation, Barten (1969) showed that the parameter estimates are invariant to the equation dropped. And an estimate of the parameters in the equation dropped can be recovered using the restrictions (19a)-(19f).

This provides a formal linkage between the benefit function and inverse Marshallian demand. In the process of estimating the inverse Marshallian demands (18), the estimated parameters provide all the information necessary to evaluate the benefit function $B_t(\cdot)$in (13). In turn, the estimated benefit function can then be used in the analysis of consumer behavior and of consumer welfare (Luenberger 1996).

5. Estimation

Our proposed approach is applied to the analysis of US consumption. It relies on annual data obtained from the Bureau of Economic Analysis (BEA, US Department of Commerce) who report prices, quantities and consumer expenditures at the national level for various years. Our analysis covers the period 1948-2010 and examines four consumption groups: 1/ food, 2/ durable, 3/ nondurable (excluding food), and 4/ services. The analysis is conducted on a per capita basis, all BEA quantity indexes being divided by civilian US population. In the benefit function approach, the reference bundle g is chosen to be the average of per capita consumption in the data. Then, all prices are normalized and defined as $p_{it}^* = p_{it}/(\Sigma_j p_{jt} g_j)$ where p_{it} is the BEA price for the i-th commodity at time t. With this normalization rule, the price of one unit of the average bundle g is equal to 1. In this context, the benefit function is measured in number of units of average per capita consumption. Using these data, the econometric model in (18) is estimated. As discussed above, given the singularly of the variance of e_t in (18), we drop the fourth equation (service) and proceed with the estimation of three price equations p_{it}^*: food (i = 1), durable (i = 2) and nondurable (i = 3).

Our econometric analysis starts with a series of statistical tests on the model. First, we examine the issue of potential endogeneity for the quantities x_t in model (18). Assuming that supply adjustments are slow, we consider two-period lagged quantities as instruments for x_t. These lagged values are found to be statistically significant in an estimation of the reduced form for x_t, indicating that they are valid instruments. We implement a Durbin-Wu-Hausman (DWH) test for the endogeneity of x_t.[15] Under the null hypothesis of independence between x_t and e_t, the DWH test statistics has a chi-square distribution and a p-value of 0.177. Thus, we conclude that there is no strong statistical evidence that the quantities x_t in (18) are endogenous. This indicates that the determination of quantities and prices is recursive. Applied at the aggregate level, this is consistent with the following interpretation: supply adjustments being slow, quantities x_t are determined first from the supply side of the market, while prices p_t are determined by the demand side of the market conditional on x_t. As noted by Barten and Bettendorf (1989), this is a scenario which helps motivate the price-dependent approach to demand analysis. On that basis, we proceed with our econometric analysis treating quantities x_t as predetermined variables.

Second, we evaluate the potential for serial correlation for e_t in (18). We implement a Breusch-Goldfrey (BG) test of first-order serial correlation for e_{it}, i = 1, 2, 3. Under the null hypothesis of no first-order serial correlation in $e_t = (e_{1t}, e_{2t}, e_{3t})'$, the BG test statistics has a chi-square distribution with 9 degrees of freedom.[16] Its corresponding p-value is 0.656. We conclude that there is no strong statistical evidence that e_t in (18) are serially correlated. We interpret this result as evidence that the lagged quantities x_{t-1} and lagged prices p_{t-1} in (18) appropriately capture the dynamics of demand.

Third, we examine whether the structure of demand has changed over time. In (18), this corresponds to testing the null hypothesis: H_0: $a_T = 0$. Using a Wald test, the test statistic has a chi-square distribution under the null hypothesis. The associated p-value is 0.181. Thus, we fail to reject the null hypothesis and conclude that there is no strong evidence of time-varying demand. As discussed in section 2, time-varying demand can come from structural changes in the function $u_t(\cdot)$ or $f_t(\cdot)$. Our test results indicate that these functions do not change over time in significant ways. On that basis, our analysis proceeds after imposing the restriction $a_T = 0$.

Treating x_t as predetermined variables and e_t as serially uncorrelated, the maximum likelihood estimation of equation (18) provides consistent and asymptotically efficient estimates of the parameters. Assuming $a_T = 0$, the maximum likelihood estimates of model (18) are presented in table 2. As noted above, the maximum likelihood estimates are invariant to the equation dropped (Barten, 1969). The model has very good explanatory power: the R-squares are 0.99 for the food equation, 0.99 for the durable equation and 0.98 for the nondurable equation. And a number of parameters are found to statistically significant. First, the diagonal elements of the matrix a_x (a_{xii}: i = 1, 2, 3) are all negative and significant at least at the 10 percent level. Since

a_{xii} measures the (partial) effect of x_{it} on p_{it} in (18), this contributes to x_{it} having negative effects on p_{it} and on the associated marginal benefit (see below).

The estimation of (18) provides strong statistical evidence of dynamics in demand. It takes two forms: the lagged quantity effects captured by the parameters a_L, and the lagged price effects captured by the parameters a_p. We first test the null hypothesis: H_0: $a_L = 0$. Using a Wald test, the test statistic is 65.93 with 3 degrees of freedom. The associated p-value being 2×10^{-9}, we strongly reject the null hypothesis and conclude that demand exhibits important lagged-quantity effects. Two of these effects are noteworthy: $a_{L11} = 1.545$ and $a_{L33} = 1.362$. Both effects are statistically significant at the 1 percent level. For both food (i = 1) and non-durable (i = 3), they indicate that increasing consumption in the previous period has a positive and significant impact on the marginal benefit of consumption in the current period. A similar result applies to durable (i = 2): the parameter $a_{L22} = 0.569$ is positive and statistically significant at the 10 percent level. This suggests that habit formation is an important feature of demand behavior. In addition, the estimates find evidence of lagged quantity effects across commodities. For example, the parameter $a_{L12} = 0.527$ is positive and statistically significant at the 5 percent level, indicating that lagged consumption of durable has a positive effect on the marginal benefit of food. And the parameter $a_{L23} = -1.124$ is negative and statistically significant at the 5 percent level, showing that lagged consumption of service has a negative effect on the marginal benefit of durable.

Next, we test the hypothesis: H_0: $a_p = 0$. Using a Wald test, the test statistic is 404.9, with 3 degrees of freedom. The associated p-value being very close to 0, we strongly reject the null hypothesis and conclude that demand exhibits important lagged-price effects. First, the own lagged price effects are all positive (but less than 1) and statistically significant at the 1 percent level. They are: $a_{p11} = 0.853$ for food (i = 1), $a_{p22} = 0.757$ for durable (i = 2) and $a_{p33} = 0.408$ for nondurable (i = 3). This indicates that slow adjustments in marginal benefits are a basic characteristic of consumption behavior. The lagged-price estimates also document the presence of cross-commodity effects. For example, the parameter $a_{p12} = 0.151$ is significant at the 5 percent level and establishes that lagged marginal benefit of durable (i = 2) has a positive impact on the marginal benefit of food (i = 1). Similarly, the parameter $a_{p31} = 0.360$ is significant at the 1 percent level and shows that lagged marginal benefit of food (i = 1) has a positive impact on the marginal benefit of nondurable food (i = 3). This demonstrates the presence of important dynamics in consumption.

Note that model (18) represents price dynamics as an autoregressive process of order one: $p_t^* = A_t(\cdot) + a_p\ p_{t-1}^*$, where $p_t^* = (p_{1t}^*, p_{2t}^*, p_{3t}^*)'$, $A_t(\cdot)$ being a vector of intercepts, and a_p being a (n-1)×(n-1) matrix. This process is stationary if and only if the Eigen values of a_p are all inside the unit circle (e.g., Hayashi, 2000, p. 374). Given the estimates reported in table 2, the dominant Eigen value of a_p is $0.869 < 1$. This indicates that the estimated dynamic process in (18) is stationary. The nature and magnitude of these dynamic adjustments are discussed in more details in section 6 below.

As showed in (14), the parameters b and c capture the effects of utility U_t on marginal benefit. First, we test the null hypothesis: $H_{0:}\ b = 0$. Using a Wald test, the test statistic is 2.916 with 3 degrees of freedom. The associated p-value being 0.404, we fail to reject the null hypothesis. We conclude that $\beta(x_t) = \exp(x_t{}'\ b) = 1$ and that term $\frac{\partial\beta(x_t)}{\partial x_t}/[U + \gamma(x_t)]$ in (14) is not statistically different from zero. Second, we test the null hypothesis: H_0: c = 0. Using a Wald test, the test statistic is 10.371 with 3 degrees of freedom. The associated p-value being 0.015, we reject the null hypothesis. This provides evidence that $\gamma(x_t) = x_t{}'\ c \neq 0$ and that the terms

$\frac{\partial \gamma(x_t)}{\partial x_t}$ $\beta(x_t)/[(U + \gamma(x_t)^2]$ in (14) are non-zero. It indicates the presence of utility effects captured by the expression $\gamma(x_t)$ in (13). Such effects are due in large part to the parameter $c_1 = 0.195$, which is statistically significant at the 1 percent level. From (14), it means that the marginal benefit for food ($i = 1$) declines with utility U_t (or equivalently that the marginal benefit for food is high under low utility U_t). This is consistent with food being a necessary good to sustain life.

Finally, our analysis has relied on the duality relationship given in equation (11b). As noted in section 3, this relationship requires the quasi-concavity of $v_t(x_t, \cdot)$ (assumption A4) or equivalently the concavity of the benefit function $B_t(x_t, \cdot)$. Since our parameter estimates provide a basis to evaluate the benefit function $B_t(x_t, \cdot)$, we check whether our estimates are consistent with $B_t(x_t, \cdot)$ being concave in x_t. For that purpose, we evaluated the Luenberger matrix $\frac{\partial^2 B_t}{\partial x_t^2}$ and its Eigen values. Under assumption (A4), we expect the matrix $\frac{\partial^2 B_t}{\partial x_t^2}$ to be negative semidefinite, implying that its Eigen values are all non-positive. When evaluated at sample means and using (13) and (17), the Eigen values of $\frac{\partial^2 B_t}{\partial x_t^2}$ are: (0, -0.684, -0.778, -.2.293). This shows that our estimated benefit function is concave at sample means.[17] To check whether this finding applies globally, we also calculated the Eigen values of $\frac{\partial^2 B_t}{\partial x_t^2}$ at different evaluation points within the range of the data. We found that the Eigen values are non-positive at all evaluation points, indicating that our estimated benefit function is globally concave. In other words, the parameter estimates reported in table 2 are consistent with a benefit function $B_t(x_t, \cdot)$ that is concave in x_t, i.e. a benefit function exhibiting diminishing marginal benefits.

6. Implications for Dynamics and Economics

As noted above, model (18) represents price dynamics as an autoregressive process: $p_t^* = A_t(\cdot) + a_p\ p_{t-1}^*$. The Eigen values of the (3×3) matrix a_p provide useful information on the nature of dynamics. Given our estimates, these Eigen values are: 0.869, and $0.575 \pm 0.139\sqrt{-1}$. As discussed above, the process is stationary as all roots are within the unit circle. The two complex roots have a modulus of 0.592 and identify cyclical patterns. The period of the cycle is [2 × π × arctan(0.139/0.575)] = 1.49 years. Thus, our analysis of consumption dynamics uncovers evidence of cycles. Where do these cycles come from? They come from the cross-commodity effects of lagged prices in (18). Two of these effects were noted above: the parameter a_{p12} = 0.151 capturing the effect of lagged marginal benefit of durable (i = 2) on the marginal benefit of food (i = 1); and the parameter a_{p31} = 0.360 reflecting the effect of lagged marginal benefit of food (i = 1) on nondurable food (i = 3). Both effects are statistically significant and act to increase marginal benefits. Importantly, the identified cycles would not exist without these cross commodity effects. Thus, these positive dynamic cross-commodity effects as the source of cyclical patterns in demand behavior.

From (18), the dynamics of the price equation $p_t^* = A_t(\cdot) + a_p\ p_{t-1}^*$ can be illustrated using dynamic multipliers. We evaluate the dynamic multipliers M_{ijk} which measure the marginal impact of a one-unit exogenous shock in p_{it} on $p_{j,t+k}^L$, the shock persisting over k consecutive periods. When k = 0, $M_{ij0} = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$ when i $\begin{Bmatrix} = \\ \neq \end{Bmatrix}$ j, reflecting the initial shock in p_{it}. When k = 1, $M_{ij1} = a_{pij}$ is the short run multiplier effect of p_{it} on next-period price $p_{j,t+1}$. When $1 < k < \infty$, M_{ijk} is the intermediate run multiplier, measuring the cumulative impact on $p_{j,t+k}$ of a shock in p_{it}

persisting over k years. And when $k \to \infty$, $M_{ij\infty}$ is the long run multiplier capturing the effect of a permanent change in p_{it} on $p_{jt'}$ as $t' \to \infty$. The dynamic multipliers are reported in figure 1 for own-price effects ($i = j$) and in figure 2 for cross-price effects ($i \neq j$).

Figure 1 illustrates the importance of own-price dynamics in (18). It shows that the long run own-multiplier for food is very large: $M_{11\infty} = 5.53$. It means that a permanent one-unit shock in the marginal benefit of food has a long run impact that is 5.53 times larger. The dynamics of this impact is revealing. The intermediate run multiplier M_{11k} increases with the length of the run k: starting with $M_{110} = 1$, it reaches $M_{11k} = 3$ in 3 years, and $M_{11k} = 5$ in 13 years. This documents that the price dynamics plays an important role in the demand for food. From figure 1, the long run own-multipliers are also large for durable ($M_{22\infty} = 4.54$) and nondurable ($M_{33\infty} = 1.80$). Again, the intermediate run multipliers are found to increase with the length of the run. This provides evidence that the price dynamics in (18) is an important part of demand dynamics.

Figure 2 reports cross-price dynamics in (18). The long run multipliers between food ($i = 1$) and nondurable ($i = 3$) are very large: $M_{31\infty} = 3.44$ and $M_{13\infty} = 3.36$. This identifies strong dynamic cross price effects between food and nondurable. In the long run, a permanent one-unit shock in the marginal benefit of food (nondurable) has an impact on the marginal benefit of nondurable (food) that is about 3 times larger than the original shock. Also notable is the long run multiplier $M_{32\infty} = 2.02$, indicating the presence of strong dynamic effects in marginal benefits between nondurable and durable. From figure 2, all long run cross-multiplier effects are found to be positive. Thus, at least in the long run, marginal benefits tend to move together across commodities. For $i \neq j$, the intermediate run multipliers M_{ijk} often increase with the length of the run k (e.g., for nondurable). However, the patterns are different for M_{12k} (food price impact on durable price) and M_{31k} (nondurable price impact on food price). These two multipliers are first negative for small k but then turn positive when k becomes large (M_{12k} becomes positive for k > 5, while M_{31k} turns positive for k > 11). This documents complex dynamic interactions in the determinants of marginal benefits across commodities.

Next, we evaluate the implications of our analysis for patterns of demand both in the short run and in the longer run. We define the short run as a situation where all lagged values (for both quantities and prices) as taken as given. In the context of equation (18), this means holding (x_{t-1}, p_{t-1}) constant. Then, the effects of x_t on Luenberger prices p_t^b can be evaluated using our parameter estimates. This is done by assessing the elasticities of marginal benefits with respect to x_t: $\partial \ln(p_t^b)/\partial \ln(x_t)$, sometimes called "price flexibilities". These price flexibilities measure the percentage change in p_t^b (or marginal benefit) due to one percent change in x_t, given (x_{t-1}, p_{t-1}, U_t). Evaluated at sample means, these short run price flexibilities are reported in table 3A.[18]

As anticipated, under a concave benefit function, the own-quantity flexibilities $\partial \ln(p_{it}^b)/\partial \ln(x_{it})$, i = 1, …, 4, are all negative: increasing any quantity x_{it} decreases the associated marginal benefit. The short run own-quantity flexibilities are: -0.719 for food (i = 1), -0.090 for durable (i = 2), -0.321 for nondurable (i = 3) and -0.190 for service (i = 4). Thus, the own-quantity effect is strongest for food: a 1 percent increase in food consumption implies a 0.719 decline in the Luenberger price from food. The own-quantity effect is weakest for durable: a 1 percent increase in durable goods implies a 0.090 decline in the Luenberger price from durable. In this case, changing durable consumption has only a small impact on the marginal benefit of durable. Nondurable and service goods exhibit intermediate response to quantity changes.

The cross-quantity flexibilities are all positive: $\partial \ln(p_{it}^b)/\partial \ln(x_{jt}) > 0$ for all $i \neq j$. This reflects that, in the short run, all goods behave as complements: an increase in x_{jt} has a positive effect on the marginal benefit of x_{it} for $i \neq j$ (Hicks 1932; Baggio and Chavas 2009). This result seems reasonable given that our analysis is applied to broad categories of consumer goods. The cross flexibility effects are particularly strong when considering the impact of service on the marginal benefit of food, $\partial \ln(p_{1t}^{b})/\partial \ln(x_{4t}) = 0.519$. This result shows that, in the short run, the marginal benefit of food varies with fluctuations in non-food consumption.

Since our analysis has identified the role of dynamics, we examine its economic implications. Rewrite equation (18) as $p_t(x_t, x_{t-1}, p_{t-1})$. Given p_1 and under stationary conditions where $x^e = x_t = x_{t-1}$ for all t, consider the forward path for prices: $p_2^e(x^e) = p_2(x^e, x^e, p_1)$, and $p_k^e(x^e) = p_k(x^e, x^e, p_{k-1}^e(x^e))$ for k = 3, 4, …, with $p_\infty^e(x^e) = \lim_{k\to\infty} p_k^e(x^e)$ being the long run equilibrium prices. Writing the short run benefit function as $B(x_t, x_{t-1}, p_{t-1})$, we can evaluate how its dynamic properties change over time. Let $B_k(x^e)$ be an "<u>intermediate run</u>" benefit function defined as the benefit obtained when the quantities x^e persists over (k+1) successive periods, k denoting the length of the run. When k = 0, then $x^e = x_t$ and $B_0(x^e)$ reduces to the short run benefit function discussed above: $B_0(x^e) = B(x^e, x_{t-1}, p_{t-1})$. When k = 1, then $x^e = x_t = x_{t-1}$ and $B_1(x^e) = B(x^e, x^e, p_{t-1}(x^e, x_{t-2}, p_{t-2})$. Note that x^e has now three effects on $B_1(\cdot)$: the first is the short-term quantity effect $B(x_t, \cdot, \cdot)$; the second is the lagged quantity effect $B(\cdot, x_{t-1}, \cdot)$; and the third includes the lagged price effect $B(\cdot, \cdot, p_{t-1}(x^e, \cdot))$. When k = 2, then $x^e = x_t = x_{t-1} = x_{t-2}$ and $B_2(x^e) = B(x^e, x^e, p_{t-1}(x^e, x^e, p_{t-2}(x^e, x_{t-3}, p_{t-3}))$. Note that $B_2(x^e)$ differs from $B_1(x^e)$ as the third effect (the lagged price effect) becomes stronger as prices have more time to adjust. And the <u>long run</u> equilibrium benefit function is defined as $B_\infty(x^e) = \lim_{k\to\infty} B_k(x^e)$: it evaluates the benefit function in a situation where quantities are constant for all periods ($x_t = x^e$ for all t) and prices adjust to their long run equilibrium values $p_\infty^e(x^e)$. In this case, the third effect (the lagged price effect) is strongest: it allows for a full adjustment of prices to the long run equilibrium.

Clearly, the short run benefit $B_0(x^e)$, the intermediate run benefit $B_k(x^e)$, $0 < k < \infty$, and the long run benefit $B_\infty(x^e)$ differ. But how much do they differ? We proceed evaluating the properties of $B_k(x^e)$ for different length of run k, with x^e being evaluated at sample means. Note that, when x^e is the sample means, the marginal benefits $\partial B_k(x^e)/\partial x^e$ are unaffected by k.[19] In this context, the marginal benefits $\partial B_k(x^e)/\partial x^e$ can be interpreted as Luenberger prices p^b in the short run as well as in the longer run.

We calculate the long run flexibilities defined as $\partial \ln(p_{it}{}^b)/\partial \ln(x_{jt}) = \partial^2 B_\infty(x^e)/\partial(x^e)^2$. Evaluated at sample means, they are reported in table 3B. Contrasting the short run flexibilities in table 3A with their long run counterpart in table 3B is instructive. Table 3b shows that own flexibility for food is very large (-5.71). In the long run, it means that the marginal benefit of food would be high under low food consumption but would decline sharply under high food consumption. This appears consistent with the adverse health effects of both undernutrition and overnutrition. Importantly, this long run effect is much larger (in absolute value) than its short run counterpart (-0.719). A similar result holds for durable and service. Table 3B also reports that the long run own flexibility for nondurable is 0.369. Being positive, this implies that the long run benefit function is <u>not</u> concave. Thus, while we found that the short run benefit function is concave, our analysis shows that this property of the benefit function no longer holds in a dynamic context. Finally, table 3B reports the long run cross flexibilities. They are negative for food-durable, food-nondurable and durable-nondurable. This contrasts with the corresponding short run results (which were showed to be positive in table 3A). Here, we interpret negative cross flexibilities as evidence of <u>substitution</u>: having $\partial \ln(p_{it}{}^b)/\partial \ln(x_{jt}) < 0$ means that an increase in x_{jt} has a negative effect on the marginal benefit of substitute good x_{it}, $i \neq j$ (Hicks 1932; Baggio and Chavas 2009). Thus, we find that dynamic adjustments contribute to the rise of substitution in demand. But we also find that $\partial\ln(p_{1t}^{b})/\partial\ln(x_{4t}) = 8.56$, indicating that service and food (that were found to be complements in the short run) become even stronger complements in the long run. In general, the large differences between tables 3A and 3B show that dynamics have profound effects on demand.

Finally, it is of interest to examine the evolution of consumer behavior as one shifts from short run to long run situations. This is done by evaluating the flexibilities $\partial\ln(p_{it}^{b})/\partial\ln(x_{jt}) = \partial^2 B_k(x^e)/\partial(x^e)^2$ as the length of run k increases from short run ($k = 0$) toward long run ($k \to \infty$). Evaluated at sample means, these flexibilities are reported in figure 3 for own effects ($i = j$) and in figure 4 for cross effects ($i \neq j$). Figure 3 shows how own flexibilities evolve as dynamic adjustments take place. The own flexibility for food changes rapidly with k: it reaches -1 after 1 year, -2 after 2 years, -4 after 6 years and -5 after 13 years. Thus, the dynamic strengthening in the response of marginal benefit for food occurs very quickly. The own flexibility for nondurable starts negative in the short run (as discussed above); but it turns positive for $k \geq 1$ (i.e. after only one year). This indicates that the non-concavity of the intermediate run benefit function arises very quickly as dynamic adjustments take place. While the long run patterns vary across goods (as noted above), figure 3 shows that the evolution from short run behavior toward long run behavior tend to be smooth.

Figure 4 reports how cross flexibilities change with the length of run k. Note both the magnitude and speed of the rise in cross flexibility of service on food: $\partial\ln(p_{1t}^{b})/\partial\ln(x_{4t})$ reaches 2 after 2 years, 5 after 6 years, and 7 after 11 years. This identifies important dynamic interactions between service and the marginal benefit of food. For a number of commodities, the path from short run behavior toward long run behavior is fairly smooth. This includes the effects of food on nondurable or service, of service on nondurable and of nondurable on food. But the impact of food on durable is an exception: the cross flexibility $\partial\ln(p_{2t}^{b})/\partial\ln(x_{1t})$ first rises (as these two goods become stronger complements) until k = 3, declines between k = 3 and k = 10, and then turns negative (as the two goods become substitutes) beyond k = 11. In this case, the dynamics of adjustments interact in complex ways with demand behavior.

7. Concluding Remarks

This paper has developed an economic and econometric analysis of demand dynamics, with an application to US aggregate data over the period 1948-2010. The model builds on duality and the benefit function, which provide strong linkages with the theory. Dynamics is captured by introducing state variables that represent the effects of physical capital, human capital, health, habit and/or addiction on consumer preferences. The analysis involves the specification and estimation of dynamic price-dependent demands (representing marginal benefits). The proposed specification is consistent with consumer theory, flexible and empirically tractable. The econometric estimation uncovers strong statistical evidence of demand dynamics in US consumption, especially for food. We find that the marginal benefit of food declines with food consumption and that this effect becomes much stronger in the long run. This is consistent with the adverse health effects of both undernutrition and overnutrition. Our analysis also provides useful information on the dynamics of substitution/complementarity relationships existing among goods. While all goods are found to be complements in the short run, we uncover evidence that substitution can arise in the longer run. We also find that, while food and service are always complements, their complementarity becomes much stronger in the long run.

Our analysis could be extended in a number of directions. First, while the conceptual model developed in section 2 allows for flexible representations of time-discounting, our empirical results on dynamics have not been linked with the effects of discounting. The reason is our reliance on the utility function $v_t(\cdot)$ given in (6). As mentioned in footnote 5, this utility function does not identify the specific role played by time-discounting. Additional research is needed to explore how our dynamic analysis could be extended to allow an explicit analysis of time-discounting effects. Second, our approach builds on a general state equation (1) and on specifications (16) and (17a) which provide a fairly flexible econometric specification of the dynamics of demand. In this context, equation (17a) has the advantage of being empirical tractable even when the state variables are not observed. But it does not provide an explicit representation of the effects of particular state variables on consumption dynamics (see footnote 12). Further research is needed to refine the empirical evaluation of dynamic linkages between demand behavior and specific state variables. Third, our econometric investigation has focused on only four broad categories of consumer goods. It would be useful to apply our approach to more disaggregate consumer goods. While we found that all goods are complements in the short run, a study of more disaggregate goods would likely increase the chances of finding evidence of substitution relationships. Fourth, our empirical analysis has focused on national US data. Additional insights could be obtained by investigating the dynamics of consumer behavior using household panel data. Finally, note that our finding of significant dynamics related to food demand is consistent with the literature evaluating the economics of nutrition (e.g., Fuchs 1991; Strauss and Thomas 1998; Smith et al. 2005). More generally, establishing economic linkages between food, nutrition and the dynamics of health is also of significant interest (e.g., Grossman, 1972, 2000; Bleichrodt and Gafni 1996). While this paper has taken a constructive step in this direction, more research is needed on this topic.

Table 1: Summary statistics

Variables	Sample mean
Year (1948-2010)	1979
Population (million)	226.4
Price of food	52.9
Price of durable	81.6
Price of nondurable	55.3
Price of service	45.6
Quantity of food	4.48
Quantity of durable	3.89
Quantity of nondurable	6.47
Quantity of service	28.69

Table 2: Parameter Estimates

Parameter	Estimate	Standard error
a_{01}	-0.1439	0.1384
a_{02}	0.4742***	0.1792
a_{03}	0.1584	0.1410
a_{x11}	-2.1370***	0.4382
a_{x22}	-0.6292*	0.3635
a_{x33}	-0.7608*	0.4207
a_{x12}	0.1783	0.2877
a_{x13}	0.3260	0.3049
a_{x23}	0.0360	0.3232
a_{L11}	1.5453***	0.4861
a_{L12}	0.5273**	0.2291
a_{L13}	-0.1676	0.3688
a_{L14}	-0.1277	0.1223
a_{L21}	-0.2192	0.5346
a_{L22}	0.5692*	0.3110
a_{L23}	-1.1242**	0.4534
a_{L24}	-0.1628	0.1582
a_{L31}	0.1846	0.4375
a_{L32}	-0.1325	0.2452
a_{L33}	1.3624***	0.4250
a_{L34}	-0.2280*	0.1235
a_{p11}	0.8533***	0.0809
a_{p12}	0.1512**	0.0596
a_{p13}	-0.0891	0.0970
a_{p21}	-0.0762	0.1016
a_{p22}	0.7578***	0.0782
a_{p23}	0.1797	0.1242
a_{p31}	0.3606***	0.0827
a_{p32}	-0.0093	0.0610
a_{p33}	0.4089***	0.0992
b_1	-0.1515	0.1113
b_2	-0.0452	0.1607
b_3	0.1097	0.1245
c_1	0.1953**	0.0610
c_2	-0.0951	0.0744
c_3	-0.0121	0.0601

Note: The R-squares are 0.99, 0.99 and 0.98 for the food, durable and nondurable equations, respectively. Stars next to the parameter estimates show the level of significance: *** at the 1 percent level, ** at the 5 percent peel, * at the 10 percent level.

Table 3: Luenberger flexibilities $\partial \ln(p_{it}^b)/\partial \ln(x_{jt})$ evaluated at sample means

3A: Short run flexibilities: $\partial \ln(p_{it}^b)/\partial \ln(x_{jt})$, holding lagged ($x_t$, p_t) constant.

$p_{it}^b \setminus x_{jt}$	food	durable	nondurable	service
food	-0.719	0.052	0.146	0.519
durable	0.035	-0.090	0.011	0.043
nondurable	0.098	0.011	-0.321	0.211
service	0.110	0.013	0.066	-0.190

3B: Long run flexibilities: $\partial \ln(p_{it}^b)/\partial \ln(x_{jt})$, letting all lagged ($x_t$, p_t) adjust.

$p_{it}^b \setminus x_{jt}$	food	durable	nondurable	service
food	-5.717	-0.341	-2.193	8.568
durable	-0.233	-0.518	-0.577	0.923
nondurable	-1.469	-0.564	1.291	0.916
service	1.815	0.285	0.290	-2.387

Figure 1: Dynamic multipliers M_{iik} measuring the marginal effects of a persistent change in p_{it} on $p_{i,t+k}$ over k years, k = 0, …, 40.

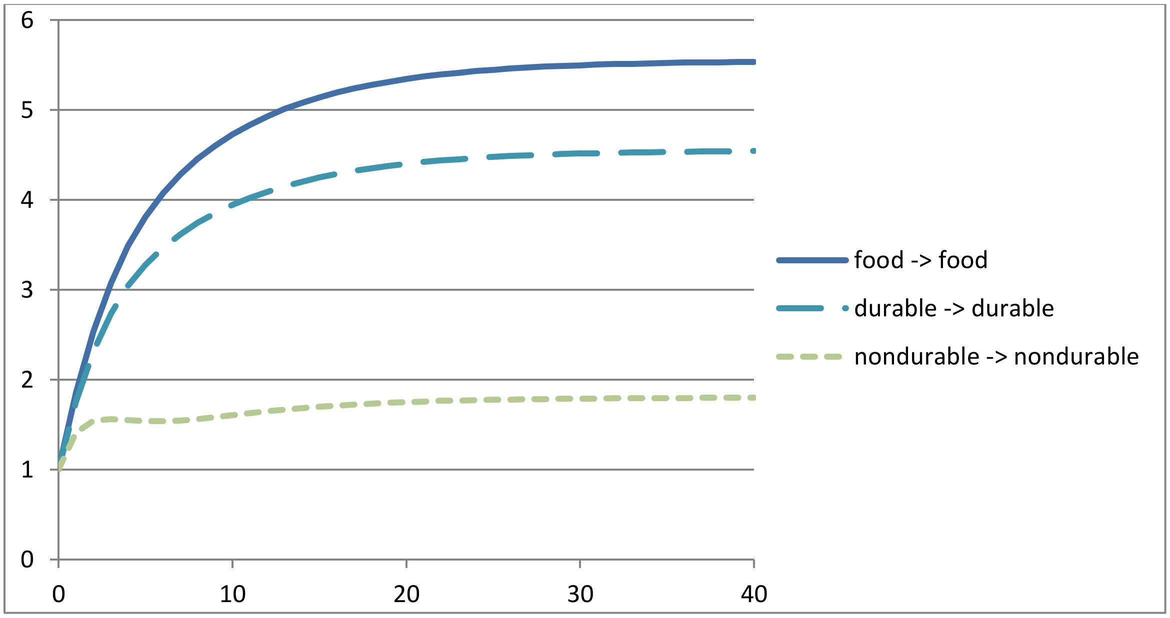


Figure 2: Dynamic multipliers M_{ijk} measuring the marginal effects of a persistent change in p_{it} on $p_{j,t+k}$ over k years, i ≠ j, k = 0, …, 40.

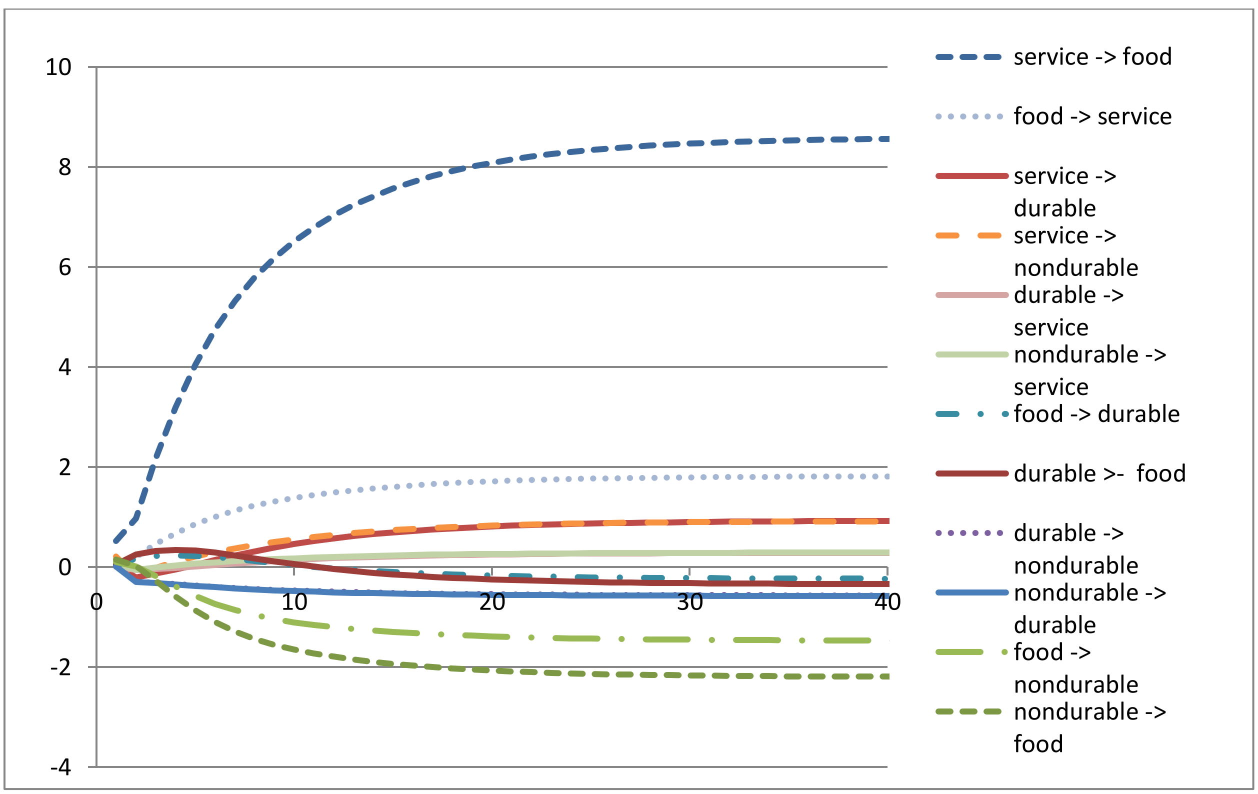


Figure 3: Elasticities of the marginal benefit of x_i with respect to a persistent change in x_i over k years, k = 0, …, 40.

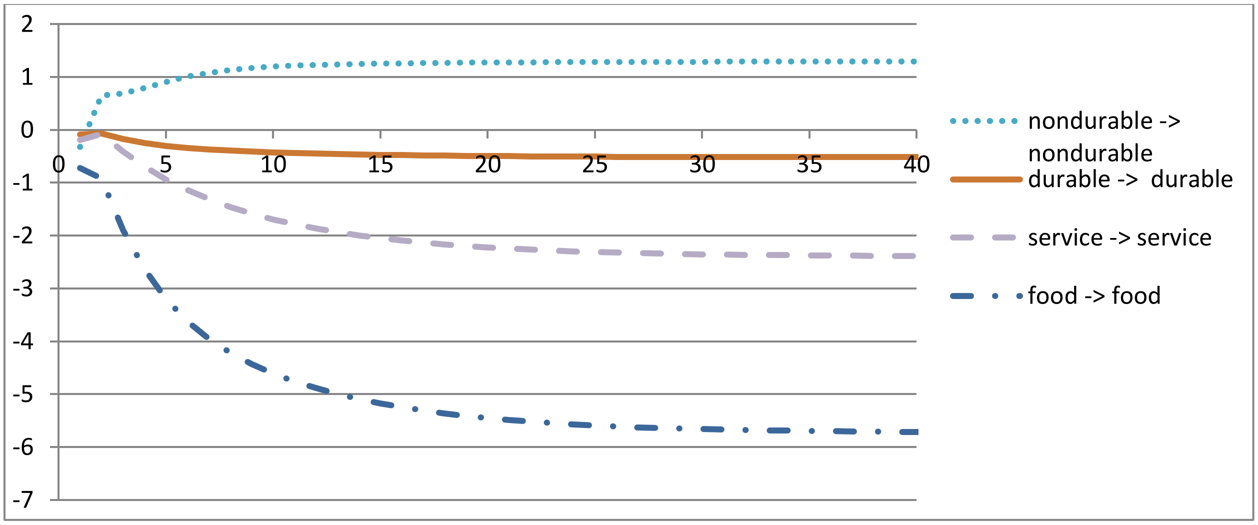
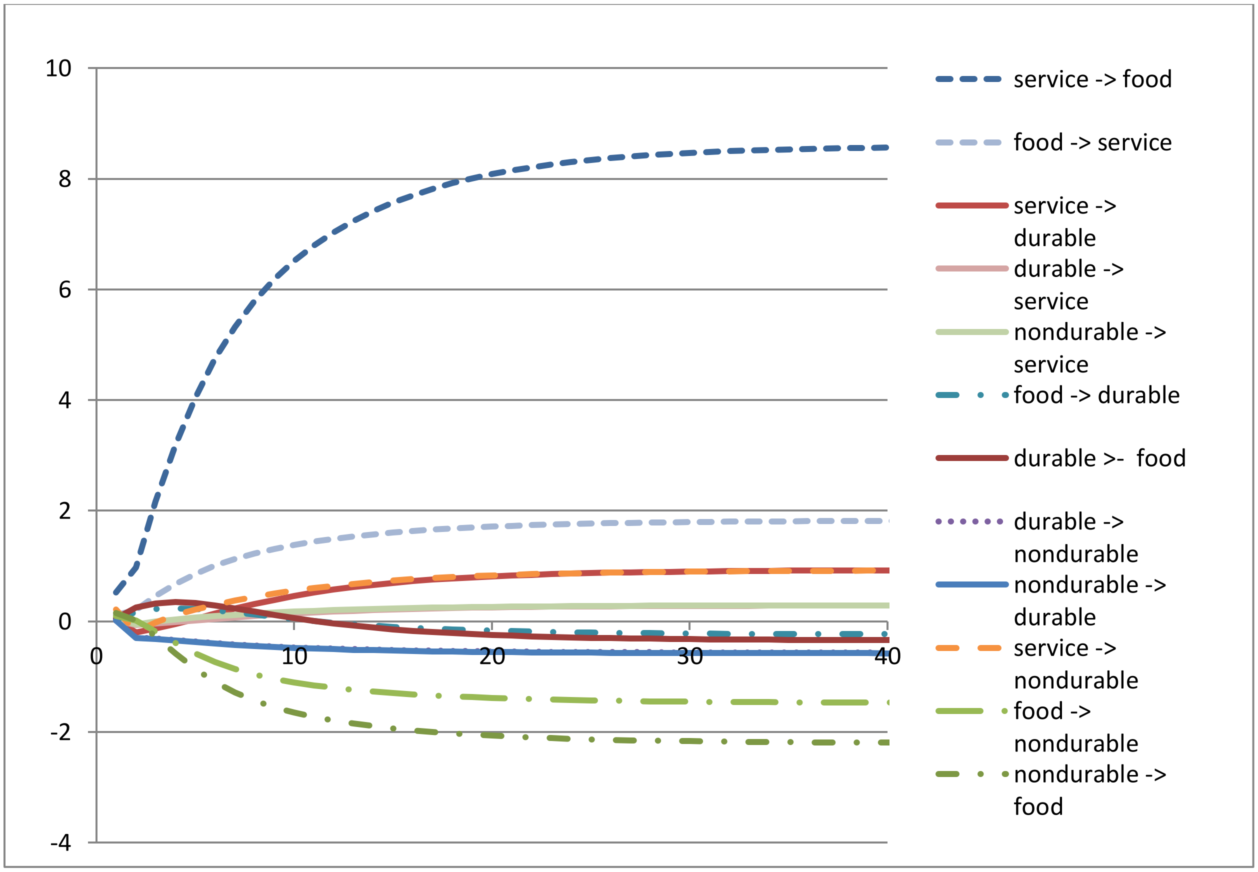


Figure 4: Elasticities of the marginal benefit of x_i with respect to a persistent change in x_j over k years, $j \neq i$, k = 0, …, 40.



References

Allais, M. (1943). *Traité d'Economie Pure*, tome I, Imprimerie Nationale, Paris.

Anderson, R.W. (1980). Some Theory of Inverse Demand for Applied Demand Analysis. *European Economic Review* 14: 281-90.

Baggio, M. and J.P. Chavas. (2009). On the Consumer Value of Complementarity: A Benefit Function Approach. *American Journal of Agricultural Economics* 91: 489-502.

Banks, J., R. Blundell and A. Lewbel. (1997). Quadratic Engel Curves and Consumer Demand. *Review of Economics and Statistics* 79: 527-539.

Barten A.P. (1969). Maximum Likelihood Estimation of a Complete System of Demand Equations. *European Economic Review* 1: 7-73.

Barten A. P. and L. Bettendorf. (1989). Price Formation of Fish: an Application of an Inverse Demand System. *European Economic Review* 33: 1509-1525.

Becker, G. and Mulligan, C. (1997). The Endogenous Determination of Time Preference. *Quarterly Journal of Economics* 112: 729-758.

Becker, G. and K. Murphy. (1988). A Theory of Rational Addiction. *Journal of Political Economy* 96: 675-700.

Bleichrodt, H. and A. Gafni. (1996). Time Preference, the Discounted Utility Model and Health. *Journal of Health Economics* 15: 49-66.

Breusch, T. (1978). Testing for Autocorrelation in Dynamic Linear models. *Australian Economic Papers* 17: 334-355.

Briec, W. and P. Gardères. (2004). Generalized Benefit Functions and Measurement of Utility. *Mathematical Methods of Operation Research* 60: 101-123.

Bureau of Economic Analysis. US Department of Commerce, Washington, D.C. http://www.bea.gov/national/consumer_spending.htm

Chambers, R.G., Y. Chung and R. Färe. (1996). Benefit and Distance Functions. *Journal of Economic Theory* 70: 407-419.

Chavas, J.P. and M. Baggio. (2010). On Duality and the Benefit Function. *Journal of Economics* 99(2010): 173-184.

Courtault, J.M., B. Crettrez and N. Hayek. (2004). On the Differentiability of the Benefit Function. *Economics Bulletin* 4: 1-6.

Deaton A. (1979). The Distance Function and Consumer Behaviour with Applications to Index Numbers and Optimal Taxation. *Review of Economic Studies* 46: 391-405.

Deaton, A. and J. Muellbauer. (1980a). *Economics and Consumer Behavior*. Cambridge University Press, Cambridge.

Deaton, A.S. and J. Muellbauer. (1980b). An Almost Ideal Demand System. *American Economic Review* 70: 312-326.

Epstein, L.G. and J.A. Hynes. (1983). The Rate of Time Preference and Dynamic Economic Analysis. *Journal of Political Economy* 91: 611-635.

Färe, R, S. Grosskpof, K.J. Hayes and D. Margaritis. (2008). Estimating Demand with Distance Functions: Parameterization in the Primal and Dual. *Journal of Econometrics* 147: 266-274.

Fuchs V. (1991). Time Preference and Health: an Exploratory Study. In Culyer, A.J. (ed.), *The Economics of Health*, Vol. I. Edward Elgar Publishing, Great Yarmouth. 93- 150.

Godfrey, L. (1978). Testing Against General Autoregressive and Moving Average Error Models When the Regressors Include Lagged Dependent Variables. *Econometrica* 46: 1293-1301.

Grossman, M. (1972). On the Concept of Health Capital and the Demand for Health. *Journal of Political Economy* 80(2): 223-255.

Grossman, M. (2000). The Human Capital Model. Chapter 7 in *Handbook of Health Economics*, Volume 1, Edited by A.J. Culyer and J.P Newhouse, North Holland, Elsevier Science, 347-408.

Gruber, J. and B. Koszegi (2001). Is Addiction Rational? Theory and Evidence. *Quarterly Journal of Economics* 116: 1261-1303.

Hayashi, F. (2000). *Econometrics*. Princeton University Press, Princeton, NJ.

Hicks J. R. (1932) *Theory of Wages*, Macmillan, London.

Jehle, G.A. and P.J. Reny. (2001). *Advanced Microeconomic Theory*. Addison Wesley, New York.

Koopmans, T.C. (1960). Stationary Ordinal Utility and Impatience. *Econometrica* 28: 287-309.

Koopmans, T.C., P.A. Diamond, and R.W. Williamson. (1964). Stationary Utility and Time Perspective. *Econometrica* 32: 82-100.

Laibson, D. (1997). Golden Eggs and Hyperbolic Discounting. *Quarterly Journal of Economics* 112: 443-477.

Lowenstein, G. and D. Prelec. (1992). Anomalies in Intertemporal Choice: Evidence and an Interpretation. *Quarterly Journal of Economics* 107: 573-597.

Luenberger, D.G. (1992). Benefit Functions and Duality. *Journal of Mathematical Economics* 21: 461-481.

Luenberger, D.G. (1995). *Microeconomic Theory*. McGraw-Hill Inc., New York.

Luenberger, D.G. (1996). Welfare from a Benefit Viewpoint. *Economic Theory* 7: 445-462.

Mas-Colell, A., M.D. Whinston and J.R. Green. (1995). *Microeconomic Theory*. Oxford University Press, New York.

McLaren, K.R. and K.K.G. Wong. (2009).The Benefit Function Approach to Modeling Price-Dependent Demand Systems: An Application of Duality Theory" *American Journal of Agricultural Economics* 91: 1110-1123.

Nakamura, A. and M. Nakamura. (1981). On the Relationships among Several Specification Error Tests Presented by Durbin, Wu and Hausman. *Econometrica* 6: 1583-1588.

Pollak, R.A. (1970). Habit Formation and Dynamic Demand Functions. *Journal of Political Economy* 78: 745-763.

Samuelson, P. (1928). A Note on Measurement of Utility. *Review of Economics Studies* 4: 155-161.

Shephard, R. *Cost and Production Functions*. (1953). Princeton University Press, Princeton, NJ.

Smith, P., Bogin, B., Bishai, D. (2005). Are Time Preference and Body Mass Index Associated? Evidence from the National Longitudinal Survey of Youth. *Economics and Human Biology* 3: 259-270.

Strauss, J. and D. Thomas. (1998). Health, Nutrition and Economic Development. *Journal of Economic Literature* 36: 766-817.

Footnotes

---

[1] Following Hicks (1932) and Baggio and Chavas (2009), substitution/complementarity relationships among consumer goods can be identified from their marginal benefits: two goods are said to be substitutes (complements) when increasing the consumption of one good has a negative (positive) impact on the marginal benefit of the other.

[2] All vectors are treated as column vectors, with x' being the $(1\times n)$ row vector denoting the transpose of x.

[3] While equation (1) is written as a first-order difference equation, it can represent a general q-th order difference equation. Indeed, the q-th order difference equation $y_t = g_t(y_{t-1}, y_{t-2}, \ldots, y_{t-q})$ can be equivalently written as the following first-order difference equation

$$Y_t \equiv \begin{bmatrix} y_t \\ y_{t-1} \\ \vdots \\ y_{t-q+1} \end{bmatrix} = \begin{bmatrix} g_t(Y_{t-1}) \\ y_{t-1} \\ \vdots \\ y_{t-q+1} \end{bmatrix} \equiv G_t(Y_{t-1}).$$

[4] Recursive preferences and their economic implications have been discussed by Epstein and Hynes (1983), Hertzendorf (1995), Kreps and Porteus (1978) and others.

[5] Note that switching from $u_t(\cdot)$ to $v_t(\cdot)$ in (6) means that we lose information about the role of discounting (which occurs only in the effects of (x_t, y_t) through V_{t+1} in (6)) We will revisit this issue in the concluding section.

[6] Prices being treated as known and given (from Assumptions A1-A2), a change in p_t in the indirect utility function $W_t(p_t, h_t(y_t), y_t)$ is to be interpreted as change in the price at time t, holding all other prices $(p_1, \ldots, p_{t-1}, p_{t+1}, \ldots, p_T)$ constant.

[7] In the AIDS and QAIDS models, Shephard's lemma in (9a) is alternatively written as $p_{it}\, x_{it}^c(\cdot)/E_t(\cdot) = \partial\ln(E_t(\cdot))/\partial\ln(p_{it})$, the empirical analysis being conducted using $\ln(E_t(\cdot))$ and expenditure shares.

[8] Throughout the paper, we use the following notation for derivatives: $\frac{\partial B_t}{\partial x_t} = (\frac{\partial B_t}{\partial x_{1t}}, \ldots, \frac{\partial B_t}{\partial x_{nt}})$ is a (1×n) vector, and $\frac{\partial^2 B_t}{\partial x_t^2} = \begin{bmatrix} \frac{\partial^2 B_t}{\partial x_{1t}^2} & \cdots & \frac{\partial^2 B_t}{\partial x_{1t}\partial x_{nt}} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 B_t}{\partial x_{nt}\partial x_{1t}} & \cdots & \frac{\partial^2 B_t}{\partial x_{nt}^2} \end{bmatrix}$ is a (n×n) matrix.

[9] The distance function D(**x**, U) is linear homogeneous and concave in **x**, and non-increasing in U (Shephard 1953; Deaton 1979).

[10] When $g = x_t$, note that $D_t(x_t, y_t, U_t) = 1/[1 - B_t(x_t, y_t, U_t)]$ establishes a non-linear relationship between D_t and B_t. This implies that the Luenberger matrix $\frac{\partial^2 B_t}{\partial x_t^2}$ differ from the "Antonelli matrix" defined as $\frac{\partial^2 D_t}{\partial x_t^2}$ and analyzed in Deaton (1979). See Chavas and Baggio (2010).

[11] Indeed, choosing $g = x_t$ has one significant drawback: in the presence of heterogeneous behavior, it implies choosing a different reference bundle for each consumer. This means that the Shephard distance function cannot be meaningfully aggregated across consumers (because proportional measurements cannot be easily added across heterogeneous consumers). In contrast, as argued by Luenberger (1992, 1995, 1996), for a a given reference bundle g, aggregate benefits can be obtained simply by summing individual benefit across consumers (even in the presence of preference heterogeneity).

[12] Equations (16) and (17a) express $\alpha_t(\cdot)$ is a form that can be estimated. This is particularly useful when the state variables y_t are not directly observed. However, while this approach is both flexible and convenient, it treats only implicitly the effects of specific state variables on consumption dynamics. We will revisit this issue in the conclusion.

[13] Note that the normalization rule p' g = 1 also implies that the variables p_{t-1}^* in (18) are perfectly collinear. On that basis, one of the prices in p_{t-1}^* must be dropped in the econometric estimation of (18). Below, we drop the n-th price $p_{n,t-1}^*$ in (18).

[14] Under the symmetry of a_x, note that, equation (19c) can be alternatively written as a_x g = 0.

[15] We implement the DWH test by introducing in (18) the error terms from the reduced form estimation of x_t and then testing whether the corresponding coefficients are statistically significant (see Nakamura and Nakamura 1981).

[16] The BG test for first-order serial correlation is asymptotically equivalent to the Durbin's h test (Hayashi, 2000, p. 149).

[17] Note from equation (13) and (17) that the benefit function is not quadratic in x_t, implying that the Eigen values of $\frac{\partial^2 B_t}{\partial x_t^2}$ are "local" (as they can change across evaluation point x_t).

[18] We also evaluated the price flexibilities $\partial \ln(p_t^b)/\partial \ln(x_t)$ at other points within the sample data. We found similar results across evaluation points.

[19] This is due to two factors: 1/ our quadratic specification for $\alpha_t(\cdot)$ in (17a); and 2/ our choice that the reference bundle g is the sample means of x.