A Differential Game Approach to Adoption of Conservation Practices

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Agricultural production can degrade water sources through leaching of nitrogen and phosphorus from agricultural land to surface and ground water sources. To minimize the pollution from agricultural production, the U.S. Department of Agriculture promotes adoption of conservation practices. Previous studies that analyzed adoption of new technologies did not incorporate the two important features of technologies that are primarily used to conserve the environment; common resource and interaction between farmers. The current study develops a conceptual framework using a differential game to analyze adoption of new technologies that impact the water quality. The results of the current study show that the single agent optimization models of the previous studies would not lead to the optimal solution of the differential game. Current study also shows that if farmers act cooperatively, they devote more capital to conserve the environment.

Keywords: Technology Adoption, Water Quality, Optimal Control, Differential Game

Good quality water for drinking, aquatic life, and agricultural production is crucial for life. According to the National Water-Quality Assessment (NAWQA) Program of the U.S. Geological Survey, agricultural production is a significant source of pollutant for underground and surface water sources (U.S. Department of Interior 1999). According to NAWQA streams and ground water in basins with extensive agricultural production have high concentrations of nitrogen and phosphorus. Nitrogen and phosphorous levels in the streams are mostly higher than the levels that cause overgrowth of plants in streams (U.S. Department of Interior 1999). Another important finding of the NAWQA is that 90 percent of nitrogen and 75 percent of phosphorus are estimated to originate from non-point sources, which are difficult to identify and monitor.

To address the concerns about water quality, the U.S. Department of Agriculture promotes adoption of conservation practices that can minimize the leaching of nitrogen and phosphorus from agricultural land to surface and ground water sources. The U.S. Department of Agriculture also provides cost share programs, such as Environmental Quality Incentives Program (EQIP) that provides funding to farmers to adopt conservation practices.

Adoption of new technologies by farmers has been analyzed in the literature. These studies developed the theories for adoption of new technologies focused primarily on profit oriented practices (Gedikoglu and McCann 2012). However, technologies, such as conservation practices, that are targeting the environmental quality and the profit oriented technologies show different structures (Gedikoglu and McCann 2012). The environment oriented technologies involve a common resource such as water quality. Hence, besides a capital and time constraint of the farmers, a common resource constraint should also be included in the analyses. Another factor that is important in adoption of environment-oriented technologies that was not involved in previous studies is the interaction among farmers (Cooper and Keim 1996). Since the actions of farmers impact the state of the common resource, adoption decision of a farmer impacted from the actions of the other farmers (Dockner et al. 2000). Actions of farmers also impact the wellbeing of each other directly through the impact on the common resources.

Since the common stock evolve over time, a dynamic optimization framework needs to be used for the adoption of environment-oriented technologies. Previous studies mentioned above included only static optimizations models. Since water quality evolves in continuous time, optimal control theory will be used in the current paper. The standard optimal control problem is structured for a representative or a single agent optimization problem (Goetz and Zilberman 2000). Hence, the solution does not involve the interaction among agents. Interaction among farmers will be incorporated into the model through the game theory. Optimal control problems that involve the game theory are called differential games (Dockner et al. 2000). These games are called "differential games" from the fact that the state variable of the optimal control problem evolves according to a differential equation. Hence, differential games include agents that interact to each other and solve an optimal control problem, and the solution to the problem is in the form of a Nash Equilibrium. Differential games have been widely used in economics and management (Ulph 1989; Khalatbari 1977; Kemp and Long 1980; Sinn 1984; Bolle 1980; Dockner et al. 2000). To our knowledge this is the first paper that uses differential games in technology adoption. The contribution of this paper is, by using the differential game framework, to provide alternative conceptual frameworks to analyze the farmers' adoption of environment-oriented technologies. The frameworks provided in the current paper will evaluate the impact of the interaction between farmers. The results of the current paper can be used by policy makers to design effective programs to promote the adoption of conservation practices.

Conceptual Framework

In the current model, farmers can spare some part of their capital or income for adoption of environment-oriented technologies, which have positive impact on the common resource or the water quality in this case. However, the capital used for adoption of conservation technologies decreases the capital available for agricultural production. We will use a linear conversion between consumption and production. Hence at time *t* for farmer *i* $c_i(t) = k_i^p - k_i^e$, where $c_i(t)$ is the consumption, k_i^p is the capital used in agricultural production and k_i^e is the capital spared for adoption of conservation practices. We will assume that there exists a group of farmers $i = 1,2...N$ whose actions impact the water quality and who make adoption decisions. The transition function for water quality then can be written as;

$$\dot{w}(t) = -\sum_{i=1}^{N} k_i^p(t) + \sum_{i=1}^{N} k_i^e(t)$$

where $\dot{w}(t)$ is the change in water quality, which is also the state variable for the optimization problem. The transition function shows that the capital devoted for agricultural production k_i^p impact the water quality negatively and the capital spared for adoption of conservation practices k_i^e impacts the water quality positively. The objective function for farmer *i* can be written as;

$$\max = \int_0^\infty e^{-rt} u(c_i(t))dt = \int_0^\infty e^{-rt} u\left(k_i^p(t) - k_i^e(t)\right)dt$$

where farmer *i* maximizes discounted stream of utility and the utility is gained from consumption, which is a linear function of capital as mentioned above. In the equation above $r \geq 0$ denotes the constant time preference rate or the discount rate.

The Cooperative Case

The benchmark will be the case where all the farmers act cooperatively. This is very similar to the single agent optimization problem. In this case, assuming that the all the farmers have the same utility function, the resulting optimal control problem can be represented as;

$$\max_{k_i^e} = \int_0^\infty e^{-rt} Nu(k_i^p(t) - k_i^e(t))dt$$
$$s.t.$$
$$\dot{w} = -N(k_i^p(t) - k_i^e(t)),\ k_i^p(t) - k_i^e(t) \geq 0,\ w(0) = w_0 > 0,\ \lim_{t \to \infty} w(t) \geq 0$$

The solution to this problem will be found using Pontyagin's maximum principle (Dockner et al. 2000). The optimal solution will be in terms of the initial value of the state variable and not the current value of the state variable. This kind of solution is called the open-loop equilibrium and the solution that includes the current value of the state variable is called the Markow-Perfect equilibrium. In general both solutions should be same given the initial value of the state variable is known (Dockner et al. 2000). The Markow-Perfect equilibrium requires farmers to observe the state of the water quality in each time period to form the optimum decision. Since this is difficult for farmers to do, we would use the open-loop equilibrium, which only requires the initial value of water quality be known. Using Pontyagin's maximum principle, the Hamiltonian can be written as;

$$H(w(t), k_i^p(t) - k_i^e(t), \lambda(t)) = Nu(k_i^p(t) - k_i^e(t)) - \lambda N(k_i^p(t) - k_i^e(t))$$

The conditions for the optimality are;

$$u'(k_i^p(t) - k_i^e(t)) - \lambda(t) \leq 0,\quad k_i^p(t) - k_i^e(t) \geq 0,$$
$$\dot{\lambda} = r\lambda$$

Solutions to these optimality conditions with using a constant elasticity of marginal utility;

$$u(c_i(t)) = \frac{Ac_i(t)^{\;1-\eta}}{1-\eta} = \frac{A(k_i^p(t) - k_i^e(t))^{1-\eta}}{1-\eta}$$

leads to

$\lambda(t) = \lambda(0)e^{rt}$ and $k_i^e(t) = k_i^p(t) - [k_i^p(0) - k_i^e(0)]e^{-rt/\eta}$

The value of $k_i^p(0) - k_i^e(0)$ can be calculated noting that an optimal path must exhaust the water quality in the sense that $\lim_{t\to\infty} w(t) = 0,$ which shows that any path that does not exhaust the water quality eventually will be dominated by a feasible path that exhausts, giving rise to more consumption over some time interval. For this reason,

$$w_0 = \int_0^\infty N(k_i^p(t) - k_i^e(t))dt = \frac{\eta N(k^p(0) - k^e(0))}{r},$$

which leads to optimum solution;

$k^e(t)^* = k^p(t) - \frac{rw_0}{\eta N}e^{-rt/\eta}$ and the time path for the state variable, water quality is

$w^*(t) = w_0 e^{-rt/\eta}$.

Also, note that $k^e(t)^* = k^p(t) - \frac{rw^*(t)}{\eta N}$, which is the Markovian strategy that requires farmers to know the value of the state at time *t* to form the optimal consumption, whereas open-loop strategy does not require farmers to know the value of the state variable at time *t*, rather it requires farmers to know the initial value of the state variable. Another important point is for any $N \neq 1,$ the single agent optimization models used in previous studies would not lead to the optimum solution for adoption of a technology that involves a common resource.

The Noncooperative Case

In the noncooperative case, each player must choose at the outset a time path of capital for adoption of conservation practices and these choices are made simultaneously and non-cooperatively. Hence, each farmer is making the optimal decision individually, but taking into account the actions of the other farmers. The outcome in the noncooperative situation or game will be analyzed for two cases $\eta \geq 1$ and $\eta < 1$ for the utility function;

$$u(c_i(t)) = \frac{Ac_i(t)^{\;1-\eta}}{1-\eta} = \frac{A(k_i^p(t) - k_i^e(t))^{1-\eta}}{1-\eta}$$

Case 1: $\eta \geq 1$

In this case $\lim_{c \to 0} u(c) = -\infty,$ hence zero time consumption over any time period would give a payoff of minus infinity. For that reason, each player will make sure that water quality will not be exhausted in finite time. For given time paths $k_j^e(.),\ j \neq i,$ player *i* must choose a time path of capital for conservation practices $k_i^e(.)$ such that $w(0) = w_o$ and

$$\dot{w}(t) = -(k_i^p(t) - k_i^e(t)) - \sum_{j \neq i} (k_j^p(t) - k_j^e(t)) \text{ and } \lim_{t \to \infty} w(t) \geq 0\,.$$

The objective function of the farmer *i* can be written as

$$\int_0^{\infty} e^{-rt} u(k_i^p(t) - k_i^e(t))dt$$

and the Hamiltonian is

$$H_i(w(t), k_i^p(t) - k_i^e(t), \lambda_i(t)) = u(k_i^p(t) - k_i^e(t)) - \lambda_i(t)\left[k_i^p(t) - k_i^e(t) + \sum_{j \neq i}(k_i^p(t) - k_i^e(t))\right]$$

The optimality conditions can be obtained as

$$u'(k_i^p(t) - k_i^e(t)) - \lambda_i(t) \leq 0, \quad k_i^p(t) - k_i^e(t) \geq 0$$
$$\dot{\lambda}_i(t) = r\lambda_i(t)$$

The solution is obtained as $k_i^e(t)^* = k_i^p(t) - \frac{rw_0}{\eta N} e^{-rt/\eta}$ for any $k_j^e(t) = k_i^p(t) - \frac{rw_0}{\eta N} e^{-rt/\eta}$ for all $j \neq i$, together with $w^*(t) = w_0 e^{-rt/\eta}$. Hence, the cooperative solution is obtained as the Nash equilibrium.

Case 2: $\eta < 1$

In this case $\lim_{c \to 0} u(c)$ is finite, hence there will be incentive for each player to exhaust the water quality in finite time, as all players try to get a bigger portion of the common stock. The consumption paths for player *i* and for all $j \neq i$ will be that

$$\int_0^\infty c_i(t)dt + \int_0^\infty \sum_{j \neq i} c_j(t)dt = \int_0^\infty (k_i^p(t) - k_i^e(t))dt + \int_0^\infty \sum_{j \neq i} (k_j^p(t) - k_j^e(t))dt \leq w_0$$

In this case player *i*'s reply to *n*-1 time paths $k_j^p(t) - k_j^e(t),\ j \neq i$ is called strictly feasible open-loop replies. In this case the cooperative outcome is obtained as the Nash equilibrium. The other possibility for consumption paths to be,

$$\int_0^{T_i} c_i(t)dt + \int_0^{T_i} \sum_{j \neq i} c_j(t)dt = \int_0^{T_i} k_i^p(t) - k_i^e(t)dt + \int_0^{T_i} \sum_{j \neq i} k_j^p(t) - k_j^e(t)dt \leq w_0$$

where $T_i = \inf\left\{ t \middle| k_i^p(s) - k_i^e(s) = 0 \text{ for all } s \geq t \right\}$, which shows that the consumption or the extraction of water quality is zero in all dates after T_i. In this case farmer *i* is allowed to exhaust the water quality in finite time and frustrate the opponent's plans by exhausting the water quality. To see this point, assume there exists a value T_i such that farmer *i* sets $c_i(t) = k_i^p(t) - k_i^e(t) = 0$ for all $t > T_i$. If for some $j \neq i$, farmer *j*'s $k_j^p(t) - k_j^e(t)$ strictly positive at some $t > T_i$, then farmer *j*'s plan frustrated by *i* as at that time *t* water quality is already exhausted by player *i*.

Hence, when farmer *i*'s opponents choose the cooperative outcome $k_j^e(t)^* = k_j^p(t) - \frac{rw_0}{\eta N} e^{-rt/\eta}$, the optimal response for farmer *i* is to deviate from cooperative outcome and exhaust the water quality in finite time. Farmers which have shorter time horizon during agricultural production may be less concerned about the sustainability of the water quality. Accomplishment of the cooperative outcome would benefit the policy makers as farmers would spare higher amount of capital for adoption of conservation practices. Success of cost share programs such as Environmental Quality Incentives Program would also be impacted from the accomplishment of the cooperative outcome.

Stock Dependent Utility

This section of the paper focuses on the modification of the previous model. For some farmers, the utility may not be only gained from consumption, but also from the current state of the water quality. This might represent the environmentally concerned farmers. Assume now the farmers obtain utility from both the consumption and the current state of the water quality. For a utility function in the form of $u_i(c_i, w) = (c_i w)^{\alpha} = [(k_i^p(t) - k_i^e(t))w]^{\alpha}$, where $0 < \alpha < 1$, the cooperative problem is to maximize, which is similar to the utility function in the previous section;

$$\int_0^{\infty} e^{-rt} N[(k_i^p(t) - k_i^e(t))w]^{\alpha}\, dt$$

$$s.t.$$

$$\dot{w} = -N(k_i^p(t) - k_i^e(t)),\ k_i^p(t) - k_i^e(t) \geq 0,\ w(0) = w_0 > 0,\ \lim_{t \to \infty} w(t) \geq 0$$

the optional consumption path for this problem is

$$k_i^e(t)^* = k_i^p(t) - \frac{rw(t)}{N2(1-\alpha)}$$

and for the noncooperative case, the optimal consumption path is given by

$$k_i^e(t)^* = k_i^p(t) - \frac{rw(t)}{2(1-N\alpha)}$$

Which constitutes a symmetric Markov perfect Nash equilibrium, provided that $N < 1/\alpha.$ The rate of exhaustion is greater in the noncooperative than the cooperative case. As previous section, for any $N \neq 1,$ the single agent optimization models in previous studies would not lead to the optimum solution for adoption of a technology that involves a common resource.

Conclusion

As the focus of the policies become water and air quality, adoption of new technologies need to be analyzed in a setting different than the ones suggested in previous studies. Current study provides the conceptual framework using the differential game scheme. The results of the current study showed that single agent optimization models, which were used in previous studies, would not lead to the optimum solution of the differential game when there is more than one farmer impacting the common resource. Current study also shows that farmers would exhaust the water quality faster if they do not act cooperatively. This result is valid also for the case where farmers are environmentally concerned or gets utility from environmental quality. As the concerns about the environmental quality increases and cost share programs such as Environmental Quality Incentives Program are developed, the policy makers will benefit from farmers acting together to devote more capital to conserve the environment. Extension programs are needed to be developed that show farmers the benefit of acting cooperatively.

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