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Estimating PIGLOG Demands Using Representative versus Average Expenditure

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Estimating PIGLOG Demands Using Representative versus Average Expenditure

Abstract-Introduction—Economists often use aggregate time series data to estimate consumer demand functions. Some of the popular applied demand systems have a PIGLOG form. In the most general PIGLOG cases the “average” demand for a good is a function of the representative consumer expenditure not the average consumer expenditure. We would need detailed information on each period’s expenditure distribution to calculate the representative expenditure. This information is generally unavailable, so average expenditures are invariably used.

Since we are estimating these demand systems using the wrong expenditure terms, our estimates may be wrong. There are special cases where the average expenditure is a perfect proxy for the representative expenditure. We do an indirect test of this case by estimating a demand system using quarterly U.S. data on three categories of consumer expenditure. To the usual price and expenditure effects we added variables that may be associated with shifts in the distribution of expenditure. These potential-expenditure distribution shifters may also be taste-demand shifters in their own right. If they are purely taste shifters, their coefficients will be a multiple of the expenditure coefficient.

One of the variables we added, the unemployment rate, was both statistically significant and acted as a pure representative expenditure shifter. While statistically significant, the unemployment rate’s coefficients imply small effects on the representative expenditure. Our tests also show that the average and representative expenditure have a 1-to-1 relationship.

PIGLOG Defined

Some demand systems use the “price-independent, generalized logarithmic” (PIGLOG) form. Two examples are Deaton and Muellbauer’s Almost Ideal Demand System (AIDS) (1980a, 1980b) and Keller and Van Driel’s CBS system (1985). In PIGLOG forms the budget shares are a function of the logarithm of expenditure

$$(1) \quad w_{i,k} = a_i + b_i \ln(x_k) .$$

In the equation above, the term $w_{i,k}$ is the budget share for product “i” and household k, a_i and b_i are coefficients and x_k is household k’s total expenditure. The budget share is the total amount spent on good “i” divided by the total amount spent on all goods. To simplify the discussion, we are assuming that all households have the same Engle curves. When we generalize from Engle curves to demand systems, the a_i term in (1) can be made a function of prices. The “price-independent” part of PIGLOG comes from the fact that the b_i term is independent of prices.

One of the advantages of PIGLOG structures is that they are consistent with non-linear aggregation over households. Suppose we construct a market “average” share for a good by (1) adding up all the households’ expenditure for that good and (2) adding up all the households’ total expenditures to get a market total, then dividing the first total by the second. Call these market shares $w_{i,M}$. With a PIGLOG structure such as (1), there is a representative expenditure x_R such that

$$(2) \quad w_{i,M} = a_i + b_i \ln(x_R) \text{ for all } i \text{ products.}$$

The representative expenditure can be the market average expenditure, x_M in only two cases.

The first case would be if all the household expenditures are the same—an unrealistic scenario.

The other case is when all the b_i coefficients are 0, the homothetic demand case. If demands are homothetic, we can use linear aggregation¹ and average quantity demanded is driven by average expenditures.

While the representative and average expenditure are generally different, it is possible that they have a simple relationship over time. One way to get this type of simple relationship is when all households’ total expenditures change at a common rate. For example, if one household’s expenditures go up by 1%, all households’ expenditures (and the market total) increase by 1% also. More realistically, there could be some “shuffling” of households in the total distribution. In cases like this, the average and representative expenditures are proportional

$$(3) \quad x_{M,t} * D = x_{R,t} \rightarrow \ln(x_{M,t}) + \ln D = \ln(x_{R,t}).$$

In (3) we have added time subscripts, the “t.” One way to generalize (3) is to make $\ln D$ a function of time itself— $\ln D_t$. We are going to use the CBS model, a differential model of demand. Differential models use the changes in variables from one period to the next

¹ Technically we can use linear aggregation if demands are all quasi-homothetic.

$$(3) \quad \Delta \ln(x_{M,t}) + \Delta \ln D_t = \Delta \ln(x_{R,t}).$$

Note that (3) implies that if the $\ln D$ is fixed over time the change in the mean expenditure is exactly equal to the change in the representative expenditure.

In order to test for the relationship between representative and mean expenditures, we are going to both make (3) more general and more specific:

$$(4) \quad \gamma_x \Delta \ln(x_{M,t}) + \gamma_0 + \sum \gamma_n \Delta z_{t,n} = \Delta \ln(x_{R,t}).$$

The added generality is that we now multiply the change in the mean expenditure by a coefficient. We have made $\Delta \ln D_t$ a function of an intercept and some “z” variables to make the relationship more specific. The change in the average expenditure will equal the change in the average expenditure if γ_x is equal to 1 and the rest of the γ are all 0.

Data

The **Data Sources** section toward the end of this report has links and a more detailed description of the data. We downloaded data from the Bureau of Economic Analysis on per capita, U.S. consumption expenditures and consumption-class deflators. We used the consumption-class deflators as our prices and divided the expenditures by their deflators to make quantities. We used quarterly data from 1980 -2012 inclusive for three broad classes of expenditures: durable goods, non-durable goods and services.

For our “z” variables used the quarterly U.S. unemployment rate and labor-force participation rate, downloaded from the Bureau of Labor Statistics. Unemployment and labor-force participation rates are likely to have an important direct effect on the distribution of income and, therefore, an effect on the distribution of expenditures. Also, Hahn (1988) used the unemployment rate in a previous study to correct for the effects of the distribution of income in a meat-demand study.

The CBS Model

Keller and Van Driel's CBS model starts with a total differential of the budget equation

$$(5) \quad \partial \left(\sum_i q_i p_i = x \right) \rightarrow \sum_i w_i \partial \ln p_i + \sum_i w_i \partial \ln q_i = \partial \ln x .$$

Here q_i and p_i are the quantity and price of good i , x and w_i are defined as above. We have dropped the household subscripts for the moment. The terms in summations equation (5) are often replaced with division price and quantity indices as defined below:

$$(6) \quad \partial P = \sum_i w_i \partial \ln p_i$$

$$(7) \quad \partial Q = \sum_i w_i \partial \ln q_i$$

The total differential that defines the CBS is:

$$(8) \quad w_i \cdot [\partial \ln q_i - \partial Q] = \sum_j c_{i,j} \partial \ln p_j + b_i [\partial \ln x - \partial P]$$

The c_{ij} coefficients show how quantity "i" react to changes in the price of "j." The b_i coefficients in (8) are equivalent to the b_i coefficients in our Engle-curve relationships. These coefficients can be used with the budget share to derive price and expenditure elasticities of demand using the following formulas:

$$(9) \quad \epsilon_{ij} = \frac{c_{ij} - b_i w_j - w_j w_i}{w_i},$$

$$(10) \quad \eta_i = 1 + \frac{b_i}{w_i}$$

In (9) and (10) ϵ_{ij} is elasticity of demand for good i with respect to price j and η_i is i 's expenditure elasticity.

In order to be consistent with optimization, the following constraints have to hold on the coefficients.

$$(11) \quad \sum_i c_{ij} = 0 \forall j$$

$$(12) \quad \sum_i b_i = 0$$

$$(13) \quad \sum_j c_{ij} = 0 \forall i$$

$$(14) \quad c_{ij} = c_{ji} \forall i, j$$

Applying the CBS to the Data

The CBS is based on a set of partial differential equations. We do not see derivatives of the demand functions. We see prices, quantities, and expenditures. When estimating the CBS demand function we take the differential equation above and use it to make a difference type equation

$$(14) \quad w_{i,t}^* [\ln q_{i,t} - \ln q_{i,t}^b - \Delta Q_t] = a_{0,i} + \sum_d a_{d,i} d_{d,t} + \sum_j c_{ij} [\Delta \ln p_{j,t}] + b_i [\Delta \ln(x_{R,t}) - \Delta P_t] + u_{i,t}.$$

In (14), the $w_{i,t}^*$ is an average of period t and $t-1$'s shares, ΔQ and ΔP are discrete versions of the quantity and price indices. The $a_{0,i}$ and $a_{d,i}$ intercepts and, in our case quarterly dummy coefficients. When added over all the goods, the “a” coefficients sum to 0 for the intercepts and seasonal dummies. The “d” terms are quarterly dummies, and $u_{i,t}$ a random error term. Note that (14) has the representative rather than the average expenditure. For estimation purposes we substitute (4) into (14)

$$(15) \quad y_{i,t} = a_{0,i} + \sum_d a_{d,i} d_{d,t} + \sum_j c_{ij} [\Delta \ln p_{j,t}] + b_i [-\Delta P_t] + \gamma_x b_i \ln x_{M,t} + b_i \gamma_0 + b_i \sum_n \gamma_n \Delta z_{t,n} + u_{i,t}.$$

In (15) we also replace the complicated term on the left-hand-side with an endogenous variable we call $y_{i,t}$. Note that the expenditure coefficient, b_i , show up in a number of different

places in (15), most notably in front of the $-\Delta P_t$. We can use the price-index term to identify the expenditure terms in the CBS.

The construction of the CBS endogenous variables is such that they should sum to 0 in every time period. This makes the covariance matrix of their error terms singular. In our case, it will have a rank of 2. Barton (1969) demonstrated that, assuming normally-distributed errors, one can implement Full-Information, Maximum Likelihood (FIML) estimates by dropping one of the equations and minimizing the determinant of the covariance matrix of the remaining errors. He also showed that these estimates are unaffected by the dropped equation. We used determinant-minimizing estimates and dropped the services equation. If the error terms are normally distributed, these estimates are FIML. Otherwise determinant minimization is equivalent to iterated generalized-least squares and will produce relatively efficient estimates.

Note that we have two intercepts in equation (15), $a_{0,1}$ and in the $\Delta \ln D$ term γ_0 . These two terms are not generally jointly identifiable. The intercepts in these differential models are generally considered taste-shifting terms. If the $a_{0,i}$ are actually all 0, then we will be able to identify γ_0 , provided that 2 or 3² of the b_i are not 0. We can have the situation where the intercepts are both driving changes in tastes and changes in the distribution of expenditures.

One of the notable features of differential demand models is the ease with which one can add taste-shifting variables to the model and still keep the model consistent with theory. Alston, Chalfant, and Piggott (2000) noted that one can add taste-shifting variables directly to the equations of differential models as we have with the intercept and seasonal dummies. For models like the AIDS, one should technically make the price and expenditure coefficients functions of the shifters. A bigger problem is dealing with the error term. Adding an error to the

² Inversely, the b_i can be identified as long as no more than 1 of the 3 is 0. Since they have to sum to 0 over the three goods, making 2 of them 0 forces the third to be 0 as well.

end of an AIDS equation means that the equation will be fully consistent with theory only when the error term is 0.

The improved ability to handle taste shifts is one of the reasons we used the CBS in this analysis. The “z” variables in (15) are written as if they are purely expenditure-distribution terms. These “z” terms could also be taste-shifters in their own right. As unemployment increases or labor-force participation decreases, we will have more households with fewer people working outside the home. The increase in forced-selected leisure time may lead to changes in what people buy. They may shift away from convenience-labor saving products.

Testing Strategies

As in the intercept case, if the z’s are purely distribution shifters, their coefficients will all be multiples of the b_i . If these variables are also taste-shifters in their own right it will be impossible to separate their taste and distribution shifting effects. We will use a two-pronged approach to testing the intercepts and z coefficients. First, do these terms matter at all? Second, if they do matter, are they consistent with being pure distribution shifters?

To implement the “do they matter at all tests” for the z variables, we replace their coefficients with a more “generic” set of coefficients, $c_{i,n}$. These have 2 degrees of freedom for each “n” as they must also sum to 0. If they do not matter at all, these $c_{i,n}$ are all 0. If they are consistent with pure distribution shifters, they can be replaced with $\gamma_n b_i$, a 1-degree-of-freedom restriction.

We also test average expenditure term. One obvious test here is that $\gamma_x=1$. We are also going to totally free up the average expenditure term by making a $c_{i,x}$. Rejecting the hypothesis that a $c_{i,x} = \gamma_x b_i$ would be particularly serious as this implies a fundamental problem with the model.

As we noted above, PIGLOG demands can be consistent with linear aggregation when all their b_i coefficients are 0. This is the homothetic demand case. In the homothetic case, the average demand is a function of the average expenditure. We will also test for homothetic demand.

Because most of our tests impose restrictions on multiple coefficients, we use a likelihood-ratio test. We estimate the model with and without the restriction. The restrictions will lower the objective. (Twice) the difference between the less-constrained and more-constrained objectives is asymptotically distributed chi-square under the null hypothesis.

Other Specification Issues: Demand Dynamics and Tests

Because we are working with quarterly data, we were concerned that demand dynamics and/or autoregression may be an issue in this data. We actually tested and restricted the demand dynamic before testing the representative-expenditure hypotheses. If we exclude important demand dynamics we can bias our tests in some undeterminable direction. Including irrelevant factors may lower the power of our tests.

Anderson and Blundell (1982) outlined a wide range of options for introducing dynamics in consumer demand systems. Our initial model had 4-quarter distributed lags for the exogenous and endogenous variables. For example, we replaced the endogenous variable on the left-hand-side of (15) with:

$$\begin{aligned}
 & y_{i,t} + \phi_1 y_{i,t-1} + \phi_2 y_{i,t-2} + \phi_3 y_{i,t-3} + \phi_4 y_{i,t-4} = \\
 (16) \quad & = a_{0,i} + \sum_d a_{d,i} d_{d,t} + \\
 & \sum_j c_{ij} [\Delta \ln p_{j,t} + \pi_1 p_{j,t-1} + \pi_2 p_{j,t-2} + \pi_3 p_{j,t-3} + \pi_4 p_{j,t-4}] \dots
 \end{aligned}$$

(16) is a truncated version of the most general equation we estimated. There is a common 4-quarter lag for the endogenous variables, the ϕ terms. The current and lagged prices are

multiplied by a common lag whose coefficients are the π . We used this same π -lag for the z 's and expenditure terms as well. The starting model also had a 2nd-order VAR error term. The starting model did not impose the $\gamma*b_i$ restrictions on neither the two z nor the average expenditure.

Table 1, below, summarizes the results of our preliminary tests. We first tested the distributed lags for the endogenous and exogenous. Each set of lags add 4 coefficients to the model. In our first set

of tests we ran all combinations of 0-4 lags for both the endogenous and exogenous variables. All the tests for dropping the lags are insignificant. Table 1 shows 3 of the 25 lag tests. The first shows what happens when we eliminate the endogenous variable's lags, the second when we eliminate the exogenous variables',

Table 1—summary of the general model structure tests^{1,2}

	<i>test</i>	<i>degrees of freedom</i>	<i>chi-square alpha</i>
<i>lag length test</i>			
endogenous lag lengths=0	1.37	4	84.91%
exogenous lag lengths=0	2.23	4	69.28%
both lag lengths are 0	8.79	8	36.07%
<i>VAR length tests</i>			
No VAR	29.93	8	0.02%
VAR is order 1	13.10	4	1.08%
second-order VAR terms are a multiple (ρ) times the first	1.10	3	77.66%
<i>VAR structure tests</i>			
durable equation driven only by services error	0.61	1	43.63%
service equation driven only by durable's error	2.04	1	15.29%
the two above together	2.65	2	26.59%
<i>intercepts and seasonal variable tests</i>			
intercepts=0	16.24	2	0.03%
seasonal variables = 0	7.25	6	29.81%

¹ Source ERS calculations based on Bureau of Economic Analysis & Bureau of Labor Statistics data

² Special highlighting for statistically-significant tests

and the third we when eliminate both sets of lags. None of these first three lag-length test statistics are significant at the 5% level. In fact, neither of the first two would be significant even if they were 1-degree-of-freedom tests. This data is consistent with complete adjustment within the quarter. We eliminated the distributed lags from the model for the rest of our analysis.

The next set of tests looked at the errors' autoregressive structure. We are using a VAR-type structure. As noted above, the errors for the system as a whole must sum to 0—they are perfectly collinear. To identify the VAR we drop one equation's lagged errors. The VAR's coefficients also have to sum to 0 over the equations. Each lag of the VAR has 4 degrees of freedom in it.

The next set of tests comes under the heading “VAR lag lengths.” For two of our restricted models we eliminated the VAR entirely and then ran a 1st-order VAR. Both of these special cases are rejected against the model with the more general 2nd-order VAR. For the last test in this series, we made the 2nd-order VAR a multiple of the first. This restriction passed.

Our use of VAR error terms in demand systems appears to be uncommon³. Analysts more commonly use scalar autoregressive structures. For example:

$$(17) \quad u_{t,i} = \rho_1 u_{t-1,i} + \rho_2 u_{t-2,i} + e_{t,i}, \text{ for all } i.$$

Note that in (17) the autoregressive terms are the same across all the equations-*i*. Making the 2nd-order part of the VAR a multiple of the 1st essentially puts us partway between the VAR specification and the more common scalar AR specification.

We would have the scalar case **if** we can make each equation's VAR a function only of its own lag. We also restrict the VAR if we can make each equation's errors a function of one

³ Barnett and Serlitas (2008) have an extensive review of the demand systems literature and note that many such estimates have unit roots in their error terms. Allowing for VAR-type error structures can allow one to incorporate cointegration in the error terms. Taha and Hahn, also presented at this conference, there is an example of a set of demand functions with a cointegrated error term. The differencing inherent in the CBS system eliminates roots=1.

other equation's lagged errors. In the next phase of the VAR testing, we made each equation's VAR a function of only its own or one of the other two equation's errors. That is 9 individual tests. In Table 1, under the heading "VAR structure tests" we show the 2 insignificant of the 9 tests and what happens when we put them together. None of the three equations are driven exclusively by its lagged errors. Lagged durable errors drive services, lagged services drive durables. The non-durable errors are functions of the both services' and durables' errors.

The final set of tests in Table 1 look at the dummy variable coefficients. All the data we used is seasonally adjusted—maybe we do not need the quarterly dummies. While we were testing the quarterly dummies we also tested the intercepts. The intercepts are significant, the seasonal dummies are not.

Is Average Expenditure a Good Proxy for Representative Expenditure?

Based on our preliminary tests, our "basic" model has complete adjustment within the quarter (i.e. no lagged exogenous or endogenous), a restricted 2nd-order VAR and no seasonal dummies. We will not have any aggregation issues if demands are homothetic. We test that hypothesis by setting the coefficients for the price index and the average expenditure to 0. This imposes 4 restrictions on the model. The test statistic for homothetic demands is 41.24. This is highly significant and we reject homothetic demands.

Given that demands are not homothetic, we then tested various restrictions on the average expenditure, intercept, and "z" variables. The results of these tests can be found in Table 2 on the next page. The first set of tests in Table 2 restricts the average expenditure coefficients.

Elimination of the average expenditure terms is rejected. Had this restriction passed, we would have had to conclude that there were issues with the model structure. The tests for making the average expenditure coefficients (1) proportional to and then (2) exactly equal to the b_i are both

insignificant. We are able to accept the hypotheses that the quarter-to-quarter changes in average expenditures are the same as the quarter-to-quarter changes in representative expenditures.

The unemployment-rate coefficients are both statistically significant

Table 2—testing adjusting data for representative expenditure^{1,2}

variable	hypothesis	test against free model	DF ³	chi-square alpha
average expenditure	$c_{ix}=0$	40.40	2	0.00%
	$c_{ix}=\gamma_x b_i$	0.54	1	46.35%
	$c_{ix}=b_i$	0.32	2	85.05%
unemployment rate	$c_{iu}=0$	6.48	2	3.91%
	$c_{iu}=\gamma_u b_i$	3.40	1	6.51%
labor-force participation	$c_{il}=0$	0.36	2	83.55%
	$c_{il}=\gamma_l^* b_i$	0.27	1	60.63%
intercept	$a_{i0}=0$	15.91	2	0.04%
	$a_{i0}=\gamma_0^* b_i$	8.26	1	0.40%
$c_{ix}=b_i, c_{iu}=\gamma_u b_i, \text{ and } c_{il}=0$		4.25	5	51.41%

¹ Source ERS calculations based on Bureau of Economic Analysis & Bureau of Labor Statistics data

² Special highlighting for statistically-significant tests, the cells with red lettering are the hypotheses accepted and used in the final model structure.

³ DF is degrees of freedom.

at the 5% level and consistent with being pure representative expenditure shifters. As a set, the labor-force participation rate is insignificant. Because the labor-force participation rate coefficients are insignificant, it is now surprising that they also pass being proportional to the b_i —we can use a $\gamma=0$ as a different method to eliminate them. We also retested the intercepts against 0. They pass again. We are able to reject the hypothesis that the intercepts act solely as representative expenditure shifters. It is entirely possible that γ_0 , the “trend” for $\ln D_t$, is not 0. It is just impossible to separate this trend for the taste-shifting trend.

Finally Table 2 shows what happens when we combine the 3 of our insignificant hypothesis tests, making a model where (1) changes in average expenditure equal changes

representative expenditure, (2) changes in the unemployment rate shift representative expenditure, and (3) the labor-force participation rate is excluded from the model entirely. This combination of restrictions is also statistically insignificant.

Implications of the Results

The estimated γ for the unemployment rate is 0.0303. This implies that increases in the unemployment rate raise the representative expenditure relative to the average expenditure. We expected to see a positive relationship between unemployment and representative expenditure. The consumers that spend the most have the largest effects on total market demand. Higher unemployment is likely to increase the relative number of low-income, low-expenditure households. Unemployment's γ coefficient measures the effect of unemployment on representative expenditure **given** some level of average expenditure. If unemployment increases but average expenditures remain the same, the loss of expenditures from the newly unemployed would have to be offset by increase in expenditures for other people. The “widening” in the distribution of expenditures would tend to raise the representative expenditure relative to the average expenditure.

In Table 3, we have taken the CBS model estimates and used them to calculate elasticities of pre-capita demand given prices, average expenditures, and the unemployment rate. All the CBS elasticities are functions of budget shares. The elasticities are evaluated at the aggregate,

Table 3—Estimated elasticities at mean budget shares¹

	durables	non-durables	services	expenditures	Unemployment rate
Durables	-1.361	1.052	0.293	0.884	-0.004
non-durables	0.387	-0.878	0.717	0.722	-0.008
Services	-0.002	0.288	-0.521	1.203	0.006

¹ Source ERS calculations based on Bureau of Economic Analysis & Bureau of Labor Statistics data

average shares for the sample.

All of the own-price effects are negative; most of the cross-price effects are positive. Durable and non-durable goods have expenditure elasticities that are less than one, but positive while services' demand is expenditure-elastic. The unemployment rate has small overall-effects on aggregate demand. As we discussed above, these unemployment elasticities are conditional on a given level of expenditure. It is likely that increases in unemployment will tend to lead to decreases in over-all expenditures. Measuring that type of effect is beyond the scope of this analysis.

We used Monte-Carlo analysis to calculate standard errors for our CBS model parameter estimates. We generated a series of new data using the final-form coefficient estimates, including the covariance matrix for the errors. We assumed that the errors were normally distributed. We used the Monte-Carlo standard deviations to calculate "z" statistics. The estimates and z-statistics are in Tables 4-6 on the next page.

The following tables have special highlighting in those cells where the z statistics are insignificant at the 5% level. Only 3 of the estimates have insignificant z values. These are all in table 4. They are the expenditure coefficient for durable goods, and the unemployment effects for durable goods and nondurable goods. The coefficients in Table 3 are the standard errors for the product $b_i\lambda_u$. The b_i z-statistics are under the "expenditure- b_i " column in Table 4 and the λ_u in Table 5.

Technically this does not imply that these coefficients are insignificant. The Monte-Carlo iterations were generated using the estimated coefficients. These z values just show that these 3 coefficients are not precisely estimated. For example, the unemployment rate's effect on nondurable demand is small relative to its standard error but is itself the product of two

significant coefficients. On the other hand, it is likely that had we tested the b_i we would have found that durable's was not significantly different from 0. Imposing that restriction on the

Table 4—CBS model parameter estimates and z statistics^{1,2,3}

		<i>price coefficients, the c_{ij}</i> ⁴					
		<i>intercepts, a_i</i>	<i>Durables</i>	<i>Non-durables</i>	<i>Services</i>	<i>Expenditure, b_i</i>	<i>UR⁵, $\gamma*b_i$</i>
Durables	estimate	-5.48E-04	-0.1647	0.1342	0.0305	-0.0153	-0.0005
	z statistic	-5.02	-22.02	31.33	3.77	-1.28	-0.97
Non-durables	estimate	1.44E-03		-0.3239	0.1897	-0.0929	-0.0028
	z statistic	7.69		-47.95	33.37	-4.47	-1.95
Services	estimate	-8.91E-04			-0.2202	0.1082	0.0033
	z statistic	-6.70			-21.00	6.83	2.00

Table 5—Single parameter estimates and z statistics^{1,2}

	<i>estimate</i>	<i>z statistic</i>
ρ multiplier for VAR	0.8156	3.97
γ for Unemployment rate	0.0303	2.22

Table 6—restricted VAR estimates and z statistics for the first-order terms^{1,2,6,7}

		<i>lagged endogenous</i>	
<i>equations</i>		<i>Durables</i>	<i>Services</i>
Durables	estimate		0.1814
	z statistic		3.15
Non-durables	estimate	-0.3889	-0.1814
	z statistic	-4.16	-3.15
Services	estimate	0.3889	
	z statistic	4.16	

Table Note for Tables 4-6

¹ Source ERS calculations based on Bureau of Economic Analysis & Bureau of Labor Statistics data

² Standard errors based on 10,000 Monte-Carlo iterations

³ Special highlighting for cells whose Z values are not significant at the 5% level

⁴ Because of symmetry only the upper triangular terms are shown

⁵ UR is the BLS unemployment rate

⁶ Blank cells denote terms restricted to 0

⁷ Second-order terms are ρ times the first-order term. See Table 5 for ρ estimates.

model would have made unemployment's effect on aggregate durable demand 0 also.

Conclusions

PIGLOG models are often used with time-series, aggregate data to estimate consumer demands. In theory, economists ought to be using representative expenditure to estimate these models. In practice, we use average expenditures. We developed and tested a method that allows us to indirectly measure changes in the difference between average and representative expenditure.

There are cases where average expenditures would be an ideal proxy for representative expenditure. For the aggregate, U.S. data we analyzed in this paper, one of these conditions was met. There is a 1-to-1 correspondence between changes in average and representative expenditures. We did, however, find at least one variable, the unemployment rate, that acted as if it were a representative expenditure shifter.

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Data Sources

Per-capita expenditures on Durables, Non-durables, and Services and their price indices were downloaded from the Bureau of Economic Analysis Website <http://www.bea.gov/>

The per capita expenditures are from *Table 7.1. Selected Per Capita Product and Income Series in Current and Chained Dollars*. The deflators-prices are from *Table 2.3.4. Price Indexes for Personal Consumption Expenditures by Major Type of Product*.

To download the unemployment and labor-force participation rates from the Bureau of Labor Statistics, go to <http://data.bls.gov/cgi-bin/srgate>

And paste the codes, LNS14000000Q and/or LNS11300000Q into the box.