Monthly Demand Relationships of U.S Meat Commodities

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Abstract

A set of price-dependent demand equations explains the interdependent nature of monthly demand relationships for 10 meat commodities The analysis uses a model with a mixed structural-time series approach, the model's forecasting capability is significantly better than that of the traditional structural model used alone.

Keywords

Price-dependent demand structure, mixed structural-time series model

Introduction

The U S consumption expenditure for red meats and poultry accounts for approximately one-third of the consumer's food budget. Because there is limited knowledge about the interdependence of the demand relationships among meat commodities in the short run, an efficient forecasting model for monthly meat prices is difficult to obtain Although previous studies have considered monthly demand behavior in the meat industry (*3*, *7*),[1] few published studies have focused on the interdependent nature of these demands This article analyzes this unexplored, yet important, facet of monthly demand and formulates a statistical model for improving the forecasting of meat prices An inverse demand system approach is adopted for specifying the monthly demand relationships of meat commodities In the statistical modeling, a mixed structural-time series model is applied, the results appear to have considerable potential to improve shortrun forecasting of meat prices

Model Specification

According to the classical demand theory, the economic problem of a representative consumer is to choose commodities under a budget constraint so that the consumer's utility function is maximized Let q denote an n-coordinate column vector of quantities, p an n-coordinate vector of their prices, $m = p'q$ the consumer's total expenditure, and $U(q)$ the utility function The primal function for consumer utility maximization is maximizing the following Lagrangian function.

$$\underset{q,k}{\text{Maximize}}\ L = U(q) - k\,(p'q - m) \qquad (1)$$

The necessary conditions for an optimum are obtained as

$$U_i'(q) = k\,p_i, \qquad i=1,2, \quad ,n \qquad (2)$$

and·

$$p'q = m \qquad (3)$$

in which $U_i'(q)$ is the marginal utility of the ith commodity By multiplying q_i in equation (2) and summing over n to satisfy the budget constraint of (3), the Lagrangian multiplier is

$$k = \sum_{j=1}^{n} q_j\,U_j'(q)/m \qquad (4)$$

Substituting equation (4) into (2) yields the Hotelling-Wold identity (*5*, *11*)·

$$p_i = m\,[U_i'(q) / \sum_{j=1}^{n} q_j\,U_j'(q)] \qquad (5)$$

$$i = 1,2,\ldots,n$$

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[1]Italicized numbers in parentheses refer to items in the References at the end of this article

This equation represents an inverse demand system in which the variation of price is a certain function of quantities demanded and is proportional to a change in income As indicated by Hicks (4), the Marshallian demands have two functions: one shows the amounts consumers will take at given prices, and the other shows the prices at which consumers will buy at given quantities The latter function, "quantity into price," is essentially what the identity expresses.

The inverse demand system has considerable appeal as applied to the shortrun demand for meat commodities. For example, beef takes about 27 months from breeding until slaughter weight, and the change of market supplies tends to be rather inflexible in the short run The aggregate quantity demanded for such a commodity becomes increasingly fixed as the time frame becomes shorter. Thus, if a monthly demand structure for meats is specified, assumptions that quantities and income are predetermined and that prices must be adjusted may be reasonable. Waugh rationalized that, in competitive markets, changes in prices are generally determined by changes in quantities marketed and changes in income, not the other way around (*10*) Most recently, Theil worked on a demand subsystem for beef, pork, chicken, and lamb by taking quantity changes as predetermined variables, on the justification that the role of meat prices is to insure that the market is indeed cleared (*9*) Thus, prices are endogenous in the demand-and-supply system.

When monthly demand relationships for U S. meat commodities are specified, questions regarding the available data sources and the functional form of the demands are of immediate concern. The available monthly data for meat commodities are rather limited, and the defined prices and quantities do not always correspond closely Monthly price observations, covering January 1964 to December 1979, for five beef products and four pork products are available from the U S. Department of Labor, Bureau of Labor Statistics. The products include sirloin steak, round steak, chuck roast, round roast, ground beef, pork chops, canned ham, bacon, and sausage Monthly price observations for broilers and the per capita quantities for meats over the same period are available from the Economic Research Service The quantities include beef, veal, pork, lamb, broilers, turkeys, and other chicken These quantities, measured in retail weight equivalents, are derived from the commercial carcass weight of various meat animals The correspondence between the price and quantity variables, although not ideal as required by the conceptual demand relations, is about as close as can be achieved with the limited data available

On the choice of functional form for the empirical fitting, the loglinear approximation of the Hotelling-Wold identity is used in this article largely for practical reasons The estimated demand parameters represent demand flexibilities which are easily interpreted The specification provides a convenient form for further elaboration in specifying the residual structure and in improving forecasting capability of the model

The statistical model for the ith price equation in terms of n quantities demanded and an autoregressive process of residuals lagged up to k months follow:

$$\log(p_{it}/m_t) = \alpha_{io} + \sum_{j=1}^{n} \alpha_{ij} \log q_{jt} + \sum_{j=1}^{11} \gamma_{ij}\, d_{jt} + u_{it} \qquad (6)$$

and:

$$u_{it} = \sum_{j=1}^{k} \beta_{ij}\, u_{i,t-j} + \epsilon_{it}, \qquad i = 1,2, \quad n \qquad (7)$$

where variables at time t are p_{it} (price of ith commodity), m_t (per capita income), q_{jt} (quantity demanded for jth commodity), and d_{jt}s (dummy variables assigned for sequential months from February to December to reflect the effects of monthly shifts), u_{it} and ϵ_{it} are respectively random disturbances in which ϵ_{it} is assumed to be normal and independently distributed, N (O, σ^2I)

In addition to the use of quantities as explanatory variables in equation (6), including monthly dummy variables in the equation reflects the possibility of seasonal variation in the demand for particular meat cuts, the possibility of seasonal difference in production costs, and the number of different graded meat animals being marketed. Shepherd and Futrell give some detailed explanations about the latter possibility (*8*). For instance, there is a rather inflexible supply of potential hog processing and marketing services during the short run in terms of plant facilities and labor supply A dwindling seasonal supply of hogs causes sharply increased buying competition among packers and forces the packers to take a lower margin for their processing and wholesaling services Consequently, the meat price in that season may not be as high as would be expected because of quantity change

Furthermore, the residual specification of equation (7) reflects a suggestion by Muth that there is little empirical interest in assuming that the disturbance term in a structural model is completely unpredictable (*6*). It is desirable to assume that past of the disturbance may be predicted based on part observations Because the expected values of the disturbance could be related to economic conditions prevailing in the past months, we may assume that the disturbance is not independent over time but follows an autoregressive process Accordingly, the model has some practical advantages for improving forecasting capability The structural component may provide forecasts for identifying the turning points of historical observations, whereas the time series component provides predictive information for the movements of random disturbance.

Estimation Procedures

The proposed statistical model on monthly demand relationships for meat commodities can be viewed as a mixed structural-time series model The model not only provides a structural explanation of meat prices in equation (6), but also replicates the past behavior of residuals by specifying an autoregressive process in equation (7) To estimate the model, one needs a three-step estimation procedure because the disturbance terms in the autoregressive process are not observable First some preliminary estimates of the structural parameters in equation (6) are obtained from ordinary least squares Within the context of serial autocorrelated errors, these estimates are known to be unbiased but inefficient, and thus a further re-estimate is required Second, the estimated residuals from the first step are used to fit the autoregressive process in equation (7) In this stage, the choice of lag order in the process can be determined by the significance of estimated coefficients in the equation Given an appropriate order for the lags, one can obtain the autoregressive coefficients in a particular equation by solving the so-called Yule-Walker equations (*1*) Third, based on the estimated autoregressive structure in equation (7), the structural parameters are reestimated by the application of an Aitken estimation procedure suggested by Gallant and Goebel (*2*)

Estimation Results

Table 1 summarizes the estimation results The values in each column express the price of a meat commodity as a function of seven meat quantities consumed, "other goods," a set of monthly dummy variables, and an autoregressive residual series The "other goods" in the consumer budget is defined as per capita a nonfood expenditure measured in constant 1967 prices. Because each equation expresses price as a function of quantities in logarithmic form, the response coefficients can be called "flexibilities "

According to the estimation results, the direct-price flexibility between the price of sirloin steak and the quantity of beef is -0 342, which indicates that if consumers make a 1-percent decrease in quantity of beef purchased, the price of sirloin steak will increase about 0 3 percent. Similar interpretation is given to the direct-price flexibility of other items The estimated direct-price flexibilities of monthly demand for various meat cuts are less than 1 in each case They are round steak (-0 377), chuck roast (-0 508), round roast (-0 332), ground beef (-0 418), pork chops (-0 581), canned ham (-0 207), bacon (-0 845), sausage (-0 457), and broilers (-0 410) The prices of round steak, chuck roast, round roast, and ground beef are functions of all beef, the prices of pork chops, canned ham, bacon, and sausage are functions of all pork

Special caution should be taken in interpreting cross-price flexibilities as the conventional view of substitution and complement between two goods Hicks distinguished the substitution relationship in the inverse demand system as q-substitutes from that in the ordinary demand system as p-substitutes, he said that "X and Y are q-substitutes when a rise in the quantity of X diminishes the marginal valuation of Y (or the price at which a fixed quantity of Y would be purchased) when the quantities of all commodities other than X are fixed, saving the

Table 1—Estımated shortrun flexıbılıties for meat commodıties

Independent varıable	Price of– Sırloın steak	Round steak	Chuck roast	Round roast	Ground beef	Pork chops	Canned ham	Bacon	Sausage	Broılers
Quantıty										
Beef	- 0 342	- 0 377	- 0 508	- 0 332	- 0 418	0 101	0 115	0 375	0 100	0 100
	(067)	(072)	(094)	(067)	(088)	(082)	(063)	(115)	(092)	(133)
Veal	- 006	- 008	- 012	- 012	- 040	- 011	- 001	008	002	008
	(010)	(011)	(015)	(011)	(014)	(014)	(009)	(018)	(014)	(022)
Pork	- 004	- 027	- 030	- 035	073	- 581	- 207	- 845	- 457	- 363
	(049)	(052)	(067)	(048)	(063)	(059)	(047)	(084)	(067)	(094)
Lamb and mutton	- 004	001	- 009	001	- 001	- 015	- 007	- 010	- 010	- 002
	(010)	(011)	(014)	(010)	(014)	(014)	(008)	(018)	(014)	(020)
Broılers	155	143	210	140	168	286	096	247	219	- 410
	(064)	(070)	(091)	(065)	(087)	(084)	(056)	(110)	(087)	(132)
Turkeys	- 006	012	003	009	014	046	011	033	009	033
	(017)	(017)	(023)	(016)	(022)	(022)	(015)	(030)	(023)	(033)
Other chıcken	045	064	075	062	031	060	- 021	044	005	095
	(022)	(024)	(031)	(022)	(030)	(030)	(019)	(038)	(030)	(044)
Other goods	- 602	- 645	- 531	- 749	- 505	- 1 188	- 614	- 1 020	- 387	- 868
	(088)	(098)	(125)	(091)	(118)	(111)	(091)	(150)	(122)	(179)
Monthly dummy										
Constant term	2 099	2 634	1 955	3 347	1 345	6 639	1 554	5 201	177	3 166
February	- 020	- 013	- 005	- 013	- 012	- 004	- 005	- 008	- 014	- 030
March	- 012	- 003	017	- 003	- 001	026	005	047	026	- 047
Aprıl	- 024	- 020	- 017	- 022	- 023	- 027	- 015	011	001	020
May	- 006	- 009	- 009	- 014	- 012	- 073	- 043	- 042	- 037	060
June	032	007	007	003	002	- 064	- 065	- 064	- 043	074
July	044	011	021	006	004	- 039	- 082	- 076	- 048	077
August	041	011	026	005	007	- 024	- 065	- 024	- 016	068
September	048	008	026	005	006	- 007	- 051	022	015	011
October	022	- 006	018	- 007	- 002	- 024	- 034	026	023	- 033
November	012	- 024	001	- 021	- 030	- 026	- 017	028	044	- 132
December	- 008	- 034	- 013	- 028	- 040	- 038	- 009	020	024	- 117
Resıdual										
Lag 1 month	467	328	346	345	286	264	618	491	471	267
Lag 2 months	115	179	131	150	168	070	060	- 066	029	129
Lag 3 months	131	247	240	262	254	274	149	237	232	342

Note The figures ın parentheses are the estımated standard errors All ıncome flexıbılıtıes are constraıned to unıtary values on the basıs of equatıon (4) quantity of money, which is adjusted so as to maintain indifference" (4)

To illustrate, considering a demand system with only two substitutable goods, we can present the system in elasticity matrix form in which the direct-price elasticities are negative and the cross-price elasticities are positive:

$$\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} -e_{11} & e_{12} \\ e_{21} & -e_{22} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} \quad \text{for positive } e_{ij}, \; i,j = 1,2$$

where p_i and q_i are, respectively, nominal price and quantity for ith goods expressed in logarithms, and where e_{ij}'s are the absolute value of demand elasticities. One can derive the inverse demand system by inverting the elasticity matrix and obtaining

$$\begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} -e_{22}/D & -e_{12}/D \\ -e_{21}/D & -e_{11}/D \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

where $D = e_{11} e_{22} - e_{12} e_{21}$ Because the direct-price flexibilities ($-e_{22}/D$ and $-e_{11}/D$) are negative for utility maximization, the value of D should be positive Consequently, the cross-price flexibilities (e_{12}/D and $-e_{21}/D$) are negative for the case of substitutable goods In other words, a marginal increase of the quantity of one good may have a substitution effect on the other goods, and the price of other goods should be lower to induce consumers to purchase the same quantity of the other goods

The relationships of substitution and complement depend on the compensated cross-price flexibilities in which the level of consumer utility is fixed Thus, in the absence of assuming a fixed utility level, the estimated cross-price flexibilities of table 1 may roughly reflect substitution of the negative sign and complement of the positive sign For example, the figure in the last column for the price of broilers associated with the quantity of pork is -0 363 which implies that the two commodities are substitutable A marginal 1-percent increase in the quantity of pork is associated with a -0 363-percent decrease in the price of broilers to induce consumers to purchase the same quantity of broilers In the same column, the figure related to the quantity of other chicken is 0.095, which may be complementary to broilers An increase in the quantity of other chicken will cause the price for other chicken to fall. Because of the complementary relationship, if the demand for broilers is to be kept constant, the price of broilers must rise

Similar interpretations can be applied to other estimated cross-price flexibilities In particular, all the cross-price flexibilities of "other goods" are negative and have relatively larger absolute values than any other flexibilities in each equation These results indicate strong substitution relationships with meats, and a marginal 1-percent increase in the "other goods" consumption induces a much larger reduction in any given meat price to keep the quantity of meats purchased constant Although some other cross-price flexibilities may not be consistent with conventional wisdom, these estimates nevertheless reflect the monthly interdependent relationships for various meat cuts that are not explored in other empirical studies

The middle section of each column in table 1 presents the effects of monthly price shifts on the various meat types The seasonal shifting pattern of meat prices is similar for commodities in the same category (for example, beef) but significantly different among categories The seasonal price pattern for beef commodities typically reaches a peak during July-August and decreases sharply in December and again in April In contrast to beef prices, pork prices typically peak in March and then bottom-out between May-July Broiler prices peak in July and are lowest in November Moreover, the estimated autoregressive process, lagged up to 3 months, is significant in all cases These results are listed at the bottom of each column

Finally, the statistical modeling in this study combines both the structural equation approach and time series analysis of residuals The mixed model, which combines the advantages of both approaches, should improve its forecasting ability To verify the forecasting performance of a model, one may conduct *ex post facto* simulation or may compare the simulated values outside the sample period with actual available data, this study considers the former approach only Because the observation of a dependent variable is stochastic, even though a model predicts perfectly well its mean value, we might risk drawing a conclusion of inefficient forecasting if a particular sample point chosen outside the sample period is far away from its mean value Two *ex post facto* simulations are evaluated; one follows the traditional approach of using the estimated structural model obtained from the estimation in the first step, and the other uses the final results from the estimation of the mixed model The ratio of the root-mean-square error of forecasts to the sample mean, expressed in percentage terms, is presented in the first two columns of table 2 for each case In the mixed model, the forecasting errors are less than 1 7 percent of the sample mean and are uniformly lower than the other model Thus, the forecasting efficiency (shown in the last column of the table) indicates that the mixed model is relatively more efficient for all monthly meat price forecasts This evidence strongly suggests that the mixed structural-time series model has greater potential for forecasting

Table 2—Ratio of root mean-square-error to sample mean for meat price forecasts

Commodity	Structural model estimated by OLS (1)	Mixed structure-time series model (2)	Relative efficiency (2)/(1)x100
		Percent	
Sirloin steak	1 43	0 90	62 9
Round steak	1 66	98	59 2
Chuck roast	2 07	1 30	62 6
Round roast	1 58	90	57 2
Ground beef	2 07	1 23	59 5
Pork chops	1 56	1 23	78 4
Canned ham	1 80	80	44 3
Bacon	2 37	1 62	68 4
Sausage	2 15	1 27	59 0
Broilers	1 86	1 31	70 4

Note The ratio of root mean square error to sample mean is calculated by

$$[\sum_{t=1}^{T} (y_t - \overset{*}{y}_t)^2/T]^{1/2} / \bar{y} \times 100,$$

in which y_t is the nominal price in the demand equation, and its predicted value and sample mean are $\overset{*}{y}_t$ and $\bar{y}$ respectively

Conclusions

I have estimated a set of price-dependent demand equations for highly disaggregated meat commodities, including five beef cuts, four pork items, and broilers. The commodity classifications closely reflect the consumer's demand in the retail market The equations depict the interrelatedness of monthly demand for meats, an area for which we have limited knowledge and an area that few empirical studies have explored

All the estimated direct-price flexibilities for these meat commodities are statistically significant and less than 1 in absolute value The estimated cross-price flexibilities demonstrate a certain economic interdependence for specific meat products and "other goods" in the short run The significance of estimated cross-price flexibilities emphasizes the importance of interdependent relationships of meat demands and underscores the possible error of ignoring these relationships

In terms of statistical modeling, a mixed structural-time series model provides both a structural explanation of meat prices and improved forecasting capability. Based on empirical results, the forecasting capability of the model is significantly better than that of the traditional structural equation approach

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For many types of assets ın most manufacturıng processes both the salvage value and the use ınterval are known and the decısion-makers have lıttle dıfficulty ın usıng net ınvestment data to calculate the rate of return needed for proposed ınvestments For other types of busıness, partıcularly sıngle proprıetorshıps, the use ınterval ıs uncertaın because of the operator's ıncomplete knowledge of hıs future wıllıngness or abılıty to operate the fırm for as long as ımplıed by the specıfied plannıng perıod In such ınstances, he needs to form estımates on the value of the asset at ınterım tıme ıntervals because of the possıbılıty that he may cease to operate the fırm and that he value of the asset may at that tıme be subject to test on the market

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