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Valuing American Options on Commodity Futures Contracts

By Gerald Plato*

Abstract

The author modified a numerical procedure developed by Cox, Ross, and Rubinstein for valuing options on stocks to value options on commodity futures contracts. The numerical procedure, unlike Black's widely used analytical approach, can include the value of early exercise in the option-premium estimates. Analysis with the numerical procedure shows that the variability in the underlying futures price is crucial in determining the value of an option.

Keywords

Commodity options, futures contracts, hedging

Introduction

Options on commodity futures contracts provide a new risk-management tool for the participants, including farmers, in the corresponding cash-commodity markets. The pricing accuracy of the markets for these options will be a crucial factor in determining their usefulness as a risk-management tool as well as their survival in the market place. Questions will inevitably arise about whether the market prices of these options lie above or below their real economic value. This article describes and evaluates a method for estimating the values of these options, illustrates its use, and examines the importance of the required parameters.

Asay (1), Figlewski and Fitzgerald (5), Gardner (6), Hoag (8), and Ramaswamy and Sundaresan (13) have examined the pricing of options on commodity futures contracts.¹ All but Ramaswamy and Sundaresan examined the pricing of European rather than American options. However, American options are being traded on U.S. exchanges.

A buyer of a European call option on a commodity futures contract can only exercise the right to buy a commodity futures contract, at the exercise price, on the option expiration date. Conversely, a buyer of a European put option can only exercise the right to sell on the option expiration date. A buyer of an American option on a commodity futures contract has the additional right of exercising the option on any date prior to the expiration date. The privilege of exercising early increases the price of an option by giving the buyer the right to immediately realize profits equal to the difference between the exercise price and the futures price. Profits from early exercise are taken only when it is advantageous to the option buyer. A major objective of this article is to examine the effect of the right of early exercise on the price of options on commodity futures contracts.

Relatively little attention has been given to put options in the recent literature on pricing options on commodity futures contracts. Gardner's article is the only one that emphasizes put options (6). Put options for farmers and storers are a substitute for taking a short position in a futures market in expectation of a later sale in the corresponding cash markets. Short selling in commodity futures markets is a major use of these markets by the participants in the corresponding cash market. Therefore, put options on commodity futures contracts have considerable potential value to many cash-market participants. This ar-

*The author is an agricultural economist with the National Economics Division, ERS. Richard Heifner and Douglas Gordon provided valuable assistance in the preparation and review of this article. Many constructive criticisms were also received from Gerald Schluter, Roger Conway, and anonymous reviewers.

¹Italicized numbers in parentheses refer to items in the references at the end of this article.

ticle examines the pricing of put options, which appear to especially interest farmers, as well as call options on commodity futures contracts. Recent publications by Paul, Heifner, and Gordon (12) and by Kenyon (10) describe alternative ways that farmers and other hedgers can use these new option markets.

The first section of this article reviews relevant parts of option pricing theory and describes a procedure for calculating the expected price or premiums of American options on commodity futures contracts. The second section illustrates the procedure for soybeans, examines the effect of the right of early exercise on option price, and examines the importance of futures price variability and the interest rate in determining option prices.

Aspects of Option Pricing Theory

Similar procedures apply in estimating the value of options, both puts and calls, on stocks, physical commodities, and commodity futures contracts. For example, the numerical procedure or algorithm derived by Cox, Ross, and Rubinstein for calculating the price of call and put options on stocks, both American and European, can be used to calculate prices of American and European options on commodity futures contracts (4). Their algorithm can also be used to calculate American and European option prices on physical commodities.

Option pricing theory is based on the concept of the perfect or riskless hedge. This hedge involves simultaneous and offsetting positions in an asset, for example, a stock, physical commodity, or commodity futures contract, and an option on the asset. One can make the hedge riskless by maintaining the ideal or perfect ratio of asset units to option units. This perfect hedge ratio balances gains on the asset position with losses on the option position or losses on the asset position with gains on the option position. Because the hedge is riskless, the equity or amount invested in the hedge is specified as earning the riskless rate of return.²

Gains and losses on the option are a function of the level of the asset price and the time remaining until

²Hedging involves simultaneous and offsetting positions in two markets. Simultaneous and offsetting positions in a cash market and the corresponding futures market is a common method of hedging in agricultural markets.

the option expires. Therefore, to keep the hedge riskless, the ratio of units of the asset to the units of options must be continually readjusted.

The method used in this article to calculate option prices on commodity futures contracts is based on the concept of the riskless hedge.

The following equation represents the perfect or riskless hedge between a call option on a stock and the stock over the time interval Δt (9)

$$H\Delta S_{\Delta t} - \Delta C_{\Delta t} + HDS_t = r(HS_t - C_t) \quad (1)$$

where

H = hedge ratio (number of stock shares per call option in the riskless hedge),

$\Delta S_{\Delta t} = S_{t+\Delta t} - S_t$
= change in stock price over the time interval Δt ,

S_t = stock price at beginning of time interval Δt ,

$\Delta C_{\Delta t} = C_{t+\Delta t} - C_t$
= change in call option price over time interval Δt ,

C_t = call price at beginning of time interval Δt ,

D = dividend rate ($D \geq 0$),

r = riskless interest rate over the time interval Δt , and

$HS_t - C_t$ = equity in riskless hedge

Equation (1) says that the net change in the value of the call option-stock combination over the time interval Δt plus any dividends equals the riskless rate of return on the equity in the combined position. As implied by the right side of equation (1), the equity in the riskless hedge consists of a long position in the stock and a short position in the call option.³ Call op-

³The plus signs in equation (1) indicate a long position and the negative signs a short position. This convention is maintained throughout this article.

tion price changes move in the same direction as the stock price changes. Therefore, opposite positions in the call option and stock are needed to make the hedge riskless.

The ratio of stock to options must be adjusted at the end of the time interval Δt to keep the hedge riskless for the next time interval. The reason is that the change in the option price depends on the level of the stock price and the time remaining until option expiration.

Black and Scholes derived a differential equation for describing the value of a call option on a nondividend stock.³ Their derivation uses an equation similar to equation (1) with the dividend, D , equal to zero.

Black used the same procedure later to derive a differential equation for describing the value of a call option on a commodity futures contract.² His derivation uses an equation similar to equation (2) which describes a riskless hedge involving call options on commodity futures contracts and commodity futures contracts.

$$H(-\Delta S_{\Delta t}) + \Delta C_{\Delta t} = rC_t \quad (2)$$

The variables and parameters in equation (2) are as defined in equation (1) except that they refer to commodity futures contracts instead of to shares of stock. New variable and parameter names were not used because one can use equation (1) to calculate call option prices on commodity futures contracts; in this situation the variables and parameters in equation (1) refer to futures contracts. The context of the discussion shows when the variables and parameters refer to commodity futures contracts and when they refer to stock.

Equation (2) says that the net change in the value of the call option-futures combination over the time interval Δt equals the riskless rate of return on the equity in the hedge. The equity in this hedge equals the value of the call option. The value of a futures position is zero at the beginning of the time interval Δt .⁴ Therefore, the level of the futures price is omitted

⁴Black assumed that the current futures price equals the expected futures price at contract expiration.² The current value of a futures position is zero under Black's assumption regardless of any previous gains or losses.

from equation (2). Equation (2) contains the call option long and the futures position short, the opposite of the call option and stock positions in equation (1).

Black recognized that his differential equation for a call option on a commodity futures contract derived from equation (2) has the same solution for the price of the call option as Merton's differential equation for a call option on a stock when the stock pays dividends at the riskless rate. Merton's differential equation can be derived from the riskless hedge in equation (1).⁵ That identical call option prices occur can be seen if one sets $D = r$ in equation (1) which, after simplifying, produces equation (2).⁵

Equation (3) describes the riskless hedge between a put option on a stock and the stock over the time interval Δt .

$$H\Delta S_{\Delta t} + \Delta P_{\Delta t} + HDS_t = r(HS_t + P_t) \quad (3)$$

P_t is the value of the put option at the beginning of the time interval Δt , and $\Delta P_{\Delta t}$ is the change in value of the put option over this interval. The other variables and the parameters in equation (3) are the same as those in equation (1). The major difference between equations (3) and (1) is that the riskless hedge in equation (3), $HS_t = P_t$, contains long positions in both the stock and put option. A change in the stock price moves the put price in the opposite direction. Therefore, long positions are needed in both the put and the stock to make the hedge riskless.

Equation (4) describes the riskless hedge between a put option on commodity futures and commodity futures over the interval Δt .

$$H\Delta S_{\Delta t} + \Delta P_{\Delta t} = rP_t \quad (4)$$

The variables are the same as those in equation (2) except that the put option, P_t , replaces the call option, C_t . As in the case of call options on commodity futures contracts, the level of the futures price is omitted because the current value of the futures

⁵Cox, Ross, and Rubinstein and also Jarrow and Rudd show stock dividend payments based on ending period stock prices, $HDS_{t+\Delta t}$, rather than on beginning period stock prices HDS_t .^{4, 9} Equations (1) and (2) with $D = r$ are not equal except in the limit as the time interval, Δt , approaches zero when the dividend payments are based on ending period futures prices.

position is assumed to be zero. Long positions are held in both the put and futures to make the hedge riskless because put-price changes move in the opposite direction from the futures-price changes.

The put-option price in equation (3) when $D = r$ is the same as the put-option price in equation (4). This can be seen if one sets $D = r$ in equation (3) and simplifies it to produce equation (4). This result is the same as that for call-option prices in equations (1) and (2) when $D = r$ in equation (1). The equivalency of option prices in equations (1) and (2) and in equations (3) and (4) when $D = r$ is the reason that the Cox, Ross, and Rubinstein algorithm for valuing stock options can also be used for valuing options on commodity futures contracts (4).

Differential equations derived from the preceding equations for describing the relationship of options to stock and to commodity futures contracts can be solved analytically if the options are European.⁶ If the options are American, then analytical solutions are possible only when early option exercise is never desirable. However, exercising American put and call options on commodity futures contracts prior to the expiration date is sometimes desirable as is shown in the analysis.

The advantage of the Cox, Ross, and Rubinstein algorithm over the analytical approach to calculating option prices is that it can handle the early exercise of American options. If the dividend rate is set equal to the riskless interest rate, then their algorithm can also estimate prices for call and put options on commodity futures contracts, both American and European. We now briefly describe how this algorithm calculates the call-option price in equation (1) and also the call-option price in equation (2) when the dividend rate is set equal to the riskless interest rate. We also discuss the modifications for calculating put-option prices.

Economists Cox, Ross, and Rubinstein, following Black and Scholes, assume that the stock price change from the current time until the option expiration date is log normally distributed. They take advantage of the fact that the log normal distribution can

be approximated as the product of a large number of binomial changes. Their algorithm enumerates these binomial changes backwards through time from the option expiration date. This procedure allows all possible stock prices and corresponding option prices to be determined prior to each binomial price change.

The Cox, Ross, and Rubinstein algorithm first divides the time remaining until option expiration into T equal intervals where t is the beginning and $t + 1$ the end of the t^{th} interval. T is set arbitrarily, but as its value is increased the approximation to the log normal distribution is improved. The option matures or expires on $T + 1$ which is the end of interval T , the last interval.

The stock price from the beginning to the end of each time interval is specified as moving up or down according to the multiplicative binomial distribution where the price at the end of an interval is $S_{t+1}^u = uS_t$ or $S_{t+1}^d = dS_t$ for a price increase and a price decrease, respectively. The parameters u and d are the possible outcomes from this binomial probability distribution over the t^{th} interval.

The algorithm next calculates all the possible stock prices on the option's expiration date using the possible outcomes from the multiplicative binomial distribution from the beginning of the first interval, $t = 1$, to the expiration date, $t = T + 1$. The possible stock prices when the option expires are represented by j equals 0 through T in equation (5).

$$S_{T+1,j} = u^{T-j}d^jS_1, \quad \text{for } j = 0, 1, \dots, T \quad (5)$$

where

- $T - j$ = number of stock price increases,
- j = number of stock price decreases, and
- S_1 = stock price at the beginning of first time interval

We can then use the stock prices to calculate all the possible option prices, call or put, on the expiration date as shown below.

For calls

$$C_{T+1,j} = \text{Max}(0, S_{T+1,j} - K), \text{ for } j = 0, 1, \dots, T$$

For puts

$$P_{T+1,j} = \text{Max}(0, K - S_{T+1,j}), \text{ for } j = 0, 1, \dots, T$$

⁶Asay discusses the relationships among the analytical solutions to these differential equations for European call options on stocks, commodity futures contracts, and physical commodities (7).

The exercise price, K , is the stock purchase price for a call option and the stock selling price for a put option

The algorithm's next step is to calculate all the possible stock prices at T , the beginning of the last time interval. Here the stock prices are calculated as before except that $T - 1$ replaces T and T replaces $T + 1$ in equation (5). Next, we calculate the option prices at the beginning of the last time interval.

We now explain the procedure for calculating each of these option prices. The time subscripts are in terms of the typical time interval, t , because the procedure for calculating option prices is the same for the beginning of each time interval.

We derive equation (6) by adding the equity in the riskless hedge for call options on stocks, $HS_t - C_t$, to both sides of equation (1)

$$HS_{t+1} - C_{t+1} + HDS_t = (r + 1)(HS_t - C_t) \quad (6)$$

Equation (6) shows that the net value of the stock and call option positions plus the dividend at the end of the current time interval equals the original equity in the hedge increased at the riskless interest rate. In this formulation, the equity in the riskless hedge is equivalent to a riskless bond over the interval t to $t + 1$. Thus equation (6) can be rewritten as

$$HS_{t+1} + HDS_t - C_{t+1} = (r + 1)B \quad (7)$$

where $B = HS_t - C_t$ and B is a riskless bond worth $(r + 1)B$ at $t + 1$. The stock price, S_{t+1} , given S_t , can have two possible values because the stock price is specified as following the multiplicative binomial distribution. The two stock price outcomes given S_t are shown in the following two equations

$$HS_{t+1}^u + HDS_t - C_{t+1}^u = (r + 1)B \quad (8)$$

$$HS_{t+1}^d + HDS_t - C_{t+1}^d = (r + 1)B \quad (9)$$

where

- S_{t+1}^u = price after upward movement,
- S_{t+1}^d = price after downward movement,
- C_{t+1}^u = option price associated with upward stock price movement, and
- C_{t+1}^d = option price associated with downward stock price movement

At this point the values of the stock prices and call option prices shown in the preceding list have been previously calculated. The algorithm next solves equations (8) and (9) simultaneously for H and B . Because $B = HS_t - C_t$, C_t can easily be calculated because the stock price, S_t , is already known.⁷

If the value of exercising the option immediately is greater than the value found by use of the preceding equations, then the value of an American option at the beginning of the t^{th} time interval is the immediate exercise value. The immediate exercise value is $S_t - K$ for calls and $K - S_t$ for puts where S_t is the price at the beginning of the t^{th} time interval and K is the exercise price. This last step is omitted for European options.

Next, the algorithm uses the option prices just calculated for the beginning of interval T to calculate the option prices for the beginning $T - 1$. The procedure is the same as described for using the option prices at $T + 1$ to calculate the option prices at the beginning of interval T . The algorithm continues by calculating all the option prices for the beginning of the previous time interval using the option prices calculated for the beginning of the current interval. This procedure is continued until the option price for the beginning of the first time interval is calculated.

The Cox, Ross, and Rubinstein algorithm can be modified to solve the price of American options on commodity futures contracts without resorting to solving for the value of a stock option with the dividend rate set equal to the riskless interest rate. The modification involves replacing equations (8) and (9) that describe the value of a call option on a stock with equations that describe the value of a call option on a commodity futures contract.

We derive equation (10) by adding the equity in the riskless hedge for call options on commodity futures contracts, C_t , to both sides of equation (2)

$$H(S_t - S_{t-1}) + C_{t-1} = (r + 1)C_t \quad (10)$$

This equation says that the change in the value of the futures position over the t^{th} time interval plus

⁷One can derive a set of simultaneous equations for put options on stocks from equation (3) using the procedures shown for deriving the simultaneous equations for calls in (8) and (9) from equation (1). The simultaneous equations for puts can be solved for the ratio of stocks to put options H and for the price of the put option P_t .

the value of the call position at the end of this interval equals the equity in the riskless hedge increased at the riskless interest rate.⁸ As in the stock option case shown in equation (6), S_{t+1} given S_t can have two values as shown in equations (11) and (12)

$$H(S_t - S_{t+1}^u) + C_{t+1}^u = (r + 1)C_t \quad (11)$$

$$H(S_t - S_{t+1}^d) + C_{t+1}^d = (r + 1)C_t \quad (12)$$

The value of the equity in the riskless hedge, C_t , can also be thought of as equivalent to a riskless bond, B , that is worth $(r + 1)B$ at the end of the t^{th} time interval. One can find the value of the call, C_t , by solving the preceding two equations simultaneously for C_t and H . These equations produce the same call value as equations (8) and (9) when the dividend rate is set equal to the riskless interest rate.⁹

The solutions for C_t and H in equations (11) and (12) are shown in equations (13) and (14)

$$C_t = \frac{[(1 - d)/(u - d)C_{t+1}^u + (u - 1)/(u - d)C_{t+1}^d]}{(r + 1)} \quad (13)$$

$$H = (C_{t+1}^u - C_{t+1}^d)/(d - u)S_t \quad (14)$$

A computer program for calculating prices of American options on commodity futures contracts is shown in the appendix. The program is based on the preceding explanation. It is written in Microsoft Basic and was implemented on an IBM Personal Computer.

Estimates of the Value of American Options on Soybean Futures Contracts

The analysis includes an examination of (1) the right or early exercise on the price of options on commodity futures contracts, and (2) the sensitivity of option

⁸One can rewrite the terms inside the first parenthesis as $-(S_{t+1} - S_t)$ or $-\Delta S_{\Delta t}$ and interpret as minus the change in the futures price over the time interval Δt . The preceding minus signs designate a short futures position.

⁹One can derive a set of simultaneous equations for put options on commodity futures from equation (4) using the procedures shown for deriving the simultaneous equations for calls in (11) and (12) from equation (2). The simultaneous equations for puts can be solved for the ratio of commodity futures contracts to put options on commodity futures contracts, H , and for the price of the put option, P_t .

prices to the variability of the underlying futures price and to the level of the riskless interest rate. I will describe the parameters used in the algorithm to calculate soybean option prices before presenting the analysis.

Description of the Parameters

One needs values of u and d in equation (5) to implement the algorithm described in the previous section to calculate the prices of options on commodity futures. Cox, Ross, and Rubinstein showed that

$$u = e^{\sigma\sqrt{\tau}/T} \quad (15)$$

$$d = e^{-\sigma\sqrt{\tau}/T} \quad (16)$$

where σ is the standard deviation of the rate of change in the stock or futures price for 1 year, τ is the fraction of a year until the option expires and is partitioned into T equal time intervals.

Table 1 shows estimates of the standard deviation of the daily rate of change in the closing futures price. I chose the November and March soybean futures contracts to examine option prices during the growing season when supplies are frequently scarce and option prices after harvest when supplies are usually plentiful. Gordon found that the variability of futures price changes for crops is generally highest during the growing season.⁷ The estimates in table 1 agree with this finding.

Table 1—Estimated standard deviations in the daily rate of change in the closing soybean futures price

Crop year	Futures contract ¹	
	November	March
1983	0.0220	0.0115
1982	0.111	0.079
1981	0.139	0.107
1980	0.185	0.198
1979	0.178	0.156
1978	0.137	0.118
1977	0.231	0.125
1976	0.218	0.155
1975	0.206	0.097
1974	0.223	0.207
1973	0.378	0.192

¹November estimates are based on daily closing prices from June 1 to October 15, and March estimates on daily closing prices from November 1 to February 15.

Two findings implied that one should make a large number of estimates to understand the significance of the variability of the soybean futures price on option prices. First, preliminary estimates of option prices using the option pricing algorithm suggested that option prices are highly sensitive to the underlying variability of the soybean futures price. Second, the initial estimates for the 1982 and 1983 crop years suggested that the daily variability of the soybean futures price may vary considerably among crop years.

The analysis primarily uses the variability estimates for the 1982 and 1983 crop years. The estimates in table 1 suggest that this choice provides both a high variability year and a low variability year for the soybean futures price. I examined the sensitivity of option prices to soybean price variability using changes in price variability from the 1982 and 1983 levels.

I made the estimates in table 1 by taking the natural logs of the daily closing prices and then calculating the standard deviation of the first differences of the

natural logs. The estimates for the November futures contract are based on daily prices from June 1 to October 15. The first date represents a typical soybean planting date and the second date the option expiration date on the November futures contract. The estimates for the March contract were based on daily prices from November 1 to February 15. The first date represents a date on which most of the current harvest is completed, and the second date represents the option expiration date on the March futures contract.

An estimate of the riskless interest rate is also needed to implement the algorithm. The estimates used in the analysis are shown in table 2. I calculated these estimates from the bid and asked discount rates for U.S. Treasury bills. I chose the current and expiration dates on the Treasury bills to correspond with the beginning and expiration dates of the four cases shown in table 2. Estimates of the riskless interest rate were only calculated for the 1982 and 1983 crop years as preliminary analysis showed that option prices are much less sensitive to the interest rate than to the variability of the futures price.

Table 2—Estimated futures price variability and riskless interest rate and specifications of option time periods used in calculating option prices on commodity futures contracts

Case ¹	Futures contract option period	tau ²	Futures variability		Annualized riskless interest rate ⁴
			Daily	Annualized ³	
		<i>Months/year</i>	<i>Standard deviation</i>		<i>Percent</i>
I	November 82 June 1-Oct 15	4 5/12	0 0111	0 1755	12 80
II	November 83 June 1-Oct 15	4 5/12	0220	3479	9 33
III	March 83 Nov 1-Feb 15	3 5/12	0079	1249	8 56
IV	March 84 Nov 1-Feb 15	3 5/12	0115	1818	9 14

¹The case numbers designate combinations of times remaining until option expiration, standard deviations of the rate of change in the soybean futures price, and riskless interest rates used in calculating the option prices shown in table 3 and in figures 1 and 2.

²Time remaining until option expiration.

³The annual standard deviations of the rate of change in the soybean futures price were calculated by multiplying the estimated daily standard deviations by the square root of 250, a year was assumed to contain 250 trading days.

⁴The annual riskless interest rates were calculated from the average of the bid and asked discount rates for U.S. Treasury bills on June 1, 1982, and 1983, and on November 1, 1982, and 1983. The maturity dates were chosen to correspond with the option expiration dates for November and March options.

Table 2 also shows the four combinations of futures price variabilities and interest rates used in the analysis. The values of tau, or fraction of a year remaining until option expiration, are also shown in table 2. The accuracy in calculating option prices is determined in part by the number of time intervals, T, used in partitioning tau. A value of T equal to 75 was chosen. This selection is discussed further in the analysis.

The Analysis

Table 3 shows estimated American and European option premiums and hedge ratios for exercise prices at and close to the assumed \$8 per bushel soybean futures price. The case or parameter descriptions for

these estimates are provided in table 2.

An important result in table 3 is that American and European option prices are essentially the same when the exercise price is within 50 cents per bushel of the current futures price. The American option prices were at most only 9/10 of a cent above their European counterparts.

The largest premium differences between American and European options in table 3 occur at the \$8.50 exercise price for the puts and at the \$7.50 exercise price for the calls. American options in these two situations in table 3 have the largest probability of early exercise because the \$8 futures price is closest to the level at which early exercise occurs. The \$8 futures price is farthest from the level at which ear-

Table 3—Estimated premiums and perfect hedge ratios for American and European put and call options on soybean futures with exercise prices ranging from ± \$0.50 from an assumed \$8 futures price

Exercise price	Option premiums and hedge ratios ¹							
	Case I		Case II		Case III		Case IV	
	Amer	Eur	Amer	Eur	Amer	Eur	Amer	Eur
Puts								
\$8.50	0.638 (.68)	0.629 (.67)	0.955 (.56)	0.947 (.55)	0.548 (.80)	0.543 (.79)	0.622 (.71)	0.617 (.70)
\$8.25	.472 (.58)	.466 (.57)	.800 (.51)	.794 (.50)	.360 (.66)	.358 (.65)	.450 (.60)	.447 (.59)
\$8.00	.332 (.466)	.329 (.46)	.663 (.45)	.659 (.44)	.212 (.48)	.211 (.48)	.308 (.47)	.306 (.47)
\$7.75	.219 (.35)	.217 (.35)	.536 (.39)	.532 (.39)	.108 (.30)	.108 (.30)	.195 (.35)	.194 (.35)
\$7.50	.135 (.25)	.134 (.24)	.426 (.33)	.423 (.33)	.046 (.16)	.046 (.16)	.115 (.24)	.114 (.23)
Calls								
\$8.50	.152 (.29)	.151 (.29)	.466 (.42)	.463 (.42)	.055 (.19)	.055 (.19)	.130 (.28)	.130 (.28)
\$8.25	.229 (.39)	.227 (.39)	.556 (.47)	.552 (.47)	.114 (.33)	.114 (.33)	.204 (.39)	.203 (.39)
\$8.00	.332 (.51)	.329 (.50)	.663 (.53)	.659 (.52)	.212 (.51)	.211 (.50)	.308 (.51)	.306 (.47)
\$7.75	.462 (.62)	.456 (.61)	.780 (.59)	.774 (.58)	.354 (.68)	.352 (.68)	.441 (.64)	.438 (.63)
\$7.50	.621 (.73)	.612 (.71)	.915 (.65)	.907 (.64)	.540 (.83)	.535 (.82)	.607 (.75)	.602 (.74)

¹The four cases are explained in table 2 and the accompanying text. The perfect hedge ratios are shown in the parentheses.

ly exercise occurs for puts at the \$7.50 exercise price and for calls at the \$8.50 exercise price. In these two situations, the probability of early exercise is the smallest in table 3, and the price increments of American over European options are the smallest. The maximum price difference in these two situations is 3/10 of a cent. The difference is less than 1/10 of a cent for case III.

The small premium differences between American and European options in table 3 suggest that Black's analytical approach to valuing European options provides a close approximation to valuing American options for exercise prices near the current futures price (2).

The perfect hedge ratios also differ little between the American and European options in table 3. As explained previously, the hedge ratio is the ratio of futures contracts to options contracts, and the perfect hedge ratio keeps the combination of futures and options riskless for the current time period, t to $t + 1$. That the hedge ratios differ little between the American and European options in table 3 is to be expected because their prices differ little.

I used Black's approach to determine a suitable number of intervals, T , in which to partition the time to option expiration, τ . Black's approach is equivalent to specifying an infinite value for T . The procedure was to compare European option prices from the algorithm with those from Black's approach for the four cases in table 3 at the \$8 exercise price. Comparisons showed that with T equal to 75 the algorithm estimates were from 7/100 to 21/100 of a cent more than with Black's approach. One can attain additional accuracy by specifying a larger value for T . However, T equal to 75 provides sufficient accuracy to satisfy the objectives of this study.

The European put and call option prices for each case in table 3 are equal when both the exercise and futures prices equal \$8 per bushel. This result is in accord with the put-call parity theorem derived by Stoll.

$$C_t - P_t = e^{-R(tau)}(S_t - K) \quad (17)$$

which shows that the call price, C_t , equals the put price, P_t , when the futures price, S_t , and the exer-

cise price, K , coincide (14).¹⁰ The put-call parity relationship in equation (12) also applies at each of the other exercise prices for the European options for each case in table 3. The put-call parity theorem does not hold exactly for American options. However, it does closely approximate the relationship of American put and call option prices in table 3.

I examined the influence of futures price variability by estimating option prices for cases II and IV after increasing the standard deviation of the futures price by 1 percent for these two cases. I then calculated percentage changes in the option prices from those originally estimated for cases II and IV. I also used the same procedure to examine the influence of a 1-percent increase in the interest rate on option price.

The results showed that the option price is highly sensitive to the variability of the futures price and insensitive to the interest rate. The 1-percent increase in the standard deviation of the futures price increased the option prices in cases II and IV from 0.4 to 2.0 percent. Each option price is increased because the probabilities of large favorable changes in the futures price are increased.

The 1-percent increase in the interest rate decreased each of these same option prices by less than 0.03 percent. Option prices are decreased by the larger discounting in equation (13).

There is considerable interest in the cost of put options for hedging the production outcome over the growing season. American put option prices for a growing season's production hedge were estimated to be 33.2 and 66.3 cents per bushel at the \$8 exercise price for cases I and II, respectively. These option prices are 4.2 and 8.3 percent of the \$8 per bushel soybean futures price.

Investigating the influence of the interest rate leads one to suspect that the put option price differences between cases I and II are largely due to the difference in futures price variability. I confirmed this suspicion by switching the interest rate in cases I and II and reestimating the option prices. Using the

¹⁰R equals the annualized interest rate (table 2)

case II interest rate in case I increased the put option prices by at most only 0.9 percent. Using the case I interest rate in case II decreased the put option prices by at most only 1 percent.

Richard Heifner, in a personal communication, has suggested that the ratio of the option price to the futures price is determined by the ratio of the exercise price to the futures price. For example, using futures and exercise prices of \$6 rather than \$8 in cases I and II also produces put option prices that are 4.2 and 8.3 percent of the futures price, respectively.

Cases I and II were chosen to represent growing seasons with low and high futures price variabilities. Therefore, the ratios of the put option to futures prices for cases I and II provide estimates of those expected in low and high variability years regardless of the actual futures price level. This conclusion is based on the small influence of the interest rate on option price and on the finding that the ratio of option to futures price is determined by the ratio of exercise to futures price.

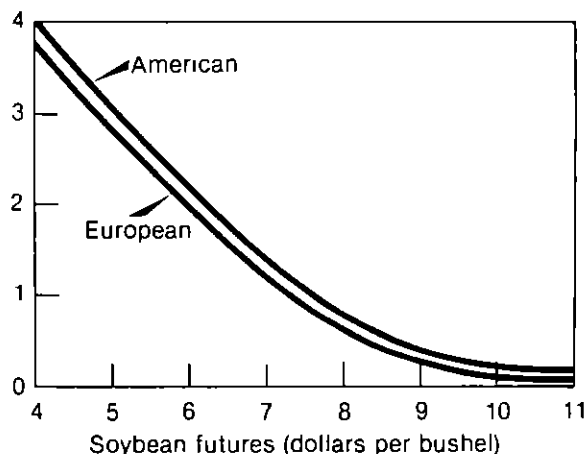
Figure 1 compares the prices of American and European put options for soybean futures prices between \$4 and \$11 per bushel for case II. The exercise price is \$8 per bushel. Figure 1 draws out the price difference between American and European options when the exercise price is not close to the futures price.

The American put option price increases relative to the European put option price as the soybean futures price decreases. For example, as the futures price decreases from \$8 to \$5.25, the price of the American option rises from about 0.5 to 8 cents per bushel relative to its European counterpart. This relative price increase for the American option reflects the increasing probability that it will be exercised prior to the expiration date. For all futures prices below \$5.25 per bushel, the optimal decision is to exercise immediately. The slope of the American put option curve below \$5.25 is minus 1. As shown in figure 1, the American curve approaches the European curve as the soybean futures price increases above \$8 per bushel. This result reflects the decreasing probability that the American put option will be exercised prior to expiration. Although not shown, each curve approaches a zero slope and the horizontal axis as the futures price rises above \$11.

Figure 1

Premiums for American and European Put Options on Soybean Futures Contracts¹

Call option (dollars per bushel)



¹Exercise price is \$8 per bushel, soybean futures range from \$4 to \$11 per bushel, and Case II assumptions apply (table 2).

The distance of the European curve from the American curve is not drawn to scale.

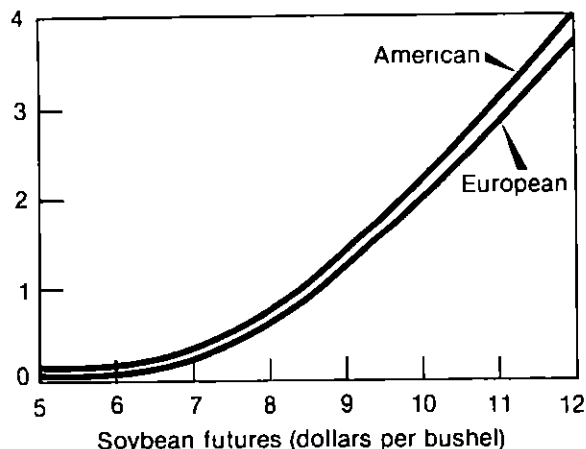
Figure 2 compares the prices of American and European call options for soybean futures between \$5 and \$12 per bushel for case II. The exercise price is \$8 per bushel. As in the previous figure, one can draw out price differences between American and European options when the exercise price is not close to the futures price.

The price of the American call option price increases relative to its European counterpart as the soybean futures price increases. For example, as the futures price increases from \$8 to \$12 per bushel, the price of the American call option rises from about 0.5 cent to about 11 cents per bushel relative to its European counterpart. As in the put-option comparison, this relative price increase reflects the increasing probability that the American call option will be exercised before the expiration date. For all futures prices above \$12 per bushel, the optimal decision is to exercise immediately. The slope of the American option curve is plus one above the \$12 futures price. As shown in figure 2, the American curve approaches the European curve as the futures price decreases below \$8 per bushel. This result reflects the decreasing probability that the American call option will be exercised prior to expiration. Although not shown, each curve approaches a zero

Figure 2

Premiums for American and European Call Options on Soybean Futures Contracts¹

Call option (dollars per bushel)



¹ Exercise price is \$8 per bushel, soybean futures range from \$5 to \$12 per bushel, and Case II assumptions apply (table 2)

The distance of the European curve from the American curve is not drawn to scale

slope and the horizontal axis as the futures price decreases below \$5¹¹

Early exercise is frequently complicated for hedgers because the timing of hedge removal is frequently dictated by business circumstances. For example, a farmer using a put-option production hedge for soybeans would not generally remove the hedge until the crop is harvested even when the soybean futures price falls sufficiently for the option to be exercised early. However, a farmer in this situation might choose to gain immediate access to the funds in the put option position while maintaining a hedge against further price declines. The farmer can do so by exercising the put option and buying another with the same expiration date but with an exercise price equal to the current lower futures price.

American put option prices are more than 3 cents per bushel higher than their European counterparts when the soybean futures price is more than \$2 below the exercise price (fig. 1) Conversely, American call option prices are more than 3 cents

per bushel higher than the European option prices when the soybean futures price is more than \$2 above the exercise price (fig 2) Put and call options are referred to as being deep in the money in this situation Most option hedges are placed at exercise prices close to the current futures price. In this situation, American and European options have essentially the same prices (table 3) However, an option may be deep in the money when an option is lifted or removed prior to expiration In this situation, American options provide a larger return to hedgers The higher returns for American options stem from the ability to gain immediate access to the funds represented by the difference between the futures price and the exercise price Access to these funds for European options is delayed until the expiration date

This advantage is examined by simulating hedge removal based on American and European options that are close to their expiration dates Option hedges will generally be based on contracts that expire close to the future date of intended cash market transaction There is no need to pay for price protection beyond that date

I compared American and European options for two hedge removal situations

Removal of a put option production hedge on the November futures contract on October 1 using the case II futures price variability and interest rate, this date is 1/2 month prior to option expiration on the November futures contract

Removal of a put option storage hedge on the March futures contract on January 15 using the case IV futures price variability and interest rate, this date is 1 month prior to option expiration on the March futures contract. The options have an \$8 exercise price

On October 1, the American and European put option prices are estimated to be \$1.50 and \$1.495 per bushel, respectively, if the soybean futures price falls to \$6.50 per bushel In this situation, the American put option provides a \$25 larger return on a 5,000-bushel options contract. On January 15, the American and European put options are estimated to be \$1.50 and \$1.489 per bushel. In this situation, the American put options provides a \$55 larger return on a 5,000-bushel options contract

¹¹The hedge ratios, like those shown in table 3, are equal to the absolute value of the slope of the appropriate option price-soybean price curve The absolute values of these slopes range from 0 to 1

The ratio of the American to European option prices in each case equals 1 plus the riskless rate of return that can be earned over the time remaining until option expiration. This result means that the difference between the American and European option prices in each case represents the opportunity costs of not being able to exercise the European option immediately to gain access to the difference between the futures price and the exercise price. The opportunity costs are relatively small in the two hedge-removal examples since only 1/2 month and 1 month remain until option expiration.

The strategy of exercising or selling put options early to receive funds immediately and maintaining the hedge by buying another put option at a lower exercise price may offer a means of increasing the returns from hedging with put options. Similarly, exercising or selling a call option early and buying another at a higher exercise price may also offer a means of increasing the returns from hedging with options. However, examination of this strategy requires detailed budgeting of an individual's business situation and is beyond the scope of this article. As figures 1 and 2 suggest, American options can sometimes provide considerably higher returns than European options with this strategy.

Conclusions

I have described and evaluated a method for estimating premiums for options on commodity futures contracts which takes into account the value of early exercise. In the examples examined, this method produced premium estimates only slightly larger than Black's formula when the futures price is close to the exercise price. However, Black's formula may undervalue premiums for American options deep in the money by as much as the percentage represented by the riskless interest rate over the time remaining until option expiration. For example, for an option deep in the money with 3 months until expiration, the undervaluation by Black's formula can be as much as the percentage available on US Treasury bills that also mature in 3 months.

The algorithm used in this article has several advantages over Black's formula beyond its ability to include the value of early exercise. It can include the effects of a support price on the price of a put option by limiting maximum future values of the option to

the exercise price minus the support price. It can also include changes in the level of futures price variability over the period prior to option expiration. A recent study by Gordon indicates that the variability of futures prices generally increases toward the end of futures contracts (?).

The analysis suggests that the prices of options on commodity futures contracts are sensitive to the variability of futures prices and insensitive to the interest rate. These results imply that considerable effort is needed to estimate futures price variability before evaluating the market prices of these options.

The analysis of futures price variability in this article is entirely after the fact. However, the market prices of options on commodity futures contracts reflect prior market estimates or expectations of variability. One interesting area of investigation is to examine the likely factors that influence market expectation of future price variability. A likely candidate for this investigation is the level of market stocks.

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Appendix

10 REM Program for Calculating Prices of
American Put Options
20 REM on Commodity Futures Contracts
30 REM (see 940 through 970 for modifying the
program to

```

40 REM calculate call option prices, see 980 for
    modifying
50 REM the program to calculate European option
    prices)
60 REM
70 DIM PUTT (100), SAVEPUT (100)
80 READ S, K, R, T, TAU, SIGMA
90 LPRINT, S, K, R, T, TAU, SIGMA
100 REM
110 REM Calculate riskless interest rate for one
    time interval, RR
120 REM
130 RR = (1 + R)^(TAU/T)
140 RR = RR-1!
150 REM
160 REM Calculate one plus the rate of futures
    price increase, U,
170 REM and decrease, D, respectively, over one
    time interval
180 REM
190 U = EXP (SIGMA*((TAU/T)^.5))
200 D = EXP (- SIGMA*((TAU/T)^ 5))
210 LPRINT RR, U, D
220 JJJ = T + 1
230 IX = JJJ - 1!
240 IY = 0!
250 REM
260 REM For loop 290 to 380 calculates all possible
    put option prices
270 REM on the expiration date
280 REM
290 FOR I = 1 TO JJJ
300 RATE = (U^IX)*(D^IY)
310 PRICE = S*RATE
320 PUTT (I) = K-PRICE
330 IF PUTT (I), <0! THEN PUTT (I) = 0!
340 PRINT PUTT (I), PRICE, RATE, JJJ, I, IX, IY
350 IY = IY + 1
360 IX = IX - 1
370 SAVEPUT (I) = PUTT (I)
380 NEXT I
390 JJJ = JJJ - 1
400 IX = JJJ - 1
410 IY = 0
420 PRINT JJJ
430 REM For loop 450 to 690 calculates all possible
    put option prices
440 REM for the beginning of the current time
    interval
450 FOR I = 1 TO JJJ

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```

460 RATE = (U^IX)*(D^IY)
470 PRICE = S*RATE
480 II = I + 1
490 REM
500 REM The following two equations calculate a
tentative option price
510 REM and hedge ratio (these equations corres-
pond with equations 13
520 REM and 14 in the text)
530 REM
540 PUTT (I) = (((1 - D)/(U - D))*SAVEPUT (I)
+ ((U - 1)/(U - D))*SAVEPUT (II)/
(RR + 1))
550 H = (SAVEPUT (I) - SAVEPUT (II)/
(D - U)*PRICE)
560 REM Calculate value of exercising immediately
570 REM
580 TEST = K-PRICE
590 REM Put option price equals exercise value
when it is greater than
600 REM or equal to the value calculated in state-
ment 540
610 REM
620 IF TEST >= PUTT (I) THEN PUTT (I) = TEST
630 IF JJJ > 10 THEN GOTO 660
640 PRINT PUTT (I), H, PRICE, I, IX, IY
650 IF JJJ = 1 THEN LPRINT, K, PRICE, PUTT
(I), H, I, IX, IY
660 SAVEPUT (I) = PUTT (I)
670 IX = IX - 1
680 IY = IY + 1
690 NEXT I
700 REM
710 REM If JJJ > 1 calculate all possible option
prices for the beginning
720 REM of the previous time interval
730 REM If JJJ = 1 the program is completed (the
option price has been
740 REM calculated for the beginning of the first
time interval)
750 REM
760 If JJJ > THEN GOTO 390
770 DATA 800 00, 800 00, 0 1280, 75, 0 375, 0 1755
780 END
790 REM
800 REM S = Futures price at the beginning of the
first time interval
810 REM K = Strike or exercise price
820 REM R = Annualized riskless interest rate
830 REM T = Number of equal time intervals until
option expiration date
840 REM TAU = Fraction of year until option ex-
piration date
850 REM SIGMA = Standard deviation of the rate
of change in the futures
price for 1 year
860 REM
870 REM PUTT (I) = Put option prices for the cur-
rent time interval
880 REM SAVEPUT (I) = Put option prices for the
following time interval
890 REM JJJ = Number of possible futures prices
for current time interval
900 REM IX = Number of futures price increases
since the beginning of the
first time interval
910 REM
920 REM IY = Number of futures price decreases
since the beginning of the
first time interval
930 REM
940 REM replace statements 320 and 580 with those
shown below to
950 REM calculate call option prices
960 REM 320 PUTT (I) = PRICE - K
970 REM 580 TEST = PRICE - K
980 REM Delete statement 620 to calculate Euro-
pean option prices

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In Earlier Issues

Those of us involved in economic research should care about methodology because it is the floor we walk on. If economics is to claim a scientific stature, then the profession must formally define a program for rejecting or accepting a proposed economic theory and, on the basis of accepted theory, produce accurate and pertinent predictions that, in principle, can be empirically tested.

Roger Conway
Vol 34, No 1, January 1982
