Journal of Agricultural and Applied Economics, 45,2(May 2013):217–227

Consumer Welfare Effects of Quantity Changes in Demand

Kuo S. Huang

Information about consumer welfare effects associated with quantity changes in demand is important for agricultural and food policy decision-makers because many policy options are directly related to controlling supplies as a means to stabilize or raise commodity prices and farmers' income. A new method is developed to measure the consumer welfare effects by using the estimates of an inverse demand system and a modified quantity-adjusted Malmquist index to represent the efficiency in quantity metric welfare. The methodology is validated by applying it to a U.S. inverse food demand system consisting of 13 food groups and a nonfood sector.

Key Words: consumer welfare, inverse demand system, Malmquist quantity index

JEL Classification: D12

The consumer welfare effects of quantity changes in demand are important information for agricultural and food policy decision-makers because many policy options are directly related to changes in quantities available in the market. The initiatives of some public regulatory agricultural policies and programs are geared toward controlling supplies for stabilizing or raising commodity prices and farmers' income. Some examples are a government acreage control program, the stabilization programs of storable crops, and marketing agreements that restrict availability through quotas. Also, agricultural economists such as Fox (1953), Houck (1965), and Waugh (1964) have long recognized that lags between farmers' decisions on production and commodities marketed may predetermine the quantities with price adjustments providing the market-clearing mechanism. Therefore, quantities rather than prices in an inverse (price-dependent) food demand system are appropriate instruments in some situations for evaluating agricultural policy and programs that affect consumer welfare.

Most studies of consumer welfare, however, concern the welfare effects of price changes. Examples can be found in Devadoss and Wahl (2004), Freund and Wallich (1996), Huang (1993), Huang and Huang (2012), and Tolley, Thomas, and Wong (1982). Thus far only a few studies have proposed measuring consumer welfare in response to quantity changes in demand. One example is Kim (1997), who used an AIDS (Almost Ideal Demand System) type distance function to measure compensating and equivalent variations of expenditures for quantity changes. Another example is McLaren and Wong (2009), who applied a benefit function originated by Luenberger (1996) to examine consumer welfare associated with quantity changes.

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I thank Darrell Bosch and two anonymous reviewers for helpful comments.

The views expressed here are those of the author and cannot be attributed to the ERS or the USDA.

In light of sound theoretical reasoning as explored in Hicks (1956), an inverse demand system is desirable for measuring the consumer welfare effects of quantity changes, an area in which research has been relatively sparse. In contributing to fill this gap in literature, a method is developed for evaluating the consumer welfare effects by applying the Malmquist (1953) quantity index to represent the efficiency in quantity metric welfare. The Malmquist quantity index has been extensively used in index numbers and tax analysis but in few applications to welfare analysis. The major research effort of this study is empirical modeling of the conceptual Malmquist quantity index with a linkage to an inverse demand system that serves as a framework for welfare measurement. The developed methodology is then validated in evaluating the consumer welfare effects of quantity changes for some food groups in a U.S. inverse food demand system, which consists of 13 food groups and a nonfood sector.

Methodology

The Malmquist quantity index has been modified in this study for empirically measuring the consumer welfare effects of quantity changes in demand. Some basic concepts of the Malmquist quantity index and their relationships to a distance function are explained first. Then the focus is on developing a linkage between the modified Malmquist quantity index and an inverse demand system for consumer welfare measurement.

Malmquist Quantity Index

A distance function, $d(u, \boldsymbol{q})$, on utility u and quantity vector $\boldsymbol{q}$ gives the amount by which $\boldsymbol{q}$ must be divided to bring it onto the indifference curve u. Mathematically, the distance function is defined by the following equation:

$$(1) \qquad u = u\left[\frac{q}{d(u,q)}\right]$$

Because the distance function is dual and symmetrical to the cost function $c(u, \boldsymbol{p})$, the distance function can be rewritten as finding a price vector $\boldsymbol{p}$ that will minimize the ratio of expenditure $(\boldsymbol{p}'\boldsymbol{q})$ to cost function $c(u, \boldsymbol{p})$:

$$(2) \qquad d(u,q) = \min_p\left[\frac{(p'q)}{c(u,p)}\right]$$

Therefore, similar to the cost function being used widely for measuring the welfare effects of price changes, the distance function can be used to measure the welfare effects arising from quantity changes (Deaton, 1979).

By using the concept of distance function, the commonly known Malmquist quantity index (MQ) is defined as the ratio of two distance functions $d(u^0, \boldsymbol{q}^1)$ and $d(u^0, \boldsymbol{q}^0)$ to represent the constant utility quantity index for quantity changes from $\boldsymbol{q}^0$ to $\boldsymbol{q}^1$ while both the quantity vectors are scaled down to provide the consumer with a given utility level u^0 as follows:

$$(3) \qquad MQ(q^1,q^0;u^0) = \frac{d(u^0,q^1)}{d(u^0,q^0)}$$

A graphic presentation of the Malmquist quantity index for the quantity vectors $\boldsymbol{q}^1$ and $\boldsymbol{q}^0$ with a given utility level u^0 is shown in Figure 1:

In the figure, the distance function $d(u^0, \boldsymbol{q}^1)$ is defined as the segment ratio of $\frac{OB}{OA}$ to represent that the quantity vector $\boldsymbol{q}^1$ is scaled down to provide the consumer with the utility level u^0. Similarly, the distance function $d(u^0, \boldsymbol{q}^0)$ is defined as the segment ratio of $\frac{OD}{OC}$. Accordingly, an alternative definition of MQ as shown in the figure is represented as follows:

$$(4) \qquad MQ(q^1,q^0;u^0) = \frac{\left(\frac{OB}{OA}\right)}{\left(\frac{OD}{OC}\right)}$$

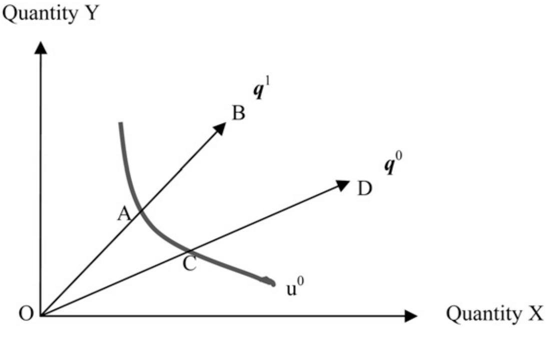

Figure 1. Quantity Metric Utility

Thus, the Malmquist quantity index *MQ* can be regarded as a measure of efficiency in quantity metric welfare (Deaton, 1979). If *MQ* is greater than one, it implies an inefficiency to achieve the same utility level as that achieved before the quantity changes from $\boldsymbol{q}^0$ to $\boldsymbol{q}^1$ and thus a decrease in consumer welfare. By contrast, if *MQ* is smaller than one, it implies an increase in efficiency and thus a gain in consumer welfare. Therefore, the quantity-adjusted *MQ* is a proper consumer welfare indicator to represent that a change in efficiency is needed to achieve the same utility level after the quantity changes. By assuming separable distance functions or homothetic preferences, the Malmquist quantity index is independent of the reference utility level, and thus it is easily computable by using only observed data (Chambers and Färe, 1998).

Empirical Modeling

For empirical modeling of the Malmquist quantity index for consumer welfare measurement, the quantity index is modified in the following two ways to provide some analytic advantage. First, the quantity index is specified as a difference form between two distance functions and is far better suited to a linear inverse demand system implemented in this study than a ratio form as shown in equation (3). Second, because the distance function in the quantity index is expressed as a normalized index number, the notion of compensating variation in expenditures is adopted in this study by multiplying the expenditures in the base period, ($\boldsymbol{p}^{0\prime}\,\boldsymbol{q}^0$) by the distance functions and obtaining a nonnormalized monetary measure of consumer welfare. All subscripts of variables and summation throughout this report refer to a total of *n* commodities in a demand system; that is, the variable index *i* or *j* representing 1, 2, ……, *n*.

Accordingly, the modified Malmquist quantity index (*MQ**) is defined as follows:

$$(5)\quad MQ^*(q^1,q^0;u^0) = [d(u^0,q^1) - d(u^0,q^0)](p^{0\prime}q^0)$$

For a given utility level u^0, this quantity index *MQ** represents the compensating changes of expenditures associated with the two distance functions for changes of quantity vectors from $\boldsymbol{q}^0$ to $\boldsymbol{q}^1$. Furthermore, by assuming that $\boldsymbol{q}^0$ represents the quantity vector in the base period and reaches the initial utility level u^0, that is the distance function $d(u^0, \boldsymbol{q}^0) = 1$, then the measure of *MQ** becomes

$$(6)\quad MQ^*(q^1,q^0;u^0) = d(u^0,q^1)(p^{0\prime}q^0) - p^{0\prime}q^0$$

The quantity index *MQ** of equation (6) is a measure of compensating variation in expenditures reflecting the changes of expenditures from the base period necessary to compensate consumers for the effects of quantity changes from $\boldsymbol{q}^0$ to $\boldsymbol{q}^1$. A positive *MQ** implies an increase in spending to achieve the same utility level u^0 as that achieved before the quantity changes in the base period and thus a decrease in consumer welfare. By contrast, a negative *MQ** implies a reduction in spending comparable to the base period and thus a gain in consumer welfare.

Now the key question is how to measure *MQ** by making use of the estimates from an inverse demand system. Because the term $d(u^0, \boldsymbol{q}^1)\,(\boldsymbol{p}^{0\prime}\,\boldsymbol{q}^0)$ of equation (6) represents the expenditures for a compensated change of quantities from $\boldsymbol{q}^0$ to $\boldsymbol{q}^1$ at the base period utility level u^0, this amount of expenditures can be quantified as the product of a compensated price vector, $\boldsymbol{p}^{\mathrm{h}}\,(u^0, \boldsymbol{q}^1)$, and a quantity vector $\boldsymbol{q}^1$, in which the compensated prices can be measured from an estimated inverse demand system. Accordingly, the *MQ** can be expressed as follows:

$$(7)\quad MQ^*(q^1,q^0;u^0) = p^{\mathrm{h}}(u^0,q^1)'q^1 - p^{0\prime}q^0$$

By defining $d\boldsymbol{q} = \boldsymbol{q}^1 - \boldsymbol{q}^0$ as a vector of quantity changes, and $d\boldsymbol{p}^{\mathrm{h}} = \boldsymbol{p}^{\mathrm{h}}\,(u^0, \boldsymbol{q}^1) - \boldsymbol{p}^0$ as a vector of compensated price changes, equation (7) for *MQ** is then transformed into the following equation:

$$(8)\quad \begin{aligned} &MQ^*(q^1,q^0;u^0) \\ &\quad = (p^0 + dp^{\mathrm{h}})'(q^0 + dq) - p^{0\prime}q^0 \\ &\quad = \Sigma_{\mathrm{i}}\left[\frac{dq_{\mathrm{i}}}{q_{\mathrm{i}}{}^0} + \frac{dp_{\mathrm{i}}{}^{\mathrm{h}}}{p_{\mathrm{i}}{}^0} + \left(\frac{dq_{\mathrm{i}}}{q_{\mathrm{i}}{}^0}\right)\left(\frac{dp_{\mathrm{i}}{}^{\mathrm{h}}}{p_{\mathrm{i}}{}^0}\right)\right](p_{\mathrm{i}}{}^0 q_{\mathrm{i}}{}^0) \end{aligned}$$

Given the initial quantities $\boldsymbol{q}^0$, prices $\boldsymbol{p}^0$, and various scenarios assigned for the quantity vectors of $\boldsymbol{q}^1$ and $d\boldsymbol{q}$, the question for computing MQ^* is how to determine the amount of changes in compensated prices $d\boldsymbol{p}^h$. For an estimated compensated inverse demand system, the changes in the compensated prices of the ith good dp_i^h can be measured as a function of quantity changes (dq_j) as follows:

$$(9) \qquad \frac{dp_i^h}{p_i^0} = \sum\nolimits_j f_{ij}^* \left(\frac{dq_j}{q_i^0}\right)$$

where f_{ij}^*s are the compensated price flexibilities of the ith commodity with respect to a quantity change of the jth commodity. Thus, all estimated direct- and cross-price flexibilities representing the interdependent relationships of an inverse demand system are incorporated into a consumer welfare measurement.

The compensated price flexibilities (f_{ij}^*'s) of equation (9) can be estimated from an inverse demand system as developed by Huang (1988), in which a differential form compensated inverse demand system is specified as follows:

$$(10) \qquad \frac{dr_i}{r_i} = \sum\nolimits_j f_{ij}^* \left(\frac{dq_j^*}{q_j^*}\right) + g_i\left(\frac{ds}{s}\right)$$

where r_i is the normalized price using per capita income to deflate the price of ith commodity, f_{ij}^* is the compensated price flexibility of the ith commodity with respect to a quantity change of the jth commodity, and g_i is the scale flexibility showing the effect of the ith commodity price on the proportional change in all quantities demanded.

This differential form demand system is a general approximation of conceptual inverse demand relationships in relating to some small change from any given point on the commodity demand surface without imposing any rigid functional form of utility structure. The merit of the demand system is that the estimates can be interpreted as commonly used price flexibilities, and the variables defined in the demand system are the relative changes of prices and quantities, which can be obtained from the available data usually expressed in index numbers. Other inverse demand models such as Eales and Unnevehr (1994), Grant, Lambert, and Foster (2010), and Moschini and Vissa (1992) specified some forms of demand systems under the AIDS framework. Although their demand models are capable of generating price flexibilities, the generated price flexibilities may be unstable inasmuch as the price flexibilities are functions of expenditure shares, which are innate stochastic variables in these models. Also, these demand models require the time series data of expenditure shares, which are not easily available.

For empirical estimation, the variables in equation (10) are defined as the amounts of relative changes of the preceding year. The scale variable (s) is calculated first as the geometric expenditure-weighted average quantity indices of all groups in the demand system; that is $\log s = \sum_j w_j \log q_j$, where w_j is the expenditure share of the jth commodity. Then a reference quantity vector $\boldsymbol{q}^*$ is obtained by using the scale variable to deflate a quantity vector $\boldsymbol{q}$.

All demand equations in the differential form demand system are estimated simultaneously by incorporating the parametric constraints of homogeneity ($\sum_j f_{ij}^* = 0$), symmetry $\left(\frac{f_{ji}^*}{w_i} = \frac{f_{ij}^*}{w_j}\right)$, and scale aggregation ($\sum_i w_i g_i = -1$), where w_i is the expenditure share of ith commodity taken at the sample mean. The negativity condition ($f_{ij}^* < 0$), however, is not incorporated, partly because there is no reduction in the number of parameters to be estimated and thus no gain in asymptotic efficiency of the estimates and partly to avoid introducing parametric inequality constraints that would increase the complexity of estimation. Now the estimated compensated price flexibilities from the inverse demand system can be used as input information for measuring the consumer welfare effects of quantity changes in equations (8) and (9).

Application

The developed methodology for measuring the consumer welfare effects of quantity changes is applied to a U.S. compensated inverse food demand system consisting of 13 food groups and a nonfood sector. The compensated price flexibilities from the estimated demand system are then used as input information for the welfare measurement.

Data Sources

The time series data required for estimation of an inverse food demand systems include per-capita food quantities provided by the Economic Research Service (ERS, U.S. Department of Agriculture, 2009), prices and expenditure shares by the Bureau of Labor Statistics (BLS, U.S. Department of Labor, 2009), and per capital income by the Bureau of Economic Analysis, U.S. Department of Commerce, 2009. All data except for food quantities can be used because they are from the original data sources without any compilation. The per-capita food quantity data provided by ERS, however, need to be aggregated from the original 131 items into 13 groups. The reason for aggregating into 13 food groups is to make the food groups in the quantity data able to match the available food groups of the aggregate consumer price index (CPI) from BLS.

The raw quantity data for per-capita food consumption consisting of 131 food items covering 1953–2008 are aggregated into 13 food groups in the following two steps. First, all quantity data of 131 individual food items are aggregated into 38 food categories by summing up their quantity weights of similar items such as oranges and tangerines into a category. These 38 food categories are conformable with the expenditure share data available from BLS. Second, the quantity data of the 38 food categories are further aggregated into 13 food groups of the weighted-average Laspeyres quantity index series using the expenditure shares from BLS as weights. Ideally, any aggregation over commodities should satisfy the Hicksian composite commodity theorem (Hicks, 1936) that some commodities in consumer budgeting may be aggregated into a single composite commodity if the prices of all those items move in exact proportion over the data sample. The assumption required to justify aggregation, however, is quite stringent and is almost never satisfied for generating the data for use in this study.

In this study, the Laspeyres quantity index series for a food group are calculated as a weighted average index of some 38 food categories within the food group with the expenditure shares of each food category in the base period as weights. Specifically, the quantity index for a food group at time t in the base period can be calculated as a weighted average of individual quantity indexes as follows:

$$(11) \qquad \frac{Q_t}{Q_0} = \sum\nolimits_i w_0^i \left(\frac{q_t^i}{q_0^i}\right),$$

where w_0^{i} is the expenditure share in the base period of the ith food category. The Laspeyres quantity index series can reflect the importance of expenditures spent for each food group without losing information in the aggregation process. In addition, this process of aggregation for obtaining the quantity index is consistent with the process in measuring CPI by BLS. Therefore, the generated Laspeyres quantity index series for food groups can be matched closely with the price response in CPI.

The selected 13 food groups are 1) meats including beef, veal, and pork; 2) poultry products including chicken and turkey; 3) fish including fresh, frozen, and canned fish; 4) eggs; 5) dairy products including milk and other dairy products; 6) fats including added fats of butter, margarine, and other fats and oils; 7) fresh fruits; 8) fresh vegetables; 9) processed produce, which also includes fruit and vegetable juices and tree nuts; 10) wheat flour; 11) starchy foods including potato, rice, corn flour, and oat products; 12) sugar including all added sugars and other sweeteners; and 13) nonalcoholic beverage including coffee, tea, and cocoa but not including other drinks like carbonated beverages, sports drinks, fruit drinks and other sweetened fruit-flavored drinks for lack of consistent times series for these products.

U.S.-Compensated Inverse Food Demand System

A U.S.-compensated inverse food demand system of equation (10) is estimated by applying the seemingly unrelated regression method available in the SAS program—SYSLIN procedure from SAS Institute Inc. Because the dependent variables expressed in the relative changes of prices are not constrained across equations, the variance–covariance matrix of the error terms in the demand system is not singular, and thus all demand equations can be estimated simultaneously. It is worth noting that some other demand models such as the AIDS model are specified by using expenditure shares as dependent variables in the demand system. Consequently, the sum of expenditure shares across equations is constrained to equal one, causing the variance–covariance matrix of the error terms in the demand system to be singular and requiring the deletion of one of demand equations from direct estimation.

All compensated price and scale flexibilities in the demand system are estimated simultaneously, whereas the parametric constraints of homogeneity, symmetry, and scale aggregation across demand equations are incorporated into the estimation. Although the constraints derived from individual consumer behavior may not hold exactly in the market demand analysis, the potential bias in aggregation is assumed to have a negligible effect on the extension of demand theory from an individual to a market.

The estimates of the inverse food demand system are compiled in Table 1. Each entry shows the compensated price flexibilities of a food group in the left column with respect to their reference quantities and the scale of quantities at the top of the table. The estimated own-price compensated flexibilities listed in the diagonal entries of the table suggest how much group price must change to induce the consumer to absorb marginally more of that group while maintaining the same utility level in the base period. For example, the compensated own-price flexibilities for meats and poultry products are –0.7003 and –0.8924, respectively. These measures indicate that, for a given utility level, a marginal 1% increase in the quantity of meats would require a price decrease of 0.7%, and the same quantity increase in poultry products would require a price decrease of 0.89%. Among the estimated own-price and scale flexibilities, 23 of 28 estimates are statistically significant at the 5% probability level.

The estimated compensated cross-price flexibilities showing the substitute or complementary demand relationships are listed in the off-diagonal entries of Table 1. If a marginal increase of the quantity of one good has a substitution effect on the other goods, the price of the other goods should be lower to induce consumers to purchase the same quantity of that other goods. For example, the compensated cross-price flexibility between the price of meats and the quantity of poultry products is –0.1726, which implies that the two food groups are substitutes. A marginal 1% increase in the quantity of poultry products is associated with a 0.17% decrease in the price of meats to induce consumers to purchase the same quantity of meats instead of substituting poultry products. On the contrary, the compensated cross-price flexibility between the price of fruits and the quantity of vegetables is 0.044, indicating a complementary relationship between these two food groups. Accordingly, in contrast to the cross-price elasticities in a quantity-dependent demand system, the cross-price flexibilities in the table reflect substitution if the sign is negative and complementary if the sign is positive.

The estimated scale flexibilities in Table 1 show the potential response of a food group price to a proportionate increase in the quantities of all groups. For example, the scale flexibility for meats is –1.8649, which indicates that a proportionate increase in the quantities of all groups by 1% would decrease the price of meats by 1.86%. All estimated scale flexibilities are negative and larger than one in absolute value as expected. The scale flexibility, although not income-related, serves as a linkage between compensated and uncompensated inverse demand systems. All income flexibilities in an inverse demand system showing the effects of changes in income on prices are constrained to be unitary values because, for given all quantities demanded, an increase of income would cause all prices to increase at the same rate.

The estimated intercepts are listed in the next to last column. Because the variables defined in the demand system are expressed in relative changes, these intercepts reflect the

Table 1. U.S.-Compensated Inverse Food Demand System, 1953–2008

Price	Reference Quantity of														Scale	Constant	Root Mean Square
	Meats	Poultry	Fish	Eggs	Dairy	Fats	Fruits	Vegetables	Processed Produce	Flour	Starch	Sugar	Nonalcoholic Beverages	Nonfood			
								Percent									
Meats	**–0.7003**	–0.1726	–0.0135	–0.0330	0.0109	–0.0393	–0.1044	–0.0982	0.0529	–0.0408	–0.0261	0.0288	0.0570	1.0785	–1.8649	–0.2977	3.84
	(0.1059)	(0.0394)	(0.0197)	(0.0252)	(0.0442)	(0.0238)	(0.0417)	(0.0433)	(0.0309)	(0.0655)	(0.0245)	(0.0294)	(0.0347)	(0.2214)	(0.3006)	(0.6065)	
Poultry	–0.6100	**–0.8924**	0.0164	–0.1347	–0.1282	0.0224	–0.0608	0.0131	0.0469	–0.1517	–0.0156	–0.2141	0.0167	2.0919	–2.1338	–1.8399	5.38
	(0.1393)	(0.1272)	(0.0512)	(0.0776)	(0.1339)	(0.0541)	(0.0838)	(0.1132)	(0.0504)	(0.1624)	(0.0565)	(0.0870)	(0.0766)	(0.3809)	(0.4876)	(0.9509)	
Fish	–0.0630	0.0218	**0.0177**	–0.0105	–0.2475	–0.0634	–0.0178	–0.0656	0.0478	0.1091	–0.0027	0.0775	–0.0876	0.2841	–0.7532	–0.3127	2.66
	(0.0922)	(0.0677)	(0.0552)	(0.0585)	(0.1084)	(0.0405)	(0.0592)	(0.0841)	(0.0333)	(0.1239)	(0.0392)	(0.0654)	(0.0566)	(0.2484)	(0.3049)	(0.5884)	
Eggs	–0.4717	–0.5457	–0.0321	**–1.8406**	–0.2630	0.3276	–0.1500	–0.7643	0.0885	–0.4956	0.0199	–0.3131	0.1658	4.2742	–4.8222	–3.1753	10.76
	(0.3602)	(0.3141)	(0.1792)	(0.5270)	(0.7749)	(0.1711)	(0.2343)	(0.3834)	(0.1244)	(0.5534)	(0.1614)	(0.3367)	(0.2226)	(1.1648)	(1.3439)	(2.2766)	
Dairy	0.0193	–0.0639	–0.0933	–0.0324	**–0.1888**	0.0259	–0.0160	–0.0292	–0.0353	0.0056	–0.0211	0.2325	0.1215	0.0752	–1.0436	0.0025	2.31
	(0.0779)	(0.0668)	(0.0409)	(0.0954)	(0.2727)	(0.0385)	(0.0497)	(0.0877)	(0.0268)	(0.1321)	(0.0342)	(0.0791)	(0.0484)	(0.3019)	(0.3241)	(0.5160)	
Fats	–0.2208	0.0356	–0.0762	0.1287	0.0828	**–0.2395**	–0.1196	–0.0041	–0.0605	–0.4295	–0.1001	–0.1708	–0.1367	1.3106	–2.7076	0.5636	5.97
	(0.1339)	(0.0862)	(0.0486)	(0.0672)	(0.1227)	(0.0742)	(0.0830)	(0.1059)	(0.0559)	(0.1672)	(0.0524)	(0.0814)	(0.0761)	(0.3852)	(0.4957)	(1.0284)	
Fruits	–0.2787	–0.0460	–0.0101	–0.0280	–0.0241	–0.0568	**–0.5051**	0.0440	–0.0956	0.2969	0.0013	0.0472	–0.0105	0.6653	–1.5980	0.8196	3.90
	(0.1114)	(0.0633)	(0.0338)	(0.0437)	(0.0752)	(0.0394)	(0.0976)	(0.0768)	(0.0437)	(0.1126)	(0.0421)	(0.0521)	(0.0594)	(0.2913)	(0.3828)	(0.7741)	
Vegetables	–0.3513	0.0132	–0.0502	–0.1911	–0.0592	–0.0026	0.0589	**–0.2691**	–0.0878	–0.4698	–0.0029	0.0205	–0.1408	1.5321	–1.5299	–1.1027	4.81
	(0.1549)	(0.1146)	(0.0643)	(0.0958)	(0.1780)	(0.0674)	(0.1030)	(0.1957)	(0.0575)	(0.2183)	(0.0727)	(0.1081)	(0.0937)	(0.4295)	(0.5287)	(0.9951)	
Processed produce	0.0962	0.0241	0.0186	0.0112	–0.0364	–0.0196	–0.0651	–0.0446	**–0.2200**	–0.1690	–0.0568	–0.0402	0.0265	0.4750	–1.6845	0.2073	4.04
	(0.0562)	(0.0260)	(0.0130)	(0.0158)	(0.0277)	(0.0181)	(0.0298)	(0.0292)	(0.0340)	(0.0493)	(0.0173)	(0.0214)	(0.0242)	(0.1567)	(0.2420)	(0.6347)	
Flour	–0.0504	–0.0530	0.0288	–0.0428	0.0039	–0.0944	0.1374	–0.1622	–0.1148	**–0.4445**	–0.0315	0.0065	0.0230	0.7939	–2.2001	1.2144	3.96
	(0.0810)	(0.0568)	(0.0327)	(0.0478)	(0.0926)	(0.0367)	(0.0521)	(0.0754)	(0.0335)	(0.1442)	(0.0332)	(0.0548)	(0.0485)	(0.2422)	(0.3083)	(0.6772)	
Starch	–0.1697	–0.0287	–0.0037	0.0091	–0.0779	–0.1157	0.0032	–0.0053	–0.2032	–0.1656	**–0.1465**	–0.0181	–0.1386	1.0607	–2.2900	1.2310	6.55
	(0.1593)	(0.1039)	(0.0544)	(0.0733)	(0.1260)	(0.0605)	(0.1025)	(0.1321)	(0.0618)	(0.1745)	(0.0870)	(0.0853)	(0.0880)	(0.4406)	(0.5769)	(1.1570)	
Sugar	0.1603	–0.3373	0.0923	–0.1218	0.7344	–0.1691	0.0985	0.0319	–0.1231	0.0291	–0.0155	**–0.4185**	–0.0715	0.1101	–2.2286	3.0562	5.73
	(0.1637)	(0.1371)	(0.0779)	(0.1309)	(0.2499)	(0.0806)	(0.1087)	(0.1681)	(0.0654)	(0.2471)	(0.0730)	(0.1940)	(0.1038)	(0.4947)	(0.6012)	(1.1876)	
Beverages	0.3130	0.0260	–0.1029	0.0636	0.3785	–0.1335	–0.0215	–0.2161	0.0801	0.1021	–0.1171	–0.0705	**–0.7403**	0.4387	–0.8835	–1.1637	6.56
	(0.1904)	(0.1191)	(0.0665)	(0.0854)	(0.1510)	(0.0743)	(0.1222)	(0.1438)	(0.0730)	(0.2154)	(0.0744)	(0.1024)	(0.1485)	(0.5154)	(0.6527)	(1.2913)	
Nonfood	0.0497	0.0273	0.0028	0.0138	0.0020	0.0107	0.0115	0.0197	0.0120	0.0296	0.0075	0.0009	0.0037	**–0.1913**	–0.8222	0.0190	0.44
	(0.0102)	(0.0050)	(0.0024)	(0.0038)	(0.0079)	(0.0032)	(0.0050)	(0.0055)	(0.0040)	(0.0090)	(0.0031)	(0.0041)	(0.0043)	(0.0317)	(0.0396)	(0.0716)	
Expenditure shares	0.0378	0.0107	0.0081	0.0026	0.0214	0.0067	0.0142	0.0106	0.0208	0.0306	0.0058	0.0068	0.0069	0.8170			

Notes: For each pair of estimates, the upper part is the estimated flexibility and the lower part in parenthesis is the standard error. The bold type represents own-price compensated flexibilities.

potential time trends in association with the changes of normalized prices defined as the deflated retail prices by per-capita income. For example, the estimate for the meat group is –0.2977, indicating a downward trend of meat prices over the sample period. Likewise, the estimates for the groups of poultry products and eggs are –1.8399 and –3.1753, respectively, showing much larger downward trends than that of the meat group. The estimate for the group of sugar and sweeteners is 3.0562, however, implying that the prices of this food group had an upward trend over the sample period.

Regarding the goodness of fit of the demand system, the commonly used R^2 for each demand equation is not calculated here because all the demand equations are estimated simultaneously with parametric constraints across equations. In this case, the SYSLIN procedure, however, estimates a special single regression by stacking all demand equations and obtaining a system R^2 of being 0.8728. Because the demand system in this study is not estimated equation by equation, the calculated R^2 associated with each demand equation is not required to be between zero and one as the generally perceived R^2 measure. Accordingly, the root mean square (*RMS)* percentage errors to sample means of actual observations are calculated to represent the goodness of fit for each demand equation as the following:

$$(12) \qquad RMS = \frac{\left[\sum_t \frac{(r_t - r_t*)^2}{T}\right]^{\frac{1}{2}}}{r**} \times 100$$

where r_t, r_t*, and $r**$ are, respectively, the levels of actual, projected, and sample mean of normalized prices for a sample period T years. As shown in the last column of Table 1, most of estimated RMS errors are less than 7% for each demand equation.

Consumer Welfare Measurements

The modified Malmquist-quantity index $MQ*$ of equation (8) is applied to measure the consumer welfare effects of quantity changes in a U.S. inverse food demand system. To evaluate the changes in consumer welfare, it requires the expenditures in the base period, that is $(\boldsymbol{p}^{0\,\prime}\,\boldsymbol{q}^0)$ of equation (5), as a basis for comparison. For this study, the real per-capita expenditures in the most recent sample period 2006–2008 are considered as the base period. The yearly average real expenditure data using prices indexed to year 2000 levels for the base period indicate that Americans on average spend $5,926 annually for foods consumed at home by each person, and these expenditures are allocated to meats ($1,224), poultry products ($346), fruits ($459), and vegetables ($342).

For illustration of measuring consumer welfare, an example is given to the quantity changes between meats and poultry products because they are major components (approximately one-fourth) of total food expenditures. The estimated welfare effects in response to the quantity changes of meats and poultry products are listed in the upper part of Table 2 with various scenarios for the combined changes in the quantities ranging from 0% to 5%. For example, because of the net effect of interdependent demand relationships, a marginal 5% increase in the quantities of meats would reduce yearly per-capita meat expenditures or incur a consumer welfare gain of $219.17, a saving of the meat expenditure by 17.89%. Similarly, a marginal 5% increase in the quantities of poultry products would reduce yearly per-capita poultry expenditures or incur a consumer welfare gain of $96.20, a saving of poultry expenditure by 27.8%. If both the quantities of meats and poultry products were increased by 5%, per-capita total expenditures would decrease by $421.02, a saving of both the meat and poultry expenditures by 26.82%.

Another example of welfare measurement is given to evaluate the quantity changes between fruits and vegetables. As obesity and being overweight continue to increase in the United States, the *2010 Dietary Guidelines for Americans* has encouraged Americans to increase fruit and vegetable consumption because diets rich in fruit and vegetables—good sources of vitamins, minerals, and fiber—are associated with reducing some chronic diseases and, more recently, lessening the problem of obesity. Thus, the consumer welfare

Table 2. Consumer Welfare Effects of Increasing Food Quantities

Example A: Quantities Increased between Meats and Poultry Products (percent)							
		Poultry Products					
		0%	1%	2%	3%	4%	5%
		Compensating Variation (MQ*) (dollars)					
	0%	0.00	–6.87	–19.93	–39.17	–64.59	–96.20
	1%	–9.53	–20.64	–37.92	–61.38	–91.03	–126.86
Meats	2%	–36.22	–51.55	–73.05	–100.75	–134.62	–174.68
	3%	–80.05	–99.61	–125.34	–157.26	–195.36	–239.64
	4%	–141.03	–164.81	–194.78	–230.92	–273.25	–321.76
	5%	–219.17	–247.17	–281.36	–321.73	–368.29	–421.02
Example B: Quantities Increased between Fruits and Vegetables (percent)							
		Vegetables					
		0%	1%	2%	3%	4%	5%
		Compensating variation (MQ*) (dollars)					
	0%	0.00	–2.74	–7.32	–13.75	–22.01	–32.12
	1%	–0.78	–3.12	–7.30	–13.32	–21.18	–30.88
Fruits	2%	–6.20	–8.13	–11.91	–17.52	–24.98	–34.28
	3%	–16.25	–17.78	–21.15	–26.36	–33.42	–42.32
	4%	–30.93	–32.06	–35.03	–39.84	–46.49	–54.98
	5%	–50.25	–50.97	–53.54	–57.94	–64.19	–72.28

Note: Per-capita expenditures are meats ($1,224), poultry products ($346), fruits ($459), and vegetables ($342).

effects of quantity changes in fruits and vegetables are of interest here. The results of estimated welfare effects in response to quantity changes in fruits and vegetables are shown in the lower part of Table 2. For example, with a marginal 5% increase in the quantities of fruits, the welfare effect would reduce per-capita fruit expenditures or incur a consumer welfare gain of $50.25, a saving of the fruit expenditure by 10.95%. Similarly, a marginal 5% increase in the quantities of vegetables would reduce per-capita vegetable expenditures or incur a consumer welfare gain of $32.12, a saving of vegetable expenditure by 9.39%. Increasing both the quantities of fruits and vegetable by 5% would decrease yearly per-capita total produce expenditures by $72.28, a saving of the produce expenditures by 9.02%.

Conclusion

In this study, a unique approach is developed to measure consumer welfare effects arising from quantity changes in demand by applying the Malmquist quantity index with a linkage to a differential form compensated inverse demand system. All compensated direct- and cross-price flexibilities from the demand system are incorporated into the welfare measurement. The methodology is useful for welfare measurement as shown in empirical application to a U.S. inverse food demand system.

Because different food groups in the demand system are interdependent and intertwined, the methodology has a general application that can be tailored to specific policy analysis by assigning various scenarios of quantity changes in any food group and evaluating their combined welfare effects. For illustration, two examples of welfare measurement are provided concerning the quantity changes between meats and poultry products and between fruits and vegetables. The results indicate that the changes in consumer welfare vary depending on how food quantity changes manifest themselves through the interdependent food demand relationships. As expected, more quantities of foods available in the market causing price decreases can save food expenditures and incur consumer welfare gains.

The focus of this study is on developing a method for measuring consumer welfare, whereas the issue of structural change in demand, although important, is not addressed within the scope of this study. Also, the welfare measurement is emphasized on the part of consumers but does not explicitly recognize the supply side of the food markets. An extension of this research to a general demand–supply equilibrium model would be helpful for understanding whether the potential gain of aggregate consumers' welfare in savings can be compensated for the loss of farmers' revenue. Perhaps Samuelson's (1965) theory of mixed demands, which expresses demand relationships as a function of a mixed set of prices and quantities, would be a starting point for the endeavor. More research, however, is needed to identify the parametric constraints that can be implemented for the estimation of a mixed demand model. In addition, determining a priori which prices and quantities are endogenous in a mixed demand system is another difficult task (Stockton, Capps, and Bessler, 2008).

[Received February 2012; Accepted February 2013.]

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