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Estimating Input Cost Shares for Agriculture Using a Multinomial Logit Framework

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Abstract

Many econometric analyses include dependent variables constrained to the interval between zero and 1 Under such conditions, simple regression procedures break down Several alternative stochastic models which avoid this problem can be defined depending on the assumed error structure Two alternative forms of the logit model are treated here The multivariate logit approach assumes that the share specification is an accurate representation of the underlying input demand structure The multinomial logit approach treats the dependent variable as a probability with a multinomial density

Keywords

Econometrics, limited dependent variable, multinomial logit, maximum likelihood

Many econometric analyses include dependent variables which are constrained to the interval between zero and 1 Typical of this type of analysis is the simultaneous estimation of input cost shares Development of the translogarithmic (translog) cost function has increased interest in estimating systems of input share equations (*10, 11*) Cost functions and underlying share equation systems have been estimated by Christensen and Green (*9*), Berndt and Wood (*6*), Denny and Pinto (*12*), and Humphrey (*18*) [1] From share equations it is possible to derive input price and substitution elasticities (*5, 20, 25*)

However, there is no implicit or explicit mechanism constraining the prediction of input shares to between zero and 1 by use of simple regression procedures Predictions may fall outside the zero-1 interval and, because of the grouped nature of the data, error terms are likely to be heteroscedastic The objective here is to outline two versions of a logit model which explicitly force predicted input shares to sum to 1 The logit is a sensible and convenient alternative to the limited dependent variable problem encountered when one estimates input share equations derived from a translog cost function

First, I describe the underlying structure of the logit model I present two alternative forms of the logit model and discuss an estimation methodology for each Finally, I cite an example of a multinomial logit model where a maximum likelihood technique is used to estimate input cost shares for agriculture

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[1] Italicized numbers in parentheses refer to items in the References at the end of this article

The Logit Model

The logit model is one approach to the analysis of a general class of problems termed discrete choice behavior These types of problems have been investigated in other fields for many years (*2, 3*) Economists have begun to analyze problems which could be cast in this framework by considering discrete choices as selections from a continuum of alternatives, thereby integrating these problems into the theory of the household and firm

Statistical economic analysis of the general population or the choice behavior of the average firm or consumer is complicated because such behavior must be described in probabilistic terms The probability of a particular choice is conditional on the explanatory variables selected for the analysis A discrete choice framework has been used to examine many problems including migration (*28*), occupational choice (*7, 30*), demand for housing (*35*), and demand for consumer durables (*16, 33*)

Interest in the logit derives from its relatively simple structural form (*29*) The logit forces estimates of the dependent variables to be between zero and 1 while summing to unity It allows for a wide array of functional forms, although certain functions are more easily applied When applied in consumption analyses, the logit allows for nonunitary income elasticities, and when applied in production analyses, it does not place *a priori* restrictions on elasticities of substitution Recent applications of the multinomial logit analyze the expenditures of firms on inputs (*20*) and of consumers on household goods (*34*)

The basic structure of the logit expenditure system is written as

$$S_i = \exp(c_i(x))/\sum_{j=1}^{k}\exp(c_j(x)) \qquad (i=1, 2, \ldots k) \qquad (1)$$

where S_i is the share or proportion of total expenditures spent on the ith good, x is a vector of explanatory variables such as prices or income, and c_i is any mathematically well-behaved function linking the explanatory variables and the dependent share variables

Two alternative forms of the logit transformation can be used for empirical analysis The multinomial form of logit, or conditional logit (*27*), treats the dependent variable as a probability with a multinomial density The multivariate logit emerges from a more *ad hoc* prediction-oriented approach which assumes that equation (1) is an accurate functional characterization of expenditures With either form, one can incorporate many factors which influence expenditures, such as family or firm size, as explanatory variables while maintaining a theoretically consistent expenditure system

Multinomial Logit

The theoretical foundation of the multinomial logit sets the decisionmaker in an environment where discrete choices of expenditures are made If it is assumed that the larger the value of an index the greater the probability that the event will occur, then one can define a monotonic relationship linking the value of the index and the probability of the event's occurrence

The decisionmaker's choices are described in a decision index of the form

$$c = c(x) + e \qquad (2)$$

where e is a random disturbance associated with a given probability distribution and c(x) is nonstochastic Faced with k alternatives, the decisionmaker will choose alternative i only if $c_i(x) + e_i > c_j(x) + e_j$ for all $j \neq i$ The probability of this event occurring is

$$P_i = \text{Prob}[c_i(x) + e_i > c_j(x) + e_j)] \qquad \text{for all } j \neq i \qquad (3)$$

and, therefore

$$P_i = \text{Prob}[e_i - e_j > c_j(x) - c_i(x)] \qquad \text{for all } j \neq i \qquad (4)$$

One must make binary comparisons of $c_i(x)$ with each alternative $c_j(x)$ Probability distributions which are closed under subtraction, or produce convenient distributions when subtracted, are particularly attractive candidates for the probability densities of e (*19*)

If $F(e_1, e_2, \ldots, e_k)$ represents the cumulative distribution function of the disturbances e_i and if F_i denotes the marginal density function of e_i (the derivative of F with respect to e_i), then the probability P_i is written

$$P_i = \int_{-\infty}^{\infty} F_i[(e_1 + c_1(x) - c_i(x), \ldots, e_k + c_k(x) - c_i(x)\, de_i \qquad (5)$$

If the errors are independent and identically distributed, then a necessary and sufficient condition for the model described by equation (4) to yield the conditional logit form is that the errors have a Weibull density (*27*) That is

$$e_i \sim \exp(-\exp(-e_i)) \qquad (6)$$

The Weibull density is a convenient way to generate the logistic distribution It is closed under subtraction and closely approximates the normal distribution while being numerically simpler McFadden (*27*) indicates that the underlying choice structure implies the independence of irrelevent alternatives axiom The independence of irrelevant alternatives condition is both a strength and a weakness of the logit expenditure model (*14*) It is a strength because introducing additional alternatives does not alter the relative odds with which previous alternatives are selected It is a weakness because it requires that the cross-elasticity of demand for each old expenditure category, with respect to an attribute of a new category, is uniform across all old categories

If the multinomial form of the logit is used to describe producer expenditures on inputs, then a set of k independent conditional probabilities are assumed to jointly determine the allocation of expenditures into k input categories The probabilities have a logistic structure and are conditional on input prices and other explanatory variables The probabilities are written as

$$P_i = \exp(c_i(x))/\sum_{j=1}^{k}\exp(c_j(x)) \qquad (i = 1, 2, \ldots, k) \qquad (7)$$

where P_i is the conditional probability of \$1 being allocated to input i, x is a vector of explanatory variables, and $c_i(x)$ is a decision index

Because the probability P_i is unobservable, the model is made operational by use of the share of the cost of production associated with input i as a proxy (*31*) Equation (7) is therefore rewritten as

$$S_i = v_i q_i/M = \exp(c_i(x))/\sum_{j=1}^{k}\exp(c_j(x)) \qquad (i = 1, 2, \ldots, k) \qquad (8)$$

where M is the sum of each input (q_i) used in production multiplied by its input price (v_i)

Before the multinomial logit input expenditure model can be estimated, a functional form must be selected for the decision index $c_i(x)$ The function may include input prices, output, and all other production or producer characteristics considered relevant Although no specific restrictions on $c_i(x)$ are necessary to estimate the multinomial logit model, estimation is easier if the functions are assumed to be linear in their parameters and invariant in structure between equations except for interequation parameter variability The functions are written as

$$c_i(x) = \sum_{j=1}^{n} B_{ij} h_{ij}(x) \qquad (i = 1, 2, \ \ , k) \qquad (9)$$

where B_{ij} are parameters and h_{ij} are functions

The expenditure of $1 on a given input is analogous to sampling with replacement from a population classified into k categories The resulting multinomial distribution is written as

$$f = \frac{M!}{E_1! \quad E_k!} P_1^{E_1} \quad P_K^{E_k} \qquad (10)$$

where E_i equals $v_i q_i$ and is non-negative For a sample of T observations, the logarithm of the likelihood function associated with equation (8) is written as

$$\ln L = \text{Constant} + \sum_{t=1}^{T} \Big(\sum_{i=1}^{k} S_{it} c_{it} - \ln \Big(\sum_{i=1}^{k} \exp(c_{it}) \Big) \Big) \qquad (11)$$

Although the functions $c_i(x)$ may take any form, functions which are linear in parameters are sufficiently flexible for most purposes A linear form also leads to simple expressions for the maximization of the likelihood function Therefore, if

$$c_{it} = \sum_{j=1}^{n} B_{ij} x_{jt} \qquad (i = 1, 2, \ \ , k) \qquad (12)$$

then, the first order conditions for the maximization of the log-likelihood function are

$$\partial \ln L / \partial B_{ij} = \sum_{t=1}^{T} x_{it} [S_{it} - P_{it}] = 0 \qquad (i = 1, 2, \ \ , k) \qquad (13)$$
$$(j = 1, 2, \ \ , n)$$

The second-order conditions for the maximization of the log likelihood function are

$$\partial^2 \ln L / \partial B_{ij} \partial B_{ij'} = \sum_{t=1}^{T} x_{jt} x_{j't} P_{it}(P_{it} - 1) < 0$$
$$(i = 1, 2, \ \ , k)$$
$$(j = 1, 2, \ \ , n)$$
$$(14)$$

where both parameters (denoted by j and j′) are in the *i*th equation and

$$\partial^2 \ln L / \partial B_{ij} \partial B_{i'j'} = \sum_{t=1}^{T} x_{jt} x_{j't} P_{i't} P_{i't} < 0 \quad (i = 1, 2, \ \ , b)$$
$$(j = 1, 2, \ \ , n)$$
$$(15)$$

where each parameter is from a different equation

The maximum likelihood estimators are invariant to monotonic transformations of the likelihood function The log likelihood function is maximized where the first-order derivatives are zero (equation (13)) and the matrix of second-order derivatives, formed by the derivatives in equations (14) and (15), is negative definite Because the first and second derivatives are nonlinear in parameters, an iterative search procedure is needed to solve for the parameters B_{ij}

The estimation procedure is complicated by an indeterminancy in the equation system which arises because the sum of the shares must equal 1 The share equations are, therefore, invariant with the addition of the same expression, lnZ for example, to each decision index The indeterminancy causes the matrix of second-order partial derivatives of the log-likelihood function to be singular

One can avoid the singularity problem by normalizing the parameters for a particular variable in the k functions (*34*) The normalization does not affect the predicted shares A straightforward approach is to divide the k − 1 equations by the *k*th equation In a logarithmic form, the share system is

$$\ln(S_i/S_k) = \sum_{j=1}^{n} (B_{ij} - B_{kj}) h_{ij}(x_j) \quad (i = 1, 2, \ \ , k-1) \quad (16)$$

Maximizing lnL with respect to c_i is equivalent to maximizing lnL with respect to $c_i - c_k$ for any k (*34*) It is only necessary to consider k − 1 equations as the information provided by the *k*th equation is constant Furthermore, regardless of the normalization employed, the predicted values of S_i are identical Therefore, equation (16) can be used to estimate input cost shares by use of linear regression One can compute the individual input cost shares, S_i, from the regression results while forcing the shares to sum to unity

Multivariate Logit

An alternative stochastic form of the logit uses the share system given in equation (8) directly. The share equations are interpreted as an accurate characterization of producer expenditures for inputs (*8, 22*). One can generate the stochastic model structure by appending error terms e_i to each share equation.

Each equation's error term represents deviations between optimal cost shares and observed cost minimizing shares. There are several reasons for the existence of the error term: the failure of input markets to clear perfectly, the aggregation or measurement error, or the randomness of human behavior. The error term associated with the *i*th cost share is assumed to be distributed normally with mean zero and variance σ_i. The variance is not assumed to be constant across shares because the variance of e_i is generally not equal to the variance of e_j.

Three covariances are generated by the error terms in the share equations. One of these is the covariance between disturbances of different observations and of the same share equation:

$$\sigma_{ii} = E(e_{it}, e_{it'}) \; (t \neq t' = 1, 2, \ldots, T) \; (i = 1, 2, \ldots, k) \quad (17)$$

The second represents the relationship between different share equations and observations:

$$\sigma_{ij} = E(e_{it}, e_{jt'}) (t \neq t' = 1, 2, \ldots, T)(i \neq j = 1, 2, \ldots, k) \quad (18)$$

Both these covariances are assumed to equal zero.

The third covariance arises from the combination of different share equations and the same observation:

$$\sigma_{ij} = E(e_{it}, e_{jt}) \; (t = 1, 2, \ldots, T) \; (i \neq j = 1, 2, \ldots, k) \quad (19)$$

This covariance is usually referred to as the contemporaneous covariance (*32*). Because the underlying production structure is estimated as a system, it is unlikely that the contemporaneous covariances are zero. By appropriately stacking the share equations, one can write the variance-covariance matrix for the disturbance term as a block-diagonal matrix with T diagonal submatrices.

This matrix represents the interdependency of the k share equations for each observation (t = 1, 2, ..., T). The off-diagonal submatrices of the error system's variance-covariance matrix are zero by assumption.

The specified system of share equations is characterized as a seemingly unrelated regression problem (*41*). Zellner's generalized least-squares procedure cannot be directly applied, however. Because the disturbances of the share equation must sum to zero, the estimated variance-covariance matrix necessary for implementing Zellner's procedure is singular. One can transform the share equations to an estimable form by normalizing with the *k*th share equation.

Parameters estimated by the Zellner generalized least-squares procedure are not invariant to the choice of common denominator share when an estimated variance-covariance matrix is employed. However, maximum likelihood estimates are invariant to which equation is deleted (*1*), and iterating the Zellner procedure leads to maximum likelihood estimates (*13, 21*). Therefore, iterating the Zellner procedure is a computationally efficient means of deriving parameter estimates. In general, the properties of maximum likelihood estimators only hold asymptotically. However, most of the maximum likelihood estimators' asymptotic properties are present in small samples (*21*).

If the same set of regressors is utilized in all k − 1 equations, then the iterative Zellner procedure and ordinary least squares give identical parameter estimates. However, estimates of the standard errors may differ.

An Example of a Multinomial Logit Estimation

A multinomial logit model is used to estimate a system of cost share equations for agricultural inputs. In addition, the price elasticity of demand for each input is calculated by use of the multinomial logit parameter estimates. Although the procedure allows one considerable flexibility in selecting a functional form and avoids the limited dependent variable problem, it is still subject to the same practical difficulties (for example, data aggregation) in applying all econometric models.

Estimated Form

The underlying model structure estimated in this example is:

$$S_i = \exp(a_i + \sum_{j=1}^{k} B_{ij} \ln v_j) / \sum_{i=1}^{k} \exp(a_i + \sum_{j=1}^{k} B_{ij} \ln v_j) \quad (20)$$

where S_i is the *i*th input share, a and B are unknown parameters, and v_j is the price of input j.

Because the model specified in equation (20) leads to a singular matrix of second-order derivatives, a maximum likelihood procedure cannot be applied. Instead, one can transform the share system to an estimable form by normalizing on the *k*th equation. After taking logarithms, the estimated form of the share system is:

$$\ln(S_i/S_k) = (a_i - a_k) + \sum_{j=1}^{k} (B_{ij} - B_{kj}) \ln v_j \qquad (i = 1, 2, \ldots, k - 1) \quad (21)$$

The parameters a_i and B_i are now defined as differences from the parameters of the *k*th equation

Data and Estimating Methodology

Production cost data were developed for land, labor, fertilizer, energy, and capital inputs for the United States Cross-sectional data for 39 continental States for 1974 are utilized (*15, 24, 38, 39, 40*)

Because the likelihood function associated with equation (21) has non-linear first and second derivatives, an interative search procedure is used to solve for the maximum likelihood estimates Tyrrell (*34*) developed the computer software used in this analysis to analyze household expenditure patterns Tyrrell employs both first and second derivatives in an extended Newton-Raphson procedure (see (*17, 26*)) The final output includes a vector of estimated coefficients, first- and second-order derivatives of the likelihood function, asymptotic standard errors, and asymptotic t-statistics

Model Results

I estimated the expenditure system by normalizing on the capital cost share Table 1 shows the estimated parameters and asymptotic standard errors of the model specified in equation (21) Because maximum likelihood estimates are asymptotically normally distributed and the standard errors are asymptotically distributed as a chi-square, the ratios of the estimated parameters to their standard errors can be interpreted as asymptotic t-statistics

The t-statistics indicate general support for the estimated parameters Of the 24 parameter estimates, 16 exceed 1 0, of these, 10 exceed 2 0 The t-statistics for the own-price parameters for each input are the highest The land and capital price parameter for each input are the lowest The low-price land and capital parameters may be associated with poor measurement of these inputs

Because the share system is indeterminant, it is not possible to identify each B_{ij} in the expenditure system The estimated parameters can only be interpreted as differences from the parameter associated with the normalizing input cost share (see equation (23))

The estimated likelihood function can be used to formulate statistical tests (*4*) The test statistic, $-2\ln\lambda$, is asymptotically distributed as a chi-square with degrees of freedom equal to the number of independently imposed restrictions when

$$\lambda = (|\Omega_R|/|\Omega_U|)^{-T/2} \qquad (22)$$

where $|\Omega_R|$ and $|\Omega_U|$ are the determinants of the restricted and unrestricted estimated variance-covariance matrices of error terms (*32*) The chi-square test allows for comparisons of different models if the restricted model is nested within an unrestricted model In this analysis, the chi-square test is used to test the null hypothesis that all coefficients equal zero The null hypothesis is easily rejected at the 1-percent level with 30 degrees of freedom

Although the estimated share system is normalized on the *k*th share equation (capital), it is possible to solve for the predicted shares for all inputs The *k*th share can be computed because

$$1/\left(\hat{S}_k + \left(\sum_{i=1}^{k-1} \exp\left(\widehat{\ln(S_i/S_k)}\hat{S}_k\right)\right)\right) = 1 \qquad (23)$$

where $\hat{S}_k$ is the projected cost share for the *k*th input (capital) and $\widehat{\ln(S_i/S_k)}$ is the estimated dependent variable associated with the *i*th equation The estimated share for the *k*th input is computed as

$$\hat{S}_k = 1/(1 + \sum_{i=1}^{k-1} \exp(\widehat{\ln(S_i/S_k)}) \qquad (24)$$

Table 2 reports the predicted and observed cost shares The model fails to predict actual cost shares with a high degree of accuracy The average absolute differences between predicted and observed input cost shares are 0 05 (land), 0 07 (labor),

Table 1—Multinomial logit parameter estimates[1]

Input	Input price					
	Land	Labor	Fertilizer	Energy	Capital	Intercept
Land	0 816 (117)	0 199 (148)	0 111 (033)	-0 390 (280)	-0 653 (726)	-7 420 (2 450)
Labor	- 009 (141)	1 233 (178)	-1 320 (358)	1 384 (346)	632 (827)	-1 040 (2 842)
Fertilizer	121 (129)	274 (166)	576 (359)	806 (322)	031 (785)	-8 330 (2 700)
Energy	- 023 (183)	987 (242)	1 249 (492)	- 496 (370)	- 569 (1 092)	-6 350 (3 546)

[1] Standard errors in parentheses

Table 2—Predicted and observed cost shares, by State

State	Inputs									
	Land		Labor		Fertilizer		Energy		Capital	
	Predicted	Observed	Predicted	Observed	Predicted	Observed	Predicted	Observed	Predicted	Observed
	Percent									
Alabama	0 227	0 214	0 197	0 151	0 185	0 214	0 068	0 072	0 323	0 350
Arizona	230	104	218	352	197	146	125	256	230	141
Arkansas	287	361	193	162	171	117	089	095	260	266
California	375	260	202	438	197	117	056	060	169	125
Colorado	267	280	159	199	188	157	129	077	258	288
Delaware	229	273	208	182	173	252	080	067	221	227
Florida	232	193	341	523	143	079	056	029	226	176
Georgia	238	222	199	162	185	288	067	075	310	252
Idaho	243	191	183	181	217	268	103	087	254	273
Illinois	428	490	106	047	168	176	062	063	225	225
Indiana	423	470	101	048	175	203	061	059	239	220
Iowa	498	506	061	039	166	172	053	063	222	218
Kansas	386	430	088	062	155	166	124	080	247	262
Kentucky	374	320	063	112	176	162	038	053	350	353
Louisiana	318	347	188	175	165	126	076	079	254	273
Maryland	205	172	239	189	192	280	076	065	288	293
Michigan	290	249	139	129	205	252	067	055	299	316
Minnesota	357	372	100	053	199	224	075	075	269	277
Mississippi	243	311	230	177	177	131	067	086	283	294
Missouri	353	416	117	062	191	168	068	068	272	286
Montana	239	393	189	103	210	135	102	087	260	281
Nebraska	353	346	105	057	181	211	111	130	250	256
New Mexico	324	168	096	252	140	114	155	174	284	292
New York	206	153	252	195	193	301	075	054	274	297
North Carolina	283	212	109	207	192	210	047	054	370	318
North Dakota	228	307	171	042	220	338	090	069	291	244
Ohio	319	345	155	086	192	219	070	058	265	291
Oklahoma	263	363	137	090	186	140	092	094	322	314
Oregon	880	121	147	298	247	204	085	064	333	314
Pennsylvania	211	162	237	198	189	233	087	069	277	338
South Carolina	176	191	204	196	196	260	056	068	369	286
South Dakota	231	359	148	056	206	189	095	107	320	289
Tennessee	334	337	101	094	183	145	044	059	337	365
Texas	265	255	140	184	150	143	131	128	314	289
Utah	216	211	186	192	226	156	070	062	302	380
Virginia	248	196	170	178	184	260	059	053	340	312
Washington	227	157	132	300	240	227	087	066	314	250
Wisconsin	290	272	158	093	196	226	067	057	289	352
Wyoming	207	236	236	167	224	199	097	121	236	277

0.05 (fertilizer), 0.02 (energy), and 0.03 (capital), whereas the average observed cost shares are 0.28 (land), 0.17 (labor), 0.20 (fertilizer), 0.08 (energy), and 0.28 (capital). If the absolute differences are compared with the observed shares, then labor and energy are the least accurate predictions and capital is the most accurate. The bad predictive capability of the model is largely attributed to the use of aggregate cross-sectional data. The appropriate unit of observation is the firm. However, no reliable set of input price and quantity data are available at the firm level. This problem is particularly true for energy.

In addition to predicting the individual cost shares, the multinomial logit model enables the analyst to derive the price elasticities implicit in the derived demand for each input. If it is assumed that total expenditure on inputs, M, is not invariant to changes in input prices, then the own-price and cross-price eleasticities are approximated by

$$E_{ii} = S_i - v_i(\partial f_i/\partial v_i - \sum_{j=1}^{k} S_j \partial f_j/\partial v_i) - 1 \qquad (i = 1, 2, \ldots, k) \quad (25)$$

and by

$$E_{ij} = S_i - v_j(\partial f_i/\partial v_j - \sum_{j=1}^{k} S_j \partial f_j/\partial v_j) \qquad (i \neq j = 1, 2, \ldots, k) \quad (26)$$

The indeterminancy caused by the adding-up constraint prevents direct calculation of the input price elasticities. However, the predicted price elasticities can be derived if the elasticities associated with the *k*th share are derived first. The predicted elasticity for the *k*th share with respect to the *i*th price is

$$\hat{E}_{ki} = \hat{S}_k - \sum_{j=1}^{k-1} \hat{S}_j (B_{ji} - B_{ki}) + -1 \text{ (if } i = k\text{, or 0 if } i \neq k) \qquad (i = 1, 2, \ldots, k-1) \quad (27)$$

and, therefore

$$\hat{E}_{ji} = \hat{E}_{ki}(B_{ji} - B_{ki}) \qquad (i \neq j = 1, 2, \ldots, k-1) \quad (28)$$

Table 3 reports the average predicted own price and cross-price elasticities. All the own price elasticities have the right sign except labor, which is close to zero. The magnitudes of the own price elasticities are similar to a translog specification with the same data (*24*). They do, however, differ significantly from translog specifications estimated with time-series data (*23*). This discrepancy suggests that the data and not the model specification are the source of the elasticity differences.

Table 3—Average price elasticities of predicted input demand

Factor	Input price				
	Land	Labor	Fertilizer	Energy	Capital
Land	-0.152	0.116	1.051	-0.559	-0.623
Labor	-.120	.006	-1.524	1.072	.519
Fertilizer	.099	.135	-.539	.583	.008
Energy	.027	.921	1.206	-1.647	-.521
Capital	-.022	-.227	-.115	-.152	-.600

Unlike a translog system, cross-price elasticities are not constrained to be symmetric. In fact, the cross price elasticities for land and labor and for land and energy do not have the same signs. The cross-price elasticities indicate that land is a substitute for labor and fertilizer in farm production, but is a complement with energy and capital (*36*). Controversy still exists over the relationship of energy to other inputs in manufacturing. The results here indicate that energy is a complement with capital and is a substitute with labor in agricultural production. Although other analyses confirm the results for capital, results for labor are ambiguous and depend on whether the model distinguishes between long- and shortrun input substitution (*23*).

Summary

The logit model provides a flexible alternative to the more popular translog approach to estimating systems of input share equations. The logit allows for a wider range of explanatory variables and functional forms than does the translog. Furthermore, the logit constrains share predictions to the interval between zero and 1. Two forms of the logit (multivariate and multinomial) can be defined depending on the assumed error structure. For input expenditure systems, the multivariate logit assumes the share specification accurately represents the underlying input demand structure. The multinomial logit treats the dependent variable as a probability with a multinomial density. Either model can be estimated with well-known techniques and can provide estimates of predicted input cost shares and price elasticities of demand. However, because of the indeterminancy of the share system, individual parameter estimates can not be identified for the multinomial logit model.

The logit model has been applied to many economic problems. Within the context of agricultural production, additional applications might include dynamic models or incorporation of the cost share system into a macromodel where agricultural production is only one of many production sectors. Because the logit's structure forces share systems to sum to unity, it is an attractive candidate for maintaining consistent predictions.

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