The Effects of Tenancy and Risk on Cropping Patterns: A Mathematical Programming Analysis

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Abstract

Most analyses of allocative efficiency under different forms of agricultural tenure—share tenancy, fixed cash tenancy, and owner cultivation—employ single product models of production These models show that risk sharing encourages share tenants to produce as much as or more than equally risk-averse owner-operators and cash tenants However, when risk and risk aversion are introduced into multiproduct linear programming models, relative allocative efficiency under share tenancy may decline The result depends on the relative production costs and the relative risk premiums of the different products

Keywords

Farm tenancy, risk, allocative efficiency

The 1978 Census of Agriculture reports that nearly 40 percent of all land in farms in the United States is rented and that approximately 86 percent of this rented acreage is owned by nonoperator landlords (*45*) [1] Studies by Reinsel and Johnson (*28*), Johnson (*21*), and Reiss (*29, 30, 31*) suggest that the contributions landlords make to the management of the land they rent out has declined in recent years An increasing percentage of tenants simultaneously rents several tracts of land from several landlords rather than a single tract from a single landlord Contributions made by individual landlords to total rented acreage operated by tenants have declined, as have opportunities and incentives for landlords to contribute entrepreneurial skills and to help finance production costs (*28, 29, 30, 31*) Tenants have expressed greater willingness to operate without assistance from landlords Many have indicated they prefer arrangements that provide them primary managerial control over farming operations so that they can coordinate their owned and rented resources effectively (*6, 7, 22, 28, 29*)

The increasing frequency of rental arrangements that allocate most managerial responsibilities to tenants has renewed interest in an issue that Alfred Marshall first raised Is the efficiency of resource allocation lower on tenant-operated than on owner-operated farmland? Many economists have argued that tenants who operate with little managerial or cost-sharing assistance from nonoperator landlords, but who pay landlords fixed cash rents, will farm just as efficiently as will owner-operators However, if these tenants pay share rents, they will farm less efficiently than will either owners or cash tenants (*1, 4, 11, 13, 14, 15, 16, 20, 23, 24, 26, 32, 33*)

Most analyses of farm tenancy have employed single-input, single-product models of production Owner-operators and fixed cash tenants receive the entire value of the marginal product from each unit of input, and they employ inputs up to an amount at which marginal value product equals marginal input cost Share tenants pay a share of the marginal value product as rent The level of input employment at which their own share of marginal value product equals marginal input cost is lower than the equilibrium level under owner operation and cash tenancy The single-input, single-product model, therefore, predicts that share tenants will produce less under equilibrium than do owner-operators and cash tenants [2]

Other studies of farm tenancy have applied multi-input, multiproduct, linear programming models Within a range of output-price ratios, share tenants maximize their own net revenues, according to these models, by producing greater amounts of lower cost products and smaller amounts of higher cost products than owner-operators and cash tenants produce Because output combinations chosen by owners and cash tenants always maximize total net revenues for the entire farm, combinations chosen by share tenants yield less than the maximum net farm revenues The discrepancy measures the loss of efficiency attributable to the share rental arrangement within the linear programming model (*10, 14, 17, 23, 25*)

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[1] Italicized numbers in parentheses refer to items in the References at the end of this article

[2] Many economists recommend that landlords share variable costs with their share tenants in the same proportion that they share output As the marginal value received by the tenants will then equal the share of input cost they must pay, they will be encouraged to employ as much of the input as owners and cash tenants employ (*1, 2, 3, 15, 16, 18, 23, 27*)

Incorporating Risk and Risk Aversion into the Single- and Multiproduct Models

Previous multiproduct programming models of farm tenancy have either ignored production risks or have assumed that all farm operators are risk neutral They have not considered how risk and risk aversion might affect predictions of tenancy-related differences in farm planning

A few presentations of single-input, single-product models have addressed this question Sutinen (*34, 35*) and Baron (*5*) showed that, although production by risk-averse operators declines as risk levels increase, this reduction is smaller under share tenancy than under owner operation and cash tenancy Risk-sharing increases share tenants' production relative to owner-operator and cash tenants' production The excess of owner-operator/cash tenants' production over share tenants' production that occurs under risk neutrality is, therefore, always reduced, or possibly eliminated, under risk and risk aversion (*5*, *19*, p 24) If the negative effect of marginal product sharing under share tenancy is overcome through landlord supervision or through contractual specification of tenants' obligations to employ variable inputs, risk sharing may encourage share tenants to produce even more than owners and cash tenants produce (*35*, pp 617 19)

However, risk sharing may not have the same unequivocally positive impact on the efficiency of share tenancy relative to owner operation and to cash tenancy when it is examined within a multi-input, multiproduct programming model To introduce risk and risk aversion into such a model, several assumptions need to be made about the attitudes of farm operators toward risk and toward changes in risk levels associated with changes in production levels First, assume operators measure risk as a dispersion of gross revenue per acre of each crop around expected gross crop revenue per acre Moreover, operators assign a cost, or risk premium, to this risk level The premium equals some constant percentage of expected gross crop revenue per acre Thus, the marginal rate of increase in the risk premium that results from an increase in production will be constant at all levels of production Risk premiums for each crop will be included in the objective function as constant costs per acre of output, so that the function will remain linear

Price maps for alternative output combinations can display the results of parametric programming of the objective function Two sets of maps are presented in this study The first is derived for a risk-neutral owner-operator and cash tenant and for a risk neutral share tenant through maximization of the standard risk-neutral objective function presented in previous studies This set is compared with a second set of price maps derived for risk-averse owner-operators and cash tenants, and for risk-averse share tenants through maximization of the risk-adjusted or certainty-equivalent objective function defined here The comparison shows that the introduction of risk and risk aversion into the programming model may either increase or decrease the likelihood that an inefficient share tenant plan will be chosen, depending on the relative prices, relative input costs, and relative risk premiums per acre of soybeans and corn That is, contrary to the conclusion of other studies reviewed above, risk and risk aversion may either augment or offset the negative effect of product-sharing on the efficiency of share tenancy relative to owner operation and cash tenancy

Price Mapping Under Risk Neutrality

The programming model considers alternative production plans on a 200-acre farm in Illinois that produces soybeans or corn Cost estimates for this farm are available from the Firm Enterprise Data System (FEDS) developed by the Economic Research Service and Oklahoma State University (*12*)

The most recent FEDS estimates, based on data from the 1977 crop year, suggest that variable costs per acre of corn are approximately twice as high as those for soybeans These estimates are derived from a production function which utilized approximately 30 inputs For convenience, the programming model used here assumes that land is the only resource in limited supply

A simplified linear programming model of farm planning by a risk-neutral operator can be represented by the equation below

Maximize

$$I = (rp_1 - c)\, a_1 + (rp_2 - hc)\, a_2$$

subject to

$$a_1 + a_2 = 200 \text{ acres}$$

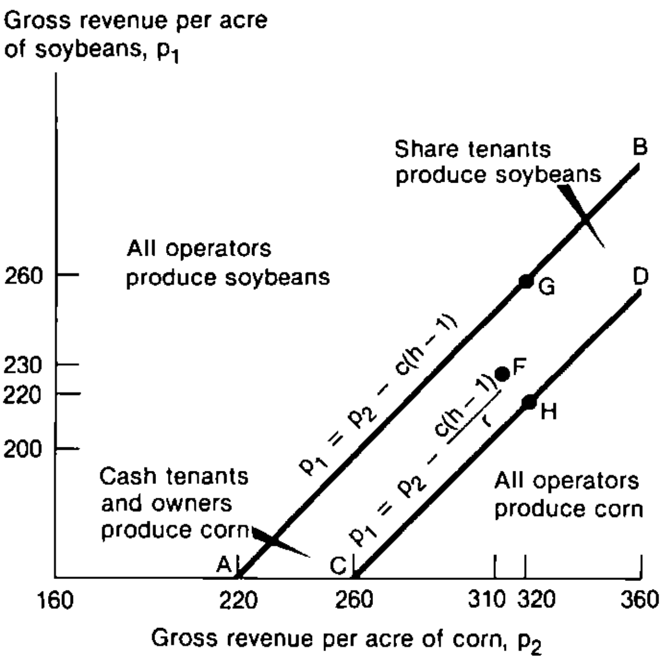
where I is net farm income, p_1 and p_2 are gross revenues per acre of soybeans and corn, respectively, r is the share rental rate, which equals 1 for owner-operators and cash tenants and some fraction less than 1 for share tenants, a_1 and a_2 are total soybean and corn acreages, c and hc are costs per acre of soybeans and corn, and h is the ratio of per acre costs of corn to soybeans The constraint equation indicates that 200 acres of land are available

Parametric variation of the coefficients, $rp_1 - c$ and $rp_2 - hc$, defines price maps which indicate the combinations of soybean and corn acreages that yield maximum profits for farm operators at any given ratio of gross soybean revenue to gross corn revenue As land is the sole resource constraint, only two plans may be optimal at any given ratio "soybeans only" or "corn only "

The price map for the risk neutral owner operator and cash tenant is represented by AB in figure 1 In this situation,

Figure 1

Price Map for Risk-Neutral Farmers



$r = 1$ The axes measure gross revenues per acre of corn and soybeans For all per-acre revenue combinations which fall to the left of AB, owner-operators and cash tenants receive higher net revenues per acre from soybeans than from corn Thus, they produce soybeans only For all combinations to the right of AB, they receive more from corn than from soybeans, and they produce corn only

For risk-neutral share tenants, assume that r equals 0 6 and that the tenants receive no cost sharing from landlords, so that their variable costs remain at c per acre of soybeans and at hc per acre of corn [3] The share tenants' price map is defined by line CD, which is the new boundary between the soybeans-only and corn-only farm plans Revenue combinations within the area bounded by AB and CD define a different optimal plan for share tenants than for owner-operators and cash tenants Combinations falling to the left of AB and to the right of CD result in the adoption of the same plan by all operators

Maps AB and CD define ratios of gross corn to gross soybean revenue at which net soybean and corn revenues are equal CD is further right than AB, indicating that share tenants require a higher ratio of corn revenue to soybean revenue before they will switch from soybeans to corn To determine why CD is further right, observe that the equality between net soybean and corn revenues per acre is

$$rp_1 - c = rp_2 - hc$$

which implies that

$$p_1 = p_2 - c(h - 1)/r \qquad (1)$$

Under owner operation and cash tenancy, $r = 1$, equation (1) indicates that gross revenue per acre of corn (p_2) must exceed gross revenue per acre of soybeans (p_1) at all points on AB by exactly the amount that cost per acre of corn exceeds cost per acre of soybeans ($c(h - 1)$) Under share tenancy, $r = 0\ 6$, equation (1) indicates that gross corn revenues per acre must exceed gross soybean revenues per acre at all points on CD by approximately 1/0 6 (or 1 67) times the cost differential

The price maps also indicate why allocative efficiency is lower under share tenancy than under owner operation and cash tenancy Net soybean and corn revenues received by owner-operators and cash tenants, $p_1 - c$ and $p_2 - hc$, are identical to corresponding net crop revenues generated for the whole farm Therefore, the farm plan that owner-operators and cash tenants select always maximizes total net farm revenues Net revenues received by share tenants, $0\ 6p_1 - c$ and $0\ 6p_2 - hc$, are less than the net crop revenues received by the entire farm Where the share tenant's farm plan differs from the owner-operator and cash tenant's farm plan, as is the case within region ABCD, total net farm revenues generated by the share tenant's plan will necessarily be less than the maximum possible As an example, consider point F Here, p_1 = \$230, p_2 = \$310, c = \$60, and h = 2 The share tenant's plan—200 acres of soybeans—yields total net farm revenues of \$34,000 A maximum possible net revenue of \$38,000 is generated by the owner-operator and cash tenant's plan—200 acres of corn

Price Mapping Under Risk and Risk Aversion

The introduction of risk and risk aversion into the linear programming model requires adjustments in the objective function Gross revenues per acre are uncertain, expected values The operator's reaction to risk is measured as reductions in expected gross soybean and corn revenues per acre Assume that these reductions, or risk premiums, equal

[3] This assumption may be challenged on grounds that proportionate cost sharing by U S landlords is quite common However, Berry (*6, 8, 9*) and Harwell and Strickland (*14*) report that, in the Great Plains Region, many share-rental landlords and tenants avoid cost sharing because they consider it a partnership arrangement which demands precisely the type of landlord participation in farm management they want to avoid These landlords and tenants typically choose share leases which assign most management responsibilities to tenants, including responsibility for financing variable costs Reiss (*29, 30, 31*) reports that even in Illinois, where cost sharing has been widely practiced, a trend has developed toward greater use of net share leases—that is, share leases which assign all variable costs to tenants Again, the motivation may be greater preference for leases which assign landlords little role in farm management or in the financing of production costs

constant percentages of expected gross revenues Thus, the risk premium from 1 acre of soybeans equals the given percentage rate multiplied by the expected gross soybean revenue per acre

With the risk premiums, the objective function becomes

$$I = rp_1a_1 - ca_1 - r\theta_1p_1a_1 + rp_2a_2 - hca_1 - r\theta_2p_2a_2 \quad (2)$$

Risk premiums per acre of soybeans and corn are $\theta_1 p_1$ and $\theta_2 p_2$, respectively θ_1 and θ_2 are the risk premium coefficients, they indicate the percentages of expected gross soybean and corn revenues deducted to account for risk

The assumption that risk premiums per acre remain constant as gross crop revenues per acre and total acreages increase may appear restrictive Risk premiums per acre may in fact be highly correlated with gross crop revenues Moreover, if covariance between gross soybean and gross corn revenue is nonzero, risk premiums per acre may vary as output ratios vary

However, if risk premiums per acre are expressed as functions of gross revenues and as functions of output ratios, the objective function will be nonlinear This nonlinearity will make the analysis more complicated, but little will be gained conceptually The linear objective function of equation (2) is sufficiently complex to establish the fundamental point of the analysis—namely, that share tenancy may compare even less favorably with owner operation and cash tenancy under risk and risk aversion than under risk neutrality

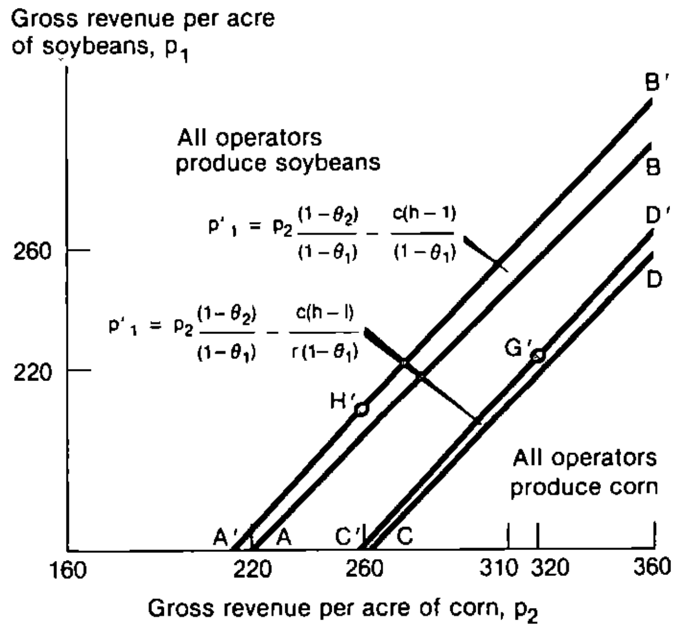
Parametric variation of certainty-equivalent net soybean and corn revenues per acre, $rp_1 - c - r\theta_1p_1$ and $rp_2 - hc - r\theta_2p_2$, defines price maps for risk-averse farm operators Two sets of these maps are derived In the first, θ_1 is assumed to be greater than θ_2, indicating that operators consider soybeans riskier than corn In the second, θ_2 is greater than θ_1, indicating that corn is riskier than soybeans

Case 1: $\theta_1 > \theta_2$

Whereas maps AB and CD (fig 1) equate expected net soybean and corn revenues per acre, price maps for risk-averse farm operators (fig 2) equate certainty-equivalent net soybean and corn revenues per acre When $\theta_1 > \theta_2$, equivalency is satisfied on map A'B' for owner-operators and cash tenants and on map C'D' for share tenants For purposes of comparison, maps AB and CD are also shown in figure 2 To the right of A'B', risk-averse owners and cash tenants receive higher certainty-equivalent net revenues from corn than from soybeans, therefore, they produce only corn To the left of A'B', they receive more from soybeans than from corn and produce only soybeans Similarly, to the right (left) of C'D', share tenants receive higher certainty-equivalent revenues from corn (soybeans) than from soybeans (corn), therefore, they produce only corn (soybeans) at all points

Figure 2

Price Map for Risk-Averse Farmers When Soybeans are Considered Riskier than Corn



Equality of certainty-equivalent net revenues per acre, $rp_1 - c - r\theta_1p_1 = rp_2 - hc - r\theta_2p_2$, implies that the value of p_1 on A'B' and C'D', hereinafter p_1', must be

$$p_1' = p_2(1 - \theta_2)/(1 - \theta_1) - c(h - 1)/r(1 - \theta_1) \quad (3)$$

This compares with a value of

$$p_1 = p_2 - c(h - 1)/r \quad (4)$$

on maps AB and CD Note that when $\theta_1 > \theta_2$, the slope of A'B' and C'D' $(1 - \theta_2/1 - \theta_1)$ is slightly greater than the slope of AB and CD, which equals 1 However, the vertical intercept of A'B' and C'D' $(-c(h - 1)/r(1 - \theta_1))$ is smaller than that of AB and CD $(-c(h - 1)/r)$ It follows that p_1' in equation (3) is greater than, equal to, or less than p_1 in equation (4) and that A'B' and C'D' therefore lie above, intersect with, or fall below AB and CD, depending on whether

$$p_2(1 - \theta_2)/(1 - \theta_1) - p_2 \gtreqless c(h - 1)/r(1 - \theta_1) - c(h - 1)/r \quad (5)$$

or, to simplify, on whether

$$p_2 \gtreqless (\theta_1/\theta_1 - \theta_2)(c(h - 1)/r) \quad (6)$$

The more θ_1 exceeds θ_2 and the smaller h is, the greater is the value of p_2 relative to $(\theta_1/\theta_1 - \theta_2)(c(h - 1)/r)$ and the greater are the values of p'_1 on A′B′ and C′D′ relative to p_1 on AB and CD, respectively

Suppose the terms in equation (6) are limited to a range of values within which $p_2 > (\theta_1/\theta_1 - \theta_2)(c(h - 1)/r)$ and, thus, $p'_1 > p_1$ For example, assume that input costs per acre, c and hc, approximate the recent FED estimates $60 for soybeans and $120 for corn Also assume that θ_1 exceeds θ_2 significantly Reasonable values might be θ_1 = 0 04 per $1 of expected gross soybean revenue, and θ_2 = 0 01 per $1 of expected gross corn revenue It follows from equation (6) that $p'_1 > p_1$ for all $p_2 > (4/3)(60/4)$ Under owner operation and cash tenancy, p'_1 on A′B′ will exceed p_1 on AB, and thus A′B′ will lie above AB (fig 2) for all cases where $p_2 >$ $80 Under share tenancy, with r = 0 6, p'_1 on C′D′ will exceed p_1 on CD, and thus C′D′ will lie above CD, for all cases where $p_2 >$ $133 33 Furthermore, given that during the 1974-80 period, the minimum reported gross corn revenue in Illinois was about $220 per acre, $p'_1 > p_1$ is the only relevant case Therefore, figure 2 shows only those portions of maps A′B′ and C′D′ that lie above AB and CD

For an intuitive explanation of why A′B′ and C′D′ lie above AB and CD, consider the equality of expected net crop revenues that occurs on AB and CD Recall that this equality implies that $p_1 = p_2 - c(h - 1)/r$ The risk premium per acre of soybeans on AB and CD is $\theta_1 p_1 = \theta_1(p_2 - c(h - 1)/r)$ Suppose c = $60, hc = $120, θ_1 = 0 04, and θ_2 = 0 01 Then $\theta_1 p_1$ is greater than $\theta_2 p_2$ (the risk premium per acre of corn) at all points on AB where $p_2 >$ $80 and at all points on CD where $p_2 >$ $133 33 At all such points on AB and CD, certainty-equivalent net revenues are lower for soybeans than for corn For certainty-equivalent net crop revenues to be equal, as is the case on maps A′B′ and C′D′, expected net soybean revenue per acre must exceed expected net corn revenue per acre to offset the higher risk premium per acre of soybeans Thus, for each combination of expected gross soybean and expected gross corn revenues per acre defined by a point on AB or CD, the point on A′B′ or C′D′ that defines the same gross corn revenue per acre must have a higher gross soybean revenue per acre

We can also show that p'_1 on A′B′ exceeds p_1 on AB by more than the amount that p'_1 on C′D′ exceeds p_1 on CD That is, for any p_2, the shift from AB to A′B′ exceeds the shift from CD to C′D′ If p_1 is expressed as in equation (4), then

$$p'_1 - p_1 = p_2(\theta_1 - O_2)/(1 - \theta_1) - \theta_1 c(h - 1)/r(1 - \theta_1) \quad (7)$$

which is greater under owner operation and cash tenancy, with r = 1, than under share tenancy, with $0 < r < 1$

Suppose gross corn revenue is $320 per acre, θ_1 = 0 04, θ_2 = 0 01, r = 0 6, and input costs per acre are again $60 for soybeans and $120 for corn Gross soybean revenue then increases from p_1 = $260 per acre on map AB to p'_1 = $267 50 per acre on map A′B′, but only from $220 per acre on map CD to $225 83 per acre on map C′D′ The risk-induced shift from AB to A′B′ is ($267 50 - $260) - ($225 83 - $220) = $1 67 greater than the shift from CD to C′D′

Thus, when p_2, c, and hc have reasonable values and when θ_1 exceeds θ_2 by enough to ensure that equal expected net corn and soybean revenues per acre imply higher risk premiums per acre for soybeans than for corn, risk and risk aversion will affect farm planning under owner operation and cash tenancy more than under share tenancy Share tenancy's standing relative to owner operation and cash tenancy should also improve Recall that, under risk neutrality, the crop plan chosen by owners and cash tenants is always optimal in the sense that it maximizes total expected net crop revenues for the entire farm At all points to the left of AB, this optimal plan is 200 acres of soybeans, at all points to the right, it is 200 acres of corn

To the left of A′B′ and to the right of C′D′ in figure 2, all risk-averse farmers do in fact choose optimal farm plans Within region A′B′AB, however, only risk-averse share tenants choose the optimal plan—200 acres of soybeans, within ABC′D′, only risk-averse owners and cash tenants choose the optimal plan—200 acres of corn

Within A′B′AB, the risk-averse share tenant's plan is more profitable to the farm than the risk-averse owner and cash tenant's plan [4] This situation contrasts with the case of risk neutrality (fig 1) Recall that the risk-neutral share tenant's plan can be equally profitable at best, but can never be more profitable than the owner and cash tenant's plan Moreover, recall that within region ABCD, the risk-neutral share tenant's plan is less profitable to the farm than the risk neutral owner and cash tenant's plan Thus, ABCD is analagous to ABC′D′, which encompasses an area where the risk-averse share tenant's plan is less profitable to the farm than the risk-averse owner and cash tenant's plan Observe that ABC′D′ is smaller than ABCD That is, the number of soybean-corn revenue combinations that result in a suboptimal share tenant's plan is smaller under risk

[4] The objective function for risk averse share tenants who receive proportionate cost share payments from landlords is 0 6 $(p_1a_1 - ca_1 - \theta_1 p_1 a_1 + p_2 a_2 - hca_2 - \theta_2 p_2 a_2)$ Maximization of this function is equivalent to maximization of the risk-averse owner operator and cash tenant's objective function at all crop revenue combinations Within ABA′B′, the crop plan chosen by risk-averse share tenants who receive proportionate cost share payments as well as by risk-averse owners and cash tenants is less profitable to the farm than the plan chosen by risk-averse share tenants who receive no cost share payments This result contradicts the claim of many economists (discussed in footnote 2) that proportionate cost sharing should always increase allocative efficiency under share tenancy

and risk aversion than under risk neutrality. The probability that the revenue combination in any given year will result in a suboptimal share tenant's plan is also smaller under risk and risk aversion than under risk neutrality.

Therefore, when risk and risk aversion are incorporated into the programming model under the assumption that soybeans are riskier than corn ($\theta_1 > \theta_2$), share tenancy should compare more favorably with owner operation and with cash tenancy than it does in the absence of risk and risk aversion. The probability that the share tenant's plan will be less profitable to the farm than the owner-operator and cash tenant's plan should decline. The probability that the share tenant's plan will be more profitable should increase from zero to some positive value.

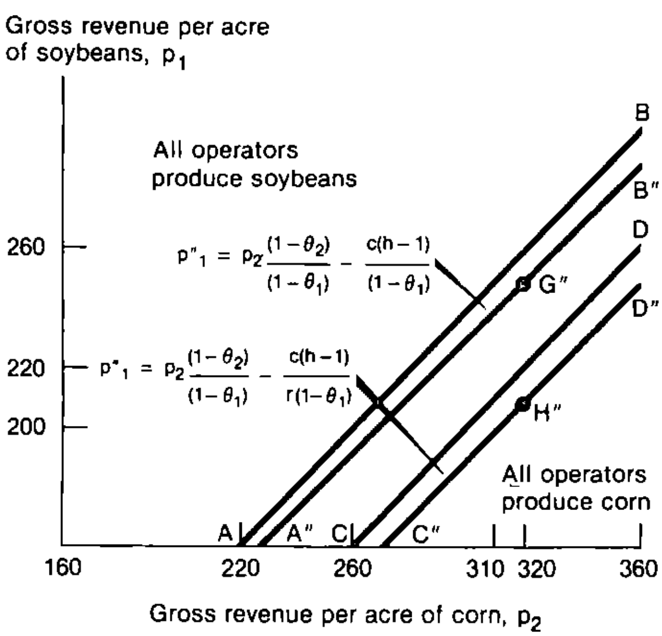
Case 2: $\theta_2 > \theta_1$

The results are much different if operators consider corn riskier than soybeans. For $\theta_2 > \theta_1$, parametric variation of certainty-equivalent net soybean and corn revenues per acre, $rp_1 - c - r\theta_1 p_1$ and $rp_2 - hc - r\theta_2 p_2$, defines price maps A″B″ and C″D″ (fig. 3). On these maps, as on maps A′B′ and C′D′, certainty-equivalent net crop revenues are equal; therefore, the value of p_1 on A″B″ and C″D″, hereinafter p_1'', is

$$p_1'' = p_2(1 - \theta_2)/(1 - \theta_1) - c(h - 1)/r(1 - \theta_1) \qquad (8)$$

Figure 3

Price Map for Risk-Averse Farmers When Corn is Considered Riskier than Soybeans



Note that with $\theta_2 > \theta_1$, the slope of A″B″ and C″D″, $(1 - \theta_2)/(1 - \theta_1)$, is slightly less than 1, the slope of AB and CD. Furthermore, the vertical intercept of A″B″ and C″D″ $(-c(h - 1)/r(1 - \theta_1))$ is less than the intercept of AB and CD $(-c(h - 1)/r)$. Thus, p_1'' will fall below p_1, and A″B″ and C″D″ will fall below AB and CD, for all p_2. Note also that the more θ_2 exceeds θ_1 and the larger h is, the more p_1 exceeds p_1'' and the greater the gap is between the price maps, for any p_2. Furthermore, p_1 on CD always exceeds p_1'' on C″D″ by more than p_1 on AB exceeds p_1'' on A″B″. That is, the shift from CD to C″D″ everywhere exceeds the shift from AB to A″B″. If p_1 is expressed as in equation (4), then

$$p_1 - p_1'' = p_2(\theta_2 - \theta_1)/(1 - \theta_1) + \theta_1 c(h - 1)/r(1 - \theta_1) \qquad (9)$$

which is higher under share tenancy than under owner operation and cash tenancy.[5]

For example, suppose gross corn revenue again equals \$320 per acre, while $\theta_1 = 0.01$, $\theta_2 = 0.04$, and input costs per acre again equal \$60 for soybeans and \$120 for corn. Gross soybean revenue must then decline from \$220 on map CD to \$209.29 on map C″D″, but only from \$260 on map AB to \$249.70 on map A″B″. The risk-induced shift from CD to C″D″ is \$0.41 greater than the shift from AB to A″B″.

Thus, given any values of p_1, p_2, c, h, θ_1, and θ_2, if $\theta_2 > \theta_1$, risk and risk aversion will always affect farm planning more under share tenancy than under owner operation and cash tenancy. Moreover, the impact of risk and risk aversion on the comparative efficiency analysis, if $\theta_2 > \theta_1$, is quite different than the impact if $\theta_1 > \theta_2$. When $\theta_2 > \theta_1$, we compare A″B″C″D″ and ABCD (fig. 3). A″B″C″D″ defines combinations of expected gross soybean and corn revenues per acre where risk-averse owner-operators and cash tenants (but not risk-averse share tenants) choose the optimal, profit-maximizing plan—200 acres of corn. Region ABCD defines gross revenue combinations where risk-neutral owners and cash tenants (but not risk-neutral share tenants) choose this same optimal plan. Because the shift from CD to C″D″ exceeds the shift from AB to A″B″, the number of soybean-corn revenue combinations falling within A″B″C″D″ exceeds the number falling within ABCD. That is, the share tenant's farm plan is less profitable to the farm than the owner and cash tenant's plan at more revenue combinations under risk and risk aversion than under risk neutrality. Everything else being equal, the probability that the revenue combination that occurs in any given year will fall within A″B″C″D″ will exceed the probability that it will fall within

[5] Note that for any $\theta_2 > \theta_1$, the excess of $\theta_2 p_2$ over $\theta_1 p_1''$ will decline, and the gap between A″B″ and AB and between C″D″ and CD will narrow as p_1'' and p_2 decline.

ABCD Thus, the probability that the share tenant will actually choose a suboptimal plan will also be higher under risk and risk aversion than under risk neutrality [6]

Comparison of Crop Risks

This analysis has shown that the impact of risk and risk aversion on the comparative efficiency analysis presented by the programming model depends on the values of the risk premium coefficients assigned to the alternative crops This result raises the question of which of the two sets of values assumed in this article better represents the attitudes of farm operators toward the riskiness of soybeans and corn

One answer is suggested by estimates of the variability of gross soybean and corn revenue that can be derived from recent average annual yield and price data provided by USDA's Crop Reporting Board (*37, 38, 39, 40, 41, 42, 43, 44*) Let us take Illinois as an example A sample of seven gross corn and soybean revenues per acre of Illinois cropland from the 1974-80 period indicates recent variations The sample standard deviation was approximately $39 per acre for corn and $33 per acre for soybeans Corn, therefore, exhibited greater variability than soybeans Thus, a typical Illinois farm operator would likely consider corn as the riskier crop The assumption that $\theta_2 > \theta_1$ appears to be more appropriate than the assumption that $\theta_1 > \theta_2$

If so, the conclusion for case 2 applies Share tenancy will likely compare less favorably with owner operation and cash tenancy under risk and risk aversion than under risk neutrality

Conclusion

Previous analyses of the comparative efficiency of different farm tenancies under risk and risk aversion have relied exclusively on single-input, single-output models When risk and risk aversion are introduced into these models, the efficiency of share tenancy relative to owner operation and cash tenancy always increases

When the comparative efficiency analysis is extended to multiproduct programming models, however, the results can be quite different Under risk neutrality, share tenancy compares unfavorably with owner operation and cash tenancy Within a certain range of crop revenue combinations, share tenants choose a crop plan which is less profitable to the farm as a whole than the plan owner-operators and cash tenants choose

To determine the impact of risk and risk aversion on this comparative efficiency analysis, we examined the implications of two different assumptions concerning reactions of farm operators to income variability Case 1 assumed that farm operators assign a higher risk premium to $1 of gross soybean revenue than they assign to $1 of gross corn revenue, even though the variable production cost per acre of corn is higher than the cost of soybeans The number of soybean-corn revenue combinations that call for a suboptimal share tenant's plan is then lower under risk and risk aversion than under risk neutrality Some revenue combinations even result in a risk-averse share tenant's plan that is more profitable than the risk-averse owner-operator and cash tenant's plan The probability that the share tenant's plan will be less profitable than the owner and cash tenant's plan is unequivocally lower under risk and risk aversion than under risk neutrality

In case 2, risk and risk aversion were introduced under the assumption that the risk premium per $1 of gross corn revenue is higher than the risk premium per $1 of gross soybean revenue The number of soybean-corn revenue combinations that result in a suboptimal share tenant's plan is now greater under risk and risk aversion than under risk neutrality Therefore, the probability that the share tenant's plan in any year will be less profitable than the owner and cash tenant's plan is also greater under risk and risk aversion than under risk neutrality

An empirical review of crop and yield data for Illinois suggests that the second of the two assumptions—that is, that the risk premium per $1 of corn is greater than the risk premium per $1 of soybeans—more accurately represents Illinois agriculture As the analysis in case 2 appears unfavorable to share tenancy, a contrast between the single-input, single-output model and the multiproduct programming model becomes obvious Although the presence of risk and risk aversion always improves share tenancy's standing relative to owner operation and cash tenancy under the first model, it probably weakened share tenancy's relative standing under the second model, at least for Illinois agriculture during the late seventies

Multiproduct models may generate different results for other regions and other time periods Where the analysis in case 1 applies, share tenancy will compare more favorably with owner operation and cash tenancy under risk and risk aversion than under risk neutrality When risk and risk aversion are relevant, the comparative efficiency of alternative farm tenancies can no longer be determined by theoretical considerations alone Comparative efficiency can be determined only after one has assigned observed or reasonable values to the model parameters

[6] It is conceivable, if improbable, that despite the greater number of revenue combinations within A″B′ C″D″ than within ABCD, the probability that the combination in any given year will fall within ABCD still exceeds the probability that it will fall within A″B′ C″D ′ The probability that the share tenant will choose an inefficient plan will then be lower under risk and risk aversion than under risk neutrality The introduction of risk and risk aversion into the model will offset, rather than augment, the distortive effect of product-sharing on the relative efficiency of share tenancy

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