Adaptive Expectations, the Exponentially Weighted Forecast, and Optimal Statistical Predictors: A Revisit

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Abstract

Relationships between adaptive expectations, the exponentially weighted moving average, and optimal univariate statistical predictors are reviewed We show that the behavioral-based adaptive expectations are a subclass of both the exponentially weighted moving average and the (0,1,1) ARIMA model The applicability of the adaptive expectations model to 25 empirical price and quantity series is investigated The adaptive expectations behavior and the optimal statistical forecasts are equivalent for 13 series- 11 on yields and 2 on prices Numerous price series, while exhibiting the general form of the adaptive expectations (a (0,1,1) ARIMA process), did not have a coefficient of expectations within the originally hypothesized range The behavior consistent with the model underlying these price series would be trend extrapolation rather than averaging (averaging the most recent observation and its forecast) Series measured at monthly or quarterly intervals were not adequately modeled by adaptive expectations or as a (0,1,1) ARIMA process

Keywords

Adaptive expectation, exponentially weighted forecast, optimal statistical predictor, Nerlovian model

A popular expectation model used in agricultural response studies is the adaptive expectations scheme Its statistical properties and theoretical motivation are given in Nerlove (*15*) [1] The model suggests that period-to-period changes in expectations of economic agents are linearly dependent on the most recent forecast error Muth notes, "its main *a priori* justification as a forecasting relation has been that it leads to correction of persistent errors, without responding very much to random disturbances" (*12*, p 299) Although this model seems to have been originally specified on *ad hoc* grounds, it can somewhat coincidentally be represented as an optimal, univariate, statistical predictor of a particular process, which, in fact, is fairly common in many agricultural series

Our purpose here is twofold (1) to explicitly review the relationships between the adaptive expectations model, the exponentially weighted forecast, and optimal statistical predictors, and (2) to present some empirical models of such a process The first part of this article is not a new contribution to the literature, these relationships were presented almost 20 years ago by Muth They have since appeared in bits and pieces elsewhere, including Box and Jenkins (*5*, p 107) and Dhrymes (*7*, p 56) Our justification for the first part of the article is that the exposition in the references cited is not particularly easy And, although the exposition given here may not be any easier or revealing to some, it may help others In addition, our empirical results and recommendations will make sense only if a knowledge of these more basic relationships is assumed

First, we review the relationship between the adaptive expectations model and the exponentially weighted moving average process Second, we extend the discussion to the class of autoregressive, integrated, moving average (ARIMA) processes Finally, we present some empirical series which can be adequately represented by such processes The new contributions to the literature are our empirical results which, along with the accumulating Monte Carlo evidence, we hope will move us to a better understanding of dynamic processes in agriculture

The Relationship Between Adaptive Expectations and the Exponentially Weighted Moving Average

The adaptive expectations model is discussed in Nerlove We present it in the following equation

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[1] Italicized numbers in parentheses refer to items in the References at the end of this article

$$Y_t^e = Y_{t-1}^e + \beta(Y_{t-1} - Y_{t-1}^e), \quad 0 < \beta \leq 1 \qquad (1)$$

where Y_t is the t^{th} observation on the variable in whose expectation we have an interest, Y_t^e represents its forecast or expectation based on the information through $(t-1)$, and β is a parameter sometimes referred to as the coefficient of expectation

The model given by equation (1) suggests that economic agents revise their expectations linearly, according to the most recent experience with their prediction accuracy The model, used by Cagan, Nerlove, and many others, is based on the economic dynamics of Hicks

Before we demonstrate the conditions under which equation (1) is equivalent to an exponentially weighted moving average of past observations on Y, it is useful to introduce the lag operator L, which is defined by $LY_t = Y_{t-1}$ Thus equation (1) can be rewritten as

$$(1 - \theta L)Y_t^e = \beta L Y_t, \quad \theta = 1 - \beta \qquad (2)$$

For illustrative purposes, we carry out the algebra in (2) to see that we do indeed have an expression equivalent to (1)

$$Y_t^e - \theta Y_{t-1}^e = \beta Y_{t-1}$$

$$Y_t^e = \theta Y_{t-1}^e + \beta Y_{t-1}$$

$$= (1 - \beta)Y_{t-1}^e + \beta Y_{t-1}$$

$$= Y_{t-1}^e + \beta(Y_{t-1} - Y_{t-1}^e)$$

Solving (2) for Y_t^e in an alternative fashion we have

$$Y_t^e = \beta(1 - \theta L)^{-1} L Y_t$$

The series, Y_t^e, can be expressed as an exponentially weighted average for $|L| < 1$, if $|\theta| < 1$ [2] That is, for $|\theta| < 1$, we have a product involving the infinite series

$$Y_t^e = \beta(1 + \theta L + \theta^2 L^2 + \ )Y_{t-1}$$

$$= \beta(Y_{t-1} + \theta Y_{t-2} + \theta^2 Y_{t-3} + \ )$$

$$= \beta \sum_{i=1}^{\infty} (1 - \beta)^{i-1} Y_{t-i} \qquad (3)$$

[2] The assignment of an absolute value to the lag operator is an operation described in Box and Jenkins (chap 3) Dhrymes (chap 2) rather extensively discusses the algebra of L that permits us to treat it as a complex variable

The required conditions on $|\theta|$ can be restated in terms of β

$$|1 - \beta| < 1 \rightarrow -1 < (1 - \beta) < 1$$

$$\rightarrow -2 < -\beta < 0$$

$$\rightarrow 0 < \beta < 2$$

This last condition suggests that for Y_t^e to be expressed as an exponentially weighted average, β, the coefficient of expectations, must fall between zero and 2

A natural question which one might then ask is, "Why did Nerlove restrict β to the unit interval?" To suggest an answer to this question, we might recall the motivating force behind his model Quoting Nerlove (*15*, p 52)

> Hicks, it will be remembered, distinguished two limiting cases an elasticity of zero, implying no effect of a change in current price upon expected future prices, and an elasticity of one implying that if prices were previously expected to remain constant, i e , were at their long-run equilibrium level, they will now be expected to remain constant at the level of current price By allowing for a range of elasticities between the two extremes, Hicks implicitly recognized that a particular past price or outcome may have something, but not everything, to do with people's notion of the normal

Thus, Nerlove was not particularly interested in modeling exponentially weighted expectations *per se*, but rather, was interested in modeling a hypothesized behavior—a behavior which suggests that economic agents change their expectations as a convex combination of the most recently observed actual and expected value of the random variable For $\beta > 1$, the expectation, Y_t^e will lie outside the interval (Y_{t-1}, Y_{t-1}^e)—a case Nerlove evidently found not very appealing

The Relationship Between Adaptive Expectations and a Class of Optimal Statistical Predictors

The relationship between the "adaptive expectations" model and an optimal statistical predictor can be obtained from the analysis of two equivalent forms of a general linear statistical model Following Box and Jenkins, we can write Y_t as equation (4)—a linear function of independent shocks—or as equation (5)—a linear function of past observation on Y_t, plus an added shock

$$Y_t = \sum_{i=1}^{\infty} \omega_i \epsilon_{t-i} + \epsilon_t, \; E\{\epsilon_t\} = 0 \qquad (4)$$

$$Y_t = \sum_{i=1}^{\infty} \pi_i Y_{t-i} + \epsilon_t, \; E\{\epsilon_t\} = 0 \qquad (5)$$

At a particular time, $t-1$, for known parameters, ω_i and π_i, and for observed shocks, ϵ_{t-i}, $i \geq 1$, we can write the equivalent expectations on equations (4) and (5) as

$$Y_t^e = \sum_{i=1}^{\infty} \omega_i \epsilon_{t-i} \qquad (6)$$

$$Y_t^e = \sum_{j=1}^{\infty} \pi_j Y_{t-j} \qquad (7)$$

Lagging equation (4) j periods, substituting each Y_{t-j} into equation (7), and rearranging terms, we have

$$Y_t^e = \sum_{j=1}^{\infty} \pi_j (\epsilon_{t-j} + \sum_{i=1}^{\infty} \omega_i \epsilon_{t-i-j}) \qquad (8)$$

Writing out the first few terms of (8), we have

$$\begin{aligned} Y_t^e = {} & \Pi_1(\epsilon_{t-1} + \omega_1\epsilon_{t-1-1} + \omega_2\epsilon_{t-2-1} + \quad) \\ & + \Pi_2(\epsilon_{t-2} + \omega_1\epsilon_{t-1-2} + \omega_2\epsilon_{t-2-2} + \quad) \\ & + \Pi_3(\epsilon_{t-3} + \omega_1\epsilon_{t-1-3} + \omega_2\epsilon_{t-2-3} + \quad) \\ & + \end{aligned}$$

Following Muth, we can rearrange terms of this last expression, paying explicit attention to like indexes on ϵ_{t-i}, that is

$$\begin{aligned} Y_t^e = {} & \Pi_1\epsilon_{t-1} \\ & + \Pi_1\omega_1\epsilon_{t-2} + \Pi_2\epsilon_{t-2} \\ & + \Pi_1\omega_2\epsilon_{t-3} + \Pi_2\omega_1\epsilon_{t-3} + \Pi_3\epsilon_{t-3} \\ & + \end{aligned}$$

Factoring out ϵ_{t-i}, we have Muth's equation (2 2)

$$Y_t^e = \Pi_1\epsilon_{t-1} + \sum_{i=2}^{\infty} (\Pi_i + \sum_{j=1}^{i-1} \Pi_j\omega_{i-j})\epsilon_{t-i} \qquad (9)$$

which is an alternative expression of our equation (6) By comparing coefficients of equations (9) and (6), we have the necessary relation between parameters ω_i, associated with the latent shocks, and Π_i, associated with the history of the process

$$\begin{aligned} \omega_1 &= \Pi_1 \\ \omega_i &= \Pi_i + \sum_{j=1}^{i-1} \Pi_j\omega_{i-j}, \quad i = 2, 3, \end{aligned} \qquad (10)$$

We can carry the analysis further by comparing the β weights of equation (3) with the results given in equation (10) This will allow us to characterize the time series for which the exponentially weighted forecast and thereby Nerlove's adaptive expectations model is an optimal statistical predictor We can substitute

$$\Pi_j = \beta(1-\beta)^{j-1}, \quad j = 1, 2,$$

of the exponentially weighted forecast into the equations given by (10) We obtain

$$\begin{aligned} \omega_1 &= \beta \\ \omega_i &= \beta(1-\beta)^{i-1} + \beta\sum_{j=1}^{i-1}(1-\beta)^{j-1}\omega_{i-j}, \quad i = 2, 3, \end{aligned}$$

It follows that

$$\omega_i = \beta, \text{ for all } i \geq 1$$

Thus, we can write (4) in terms of β, our coefficient of expectation

$$Y_t = \epsilon_t + \beta\sum_{i=1}^{\infty} \epsilon_{t-i} \qquad (11)$$

By lagging equation (11) one period and subtracting, we have

$$\begin{aligned} Y_t - Y_{t-1} &= \epsilon_t - (1-\beta)\epsilon_{t-1} \\ &= \epsilon_t - \theta\epsilon_{t-1} \end{aligned} \qquad (12)$$

which for, $-1 < \theta < 1$ (or $0 < \beta < 2$) is an integrated (differenced) moving average process of order 1 The one-step-ahead forecast, Y_t^e, based on equation (12) is given as

$$Y_t^e = Y_{t-1} - \theta\hat{\epsilon}_{t-1}$$

where $\hat{\epsilon}_{t-1}$ is given as the one-step-ahead forecast error

Equation (12) is a special case of the general, univariate, autoregressive integrated moving average (ARIMA) process of order (p,d,q) The models given by equations (4) and (5) can be represented (under rather general conditions) as

$$\phi(L)(1 - L)^d(Y_t - m) = \theta(L)\epsilon_t$$

where $\phi(L)$ is an autoregressive operator of order p, given as

$$(1 - \phi_1 L^1 - \phi_2 L^2 - \quad - \phi pLP)$$

$\theta(L)$ is a moving average operator of order q, given as

$$(1 - \theta_1 L^1 - \theta_2 L^2 - \quad - \theta qL^q)$$

Here, as above, L refers to the lag operator, d is an integer indicating the number of differences required to reduce the series Y_t to stationarity, m is the mean of the Y_t series, and ϵ_t is a white noise (random) disturbance.

More explicitly, equation (12) can be written in this form where

$$\phi(L) = 1, d = 1, \text{and } \theta(L) = (1 - \theta_1 L^1)$$

$$1(1 - L)^1(Y_t - m) = (1 - \theta_1 L^1)\epsilon_t$$

Carrying out the lag operations, we have

$$Y_t - m - Y_{t-1} + m = \epsilon_t - \theta\epsilon_{t-1}$$

$$Y_t - Y_{t-1} = \epsilon_t - \theta\epsilon_{t-1}$$

This process is often referred to as a (0,1,1) ARIMA model

We can identify and fit to time series more general ARIMA processes (different orders of p, d, q) by studying the correlation patterns of the observed series at various lags This identification process essentially allows the data to suggest which particular process "best" represents the observed data

The following relations between the autocorrelation and partial autocorrelation functions of the series, Y_t, can be used to identify the more general (p, d, q) ARIMA model

(a) For a nonstationary process, the autocorrelation function tails off slowly,
(b) For a purely autoregressive process of order p, the autocorrelation function tails off and the partial autocorrelation function has a cutoff after lag p,
(c) For purely moving average process of order q, the autocorrelation function has a cutoff after lag q and the partial autocorrelation function tails off, and
(d) For a mixed autoregressive process of order p and a moving average process of order q, the autocorrelation function is a mixture of exponential and damped sine waves after the first q-p lags, and the partial autocorrelation function is dominated by a mixture of exponential and damped sine waves after the first p-q lags

Box and Jenkins (*5*) suggest comparing the estimated autocorrelation and partial autocorrelation functions applied to a particular series with the above patterns

The results we have summarized here suggest that the adaptive expectations model is a subclass of a much larger class of optimal statistical predictors (For an explicit demonstration of this point in a minimum mean squared error sense, see Box and Jenkins) We suggest that Nerlove (*15*) was not necessarily interested in modeling optimal statistical predictors, but rather a hypothesized behavior—a behavior which suggests that economic agents form their expectations as a convex combination of their previous expectation and the most recently observed actual value

Before we move on to the consideration of particular empirical series, we might make a historical note with the purpose of better defining Hicks' role in the theoretical foundations of adaptive expectations Indeed, if we reread chapter 9 of *Value and Capital*, we note Hicks has a notion of a best or optimum representation of the stochastic process (*9*, p 126)

> Thus we shall formally assume that people expect particular definite prices, that they have certain price-expectations But we shall be prepared on occasion to interpret these certain expectations as being those particular figures which best represent the the uncertain expectations of reality

Continuing in chapter 15, we see that this notion of "best" included values of β, the coefficient of expectations, different from those specified by the adaptive expectations model For example (*9*, p 205)

> The elasticity of expectations will be greater than unity, if a change in current prices makes people feel that they can recognize a trend, so that they try to extrapolate, it will be negative if they make the opposite kind of guess, interpreting the change as the culminating point of a fluctuation

Thus, while Nerlove (*15*) relied on Hicks' work as a basis for adaptive expectations, he chose to model only a portion of Hicks' original hypothesis

As a final note in this section, we should properly recognize Nerlove's contributions to expectations modeling—a contribution that goes far beyond modeling adaptive expectations About a decade after introducing us to adaptive expectations, he made a case for modeling optimal statistical

predictors—evidently recognizing the limitations of constraining our representation of expectation to suboptimal descriptions of dynamic economic processes Recall Nerlove's words written in his seminal work, "Distributed Lags and Unobserved Components in Economic Time Series" (*14*, p 129)

> One might argue, for example, that the economic agents have a clear conception of what the stochastic mechanism really is, then determine optimal predictors, and, finally use optimal predictors directly as variables in a subsequent statistical analysis

He reiterates this point nearly 5 years later in his more popular paper, "Lags in Economic Behavior" (*17*, p 230)

> As long as the variables forecast are treated as exogenous in the behavior relationship studied, and if we assume that those economic agents whose behavior we are observing have knowledge of the underlying structure generating the time series to which they react, all manner of distributed lag relations may be developed For stationary processes, minimum mean-square-error forecasts and conditional expectations are equivalent That part of the lag structure arising from expectation formation may be estimated independently of the behavior studied from observations on the variables which are assumed to be forecast

Finally, in 1979, Nerlove argues (*16*, p 879)

> If we assume the economic agents, whose behavior we are attempting to describe, are aware of the underlying structure, quasi-rational expectations offer an approximation to fully rational expectations and a far less arbitrary, less ad hoc, approach to expectation formation than the adaptive expectations used in the basic supply response model

The above arguments support the position that in 1980 adaptive expectations are not particularly relevant—that is, a more general way of doing things exists, and this more general scheme ought to be used, whenever possible This conclusion would ordinarily be of little importance in that numerous authors have previously made similar appeals (*13*, *14*, *17*), however, a review of the agricultural economics literature suggests that this feeling is not generally held Indeed, Just (*10*, *11*) Turnovsky (*18*), Askari and Cummings (*1*), and others all wrote well after the above mentioned appeals And although we recognize that in some cases adaptive expectations may be an appropriate specification, we do not think their distinction as a particular or special type of expectation scheme deserves more than a historical footnote (albeit an illustrious one)

Analysis of Some Empirical Series

Finally, we will explore a few empirical agricultural series Our motivation for proceeding in this fashion is essentially that given by Nerlove (*17*)—namely, that our analysis of expectations formation may, under fairly general conditions, proceed independent of our analysis of the economic behavior in which we have a more fundamental interest Thus, in constructing models of agricultural supply response, for example, we can replace the future values of our exogenous variables by their minimum-mean-squared error forecasts These forecasts can be made independent of all other econometric estimation

As suggested above, this part of our article is its major contribution to the literature Even if we had been the first to argue the contents of part 1 (which we were not), the skeptic will remain unconvinced, at least until empirical data have been presented and thoroughly analyzed It is quite conceivable that the more general stochastic models represented by equations (4) or (5) have no empirical validity beyond a (0,1,1) ARIMA representation, which has a moving average parameter between zero and 1 That is, it may well be that series which we treat as exogenous in our econometric models are in fact properly modeled as adaptive expectations

We have fitted a first order moving average process to the first differences of 25 agricultural price and yield series Table 1 gives the estimated moving average parameters, θ, upper and lower bounds on 95 percent confidence intervals, and diagnostic Q statistics (and their degrees of freedom) for each series

Recall that for an optimal univariate statistical representation and for adaptive expectations behavior to be equivalent, θ must lie in the interval [0,1] In addition, the residuals associated with the fitted model must be white noise That is, adaptive expectations behavior and optimal statistical predictions (and thus Nerlove's quasirational expectations) will be equivalent if our estimated θ value is between zero and 1 and if the unexplained portion associated with the application of such a model is random from time period to time period

In table 1, notice that series 1, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, and 19 meet the conditions outlined above That is, the moving average parameters fall between zero and 1, and the diagnostic Q statistics do not indicate anything but white noise residuals for these series [3] The remaining series 2, 3, 4, 5, 7, 18, 20, 21, 22, 23, 24, and 25 fail to meet the outlined conditions

We illustrate the identification of a particular ARIMA model with the autocorrelation and partial autocorrelations on

[3] Q is a diagnostic statistic applied to the residuals from the fitted model It is described in Box and Jenkins

Table 1—Estimated (0,1,1) ARIMA processes applied to 25 agricultural series

Series number[1]	θ_L[2]	θ[3]	θ_U[4]	Q[5]	d f[6]
1	0 06	0 34	0 62	10 50	14
2	- 97	- 78	- 59	7 57	14
3	- 36	- 05	26	7 36	14
4	- 65	- 35	- 05	5 17	14
5	- 56	- 26	05	6 45	14
6	08	41	75	10 20	14
7	74	93	1 11	11 10	14
8	- 12	27	66	7 54	14
9	36	64	92	13 30	14
10	17	49	81	12 40	14
11	35	65	94	11 10	14
12	12	46	81	7 74	14
13	26	59	92	8 42	14
14	28	57	85	15 72	14
15	29	56	84	12 45	14
16	23	50	78	10 10	14
17	21	49	77	11 57	14
18	- 17	05	28	62 23*	19
19	- 16	06	29	19 11	19
20	09	30	51	47 27*	19
21	84	92	1 00	277 80*	19
22	42	61	81	194 10*	19
23	- 27	- 16	- 06	57 68	59
24	- 67	- 58	- 48	335 24*	59
25	- 17	- 08	02	256 57*	59

[1] Series names are listed in the appendix table
[2] θ_L refers to the lower bound on a 95-percent confidence interval around the estimated moving average parameter (Here L is not to be confused with the lag operator)
[3] θ refers to the estimated moving average parameter associated with the process $(1 - L)Z_t = (1 - \theta L)a_t$ of the particular series
[4] θ_U refers to the upper bound on a 95-percent confidence interval around the estimated moving average parameter
[5] Q refers to a diagnostic statistic applied to the estimated residuals from the (0,1,1) process applied to each particular series An asterisk (*) indicates that the calculated Q statistic is above the critical chi-squared value at the 5-percent level
[6] d f refers to the degrees of freedom associated with the Q statistic reported in the preceding column

1936-76 Indiana Dubois county corn yields in table 2 (series 15 of table 1) Here note that the estimated autocorrelations on the levels tail off slowly—indicative of possible nonstationary behavior The autocorrelations on the first differences of the corn yields cut off at lag 1—indicative of nonautoregressive behavior The partial autocorrelations of the different series, however, seem to tail off, suggesting a moving average behavior We fit a first-order moving average term to the differenced series

$$Y_t - Y_{t-1} = -0\ 56\epsilon_{t-1} + \epsilon_t$$

The residuals associated with this model appear to be nonautocorrelated (table 2) That is, there appears to be no further systematic structure to this series This is noted by the relatively small autocorrelations associated with various lags of the residual series

Of the 13 series which do meet the conditions, 11 are yearly yield series, the other 2 are price series Of the series which do not meet the outlined conditions, series 2, 3, 4, 5, and 23 are adequately fit by the (0,1,1) process, however, the estimated θ values are negative That is, the diagnostic Q statistics associated with the residuals from a (0,1,1) model fit to these series are acceptable, but the estimated θ values suggest nonadaptive behaviors These four series are price series Series 2, 3, 4, and 5 are measured yearly, and series 23 is measured monthly

The remaining series, which do not meet the conditions for equivalence between adaptive expectations and a (0,1,1) ARIMA process, fail on two accounts, we suspect both underfitting and overfitting Series 18, 20, 21, 22, 24, and 25 are measured quarterly or monthly The Q statistics associated with the fitted (0,1,1) model suggest the inappropriateness of the specification That is, a more complex ARIMA representation is probably required for these series However, series 7 seems to have been overfitted The moving average parameter is quite close to the invertibility upper bound (*1*) Elsewhere (*2*) we have suggested that the levels of this series are white noise Differencing will tend to introduce a nonstationary behavior, and, thus, a (0,1,1) specification is inappropriate A much simpler process is likely to give a better representation for series 7

The above results suggest that many series can be adequately modeled as adaptive expectations In particular, we find many yield series exhibit a stochastic process for which adaptive expectations behavior is optimum For these series, revision of expectations as a convex combination of the differences

Table 2—Estimated autocorrelations and partial autocorrelations on 1936-76 Dubois County, Ind , corn yields

Auto-correlation		Lags									
		1	2	3	4	5	6	7	8	9	10
Y_t[3]	A[1]	0 73	0 64	0 60	0 49	0 41	0 38	0 40	0 26	0 18	0 05
	P[2]	73	22	17	- 09	- 01	04	17	- 25	- 11	- 28
∇Y_t[4]	A[1]	- 49	- 03	07	- 11	09	- 19	28	- 13	10	- 27
	P[2]	- 49	- 34	- 17	- 24	- 13	- 37	- 05	- 09	14	- 34
ϵ_t[5]	A[1]	- 20	- 10	- 04	- 15	00	- 12	24	- 05	01	- 22

[1] A refers to estimated autocorrelations at various lags Approximate standard error is 0 16
[2] P refers to estimated partial autocorrelations of various lags Approximate standard error is 0 16
[3] Y_t refers to the levels of the yield series
[4] ∇Y_t refers to the first differences of the yield series
[5] ϵ_t refers to the residuals from the fit first-order moving average representation

between the previous observation and its forecast makes sense Drawing an analogy with the macroeconomic literature, we can then say that for these yield series a notion of permanent yield might be a useful concept (similar to Friedman's permanent income) That is, farmers forming optimal expectations on these yield series might view yield as composed of both permanent and transitory components (For more on the relationship between the (0,1,1) process and the modeling of permanent income, the reader is referred to Nerlove (*14*)) Such behavior might be justified if one notes that specific changes in yield might be viewed as permanent in that they reflect basic changes in technology (new crop varieties, pesticides, and herbicides), whereas other changes might reflect year-to-year variability in weather It is only the former which farmers will want to incorporate in their future expectations

Alternatively, we have noted other series are not adequately modeled as adaptive expectations—in particular, most of our price series The yearly series generally follow a (0,1,1) process, however, the optimal process will tend to extrapolate trends That is, many of the yearly prices we studied have an elasticity of expectation (β) greater than 1 Thus, instead of finding some middle value between the most recent observation and its forecast, the economic agent who follows an optimal statistical predictor of these series will go outside these upper and lower bounds

Finally, the series measured quarterly and monthly tend to indicate a more complex univariate ARIMA process Elsewhere, these series are modeled as seasonal autoregressive and/or moving averages (see *3*, *4*)

Discussion

We have reviewed the relationships between adaptive expectations, the exponentially weighted moving average, and optimal univariate statistical predictors We have shown that the behavioral-based adaptive expectations are a subclass of both the exponentially weighted moving average and the (0,1,1) ARIMA model These results have been known heretofore, our review here was a simple prelude to the empirical section of this article

We have investigated the applicability of the adaptive expectations model to 25 empirical price and quantity series The adaptive expectations behavior and the optimal statistical forecasts are equivalent for 13 series—11 yield series and 2 price series We suggest that historical advances in technology and seemingly random weather make this behavior on yield expectations quite reasonable

The empirical results suggest that 12 series are not appropriately modeled as adaptive expectations Numerous price series, while exhibiting the general form of the adaptive expectations (a (0,1,1) ARIMA process), did not have a coefficient of expectations within the originally hypothesized range The behavior consistent with the model underlying these price series would be trend extrapolation rather than averaging (averaging the most recent observation and its forecast)

Finally, series measured at monthly or quarterly intervals were not adequately modeled by adaptive expectations or as a (0,1,1) ARIMA process

Although adaptive expectations behavior does represent an optimum behavior for many series, it does not for all Continued use of adaptive expectations in a behavioral model will likely lead to serious specification bias (*13*) Although we have not actually measured economic agents' expectations, until we do so, it seems reasonable that modeling them as suboptimal is not recommended

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Appendix table—Series number, title, interval measure, and number of observations for 25 agricultural time series

Series number	Title	Interval	Number of observations
1	Tintner's wool prices	Yearly	47
2	California rice prices	Yearly	44
3	California corn prices	Yearly	44
4	California wheat prices	Yearly	44
5	California barley prices	Yearly	44
6	California central valley barley yields	Yearly	32
7	California central coast barley yields	Yearly	32
8	California central valley wheat yields	Yearly	32
9	California central coast wheat yields	Yearly	32
10	California central valley sugar beet yields	Yearly	32
11	California central coast sugar beet yields	Yearly	32
12	Indiana Park County soybean yields	Yearly	33
13	Indiana Tippecanoe County soybean yields	Yearly	33
14	Indiana Hancock County corn yields	Yearly	41
15	Indiana Dubois County corn yields	Yearly	41
16	Indiana Knox County wheat yields	Yearly	41
17	Indiana Ohio County wheat yields	Yearly	41
18	U S hog prices	Quarterly	86
19	U S cattle prices	Quarterly	86
20	U S broiler prices	Quarterly	86
21	U S sow farrowings	Quarterly	84
22	U S cattle on feed	Quarterly	68
23	U S hog prices	Monthly	300
24	Indiana, 1908-32 corn prices	Monthly	300
25	Indiana, 1933-57 corn prices	Monthly	300