THE RELATIONSHIP BETWEEN THE INCOME ELASTICITIES OF DEMAND AND WILLINGNESS TO PAY

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ABSTRACT

The relationship between income and willingness to pay for a collectively-provided public good is investigated. We show that while the income elasticity of willingness to pay and the ordinary income elasticity of demand are functionally related, knowledge of one is insufficient to determine the magnitude or even the sign of the other. This is because the sign and magnitude of the income elasticity of willingness to pay is influenced by a number of other factors which are usually unobservable. Examples are provided for several common preference specifications to help illustrate why and when the two income elasticities diverge. One implication of our work is that public goods, which are luxuries goods in the traditional economic usage of that term, may or may not have income elasticities of willingness to pay which are greater than one.

I. INTRODUCTION

It has often be assumed that many environmental amenities are luxury goods in the traditional economics sense. However, empirically, one rarely observes an income elasticity of willingness to pay (WTP) larger than one (Kriström and Riera, 1994). Because most of these empirical estimates are from contingent valuation surveys, some critics of contingent valuation (McFadden, 1994) have taken these estimates as evidence against the reliability of contingent valuation surveys. Other techniques used in assessing environmental benefits, such as travel cost analysis, however, also tend to exhibit income elasticities of WTP less than one (Morey, Rowe and Watson, 1993).

The question of the relationship between income and environmental amenities was first raised in other contexts. In political discourse it is frequently noted that environmental group members tend to be more educated and have higher incomes than the general American public. This has long led to charges that an environmental elite is forcing their preferences on the general public (Tucker, 1977). However, as first shown by Mitchell (1979) and subsequently confirmed many times, there are surprisingly few substantial differences in support in public opinion surveys for major environmental programs between income groups.

More recently, Grossman and Kruger (1991) have argued that environmental quality improves as per capita GDP goes up in a country. A number of recent papers (Seldon and Song, 1994; CHECK) have further explored the nature of the environmental Kuznet's curve. Less noted in these works is that the empirical results show while the income elasticity of expenditures is positive it is below one. Baumol and Oates (1988) argue that most environmental policies appear to be "pro-rich" with respect to their benefits/expenditures, but stop short of the luxury claim, asserting only that the empirical evidence suggests that environmental goods tend to be "normal" goods.

What both WTP estimates from contingent valuation studies and expenditure estimates from studies like Grossman and Kruger point out is that one observes willingness to pay for environmental amenities or expenditures on them, but the levels in question are not quantities demanded in the traditional sense; rather, provision of an environmental amenity is generally a collective action. This suggests a potential direction to look for an explanation for the divergence between the intuition that environmental goods are luxury goods and the empirical evidence which suggests that they are not. The economic intuition is with respect to the ordinary (Marshallian) income elasticity of demand, while the empirical evidence is with respect to a different quantity the income elasticity of WTP. Is it the case that the two income elasticities are equivalent for public goods?

We raise this issue because much of the evidence is concerning how income influences the willingness to pay for *the same increment* rather than the way income affects *the choice of levels*, as in the case of the income elasticity of demand. The income elasticities of demand and willingness to pay are often treated as equivalent or, at least, closely linked and similar in magnitude. This leads us to a second question: if the two income elasticities are different, is the income elasticity of WTP informative with respect to the income elasticity of demand?

These two questions are addressed by examining the relationship between the two elasticities. First, it is shown that the two income elasticities are not equivalent. This is because public goods are a special case of quantity rationed goods. The elasticity of WTP is a substantially different concept than the ordinary income elasticity of demand and is defined in the context of an inverted mixed, private and public good demand system. Further, while we show the two income elasticities to be functionally related, we also show in the general case that for any fixed value of the income demand elasticity, the income elasticity of WTP can vary from minus infinity to plus infinity. This occurs because the income elasticity of WTP depends upon both the income and substitution elasticities of demand for all of the public goods. These other elasticities will usually be unobservable so that knowledge of the income elasticity of WTP will be uninformative as to the sign and magnitude of the ordinary income elasticity of demand.

II. THE INCOME ELASTICITY OF WILLINGNESS TO PAY

The stylistic model we use to conduct our analysis is the mixed or rationed model of consumption in which consumers have convex preferences over n market goods, denoted by the n-vector X, and k public goods which will be denoted by the k-vector Q.[1] In this model, consumers have freedom of choice over the levels of market goods, but face quantity rationing in the public goods. Preferences may be represented by an increasing, quasi-concave utility function, $U(X,Q)$, which consumers maximize subject to a vector of market prices, p, an income constraint, $p \cdot X \leq y$, and the level of public goods, Q. The maximization problem generates a set of Marshallian demands, $X^m(p,Q,y)$, which represent the optimal choice of market goods, as well as an indirect utility function $v(p,Q,y) = U(X^m(p,Q,y),Q)$. Willingness to pay for a change in q_1 from an initial level q_1^0 to a new, higher level q_1^1 satisfies the equality

$$v(p,q_1^1,Q_{-1},y\text{-}WTP) = v(p,q_1^0,Q_{-1},y) \ .$$

One can also consider the dual minimization problem in which expenditures on market goods are minimized subject to a given utility level, market prices, and levels of public goods. The expenditure-minimizing bundle is the set of Hicksian (compensated) demands $X^h(p,Q,U)$ and the analog to the indirect utility function is the expenditure function $e(p,Q,U) = p \cdot X^h(p,Q,U)$. Using the expenditure minimization framework, willingness to pay can be rewritten as a difference in minimized expenditures at the previous and subsequent levels of public good one,

[1] A comprehensive discussion of these models can be found in Cornes (1992).

$$WTP = e(p,q_1^0,Q_{-1},U) - e(p,q_1^1,Q_{-1},U) ,$$

where $U = v(p,q_1^0,Q_{-1},y)$ and Q_{-1} is the k-1 vector of public goods 2 through k. The first important step in our analysis is to develop relevant point income elasticities of demand and virtual prices (marginal willingness to pay). This development allows us to analyze the income elasticity of WTP in terms of point elasticities.

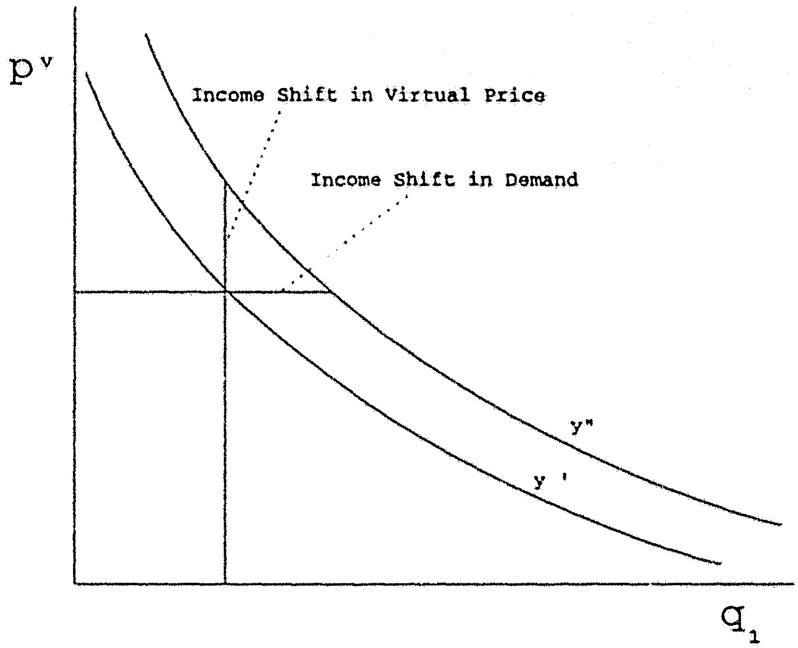
First note that following Mäler (1974), the derivative of the expenditure function with respect to q_i is the negative of the virtual price of q_i.[2] Thus, willingness to pay can be rewritten using the virtual price of q_1, p_1^v.

$$WTP = \int_{q_1^0}^{q_1^1} p_1^v(p,\ s,\ Q_{-1},\ U)ds\ .$$

The relationship between the virtual price and the level of q_1 can be represented as an inverse demand schedule. For willingness to pay, we are interested in how this curve shifts vertically when income (reflected through higher utility) is increased. In contrast when considering the income effect of demands, the focus is on how *quantity* adjusts. Figure 1 shows the differences between these two responses.

[2] The term virtual price was introduced by Rothbarth (1941) and is commonly used in the literature which deals with quantity-rationed goods such as Neary and Roberts (1980) or Cornes (1992).

Figure 1



The vector of virtual prices for Q plays an important role in our analysis because it allows us to derive elasticities of demand for the public goods. The virtual prices satisfy the tangency conditions that when both X and Q are in the agent's choice set, they would induce individuals to consume the same utility maximizing/expenditure minimizing bundle of (X,Q) as in the respective rationed problems, subject to an adjustment of income. We will refer to these as the virtual utility maximization problem and virtual expenditure minimization problem, respectively. The virtual utility maximization problem is to maximize $U(X,Q)$ in both X and Q subject to prices p and p^v respectively and subject to $e^v = y + p^v \cdot Q$ which we will refer to as virtual expenditures. The utility-maximizing bundle will result in the identical choice of goods as in the rationed goods problem,

$$\begin{bmatrix} X^m(p, Q, y) \\ Q \end{bmatrix} = \begin{bmatrix} X^m(p, p^v, e^v) \\ Q^m(p, p^v, e^v) \end{bmatrix} .$$

The analogous relationship can be derived for expenditure minimization where expenditures are minimized subject to the same level of utility,

$$\begin{bmatrix} X^h(p,\,Q,\,U) \\ Q \end{bmatrix} = \begin{bmatrix} X^h(p,\,p^v,\,U) \\ Q^h(p,\,p^v,\,U) \end{bmatrix}\ .^3$$

Using this relationship, one can derive an income elasticity of demand at the point of consumption for q_i,

$$\eta_i^d = \frac{\partial q_i^m(p,\,p^v,\,e^v)}{\partial y}\frac{e^v}{q_i^m}\ .$$

The infinitesimal counterpart in the rationed problem is the income elasticity of the virtual price of q_i,

$$\eta_i^v = \frac{\partial p_i^v}{\partial y}\frac{y}{p_i^v}\ .^4$$

In the case of a single rationed good, Hanemann (1991) uses the relationship $q_1 = q_1^m(p,p_1^v,e^v)$ to derive the marginal relationship between income and the virtual price of q_1. The virtual price and virtual expenditures are implicit functions of income and therefore, the implicit function theorem can be used. Flores (1994) extends that analysis to multiple rationed goods which will be followed here. Using the 2-public good case as an example, the relationship can be differentiated with respect to income to allow a derivation of the virtual price income elasticity.

$$0 = \frac{\partial q_1^m}{\partial p_1^v}\frac{\partial p_1^v}{\partial y} + \frac{\partial q_1^m}{\partial p_2^v}\frac{\partial p_2^v}{y} + \frac{\partial q_1^m}{\partial y}\left(1 + \frac{\partial p_1^v}{\partial y}q_1 + \frac{\partial p_2^v}{\partial y}q_2\right).$$

[3] For a thorough analysis of the utility-constant, rationed model, see Madden (1991) who provides a taxonomy of the substitution relationships between and within the sets of market and rationed goods.

[4] Randall and Stoll (1980) refer to this measure as the price flexibility of income. It is important to note that this elasticity is with respect to expenditures on market goods rather than virtual expenditures.

$$0 = \frac{\partial q_2^m}{\partial p_1^v}\frac{\partial p_1^v}{\partial y} + \frac{\partial q_2^m}{\partial p_2^v}\frac{\partial p_2^v}{y} + \frac{\partial q_2^m}{\partial y}\left(1 + \frac{\partial p_1^v}{\partial y}q_1 + \frac{\partial p_2^v}{\partial y}q_2\right).$$

The virtual price income derivatives can be factored out and terms rearranged using the Slutsky equation:

$$\begin{bmatrix} \frac{\partial q_1^h}{\partial p_1^v} & \frac{\partial q_1^h}{\partial p_2^v} \\ \frac{\partial q_2^h}{\partial p_1^v} & \frac{\partial q_2^h}{\partial p_2^v} \end{bmatrix} \begin{bmatrix} \frac{\partial p_1^*}{\partial y} \\ \frac{\partial p_2^*}{\partial y} \end{bmatrix} = - \begin{bmatrix} \frac{\partial q_1^m}{\partial y} \\ \frac{\partial q_2^m}{\partial y} \end{bmatrix}.$$

Operating under the assumption that the matrix of substitution terms which pertain to the goods q_1 through q_k is invertible, the income derivatives of virtual prices can be deduced:

$$\begin{bmatrix} \frac{\partial p_1^v}{\partial y} \\ \frac{\partial p_2^v}{\partial y} \end{bmatrix} = - \begin{bmatrix} \frac{\partial q_1^h}{\partial p_1^v} & \frac{\partial q_1^h}{\partial p_2^v} \\ \frac{\partial q_2^h}{\partial p_1^v} & \frac{\partial q_2^h}{\partial p_2^v} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial q_1^m}{\partial y} \\ \frac{\partial q_2^m}{\partial y} \end{bmatrix}.$$

In order to derive the virtual price income elasticities, scaling by income over the virtual price is needed:

$$\begin{bmatrix} \eta_1^v \\ \eta_2^v \end{bmatrix} = - \begin{bmatrix} \frac{1}{p_1^v} & 0 \\ 0 & \frac{1}{p_2^v} \end{bmatrix} \begin{bmatrix} \frac{\partial q_1^h}{\partial p_1^v} & \frac{\partial q_1^h}{\partial p_2^v} \\ \frac{\partial q_2^h}{\partial p_1^v} & \frac{\partial q_2^h}{\partial p_2^v} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial q_1^m}{\partial y} \\ \frac{\partial q_2^m}{\partial y} \end{bmatrix} y\,.$$

By rewriting the identity matrix, the right hand side can be converted into terms involving compensated substitution elasticities:

$$\begin{bmatrix} \eta_1^v \\ \eta_2^v \end{bmatrix} = - \begin{bmatrix} \frac{1}{p_1^v} & 0 \\ 0 & \frac{1}{p_2^v} \end{bmatrix} \begin{bmatrix} \frac{\partial q_1^h}{\partial p_1^v} & \frac{\partial q_1^h}{\partial p_2^v} \\ \frac{\partial q_2^h}{\partial p_1^v} & \frac{\partial q_2^h}{\partial p_2^v} \end{bmatrix}^{-1} \begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix} \begin{bmatrix} \frac{1}{q_1} & 0 \\ 0 & \frac{1}{q_2} \end{bmatrix} \begin{bmatrix} \frac{\partial q_1^m}{\partial y} \\ \frac{\partial q_2^m}{\partial y} \end{bmatrix} y$$

$$\begin{bmatrix} \eta_1^v \\ \eta_2^v \end{bmatrix} = - \begin{bmatrix} \sigma_{11}^d & \sigma_{12}^d \\ \sigma_{21}^d & \sigma_{22}^d \end{bmatrix}^{-1} \begin{bmatrix} \frac{1}{q_1} & 0 \\ 0 & \frac{1}{q_2} \end{bmatrix} \begin{bmatrix} \frac{\partial q_1^m}{\partial y} \\ \frac{\partial q_2^m}{\partial y} \end{bmatrix} y \, .$$

$\sigma_{ij}{}^d$ is the compensated, cross-price substitution elasticity of demand for q_i and q_j. The right-hand side can then be completely converted to elasticities by scaling with virtual expenditures:

$$\begin{bmatrix} \eta_1^v \\ \eta_2^v \end{bmatrix} = - \begin{bmatrix} \sigma_{11}^d & \sigma_{12}^d \\ \sigma_{21}^d & \sigma_{22}^d \end{bmatrix}^{-1} \begin{bmatrix} \frac{e^v}{q_1} & 0 \\ 0 & \frac{e^v}{q_2} \end{bmatrix} \begin{bmatrix} \frac{\partial q_1^m}{\partial y} \\ \frac{\partial q_2^m}{\partial y} \end{bmatrix} \frac{y}{e^v}$$

$$\begin{bmatrix} \eta_1^v \\ \eta_2^v \end{bmatrix} = - \begin{bmatrix} \sigma_{11}^d & \sigma_{12}^d \\ \sigma_{21}^d & \sigma_{22}^d \end{bmatrix}^{-1} \begin{bmatrix} \eta_1^d \\ \eta_2^d \end{bmatrix} \frac{y}{e^v} \, .$$

Using the relationship that the virtual price substitution elasticities between the rationed goods equals the inverse of the compensated substitution elasticities and denoting the budget share factor of market goods as $y/e^v = S_y^v$, the virtual price income elasticities can be represented as follows:

$$\begin{bmatrix} \eta_1^v \\ \eta_2^v \end{bmatrix} = - \begin{bmatrix} \sigma_{11}^v & \sigma_{12}^v \\ \sigma_{21}^v & \sigma_{22}^v \end{bmatrix} \begin{bmatrix} \eta_1^d \\ \eta_2^d \end{bmatrix} S_y^v \, .$$ [5]

The income elasticity of a given virtual price involves more than just the corresponding demand income elasticity. It involves the income elasticities of demand for all of the other rationed goods, the corresponding cross-price demand substitution elasticities (inverted), and the share

[5] It is worth noting that while the Engel aggregation condition applies to the complete set of demand income elasticities (market and public goods), it does not apply to the set of rationed goods' virtual price income elasticities. This is true because the budget constraint does not hold in the traditional sense; the public goods are quantity constrained. Anderson (1980) shows that when *all* goods are rationed, there is an additivity condition that applies to these elasticities. Flores (1994) provides a comprehensive treatment of the mix private-public goods case.

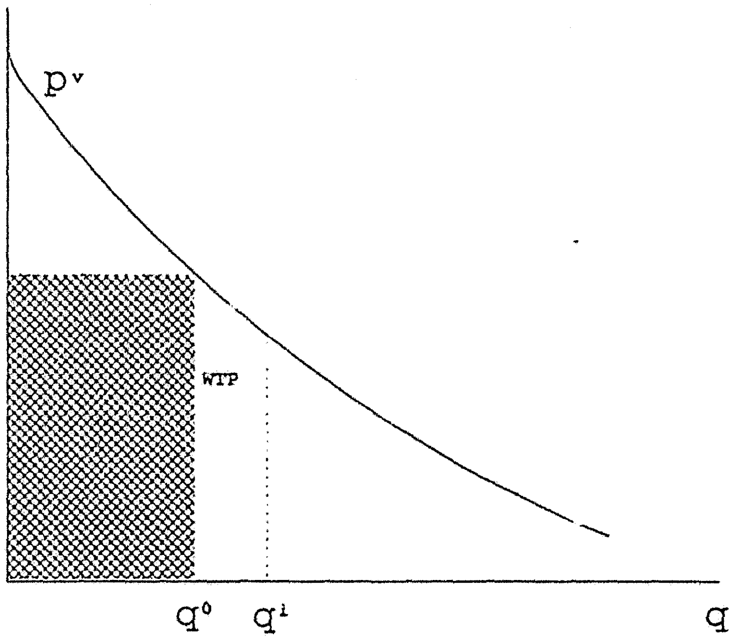
of virtual expenditures for the n goods in X. The virtual price income elasticity is basically a linear combination of the income elasticities of demand for the goods q_1 through q_k,

$$\eta_i^v = -\sum_{j=1}^{k} \sigma_{ij}^v \, \eta_j^d \, S_y^v = \sum_{j=1}^{k} \omega_{ij} \, \eta_j^d \, .$$

The virtual price income elasticity of any element of q_i may differ substantially from its income elasticity of demand and this divergence may come from any one or combination of three factors: the inclusion of other public goods' income elasticities, the pre-multiplication by the inverse substitution matrix, or multiplication by the budget share factor for market goods, S_y^v. First, we will discuss the budget share factor and then in the next section, we focus on the combined income and substitution factors. The budget share of expenditures on market goods from the virtual minimization/maximization problems is always less than one and may be quite small once all of the public goods an individual consumes are considered.

Figure 2 helps illustrate this point. Suppose that there is only one public good, q. The shaded portion of the graph then represents the amount of additional money needed to supplement the income an individual spends on the private good in order to solve the virtual minimization problem. Note that relative to willingness to pay for the increase in q from q_0 to $q1$ (the unshaded area marked WTP), this amount is quite large. If one allows for the reality of many public goods, the share of expenditures on market goods becomes relatively smaller and smaller. Therefore one important source of divergence between the income elasticity of virtual prices (as well as willingness to pay via the relation with virtual price) and demand is the reduction that occurs from multiplying later by the budget share factor which may be *much* less than one.

Figure 2



With the demand and virtual price income elasticities defined, we can now turn our attention to an analysis of a discrete change in q_1 and the income elasticity of WTP. Recall from above that willingness to pay is the integral of the virtual price over the change in q_1:

$$WTP = \int_{q_1^0}^{q_1^1} p_1^v(p,\ s,\ Q_{-1},\ U)ds\ .$$

Differentiating willingness to pay with respect to income is somewhat difficult because it actually involves a continuum of expenditures available for market goods which occurs because of the utility-constant framework. Essentially, income serves as a utility index for a given set of preferences. In our model, the virtual price income elasticity derived above measures how the virtual price changes with respect to expenditures available for market goods. Recall that virtual expenditures at the initial point were defined $e^v = y + p^v \cdot Q$. Alternatively, this relationship can be expressed as $e^v = e(p,Q,U) + p^v \cdot Q$. Market goods expenditures are functions of the initial level of income through utility. At the initial q_1, $e(p,Q,U) = y$ which when differentiated with respect to y the following conditions holds:

$$\frac{\partial e(p,\ Q,\ U)}{\partial y} = \frac{\partial e(p,\ Q,\ U)}{\partial U}\frac{\partial U}{\partial y} = 1\ .$$

However once moving away from q_1^0, the relationship changes since the expenditures on market goods no longer equal y:

$$\frac{\partial e(p,\ q_1^*,\ Q_{-1},\ U)}{\partial y} = \frac{\partial e(p,\ q_1^*,\ Q_{-1},\ U)}{\partial U}\frac{\partial U}{\partial y} \neq 1\ .$$

Therefore, the income elasticity of the virtual price is different when q_1 changes:

$$\begin{bmatrix}\eta_1^v(q_1)\\ \eta_2^v(q_1)\end{bmatrix} = -\begin{bmatrix}\sigma_{11}^v & \sigma_{12}^v\\ \sigma_{21}^v & \sigma_{22}^v\end{bmatrix}\begin{bmatrix}\eta_1^d\\ \eta_2^d\end{bmatrix}\left(\frac{y}{e^*}\right)\left(\frac{\partial e(q_1,\ Q_{-1},\ U)}{\partial U}\frac{\partial U}{\partial y}\right).$$

With this slight augmentation, bounds on the income elasticity of WTP are available. First we define the income elasticity of WTP:

$$\eta_1^{WTP} = \frac{\partial WTP}{\partial y}\frac{y}{WTP} = \left(\int_{q_1^0}^{q_1^1}\frac{\partial p_1^v(p,\ s,\ Q_{-1},\ U)}{\partial y}ds\right)\frac{y}{WTP}\ .$$

In order to relate the income elasticity of WTP to the point elasticities discussed above, we derive a set of bounds that involve the point elasticities. The strictly quasi-concave utility assumption implies that the virtual price p_1^v is decreasing in q_1 and therefore, for all q_1 in the interval $[q_1^0, q_1^1]$,

$$p_1^v(q_1^1) < p_1^v(q_1) < p_1^v(q_1^0).^6$$

Consequently, one can bound WTP:

$$(q_1^1 - q_1^0)p_1^v(q_1^1) < WTP < (q_1^1 - q_1^0)p_1^v(q_1^0)\ .$$

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[6] In order to reduce notation, we drop reference to prices, other public goods, and utility.

Using this inequality, we can develop an initial set of bounds on the income elasticity of WTP. As will be shown in the next section, the income to willingness to pay relationship may be negative (inferior) while in other cases it may be positive. These two cases must be considered separately because the bounds will differ due to our working with inequalities. We first consider the case in which the virtual price responds positively to increases in income.

$$\left[\int_{q_1^0}^{q_1^1} \frac{\partial p_1^v(s)}{\partial y} ds\right] \frac{y}{(q_1^1-q_1^0)\, p_1^v(q_1^0)} \leq \eta_1^{WTP} \leq \left[\int_{q_1^0}^{q_1^1} \frac{\partial p_1^v(s)}{\partial y} ds\right] \frac{y}{(q_1^1-q_1^0)\, p_1^v(q_1^1)} .$$

The assumed continuity and differentiability of the underlying demands combined with the compact nature of the interval $[q_1^0, q_1^1]$ imply that there exists a q_1^L such that

$$\frac{\partial p_1^v(q_1^L)}{\partial y} \leq \frac{\partial p_1^v(q_1)}{\partial y} \;\; \forall\, q_1 \in [q_1^0,\, q_1^1] .$$

Similarly, there exists a q_1^H such that

$$\frac{\partial p_1^v(q_1)}{\partial y} \leq \frac{\partial p_1^v(q_1^H)}{\partial y} \;\; \forall\, q_1 \in [q_1^0,\, q_1^1] .$$

Using these two pieces of information, new bounds are possible:

$$\frac{\partial p_1^v(q_1^L)}{\partial y} (q_1^1-q_1^0) \frac{y}{(q_1^1-q_1^0)\, p_1^v(q_1^0)} \leq \eta_1^{WTP} \leq \frac{\partial p_1^v(q_1^H)}{\partial y} (q_1^1-q_1^0) \frac{y}{(q_1^1-q_1^0)\, p_1^v(q_1^1)} .$$

Simplifying the expressions and scaling by one rewritten using the respective virtual prices gives us a workable set of bounds.

$$\eta_1^v(q_1^L) \frac{p_1^v(q_1^L)}{p_1^v(q_1^0)} \leq \eta_1^{WTP} \leq \eta_1^v(q_1^H) \frac{p_1^v(q_1^H)}{p_1^v(q_1^1)} .$$

The virtual price ratio for the lower bound (the term multiplying the virtual price income elasticity at q_1^L) is less than one and greater than one for the upper bound. Thus, the willingness to pay income elasticity will fall in an interval that is wider than the interval bounded by the smallest and largest virtual price income elasticity with the width determined by the deviations between $p_1^v(q_1^L)$ and $p_1^v(q_1^0)$; $p_1^v(q_1^H)$ and $p_1^v(q_1^1)$. At first glance, this set of bounds does not appear particularly useful. However, when one considers the relationship between the virtual price and demand income elasticities, below we show that the right hand bound can be negative in some cases and the left hand bound can be positive infinity in others. Thus, the relationship between the income elasticities of WTP and demand is essentially unrestricted.

It may be the case that both point elasticities used in the bounds are negative which implies a slightly different bound due the use of inequalities. The bounds for income inferior willingness to pay takes the form:

$$\eta_1^v(q_1^L)\,\frac{p_1^v(q_1^L)}{p_1^v(q_1^1)} \le \eta_1^{WTP} \le \eta_1^v(q_1^H)\,\frac{p_1^v(q_1^H)}{p_1^v(q_1^0)}\ .$$

The difference between the normal and inferior values is the denominator in the upper and lower bounds. In cases which are mixed (negative virtual price income elasticity for some q_1 and positive for others), the bounds for the normal values apply.

III. DIVERGENCE BETWEEN INCOME ELASTICITIES OF DEMAND AND WTP

In this section we use point elasticities to manipulate the bounds on the income elasticity of WTP. Using a single public good, we first show that the lower bound can essentially be positive infinity and the upper bound can be zero. In this simple case, the virtual price and demand income elasticities are related as follows:

$$\eta_1^v = -\frac{1}{\sigma_{11}^d}\eta_1^d S_y^v\ .$$

Note that no matter what the size of the income elasticity of demand, the income elasticity of virtual price can be driven to positive infinity by simply letting the own-price demand substitution elasticity tend to zero. Similarly, if we let this substitution elasticity get large, we can drive the virtual price income elasticity to zero.

The relationship between the single public good's income elasticity of virtual price and demand can be linked to Hanemann's (1991) result on willingness to pay and willingness to accept. Hanemann treats all market goods as a composite commodity and then uses the adding-up conditions to rewrite $-\sigma_{11}{}^{d}=\sigma_{1y}{}^{d}$ where $\sigma_{1y}{}^{d}$ is the compensated demand substitution elasticity between the public good and the composite commodity y (or income). One can further simplify by noting that the Allen elasticity of substitution between the public good and the composite satisfies the equality $\sigma_{1y}{}^{A}=\sigma_{1y}{}^{d}/S_y$. Therefore, the relationship between the virtual price and demand income elasticities is simplified to

$$\eta_1^v = \frac{1}{\sigma_{1y}^A}\eta_1^d .$$

Hanemann uses this relationship with a different set of bounds to show that willingness to pay and willingness to accept can greatly differ for an imposed quantity change.

One can think of two limiting cases of preferences with respect to substitution between q and y, which we use as a composite commodity. Leontief preferences represent one extreme and coincide with the zero substitution that yields the infinite virtual price income elasticity. Linear preferences represent the other extreme and coincide with infinite substitution that yields the zero virtual price income elasticity. In between these limiting cases are preferences that have moderately convex indifference curves such as those represented by the Cobb Douglas utility function. It is visually useful to consider "near" limiting preferences which are differentiable to demonstrate how much difference substitution effects can make. Figure 3a shows the income effect on near Leontief preferences; Figure 3b shows the income effect on preferences with moderate substitution; and Figure 3c shows the income effect on preferences which are near linear.

Figure 3a

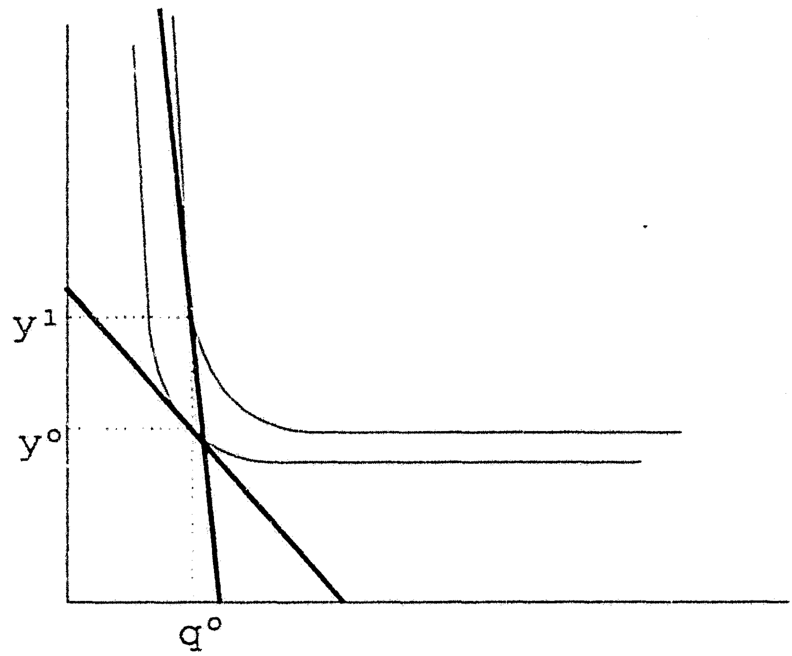


Figure 3b

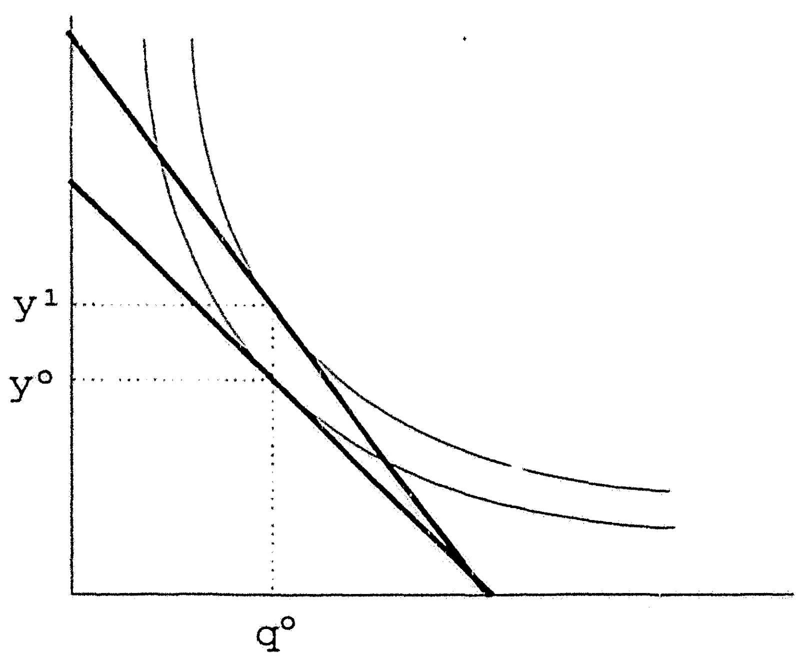
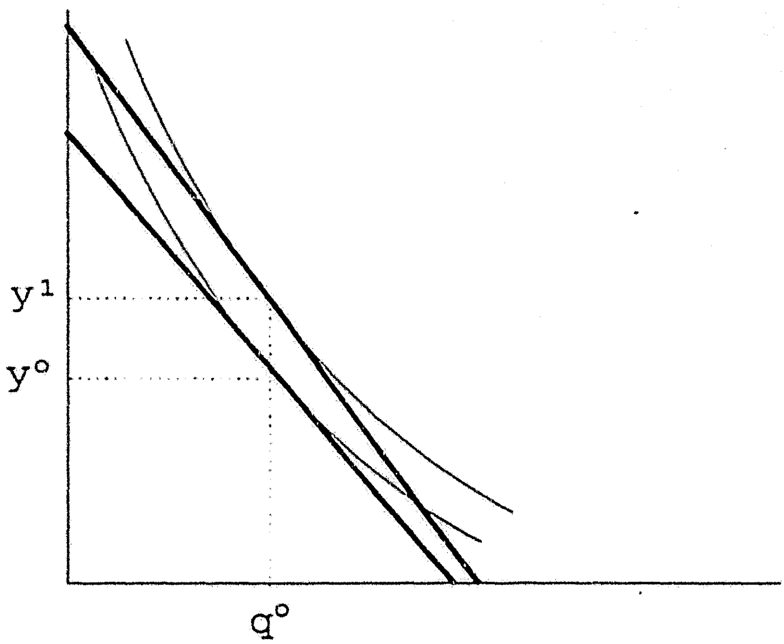


Figure 3c



What is important to note in each figure is the difference in slope of the tangent lines at the lower (y^0) and higher (y^1) income levels. In the near linear case (3c), the slopes are almost the same indicating a small difference in virtual price before and after the change due to the high degree of substitutability. In the intermediate case (3b) there is a considerable difference in the slopes of the tangent lines. Finally in the near Leontief case (3a), we see a dramatic difference in the slope of tangent lines, indicating extreme income effects.

There are specific classes of preferences for which exactly deriving the relationship between the income elasticities of virtual price and demand is straightforward. In the case of Cobb Douglas preferences, the income elasticities of demand and virtual price are restricted to both equal one. For constant elasticity of substitution preferences, the income elasticity of virtual price equals the inverse of the substitution parameter.[7] Thus only for the highly

[7] The Cobb Douglas and CES results also apply in cases of more than one public good. Like the Cobb Douglas,

restrictive, Cobb Douglas class of preferences is it true that demand and virtual price income elasticities are equal. Introducing minimal flexibility, such as the CES, introduces the possibility of significant divergence between these two elasticities.

IV. MULTIPLE PUBLIC GOODS

In the previous section we considered the case of a single public good. In reality there are a large number of other public goods from which agents derive utility and explicitly considering this possibility allows for greater range of substitution relationships. In particular, by allowing for other public goods, goods may become virtual price complements. This raises the possibility of a negative income elasticity of a given virtual price even though the income elasticity of demand is positive.

To see this suppose that all goods, private and public, are normal goods with income elasticities of demand greater than zero. Intuitively it would seem that even once quantity-rationed, the public goods should have a positive income elasticity of virtual price. After all, virtual prices are simply the inverse demand schedules which one may reason will naturally shift out due to the positive income effect. In an unrationed regime, this reasoning holds. Income effects would simply shift the demands out and the prices would remain the same. However, the problem with this reasoning is that for public goods, the quantity constraints are binding. With more income, the virtual prices for the goods must also go up in order to hold virtual choice at the constrained level. Therefore when using demand intuition, we cannot simply picture an outward shift in the inverse demand schedule because there are price changes (for the public goods) going on as well. For good one we can first picture the income shift as suggested

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the CES demand income elasticities are also restricted to one.

above. However once the virtual prices of other public goods adjust, our demand analogy must be adjusted to reflect the price changes. In a world in which there is demand complementarity, there will be an inward shift of the demand curve due to the increase in prices of the complementary public goods. This works against the demand income effect. If the complementary effect is large relative to the income effect, then a negative virtual price income elasticity may result.

This can be demonstrated by considering two public goods with no restriction on the number of market goods. Recall that for two public goods we have

$$\begin{bmatrix} \eta_1^v \\ \eta_2^v \end{bmatrix} = - \begin{bmatrix} \sigma_{11}^d & \sigma_{12}^d \\ \sigma_{21}^d & \sigma_{22}^d \end{bmatrix}^{-1} \begin{bmatrix} \eta_1^d \\ \eta_2^d \end{bmatrix} \frac{y}{e^v} .$$

$$\begin{bmatrix} \eta_1^v \\ \eta_2^v \end{bmatrix} = - \begin{bmatrix} \sigma_{11}^v & \sigma_{12}^v \\ \sigma_{21}^v & \sigma_{22}^v \end{bmatrix} \begin{bmatrix} \eta_1^d \\ \eta_2^d \end{bmatrix} S_y^v .$$

Using good one as an example

$$\eta_1^v = - [\sigma_{11}^v \eta_1^d + \sigma_{12}^v \eta_2^d] \, S_y^v .$$

Assuming all normal demands, the virtual price income elasticity for good one is negative if

$$- \left[\sigma_{11}^v + \sigma_{12}^v \frac{\eta_2^d}{\eta_1^d} \right] < 0 .$$

The simple inverse relationship in the two public good case allows us to further rewrite this bound in terms of demand substitution elasticities.

$$-\frac{\sigma_{11}^v}{\sigma_{12}^v} < \frac{\eta_2^d}{\eta_1^d} \rightarrow$$

$$\frac{\sigma_{22}^d}{\sigma_{12}^d} < \frac{\eta_2^d}{\eta_1^d} .$$

Here we must recall that both demand substitution elasticities are negative due the complementary relationship. Thus if the ratio of substitution terms is relatively smaller than the ratio of income terms, then we have sufficient conditions for negative income elasticity of virtual price even when the public goods are normal demands.

V. CONCLUDING REMARKS

One implication of our work is that public goods which are luxuries goods in the traditional economic usage of that term may or may not have income elasticity of WTP which are greater than one. Indeed, an income elasticity of one type is uninformative about the other. With respect to public goods, the ordinary Marshallian income elasticity of demand will generally be unobservable. Because the income elasticity of WTP involves scaling the ordinary income elasticity by the ratio of disposable income to virtual expenditures and that ratio is always less than one and probably considerably less than one, empirically observing an income elasticity of WTP less than one is likely to be the rule rather than the exception. This is true even if the good in question is a luxury good in the sense of having an ordinary income elasticity greater than one. The matrix of substitution terms between public goods which also enters into this equation can, however, allow the income elasticity of WTP to take on any value from minus to plus infinity for any given value of the ordinary income elasticity. As a result, it is misleading even to use the terms luxury, necessity, and inferior good to refer to public goods with income elasticities of WTP greater than 1, between 1 and 0, and less than 0, respectively. The economic intuition behind our results can be expressed simply: the rich person may want to buy proportionately more loaves of bread than the poor person, but this does not imply that the rich person is willing to pay proportionately more for the same loaf of broad.

Our results suggests that there may not be a divergence between the intuition that some environmental goods are luxuries and the frequent empirical observations that they have income elasticities of WTP substantially less than one. From a practical standpoint, our results have implications for applied policy analysis where values for elasticities are often assumed on the basis of what appear to be plausible values rather than empirically estimated. Here, intuition based on experience with the typical demand systems estimated for consumer goods will fail.

This problem manifest itself in another way when environmental benefit estimates are transferred from one country to another. Something which is a common practice, particularly with respect to work in developing countries were original country specific estimates are infrequently available. Here the usual practice is to scale the original benefit estimate by some ratio measure of the income in the two countries (Eskeland and Kong, 1994). This practice embodies in it the assumption that the income elasticity of WTP is one. This maintained assumption is unlikely to be either an innocuous or reasonable assumption (Alberini *et al.*, 1995).

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