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Abstract

The question of rail freight pricing for coal and wheat is discussed in this paper. As bulk goods, wheat and coal are moved in large quantities by both road and rail. Transport by rail frequently suffers from the difficulty of recurring deficits. The pricing of rail services so as to cover these deficits is therefore important. An adapted spatial equilibrium model is used to derive a set of economically efficient Ramsey prices for movement of both commodities by rail in New South Wales. Ramsey prices represent a set of second-best prices in the presence of unallocated overhead costs, and a constraint precluding deficits. The technique used may also be suitable for application to a wide variety of situations where joint costs must be attributed across a variety of outputs in a spatial context.

* Early work on this project began as a fourth year research project of the fourth author as part of the requirements for the Bachelor of Agricultural Economics degree at the University of Sydney. This author is now a member of the staff of the Industry Commission, Canberra.

Introduction

Economists have long focussed their attention on the challenge faced by railways (and other decreasing cost industries) of how to price efficiently without incurring perpetual deficits. A number of approaches have been adopted to overcome this problem, including attempts to specify efficient prices, as well as rate of return regulation. In the case of Australian railways up to this point, pricing policies appear to have been largely ad hoc and piecemeal in nature. There has been a tendency to respond to immediate financial and political considerations rather than the underlying economic considerations. Developments during the 1980s such as commercialisation of government enterprises and micro-economic reform generally have underlined the inadequacy of this approach to the pricing problem of state-owned railways, and precipitated renewed interest in economically efficient pricing rules. In this paper, some of the issues related to pricing of services faced by New South Wales railways in the movement of grain and coal are examined.

The objective for this paper is to derive a theoretical solution to the problem of optimal pricing for rail transport in the face of decreasing costs. The issue of inter-modal competition will be considered at a later stage. Use will be made of the spatial equilibrium approach. In the first section, the empirical and theoretical background to the problem presented by decreasing cost railways is discussed. Particular emphasis is given to the concept of Ramsey pricing, and the solution it affords to the problem presented by a decreasing cost transport industry handling multiple commodities. In the next section, a spatial equilibrium model is developed to solve the problem of setting optimal prices in the face of decreasing costs, a revenue constraint and inter-modal competition. Although Ramsey optimal prices have been derived elsewhere in the literature, the method developed in this paper applies the technique to a network model and thereby offers a practical tool for the application of Ramsey pricing. A practical example of the use of Ramsey pricing is then presented. The paper is concluded by examining the situations in which the technique in this paper may be used along with comments on areas for further development of the approach.

Pricing of Transport Services

Analysis of the transport pricing problem can be traced back to the early to mid 1800s (Winston 1985, p. 78). Although the initial interest was in appropriate pricing rules for urban roads in an attempt to overcome congestion, economists were critical of rate setting in railroads, especially the 'value-of-service' or 'charging what the traffic will bear' approach adopted during the later half of the nineteenth century (Jannson 1984, p. 252). This criticism can be attributed to concern over exploitation of the monopolistic position occupied by railroad operators. These concerns have to some extent become muted with the development of alternatives to rail transport, especially road transport. Ironically, however, the principles embodied in the 'value-of-service' approach have become readily accepted within the economics profession as a legitimate and welfare enhancing approach to the problem posed by decreasing cost industries such as railways.

The difficulty posed by railways, and decreasing cost industries more generally, has been extensively set out in the welfare literature.[1] In short, if welfare maximising marginal cost pricing rules are to be adopted, an industry characterised by decreasing average costs will suffer a deficit. This feature has in the past been relied on to justify public sector activity in industries such as railways. However, public sector involvement in the industry raises the additional question of whether and how the revenue deficit should be made up. A first best solution would require the imposition of poll taxes of some form. The difficulty (or even impossibility) of imposing taxes in this f rm focuses attention on the use of the appropriate pricing rule.

It is clear that an infinite number of solutions exist to the problem of requiring a firm in the face of decreasing average costs to meet a profit constraint (Allen 1986, p. 300). Two part tariffs and average cost pricing represent some of the better known rules. Ramsey pricing, a form of differential pricing, has been shown to represent a second-best solution to the welfare maximisation problem in the face of decreasing average costs and a profit constraint (Baumol and Bradford 1970, pp. 267-71; Lipsey and Lancaster 1956). However, Ramsey pricing has not been immune from criticism from the point of view of its welfare implications and its practicality as a pricing rule (Allen 1986, pp. 294-96).

In the Australian context, government budgetary constraints and a desire to commercialise the operations of government business enterprises have focussed attention on the pricing policies of a range of government instrumentalities over the past decade[2]. From a broader perspective, an efficient approach to pricing of freight transport services is central to ensuring the competitiveness of a range of Australian export industries.[3] This is particularly relevant for agricultural and mineral industries in which the place of production is determined by climatic and or geographic considerations, and is often far removed from regions where demand for the product exists. In addition, analysis suggests that the cost of

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[1] It will be assumed for the purposes of this paper that the railways under consideration exhibit economies of scale, at least over the relevant range of output. It is clear from the litereature, however, that this is not unambiguously accepted (Winston 1985, pp. 61- 66; Freebairn and Trace 1988, pp. 42- 43).

[2] Numerous analyses have been undertaken by the Federal and State governments on the operation of government trading enterpises. Pricing policies adopted by these enterprises have been reviewed in most of the studies and these reviews have formed the basis for subsequent pricing policy reform. Typical of these reports is that of the Steering Committee on Government Trading Enterprises (1988).

[3] For example, the Royal Commission into Grain Storage, Handling and Transport (1988), estimated that storage, handling and transport charges represented 20 percent of the average free-on-board price for wheat. In 1988-89, the average freight rate as a percentage of the average free-on-board price of coal, was 15.5 percent for New South Wales (Industry Commission, 1991b).

inefficient rate setting to those industries affected directly, and to the Australian economy generally, is immense (Freebairn 1988).

Ramsey Pricing

Although usually associated with Frank Ramsey's (1927) treatment of the problem, differential pricing on the basis of demand elasticities has a long tradition in the economics literature (Winston 1985, p. 80; Baumol and Bradford 1970, pp. 277-78). Moreover, Ramsey's original solution dealt with the problem of optimal taxation subject to a revenue constraint (Ramsey 1927). This problem is analogous to that faced by a decreasing cost industry required to avoid a deficit outcome by raising the charge for the good or service above marginal cost. It is irrelevant if the sum raised (to cover a deficit or to raise the required tax revenue) is obtained by way of a tax impost over and above marginal cost, or if it is raised by setting prices over and above marginal cost. Since Ramsey's analysis, Ramsey pricing rules have been derived according to a number of different formulations and its 'second best' character has been made explicit.

Although widely discussed in the theoretical literature on optimal pricing, Ramsey pricing has generally been ignored for operational reasons because of the empirical demands in deriving a set of Ramsey prices. Nevertheless, a number of attempts have been made to empirically determine Ramsey prices for rapid-rail and buses (Train 1977), as well as the prices for rail in the context of inter-modal rail and road competition (Levin 1981a and 1981b). In the Australian context, Ramsey prices have been derived for road and rail transport (Taplin and Waters 1985; Taplin 1980).

Ramsey pricing involves setting prices so as to cover costs or meet a budget constraint such that the distortions resulting from a deviation from marginal cost pricing are minimised. The rule exploits the differences in elasticities of demand by charging a higher rate to consumers with the most inelastic demand (Terry, Jones and Braddock, 1988). Described in this way, it is clear that Ramsey pricing represents a form of differential pricing where charges are determined according to the demand for the service, and raised to the point where the aggregation of all charges meets a net revenue constraint (Freebairn and Trace, 1988). In its simplest form, where cross-price elasticities for the goods are zero, the Ramsey pricing rule collapses to the well recognised 'inverse elasticity rule' requiring prices be set according to the inverse of the elasticity of demand for each commodity (or consumer), service or product.

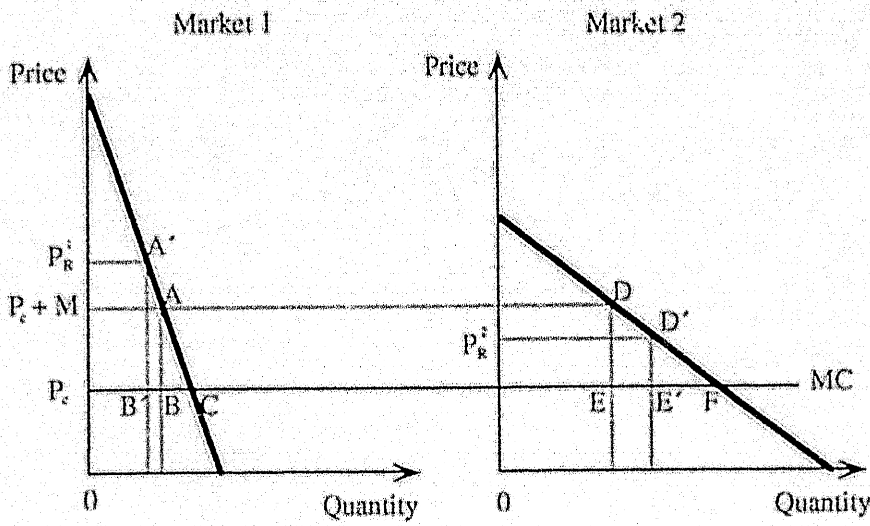

FIGURE 1—Ramsey Pricing in Two Markets.

The rationale for Ramsey pricing is demonstrated in Figure 1. Suppose that a firm, serving two markets, faced a loss from fixed joint costs of production if the competitive solution was adopted and price was set equal to short run marginal or operating cost ($P_c = MC$). To recover joint or unattributable costs over the system, it is necessary to set price greater than short run marginal cost. Suppose that an equal mark-up over marginal cost of M in both markets was sufficient to recover the joint costs. The total deadweight economic loss across markets 1 and 2 is then given by the areas ABC (market 1) and DFF (market 2). From the diagram, it is clear that the loss in surplus in market 2 (the market with relatively elastic demand) exceeds the loss in market 1 (the market with relatively inelastic demand). Further, it is clear that the total loss in welfare can be reduced if the mark-up in market 1 exceeds M, and the mark-up in market 2 is reduced to less than M. Prices consistent with this approach are shown in the diagram as P_R^1 and P_R^2. These 'Ramsey approximate prices' highlight the need for the mark-up over marginal cost to be greater in the inelastic market (market 1), than the elastic market (market 2). Hence, the mark-up over marginal cost should be inversely related to the price elasticity of demand.

The rationale for Ramsey pricing is intuitively appealing. If marginal cost pricing does not cover costs, additional revenue should be obtained by higher charges to users whose demand is the least sensitive to prices. Such an approach will cause the least distortion from the first-best policy of marginal cost pricing by minimising the output-consumption deviations from the first-best outcome. Although the Ramsey rule can be formulated in various ways, if the assumption of zero cross-price elasticities is discarded its application becomes significantly more complex as cross-price elasticity terms enter into the rule (Baumol and Bradford 1970, p. 270)[4].

However, it is suggested that as a form of price discrimination, Ramsey pricing tends to create independence of demand by segmenting the various markets for goods or services. The conditions under which the inverse elasticity rule can be applied are thereby enhanced (Jansson 1984, p. 252). Moreover, in the case of demand for transport services provided by railways, cross-price elasticities are likely to be low anyway (Taplin 1980, p. 200; Taplin and Waters 1985, p. 339).

Ramsey pricing represents a formalised version of the principle of 'value-of-service' pricing. For transport services this involves charging a higher price for commodities with a relatively inelastic demand for transport compared to those commodities with more price elastic demands. Price is charged according to the value placed on the service by the consumer. For example, high value commodities will tend to be less sensitive to changes in transport charges and shall thus be relatively more price inelastic in the demand for transport than lower valued commodities. Similarly, the price elasticity of demand for the product transported will influence the elasticity of demand for the transport of that good. Although criticised in the early part of railway history as an abuse of monopolistic power and reflecting a lack of cost awareness, the 'value of service' rule or 'charging what the traffic will bear' is now a well respected and widely practised pricing rule in various modes of transport such as airlines (Jansson 1984, p. 252).

The conditions under which Ramsey pricing can be applied as a second-best rule are as follows (Kamerschan and Keenan, 1983, p. 198):

- joint outputs with common inputs;
- scale economies exist such that marginal cost pricing would result in a deficit;
- costs must be covered by the firm using a uniform cost recovery pricing policy;
- the regulated firm is a price taker for its inputs;
- the services do not exhibit consumption externalities;
- the rest of the economy is perfectly competitive.

All of these conditions are assumed to be adequately met in the circumstances considered in the present paper. These conditions highlight the aim of Ramsey pricing, that is, deriving a set of prices for a range of goods or services so that any deficit which would accrue from economically efficient marginal cost pricing is met. The deficit which would accrue arises

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[4] Baumol and Bradford (1979, p. 268) specify and derive four formulations with respect to prices, some of which are more general than others.

from the fact that the goods or services use common inputs so that 'joint' or 'unattributable' costs are thereby generated. In the presence of marginal cost pricing, total revenue is inadequate to cover total costs. For example, a number of railway services may use a common section of track for which the annual maintenance must be shared amongst the various services. If price were set at the operating or marginal level for each service, total revenue would be insufficient to cover total costs. Ramsey pricing provides a mechanism to cover the joint or unattributable cost and set prices which minimise the distortion from first best levels.

Although Ramsey pricing dominates all other pricing rules on a strict welfare basis, it has been criticised for its distributional consequences (Allen 1986, p. 296; Jansson 1984, pp. 258-60). In addition, the practical considerations involved in deriving estimates of all the relevant elasticities so as to be able to implement Ramsey pricing have in the past been considered onerous. In the following section, a more generalised approach to the use of the Ramsey pricing rule is set out.

The Spatial Equilibrium Model

As indicated above, the aim of this analysis is to incorporate a Ramsey pricing constraint into the framework of a spatial equilibrium model, and then use quadratic programming to solve for the optimal freight rates under such a pricing policy. The use of a spatial equilibrium model is significant because in the past, the spatial nature of transport has not been fully included in the development of pricing rules. However, as Winston (1983) notes the transportation service can be characterised as an output from a multiple product firm, with each different 'commodity trip' having a different origin and destination. This requires the simulation of each trip as a separate output.

The mathematical programming form of the spatial equilibrium model originated with Samuelson (1952), and was more fully developed by Takayama and Judge (1964) using quadratic programming. The framework developed by Takayama and Judge (1971) widened the range of problems to which spatial equilibrium models could be applied and practically solved. Martin (1981), provided a simplified explanation of the modelling techniques developed by Takayama and Judge (1964).

The model presented in this paper represents a short-term, single-period quadratic spatial equilibrium model with emphasis placed on the pricing policy for rail services and a certain degree of cost recovery. A single-period representation was chosen to keep the problem from becoming too complex although in principle there would be no difficulty in extending the model to multiple time periods. It has not been attempted to show a detailed and comprehensive analysis of a transport system. However, in the example presented later, estimates of the required demand and supply curves are simply based on elasticities from various sources. These are considered appropriate for the purposes of this analysis.

The spatial equilibrium problem is generally described and set out as below (Martin, 1981). Two or more regions with known supply and demand functions produce and consume a homogenous product. The regions are separated but not isolated by known transfer costs. The problem is to determine the equilibrium levels of production, consumption, trade and prices in each region (Martin 1981, p. 22). It is assumed that there are n producing and consuming regions (i = 1, 2, ...n), with known supply and demand functions of the following form:

(1) $$y_i = \alpha_i - \beta_i p_i, \quad i = 1, \dots, n$$

(2) $$x_i = \theta_i + \gamma_i p^i, \quad i = 1, \dots, n$$

where p_i and p^i are demand and supply prices respectively, y_i and x_i are demand and supply quantities, α_i and θ_i are the intercepts of the demand and supply functions such that $\alpha_i > 0$, and β_i, γ_i are the slope coefficients of the demand and supply functions such that $\beta_i, \gamma_i > 0$.

For a set of n regions, the demand and supply functions can be expressed in matrix form as:

(3) $$\begin{aligned} y &= \alpha - B p_y \\ &= \alpha - B(\rho_y - w) \end{aligned}$$

(4) $$\begin{aligned} x &= \theta + \Gamma p_x \\ &= \theta + \Gamma(\rho_x + v) \end{aligned}$$

where α and θ are (n x 1) vectors of demand and supply intercepts respectively; B is an (n x n) matrix of the demand slope coefficients β_i and Γ is an (n x n) matrix of the supply slope coefficients γ_i (both may be asymmetric as shown by Takayama and Judge (1971) and Takayama and Uri (1983)); p_y and p_x are (n x 1) vectors of unrestricted demand and supply prices for n regions; y and x are (n x 1) vectors of demand and supply quantities; and v and w are non-negative vectors used to handle the irregular cases as outlined by Takayama and Judge (1971, p. 156). Within the context of the mathematical programming problem the quantities y and x are restricted to be non-negative.

Defining the transport cost between each pair of regions as t_{ij} (that is, the cost of transferring the good from region i to j), and the trade flow between the regions as x_{ij}, the total transport costs can be expressed as:

$$(5) \quad \sum_{i=1}^{n} \sum_{j=1}^{n} t_{ij}x_{ij} = T'X,$$

where $T = [t_{11}, t_{12}, \ldots t_{1n}; t_{21}, t_{22}, \ldots t_{2n}; \ldots ; t_{n1}, t_{n2}, \ldots , t_{nn}]$
and $X = [x_{11}, x_{12}, \ldots x_{1n}; x_{21}, x_{22}, \ldots x_{2n}; \ldots ; x_{n1}, x_{n2}, \ldots , x_{nn}]$.

The standard price form of the spatial equilibrium problem can then be expressed as a quadratic programming problem using a net social monetary gain objective function as below (Takayama and Judge 1971, p. 256). The net social monetary gain objective function consists of the social monetary gain $p_y y$ less the total social production cost $p_x x$ less the total transport cost T'X. This may be conveniently referred to as a net revenue objective function. Thus:

$$(6) \quad \text{Net revenue} = p_y'y - p_x'x - T'X \ .$$

For the price form of the spatial equilibrium model the supply and demand quantities in equation (6) are replaced by the Walrasian market supply and demand function equations (3) and (4) while for the quantity form of the model the prices are replaced by the Marshallian market supply and demand functions (the inverted form of equations (1) and (2)).

The set of constraints to the spatial equilibrium problem need to ensure that the characteristics of a competitive spatial equilibrium are defined such that the supply and demand functions must hold, that the supply and demand quantities and the quantities traded balance, and that the spatially competitive price arbitrage conditions hold (Takayama and Judge 1971; Martin 1981). The primal-dual form of the model in the price domain can be defined as follows:

$$(7) \quad \text{Maximise } G(\rho_y, \rho_x, X, v, w) = (\alpha - B(\rho_y - w))'(\rho_y - w) - (\theta + \Gamma(\rho_x + v))'(\rho_x + v) + X\{T - G'\begin{bmatrix}\rho_y \\ \rho_x\end{bmatrix}\}$$

subject to

(8)	$\alpha - B(\rho_y - w) - G_yX \leq 0$	$(\alpha - B(\rho_y - w) - G_yX)'\rho_y = 0$	Demand function
(9)	$-(\theta + \Gamma(\rho_x + v)) - G_xX \leq 0$	$(-(\theta + \Gamma(\rho_x + v)) - G_xX)'\rho_x = 0$	Supply function
(10)	$-(\alpha - B(\rho_y - w)) \leq 0$	$-(\alpha - B(\rho_y - w))'w = 0$	Non-negative demand price
(11)	$-(\theta + \Gamma(\rho_x + v)) \leq 0$	$-(\theta + \Gamma(\rho_x + v))'v = 0$	Non-negative supply price

(12) $T - G'\begin{bmatrix}\rho_y\\ \rho_x\end{bmatrix} \geq 0$ $\qquad (T - G'\begin{bmatrix}\rho_y\\ \rho_x\end{bmatrix})'X = 0$ Arbitrage condition

(13) $(\rho_y'\ \rho_x'\ X'\ v'\ w') \geq 0$ Non-negativity

The matrices B and Γ are (n x n) matrices of demand coefficients , β_{ij}, and supply slope coefficients, γ_{ij}. These matrices may include non-zero off-diagonal elements and may be non-symmetric. G_x and G_y are (n x n^2) matrices designed to ensure that the sum of product shipments into or out of a region can be equated to the supply and demand quantities and are of the following form:

$$(11)\quad G_y = \begin{bmatrix} 1 & & & 1 & & & & 1 & & \\ & 1 & & & 1 & & & & 1 & \\ & & \ddots & & & \ddots & \cdots & & & \ddots \\ & & & 1 & & & 1 & & & & 1 \end{bmatrix}, \quad (n \times n^2)$$

$$(12)\quad G_x = \begin{bmatrix} -1\ -1 \ldots -1 & & & \\ & -1\ -1 \ldots -1 & & \\ & & \ddots & \\ & & & -1\ -1 \ldots -1 \end{bmatrix}. \quad (n \times n^2)$$

In the constraints (8) to (12), the second set of conditions are 'complementary slackness' conditions ensuring that if the variable concerned is not zero, then the marginal condition contained in the brackets is equal to zero (Lee, Moore and Taylor 1981, p. 130 and pp. 696-698; Takayama and Judge 1971). These conditions are automatically incorporated into the solution algorithm of quadratic programming problems.

The standard problem can be represented in matrix form as follows:

Find ($\bar{\rho}_y'\ \bar{\rho}_x'\ \bar{X}'\ \bar{v}'\ \bar{\omega}'$) $\geq 0'$ that maximises

$$(13)\quad \left[\begin{bmatrix}\alpha\\ -\theta\\ -T\\ -\alpha\\ -\theta\end{bmatrix} - \begin{bmatrix} B & 0 & G_y & -B & 0\\ 0 & \Gamma & G_x & 0 & \Gamma\\ -G_y' & -G_x' & 0 & 0 & 0\\ -B & 0 & 0 & B & 0\\ 0 & \Gamma & 0 & 0 & \Gamma \end{bmatrix}\begin{bmatrix}\rho_y\\ \rho_x\\ X\\ w\\ v\end{bmatrix}\right]' \begin{bmatrix}\rho_y\\ \rho_x\\ X\\ w\\ v\end{bmatrix}$$

subject to

$$(14)\qquad \begin{bmatrix} \alpha \\ -\theta \\ -T \\ -\alpha \\ -\theta \end{bmatrix} - \begin{bmatrix} B & 0 & G_y & -B & 0 \\ 0 & \Gamma & G_x & 0 & \Gamma \\ -G_y' & -G_x' & 0 & 0 & 0 \\ -B & 0 & 0 & B & 0 \\ 0 & \Gamma & 0 & 0 & \Gamma \end{bmatrix} \begin{bmatrix} \rho_y \\ \rho_x \\ X \\ w \\ v \end{bmatrix} \leq 0$$

and

$$(15)\qquad (\rho'_y\ \rho'_x\ \lambda'\ w'\ v') \geq 0'$$

where

$$\rho_y = \begin{bmatrix} \rho_1 \\ \rho_2 \\ \cdot \\ \cdot \\ \cdot \\ \rho_n \end{bmatrix} \geq 0 \qquad \text{and} \qquad \rho_x = \begin{bmatrix} \rho^1 \\ \rho^2 \\ \cdot \\ \cdot \\ \cdot \\ \rho^n \end{bmatrix} \geq 0$$

are non-negative demand and supply price vectors each (n x 1).

The arbitrage condition (12) in this form of the model is the standard competitive arbitrage constraint that the demand price less the supply price must be less than or equal to the cost of transport between the two regions:

$$(16)\qquad p_j - p^i \leq t_{ij}\,.$$

To incorporate Ramsey pricing for transport services this condition must be modified. It will be necessary to price the transport services so that the cost recovery target is met taking into account the derived elasticity of demand for transport services.

The Ramsey Arbitrage Constraint

In the case of independent demand functions, the Ramsey pricing rule can be expressed as follows (Terry, Jones and Braddock 1988, p.265; Baumol and Bradford 1970, p. 270):

$$(16)\qquad (P_1 - MC_1)/(MR_1 - MC_1) = (P_2 - MC_2)/(MR_2 - MC_2) = k$$

where P_i are the prices, MC_i are the marginal costs and MR_i are the marginal revenues and k is a negative constant determined by the size of the cost recovery target. When k = 0, price is equal to marginal cost and the Ramsey rule corresponds to the competitive outcome. This form of the Ramsey rule is equivalent to the pricing rule specified by Braeutigam for his partially regulated second best solution (Braeutigam 1979, p. 44; Baumol and Bradford 1970, p. 270).

To apply the Ramsey pricing rule, it is necessary to derive expressions for the marginal revenue and marginal cost of the transport services. An expression for total revenue and hence marginal revenue from trade can be derived in terms of the spatial prices for the goods being shipped. Consider first a two-region case in which the only trade route considered in this algebraic representation is x_{21}, representing the trade flow from production region 2 to consumption region 1. The excess supply (ES_i) in the producing region is the difference between that region's domestic supply and demand for the commodity, and is represented by ES_2. The excess demand (ED_i), is simply the difference between the domestic demand in region i and its supply.

The excess demand and supply functions for regions one and two may be represented as follows:

(17) $ED_2 = y_2 - x_2 = \alpha_2 - \beta_2 p_2 - (\theta_2 + \lambda_2 p^2)$

(18) $ES_1 = x_1 - y_1 = \theta_1 + \lambda_1 p^1 - (\alpha_1 - \beta_1 p_1)$

where, α_i and θ_i are the intercepts and, β_i and λ_i are the absolute slope coefficients for the two regions. In the two region case, $y_2 - x_2 = x_{12}$ and $x_1 - y_1 = x_{12}$. Thus the excess supply and demand functions can be expressed in terms of the trade flow (x_{12}) as follows:

(19) $x_{12} = \alpha_2 - \beta_2 p_2 - (\theta_2 + \lambda_2 p^2)$ (ED_2)

(20) $x_{12} = \theta_1 + \lambda_1 p^1 - (\alpha_1 - \beta_1 p_1)$ (ES_1) .

In equilibrium the supply and demand prices for each region will normally be equal to each other so it is then possible to write the two equations with their own regional prices. The excess supply function can then be interpreted as containing a supply price, and the excess demand function interpreted as containing a demand price. The two equations may then be rewritten as:

(21) $x_{12} = \alpha_2 - \beta_2 p_2 - (\theta_2 + \lambda_2 p_2)$

(22) $x_{12} = \theta_1 + \lambda_1 p^1 - (\alpha_1 - \beta_1 p^1)$.

These equations may be solved individually for p^1 and p_2 and then used to determine the revenue obtained from the transfer of the goods by the regulated operator of the transport network. Solving in this way:

(23) $p_2 = (\alpha_2 - \theta_2 - x_{12})/(\beta_2 + \lambda_2)$

(24) $p^1 = (\alpha_1 - \theta_1 + x_{12})/(\beta_1 + \lambda_1$.

The revenue from transport between the two regions may be expressed as:

(25) $R = (p_2 - p^1)\, x_{12}$

and the relationships (23) and (24) substituted to obtain:

(26) $R = [(\alpha_2 - \theta_2 - x_{12})/(\beta_2 + \lambda_2) - (\alpha_1 - \theta_1 + x_{12})/(\beta_1 + \lambda_1)]\, x_{12}$

The marginal revenue, MR, may then be derived as

(27) $MR_{12} = p_2 - p^1 - [1/(\beta_1 + \lambda_1) + 1/(\beta_2 + \lambda_2)]\, x_{12}$.

As a matter of convenience and consistent with the empirical model presented later in the paper the marginal cost of transport may be written as a constant

(28) $MC_{12} = \sigma_{12}$.

By rearranging the Ramsey condition (equation (16)) and combining it with the standard arbitrage condition (equation (15)), the arbitrage condition may be re-expressed as:

(29) $p_2 - p^1 \leq k(MR_{12} - MC_{12}) + MC_{12}$.

Substitution for the marginal revenue and marginal cost provides a new price setting condition, thus:

(30) $p_2 - p^1 \leq \sigma_{12} - \frac{k}{(1+k)}\left[\frac{1}{\beta_1 + \lambda_1} + \frac{1}{\beta_2 + \lambda_2}\right] x_{12}$.

To simplify this expression it may be written as

(31) $p_2 - p^1 \leq \sigma_{12} - m\, x_{12}$

where $m = \frac{k}{(1+k)}\left[\frac{1}{\beta_1 + \lambda_1} + \frac{1}{\beta_2 + \lambda_2}\right]$ and $-\infty \leq k < 0$.

From the inequalities (29) and (30) it is clear that the price difference between the two trading regions will be the marginal cost of transport plus a term dependent on the value of k, the slopes of the relevant supply and demand functions (note that k was originally defined as a negative number) and the volume of the good traded from region 1 to region 2.

For the more general case of Ramsey pricing for each possible trade flow then the inequality (31) may be expressed in matrix form as:

$$(32) \quad [G'_y \;\; G'_x]\begin{bmatrix} p_y \\ p_x \end{bmatrix} + MX \leq S$$

where

$$S = [\sigma_{11}, \ldots, \sigma_{nn}]'$$

$$M = \begin{bmatrix} m_{11} & & & & \\ & m_{21} & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & m_{nn} \end{bmatrix}$$

and typically $\sigma_{ii} = 0$ and $m_{ii} = 0$.

Ramsey Pricing in the Spatial Equilibrium Model

The standard spatial equilibrium model and the corresponding price equilibrium is illustrated in Figure 2 with the excess supply and demand functions ES1 and ED2. Transport costs are given as t_{12}. The equilibrium prices after trade takes place are indicated as p_1 and p_2 and since trade flows from region 1 to region 2 arbitrage will ensure that the price, p_2 exceeds the price p_1 by the transport cost t_{12}. The trade from region 1 to region 2 is indicated as x_{12} and is equal to the difference $(x_1 - y_1)$ or $(y_2 - x_2)$, where x_i is the quantity supplied and y_i is the quantity demanded for region i.

The vertical difference between the excess supply and demand functions represents the demand for transport services shown in the second pane of Figure 2. The demand for transport services is a derived demand. Although it is not necessary to assume that the marginal operating cost of transport is constant this is made consistent with the empirical model reported later in the paper and is shown as σ_{12}. Ramsey pricing adds a mark-up to this marginal cost which is a function of the volume shipped x_{12}, the slope of which is determined by the value of k and the coefficients of the commodity supply and demand functions (as reflected in equation (30)). To reflect different levels of cost recovery, the transport charge is indicated at two levels in the second panel of Figure 2 (at k and k'). The point of intersection of these lines with the demand for transport services or the average revenue from transportation provides the equilibrium level of trade and transport charges represented by points a and b on the vertical axis. It is clear that as the level of recoveries increases the volume of trade will decrease.

In the third panel of Figure 2 the solutions obtained for a competitive market and for a monopolist trader are indicated as 'c' and 'm' assuming that for the competitively traded commodities that the transport cost would be set at the marginal cost. It is also assumed that the monopolist trader faces a large transport sector and therefore a given transport cost. With a regulated transport sector charging Ramsey prices as the value of k is made a larger negative number from zero to negative infinity so will the net revenue extracted from the commodity traders to pay for the unallocated costs of the transport sector increase. At a very large negative k value the regulated transport network would receive the same net revenue as a monopolist trader. It is quite apparent, however, that the exploitation of such revenue will be limited by the possibility of the use of alternative modes of transport.

The modified spatial equilibrium formulation for the inclusion of Ramsey pricing can be obtained by substituting the relationship (32) into the competitive formulation by replacing the appropriate arbitrage conditions so that the new problem becomes:

Find $(\bar{y}' \; \bar{x}' \; \bar{X}' \; \bar{\rho}_y' \; \bar{\rho}_x') \geq 0'$ that maximises

$$(33) \quad Z_1 = (y' \; x' \; X' \; \rho_y' \; \rho_x') \left\{ \begin{bmatrix} \lambda \\ v \\ -S \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} \Omega & 0 & 0 & I & 0 \\ 0 & H & 0 & 0 & -I \\ 0 & 0 & -M & -G_y' & -G_x' \\ -I & 0 & G_y & 0 & 0 \\ 0 & I & G_x & 0 & 0 \end{bmatrix} \begin{bmatrix} y \\ x \\ X \\ \rho_y \\ \rho_x \end{bmatrix} \right\}$$

subject to

$$(34) \quad \begin{bmatrix} \lambda \\ -v \\ -S \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} \Omega & 0 & 0 & I & 0 \\ 0 & H & 0 & 0 & -I \\ 0 & 0 & -M & -G_y' & -G_x' \\ -I & 0 & G_y & 0 & 0 \\ 0 & I & G_x & 0 & 0 \end{bmatrix} \begin{bmatrix} y \\ x \\ X \\ \rho_y \\ \rho_x \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$(36) \quad \text{and} \quad (y' \; x' \; X' \; \rho_y' \; \rho_x') \geq 0' .$$

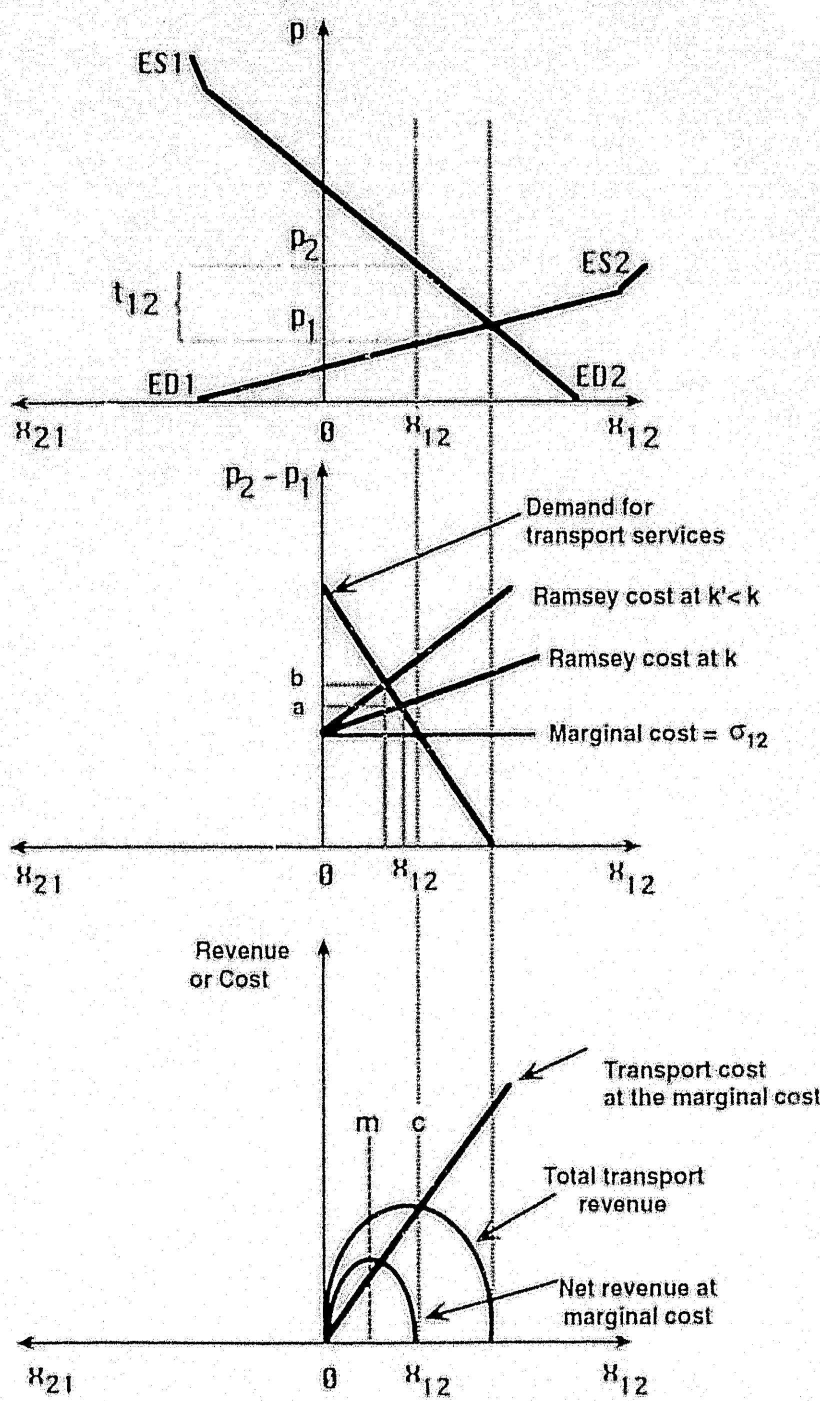

FIGURE 3—Representation of the Derived Demand for Transfer Services and Ramsey Pricing.

For most practical models it is possible to reduce the size of the above formulation by omitting the variables w and v provided that all of the regional prices will remain non-negative for all of the model solutions.

Application of the Model

The model derived above has been applied to the North-West rail line, stretching from Moree to the port of Newcastle, a physical distance of almost 500 kilometres. This railway line was chosen as both wheat and coal compete for rail space along it. Wheat transportation and coal transportation being the two major services supplied by the rail authority. Zero cross-price elasticities of demand for the services were assumed. That is, the demand for wheat transport services is assumed to not be affected by the price of coal transport services and vice versa. It was also assumed there were no capacity constraints on the rail line.

It has been assumed that the supply of coal at the mine level and wheat at the farm level is exogenous. Further, it is also assumed that there is no demand for either commodity in the production region. Hence, the supply curve for each region is an excess supply. Three regions are engaged in the production and trade of wheat, namely Moree, Gunnedah and Werris Creek, and two regions produce coal, namely the Hunter and Newcastle.

Similarly, the demand for each commodity at the port of Newcastle is assumed to be exogenous. Demand at the port of Newcastle is represented by the export demand for each commodity. As there is no production of coal or wheat in Newcastle, the export demand curve will represent the excess demand for that region. The direction of the trade flow for both commodities is thus one way, from the producing regions to the consuming region (the port of Newcastle).

The constraints implied by inequalities (8) and (9) above ensure that the quantities supplied must equal or exceed the given demand quantity, and that the quantities shipped cannot exceed the available supply.

In order to undertake the analysis several basic items of data were required. These included:

- estimates of per tonne operating costs for the rail transport of wheat and coal;
- the average f.o.b. price for wheat and coal at the port of Newcastle;
- elasticity estimates for the supply and export demand for wheat and coal;
- quantities supplied from each of the producing regions considered for each commodity.

The demand for transport in this model is given by the difference between the export demand for the commodity and its regional supply.

• *Rail costs*—The estimation of per tonne rail costs was based on cost estimates provided in Blyth *et al.* (1987). The technical information required to determine the total cost levels were obtained from the Bulk Freight section of the State Rail Authority (personal communication, John Dawes, Bulk Freight). Two locomotives hauling thirty-nine wagons, each with a capacity of fifty-five tonnes was the standard train configuration used for grain haulage. This varied for coal transport where the train size was eighty-four wagons with a load capacity of seventy-four tonnes per wagon. A crew shift time of eight hours was assumed in determining the number of crew shifts per journey. For coal, the average distance of the five largest mines in each region to Newcastle was used to derive the distance to port and hence the round trip time (personal communication, Emma Goddard, Freight Rail). It was assumed that there were two train drivers on each journey.

Locomotives and wagons are fairly well associated with the transport of a specific commodity and so these costs can be directly allocated to a specific traffic flow. Track costs are a joint cost which cannot be allocated exactly to a specific traffic flow. Nevertheless, an estimate of 70 cents per thousand gross kilometres for track maintenance was taken into account (Royal Commission into Grain Storage, Handling and Transport 1988). Due to the lack of available cost information for rail transport of coal, the costs for wheat transport, as estimated by Blyth *et al.* (1987) and the Royal Commission (1988) were considered to be applicable. This assumption is based on the fact that each commodity has similar volumes of per cubic metre. The rail cost estimates were used to determine farm and mine level prices for wheat and coal, and also to derive the marginal cost curves for rail transport.

The estimated total rail transport cost equations for each region per trip are as follows:

Moree:	$TC_{w3} = 7691.1 + 6.5852\, x_{ij}$
Gunnedah:	$TC_{w2} = 5457.5 + 4.2692\, x_{ij}$
Werris Creek:	$TC_{w1} = 3577.8 + 3.1908\, x_{ij}$
Hunter region:	$TC_{c2} = 2239.9 + 1.4742\, x_{ij}$
Newcastle region:	$TC_{c1} = 1371 + 0.81863\, x_{ij}$

The average transport costs per tonne for a fully laden train between the production region and Newcastle were derived from the above as follows:

$t_{w31} = \$\ 10.185$

$t_{w21} = \$\ 6.823$

$t_{w11} = \$\ 4.864$

$t_{c21} = \$\ 1.839$

$t_{c11} = \$\ 1.042$.

•*Prices*—The price per tonne of wheat and coal at each supply region was assumed to be the f.o.b. price less wharfage charges and estimated transport costs. The f.o.b. prices were assumed to be $175 and $52.84 respectively for wheat and coal (MacAulay, Batterham and Fisher 1989; Joint Coal Board 1991). These are average per tonne prices and do not distinguish between different types of coal or grades of wheat. The prices for each supply region were determined by subtracting estimated transport costs and wharfage charges from the f.o.b. price at Newcastle.

•*Quantity*—The Australian Wheat Board provided information on the receivals at all sites supplying the port of Newcastle in a 'representative' year. The production from smaller towns surrounding Moree, Gunnedah and Werris Creek, were aggregated to represent one production figure for each respective region. These production data were used to represent quantities for the purpose of estimation of linear demand and supply functions. The Newcastle figure represents out loadings (exports), rather than receivals, at the port of Newcastle.

Domestic consumption was assumed to represent 15 percent of total production, hence this quantity was subtracted and not used in the model (personal communication, Australian Wheat Board). More detailed information regarding stocks held and mill demands was not available due to its commercial sensitivity. Although the data may not be particularly accurate, it does serve the purpose of enabling an export demand and regional supply curves to be approximated.

For coal, the receivals recorded at the port of Newcastle represent the export quantity. Production from mines within the Hunter and Newcastle regions were aggregated to obtain a total production figure for each region.

•*Elasticities*—The elasticity of the supply of wheat at the port of Newcastle was taken from an average of estimates determined in four earlier studies (Wicks and Dillon 1987; Powell and Gruen 1966; Myers 1982; Hall and Menz 1985). The elasticity for coal used was 0.4 (Beck, Jolly and Loncar 1991). The demand elasticity for wheat at Newcastle is from an estimate used by MacAulay, Batterham and Fisher (1989). Finally, the demand elasticity for coal was assumed to be -5.0, based on an estimate from the Industry Commission (1991b). The data used in the derivation of the supply and demand curves is set out in Table 1.

TABLE 1
Basic Data Used for the Estimation of Supply and Demand Functions

Commodity	Site	Receivals (000 tonnes)	Price[b] ($/ tonne)	Elasticity
		Export Demand[a]		
Wheat	Newcastle	1581	173.22	-48.9
Coal	Newcastle	36604	51.06	-5.0
		Regional Supply		
Wheat	Moree	329	163.03	0.8
	Gunnedah	366	166.40	0.8
	Werris Ck	886	168.36	0.8
Coal	Hunter	23557	49.22[c]	0.4
	Newcastle	13047	50.02[c]	0.4

[a] Represents total exports from the port of Newcastle.
[b] F.o.b. price of wheat and coal at the port of Newcastle, was assumed to be $175 and $52.84, respectively. The wharfage charge for wheat is assumed to be $1.78.
[c] As wharfage rates for coal where not available the wharfage charge for wheat has been applied.

The estimated linear excess supply functions for wheat and coal derived for each producing region take the form $x_{12} = \alpha_2 - \beta_2 p_2$ and are shown below:

Moree:	$Q_{w3} = 65.8 + 1.61p^{w3}$
Gunnedah:	$Q_{w2} = 73.2 + 1.76p^{w2}$
Werris Creek:	$Q_{w1} = 230.8 + 5.48p^{w1}$
Hunter Region:	$Q_{c2} = 14134.2 + 191.44p^{c2}$
Newcastle Region:	$Q_{c1} = 7828.2 + 104.34p^{c1}$

where the superscripts refer to wheat (w) and coal (c). For each commodity, the supply regions face the same demand function, namely the export demand for the commodity at the port of Newcastle:

Wheat:	$Q_W = 78392.2 - 443.49p_{w1}$
Coal:	$Q_C = 219624 - 3584.41p_{c1}$.

Results

As noted previously, when the Ramsey number (k) is set equal to zero the Ramsey pricing rule is equivalent to marginal cost pricing. For lower values of k, total revenue from the transport services offered increases. Moreover, total revenue generates a surplus over the short-run marginal or operating costs. This surplus can be used to cover the system wide joint or unattributable costs of the services offered.

The output from the model for a value of k = -0.003 is shown in Table 2.

TABLE 2
Solution of Spatial Equilibrium Model with Ramsey Pricing and the Value of k = -0.003[a]

Item	Wheat			Coal		Total
	Moree	Gunnedah	Werris	Hunter	Newcastle	
Ramsey quantity (tonnes)	333.78	369.34	880.19	23553.63	13030.78	
Ramsey supply price ($/tonne)	165.99	168.30	169.37	49.20	49.86	
Ramsey demand price at port ($/tonne)	173.20	173.20	173.20	51.07	51.07	
Ramsey transport charge ($/tonne)	7.21	4.90	3.83	1.86	1.20	
Marginal cost ($/tonne)	6.5852	4.2692	3.1908	1.4742	0.81863	
Operating cost ($)	2197995	1576767	2808505	34722764	10667390	51973421
Total revenue ($)	2405150	1809685	3370424	43853235	15676553	67115047
Net operating surplus ($)	207155	232917	561918	9130471	5009163	15141626
Transport demand slope	6.17E-01	5.66E-01	2.38E-01	4.94E-03	9.31E-03	
Elasticity of transport demand	3.50E-02	2.34E-02	1.83E-02	1.60E-02	9.92E-03	
Percent mark-up on MC	9.42	14.77	20.01	26.30	46.96	
Ratio of price to MC	1.095	1.1477	1.200	1.262	1.466	

[a]The quadratic programming tableau used to obtain these solutions is given in Appendix A.

Hence, for k = -0.003, a surplus of $15.141 million over and above short-run operating costs is generated. The value of k can be adjusted so as to cover various levels of the joint or fixed costs of operating the system. In Figure 3 the relationship between total revenue less operating costs as the value of k changes is shown. The joint costs of operating the system may include provision for maintenance and depreciation of jointly used assets amongst the services. Notice too, that in accordance with the Ramsey principle, the mark-up over marginal cost for the transport service is inversely proportional to the price elasticity of demand for transport.

It should be stressed that it is not suggested that the net operating surplus available to the railway could be increased indefinitely simply by increasing the value of k. Rather, the maximum profit would occur at the monopolist solution where marginal cost is set equal to marginal revenue for each service. Moreover, the price which could be set for any one service would be constrained by the charges set by competing modes. Subject to availability, road freight charges would provide an effective cap to the prices which could be charged for rail services.

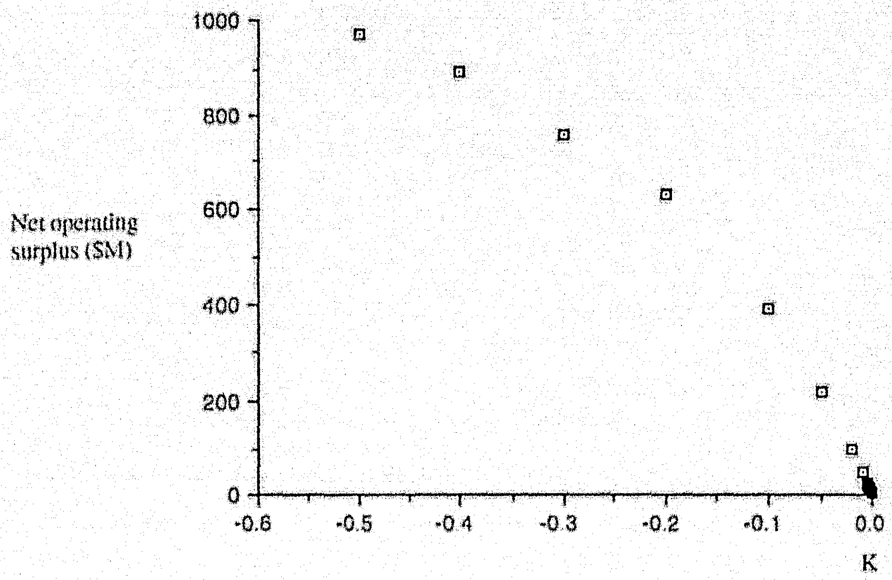

FIGURE 3—Net Operating Surplus Resulting from Ramsey Pricing.

Conclusions and Further Research

An application of Ramsey pricing to the setting of prices for rail services over a network characterised by joint costs has been developed in this paper. The prices generated are consistent with the inverse elasticity rule of Ramsey pricing. Ramsey prices have been derived on previous occasions (Taplin and Waters 1985), but not within the context of a spatial equilibrium system which takes into account a network of trade flows.

The areas for further analysis of the present approach are numerous. In particular, it is apparent that better estimates of the supply and demand elasticities are necessary, and hence improved estimates of the supply and demand equations in the spatial model. Further work is required on the specification of the costs for the railway services. Work in this respect may include the use of quadratic supply, demand and cost relationships, and adaption of the spatial system to accommodate them (MacAulay, Batterham and Fisher, 1989).

The significant advantage of the approach outlined in this paper over previous attempts to specify Ramsey prices is its flexibility. The arbitrage condition in the spatial model can be altered relatively easily to incorporate other pricing rules such as a constant or proportional mark-up on marginal cost. The effects of these policies on each of the markets can then be determined. In addition, the spatial equilibrium model can be readily applied to other situations in which joint or unattributable costs must be recovered over a network. Further work is required to adequately reflect inter-modal competition in the model which because of the assumptions made, was effectively ignored in the present paper. Work in this area requires that the model be adapted so that competitive transport rates are determined endogenously for a road system and regulated rates apply to the rail system. In addition, the possibility of non-independent demands amongst the various services offered by any given mode and amongst modes should be incorporated into the model. These changes may complicate the application of Ramsey pricing principles, but should lead to a more realistic representation of the transport system.

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Appendix A

TABLE A.1
Tableau for the Ramsey Pricing Problem with k=-0.003

	DPW1	SPW31	SPW21	SPW11	DPC1	SPC21	SPC11	XW31	XW21	XW11	XC21	XC11	REL	R-S
DPW1	443.49							1	1	1			N	
SPW31		1.6144						-1					N	
SPW21			1.75465						-1				N	
SPW11				4.1625						-1			N	
DPC1					3584.41						1	1	N	
SPC21						191.43784					-1		N	
SPC11							104.34					-1	N	
XW31	-1	1						0.00185946					N	
XW21	-1		1						0.00171137				N	
XW11	-1			1						0.00072531			N	
XC21					-1	1					1.6458E-05		N	
XC11					-1		1					2.9501E-05	N	
OBJ	-78392.9	65.8	73.2	175.2	-219624	14134.2	7828.2	6.5852	4.2692	3.1908	1.4742	0.81863	N	
RDPW1	-443.49							-1	-1	-1			L	-78392.9
RSPW31		-1.6144						1					L	65.8
RSPW21			-1.75465						1				L	73.2
RSPW11				-4.1625						1			L	175.2
RDPC1					-3584.41						-1	-1	L	-219624
RSPC21						-191.43784					1		L	14134.2
RSPC11							-104.34					1	L	7828.2
RXW31	1	-1						-0.0018595					L	6.5852
RXW21	1		-1						-0.0017114				L	4.2692
RXW11	1			-1						-0.0007253			L	3.1908
RXC21					1	-1					-1.646E-05		L	1.4742
RXC11					1		-1					-2.95E-05	L	0.81863

Note: In the column names DP refers to demand price, SP to supply price, X to shipments, W to refers to wheat, C to coal and the supply regions for coal are numbered 1 to 2 and for wheat 1 to 3. Row names are indicated with an R as the first character. Shipments between two locations are indicated with two numbers such as 21 implying shipment from location 2 to location 1. OBJ refers to the linear objective function and RHS to the right hand side. A ≤ constraint is indicated as L and N refers to an objective function row.