Measuring Input Substitution and Output Expansion Effects: A Nonparametric Approach*

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In this paper a new framework is developed for measuring input substitution and output expansion effects. The major advantages of the proposed approach include that (i) specification and/or estimation of any parametric functions are not required; (ii) the framework is operational as long as two or more observations on inputs and input prices are available; and (iii) more insightful information can be obtained in comparison with the conventional approach. An empirical application of our approach to Japanese manufacturing data yields results which satisfy a priori expectations.

1 Introduction

Studying input substitution and output expansion effects[†] is of considerable importance in production theory (Ferguson 1968; Sakai 1974), trade theory (Finger 1969; Grubel and Lloyd 1971), price theory and welfare economics (Hughes 1981) and macroeconomics (Kreinin, Ramsey and Kmenta 1971). In particular, if factors are substitutable for one another, policies can be designed to adjust resource or income allocation while maintaining or expanding total output. To implement such kinds of policies requires knowledge of the directions as well as magnitudes of the substitution and output expansion effects. Quantification of these effects is also essential for evaluating the impacts of any changes in policy and other parameters in an economy that influence input prices and producers investment behaviours. For example, the analysis of substitution and output expansion effects resulting from

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[†] The concept of output expansion effect used in this paper is alternatively termed scale or output effect elsewhere, see Nagatani (1978) and Lopez (1984).

energy price changes are important not only to a nation's well-being in the long run, but also potentially have implications for global political stability.

Previous studies on factor substitutions are likely to suffer from the many usual problems associated with model specification and estimation. As is widely known, policy conclusions drawn from such studies are sensitive to functional form specification and perhaps estimation techniques. Moreover, there is often no clear-cut basis as to which form and estimation technique are the best choices despite rapid development of various test procedures in econometric theory in the past decade or so. To overcome these deficiencies inherent in conventional approaches, a new framework is proposed in this paper, where parametric specification of production or profit functions is not required.

The paper is organised in the following sequence. In the next Section, basic concepts relevant to substitution and output expansion effects are briefly discussed. The new approach is presented in Section 3. An empirical application using Japanese manufacturing data on labour (L) and capital (K) is undertaken in Section 4. Finally, Section 5 concludes the paper.

2 Input Substitution and Output Expansion Effects: Basic Concepts

The concepts of input substitution and output expansion effects can best be explained with an aid of a diagram, such as Figure 1. In the Figure, X_r represents an input vector with prices P_r and $I(Y_r)$ denotes an isoquant with output level Y_r $(r=1,2)$. Suppose the producer under examination faces an outlay constraint C_1 or $P_1X_1 + P_2X_2 = c_1$, where c_1 is the total production budget available to the producer, the equilibrium point will be at X_A in Figure 1. Now, without loss of generality, assuming P_1 decreases relative to P_2, say becoming P_1^* and P_2^*, the new equilibrium point on $I(Y_1)$ will be moved to X_B. The increment of X_1 from the actual values x_1 to x_1^* is defined as the substitution effect. The iso-outlay curve corresponding to the new equilibrium point X_B can be represented by C_1^* or $P_1^* X_1 + P_2^* X_2 = c_1^*$, where c_1^* denotes the actual production cost at X_B. It is a rule rather than exception that $c_1^* \neq c_1 = P_1 x_1 + P_2 x_2$. For the purpose of illustration, it is assumed here that $c_1 > c_1^*$. Since the purchasing power of the producer has increased after the relative prices change (total nominal production budget remains the same), the production can be expanded to point X_D in Figure 1, where the iso-outlay line $C_1^d = P_1^* X_1 + P_2^* X_2$ is tangent to an isoquant with output level $Y' > Y_1$ and $P_1^* x_1^d + P_2^* x_2^d = P_1 x_1 + P_2 x_2 = c_1$. The difference, $x_1^d - x_1^*$, is defined as the expansion effect resulting from relative price changes without changing the nominal amount of the total outlay.

It is obvious that to identify and measure any substitution and expansion effects requires the location of points X_A, X_B and X_D in the input space. While X_A is observable as an initial input vector producing output Y_1, points X_B and X_D can not be observed in reality simply because relative input prices and the nominal outlay are most likely to change simultaneously. What can be observed is: in the initial period (or for firm 1), the producer applies inputs x_1, x_2 to produce output Y_1 with the input prices P_1, P_2 and outlay c_1; in the next period (or for firm 2), the producer applies inputs x_1'', x_2'' to produce $Y_2 \neq Y_1$ with the input prices P_1'', P_2'' and total outlay $c_2 \neq c_1$. The observed equilibrium point in the second period (or for firm 2) is, say, at X_E in Figure 1. Therefore, another concept is needed to define the change in inputs from points X_D to X_E; let this be termed the outlay effect as it is resulted from the nominal outlay changes only.

To summarise and generalise to the k-factor ($k \geq 3$) case, in practice only two points in a k-dimensional input space, namely X_A and X_E are observed. The data associated with point X_A relate to the input vector $X_A = (x_{a1}, x_{a2}, \cdots, x_{ak})$, the price vector $P_A = (p_{a1}, p_{a2}, \cdots, p_{ak})$, and output Y_1. The data associated with X_E include the input vector $X_E = (x_{e1}, x_{e2}, \cdots, x_{ek})$, the price vector $P_E = (p_{e1}, p_{e2}, \cdots, p_{ek})$, and output Y_2.

From these data, iso-outlay lines C_1 (which passes through X_A) and C_2 (which passes through X_E) can be drawn. By shifting line C_2 downward until it is tangent to $I(Y_1)$, point X_B can be obtained, only conceptually at this stage. The inputs at X_B are denoted by the vector $X_B = (x_{b1}, x_{b2}, \cdots, x_{bk})$. The input changes from X_A to X_B are the conventional substitution effects $(= X_B - X_A)$. They can be used to construct measures such as the marginal rate of technical substitution and the elasticity of substitution. From X_B to X_D, the input changes contain the usual expansion effects $(= X_D - X_B)$. They indicate the factor demand variations that result from changes in purchasing power following relative prices change, holding total nominal cost constant. Finally, the changes in inputs from X_D to X_E or $X_E - X_D$ represent the outlay effects corresponding to changes in the total nominal outlay alone. The sum of the usual expansion effects and the outlay effects is defined as the output expansion effects in this paper. Since, by construction, the relative prices at X_B, X_D and X_E are identical, this sum $(= X_E - X_B)$ reflects the effects on input demands of the overall change in the purchasing power only. It is shown in Sakai (1974) that output expansion effects are positively related to output growth. In other words, positive (negative) output expansion effects always imply output expansion (contraction). Clearly, the total changes in inputs $(X_E - X_A)$ are equal to the substitution effects $(X_B - X_A)$ plus the output expansion effects $(X_E - X_B)$.

Thus, the task reduces to obtaining X_B in an input space given observations on inputs and their prices at X_A and X_E.

3 Measuring Input Substitution and Output Expansion Effects

It is impossible to locate the precise position of X_B without knowing the *true* parametric production function underlying the production process. However, an approximation to the segment of the isoquant surface between X_A

and X_B by a hyperplane can be made. According to Varian (1984, p.317), the *best* linear approximation to the local isoquant surface near X_B is given by the following condition:

$$\nabla f(X_B) \cdot \Delta X' = 0, \tag{1}$$

or equivalently,

$$\sum_{j=1}^{k} \alpha_j (x_{b_j} - x_{a_j}) = 0, \tag{2}$$

where $\Delta X' = X_B - X_A$, $f(X)$ denotes the true but unknown production function; $\nabla f(X_B)$ denotes the gradient of the production function at X_B and α_j is the j-th directional cosine of the gradient.

Next we assume that in the short run, say from year to year, production expands along the gradient direction of the underlying production function. This assumption implies that the producer under study would expand his/her production of output at the maximum rate for any increases in the total production cost (say, from c_1^* to c_2), holding relative prices constant. Clearly, this is not a terribly unrealistic assumption. Under this assumption it must be true that

$$\frac{x_{e_j} - x_{b_j}}{\alpha_j} = \frac{x_{e_i} - x_{b_i}}{\alpha_i}, \quad i, j = 1, 2, \cdots, k. \tag{3}$$

Using the identity $\sum_i \alpha_i^2 = 1$, equations (2) and (3) can be simultaneously solved for the unknown x_{b_i} to yield

$$x_{b_i} = \alpha_i \sum_j \alpha_j (x_{a_j} - x_{e_j}) + x_{e_i}, \quad i = 1, 2, \cdots, k. \tag{4}$$

Once α_i $(i = 1, 2, \cdots, k)$ are computed, $X_B = (x_{b_1}, \cdots, x_{b_k})$ can be obtained and used for calculating the substitution and output expansion effects.

To obtain α_j, it is useful to note that X_B is at the equilibrium point on the old isoquant surface under the same relative factor prices as at X_E. Thus, we have

$$\frac{\partial f(X_B)/\partial x_{b_j}}{\partial f(X_B)/\partial x_{b_i}} = \frac{p_{e_j}}{p_{e_i}} \tag{5}$$

for $i, j = 1, \cdots, k$. Moreover, α_j is the j th directional cosine of the gradient of the production function at X_B. According to the definition of directional cosine and using (5),

$$\begin{aligned}
\alpha_j &= \frac{\partial f(X_B)/\partial x_{b_j}}{|\nabla f(X_B)|} \\
&= \frac{\partial f(X_B)/\partial x_{b_j}}{\sqrt{\sum_{i=1}^{k}(\partial f(X_B)/\partial x_{b_i})^2}} \\
&= \frac{1}{\sqrt{\sum_{i=1}^{k}(\partial f(X_B)/\partial x_{b_i})^2/(\partial f(X_B)/\partial x_{b_j})^2}} \\
&= \frac{p_{ij}}{\sqrt{\sum_{j=1}^{k} p_{ij}^2}} \qquad (6)
\end{aligned}$$

can be obtained, where $|\nabla f(X_B)|$ denotes the norm of the gradient of the production function at X_B.

4 Capital-labour (K-L) Substitution in Japanese Manufacturing

A considerable amount of work has been published on the subject of capital labour substitution. It is most likely that the results, as well as the conclusions drawn from those studies, will be altered should different functional forms or estimation techniques be employed (see Ferguson 1963; Sato 1977; Diamont *et al.* 1978). It is this problem that warrants our approach.

A set of data on labour and capital use (in current yen) in Janpanese manufacturing sector are readily available from Norsworthy and Malmquist (1983). The implicit quantity of each input is obtained by dividing the input in current yen by the corresponding price index (the base year = 1965). The data are presented in Table 1.

[Table 1 near here]

The major purpose of this section is to illustrate the use of the framework developed in the previous section. Attention will be focused on the changes in the substitutability of capital for labour over the period from 1965-66 to 1977-78. Since the input quanties are all in comparable value terms, the marginal rate of substitution is a good indicator of substitutability.

The first step is to use data on factor prices or their indices to compute α_s based on equation (6). Then X_B can be obtained according to (4). The substitution effects are then given by $X_B - X_A$, and the output expansion effects given by $X_E - X_B$.

The results, as reported in Table 2, are quite plausible. From these results, it is clear that Japan had been constantly substituting capital for labour since the mid 1960s except for 1974-75. From the marginal rates of substitution (the ratio of the substitution effects) given in the Table it is apparent that it had become increasingly difficult to substitute capital for labour in Japanese manufacturing. For example, while a yen spent on capital could be used to substitute for 0.92 yen of labour in 1965-66, it could only substitute for 0.19 yen of labour in 1977-78. There was a strong decreasing trend in the substitutability, as shown in Figure 2. This finding, together with the rising labour cost relative to capital cost in Japan can help explain why Japan has relocated many companies into less developed countries in recent decades.

[Table 2 near here]

[Figure 2 near here]

Based on the output expansion effects reported in Table 2, it can be concluded that output of Japanese manufacturing sector had been growing constantly with the exception of 1974-75. However, the increased outputs were produced by utilising increasingly capital-intensive technologies. This is indicated by the decreasing labour-capital ratios associated with the output expansion effects (Figure 3).

[Figure 3 near here]

The energy crisis resulted in the only output contraction between 1974 and 1975 in Japanese manufacturing over the sample period. This contraction was reflected by significant drops in capital input (−3657.68) and in labour input (−457.97). Positive substitutions of labour for capital ocurred immediately following the energy crisis. This is consistent with the findings of Hartman (1976), Holthausen (1976) and Ghosal (1991). These authors showed that the capital-labour ratio is negatively related to economic uncertainty. The energy crisis caused a big shock and unprecedented uncertainty in Japan, thus it is reasonable for manufacturers to substantially increase labor input and meanwhile reduce capital input. From another perspective, the unusual reversal in labor-capital substitution right after the crisis can be attributed to that labour was probably used for substituting for energy which, in turn, led to its substitution for capital. This is because energy and capital are likely to be complements, according to Berndt and Wood (1979).

To further show the plausibility of the empirical results and thus the proposed framework, the elasticity of substitution (denoted by δ) between labour and capital can be estimated parametrically. Assuming that the elasticity is constant, the estimate is simply equivalent to the slope coefficient from a simple regression where the logarithms of L/K (Table 1) are regressed on the logarithms of $\Delta L/\Delta K$ (Table 2). See Nadiri (1982, p. 441). The estimated equation has a very high $R^2 = 0.97$, which indicates that ninty-seven percent of the variation in the capital-labour ratio can be explained by the relative factor price changes (noting that the substitution effects, ΔL and ΔK, are completely cuased by relative factor price changes). The estimate of δ is 0.82 with a t-ratio of 19.23. The estimation results are consistent with the finding of Arrow et al. (1961) that the elasticity of substitution between capital and labour in manufacturing is less than unity.

5 Conclusion

A framework has been developed in this paper for quantifying input substitution and output expansion effects. When applied to the Japanese manufacturing data, the results are quite plausible and consistent with normal expectations. Apart from the sensitivity problem mentioned earlier, it is difficult, if possible at all, to obtain the same insightful results (for example, examining changes in the substitutability of inputs, analysing output expansion effects) when conventional parametric production function approaches are utilised.

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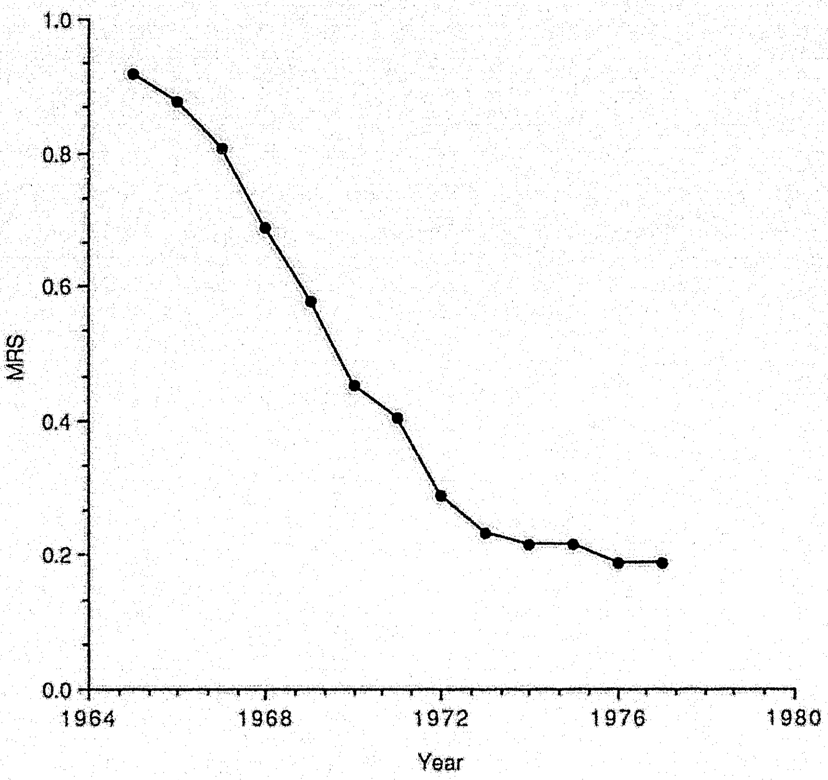

Figure 2: Marginal rate of labour-capital substitution in Japanese Manufacturing

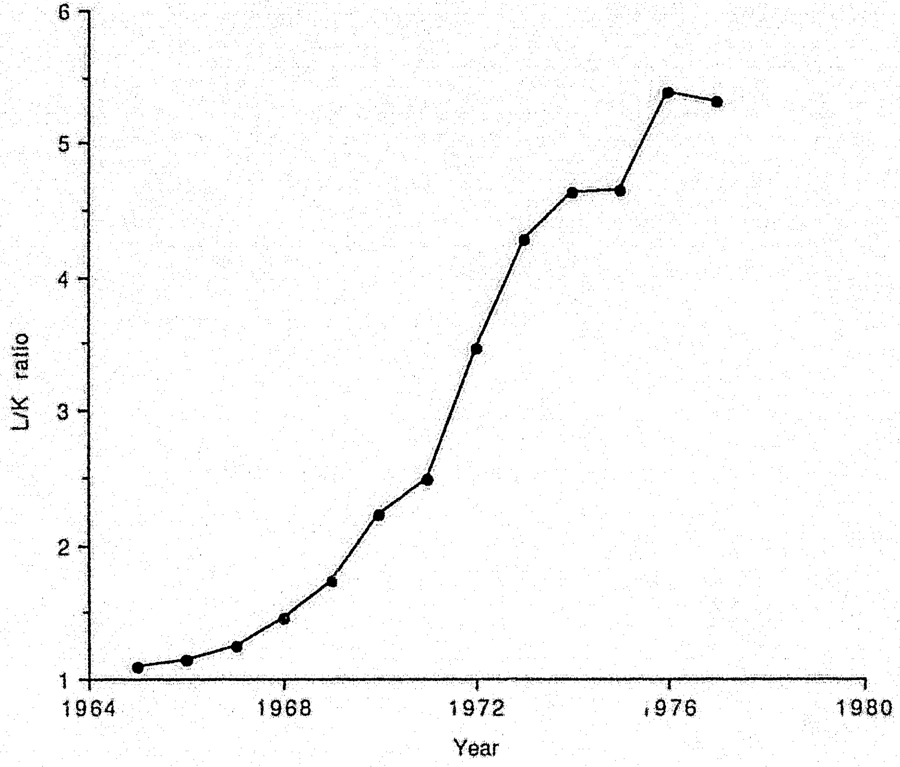

Figure 3: Labour-capital ratio acompanying the output expansion effects in Japanese manufacturing

Table 1 Labour and capital inputs in Japanese manufacturing
(in billion yen)

	Implicit Quantity		Price Index		
Year	Labour (L)	Capital (K)	Labour	Capital	L/K Ratio
1965	2627.58	5853.44	1.00000	1.00000	0.45
1966	2702.32	6735.35	1.11224	1.01786	0.40
1967	2791.65	8078.50	1.25100	1.09347	0.35
1968	2872.66	8928.63	1.45053	1.16768	0.32
1969	2957.13	10813.23	1.72047	1.18522	0.27
1970	3001.76	13251.40	2.03919	1.17285	0.23
1971	2902.67	15290.10	2.34935	1.05911	0.19
1972	2904.06	15832.72	2.71385	1.09019	0.18
1973	2932.19	21982.20	3.38159	0.97962	0.13
1974	2673.68	24761.88	4.50394	1.05282	0.11
1975	2486.11	19855.93	5.15371	1.11637	0.13
1976	2704.59	20973.50	5.59482	1.20521	0.13
1977	2622.31	21564.24	6.10874	1.13529	0.11
1978	2578.27	25534.42	6.43850	1.21577	0.10

Source: Calculated from Norsworthy and Malmquist (1983)

Table 2 Substitution and output expansion effects in Japanese Manufacturing

Year	Substitution Effects		Output Expansion Effects		MRS_{LK}
(t/t+1)	Labour (ΔL)	Capital (ΔK)	Labour	Capital	$-\Delta L/\Delta K$
1965/66	-405.17	442.73	479.90	439.18	0.92
1966/67	-626.84	717.15	716.18	625.99	0.87
1967/68	-383.40	476.28	464.41	373.85	0.80
1968/69	-853.27	1238.61	937.74	646.00	0.69
1969/70	1042.65	1812.81	1087.28	625.36	0.58
1970/71	-780.57	1731.49	681.47	307.21	0.45
1971/72	-187.50	466.74	188.90	75.88	0.40
1972/73	-1641.35	5665.85	1669.48	483.63	0.29
1973/74	-629.49	2692.96	370.98	86.72	0.23
1974/75	1006.67	-4647.27	-1194.24	-258.69	0.22
1975/76	-220.38	1023.04	438.86	94.54	0.22
1976/77	-647.80	3485.64	565.51	105.10	0.19
1977/78	-178.41	944.81	134.37	25.37	0.19

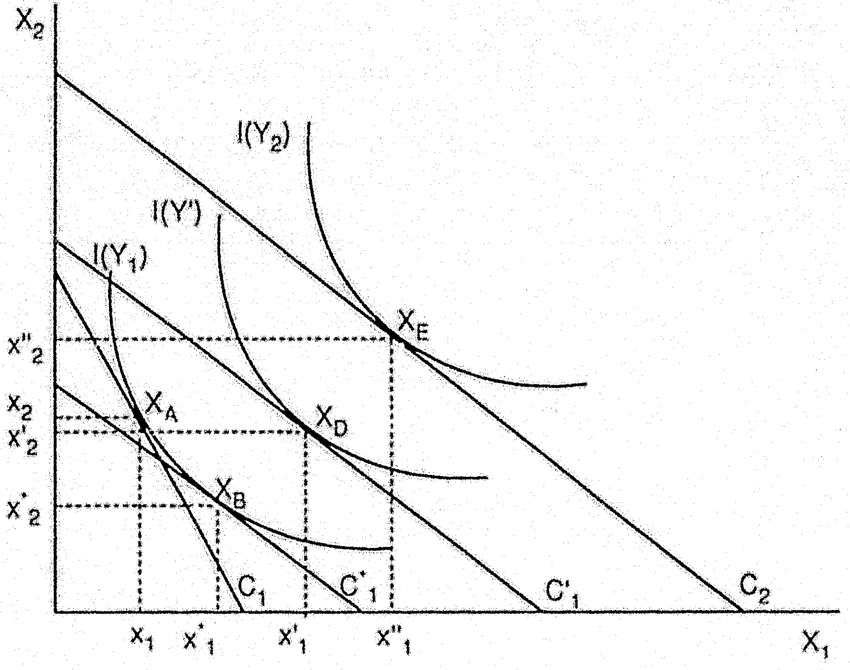

Figure 1: Substitution (X_A to X_B) and output expansion (X_B to X_E) effects