Use of Paasche and Laspeyres Variations to Estimate Consumer Welfare Change

By Dennis C. Cory, Russell L. Gum, William E. Martin, and Marie Leigh[*]

Abstract

Many measurements of consumer welfare have been proposed to reflect the income equivalent of a welfare change Measures used in policy evaluations are compensating variation, equivalent variation, Paasche variation, Laspeyres variation, and consumer's surplus This research presents an empirical test for differences among these five welfare measures in a dynamic multimarket context We argue that in many policy evaluations, adequate and rigorous information for decisionmaking is supplied by estimates for the easily calculated Paasche and Laspeyres variations, and consumer's surplus measures

Keywords

Consumer welfare change, Paasche variation, Laspeyres variation, consumer's surplus, policy evaluation

Introduction

Almost all changes in agricultural policy affect consumers directly or indirectly For example, a ban on insecticides in corn production directly affects prices and quantities of products for which corn is an input as well as prices and quantities of substitute commodities This research compares and evaluates alternative measures of consumer welfare for this example

In a classic series of articles appearing in the *Review of Economic Studies*, J R Hicks (6, 7, 9) examined the relationships among alternative measures of consumer welfare change and delineated the conditions under which a particular measure would be appropriate [1] Hicks analyzed the theoretical foundations of compensating variation (CV), equivalent variation (EV), and consumer's surplus (CS) measures of welfare change as well as their relationship to the more readily computable measures of Paasche (PV) and Laspeyres (LV) variations [2]

Theoretical interest in alternative measures of welfare change characterizes much of the welfare literature Although opinions are diverse and at times adversarial, analysts agree that CV and EV constitute a conceptually valid basis for cost-benefit analysis (14, 15) [3] In addition, Willig (20) has demonstrated that consumer's surplus can be reliably used in many applied studies as a measure of welfare change

However, a review of this literature leaves analysts with no clear-cut procedure for estimating the consumer impacts of policy proposals Although the convenience of using PV and LV in estimating consumer welfare change is appealing, theoretical considerations suggest that the additional analysis required for calculating appropriate CV and EV estimates is also necessary Much confusion still exists, although 40 years ago, Hicks considered the convenience-accuracy trade-offs and concluded

> There is no theoretical objection to this sort of adjustment, but it is a fiddling business, fortunately not likely to be of much importance (9, p 109)

He later wrote

> the distinctions we have been making will, in the vast majority of cases, be of very little importance (7, p 40)

[*]Cory is an assistant professor of agricultural economics, University of Arizona, Gum is an agricultural economist with the Natural Resource Economics Division (NRED), ESS, and an adjunct professor of agricultural economics at Oregon State University and the University of Arizona, Martin is a professor of agricultural economics, University of Arizona, and Leigh is an agricultural economist with NRED at Colorado State University

[1] Italicized numbers in parentheses refer to the references at the end of this article

[2] Each of these measures of consumer welfare change is defined in the following section

[3] Some analysts maintain, however, that only CV measures satisfy the potential Pareto improvement criterion Moreover, when post-policy quantity adjustments are impossible, analysts must use Hicks' compensating surplus measure of consumer welfare change

Our purpose here is to test empirically, in a dynamic multi-market context, the differences among the five welfare measures—PV, LV, CV, EV, and CS Using a multimarket econometric model, we estimate the change in consumer welfare resulting from a ban on the use of insecticides in corn production, and trace the impacts through the price and quantity changes for 11 agricultural products over the 1980-85 period We calculate differences among these five measures of consumer welfare change by commodity, and we explore the implications for policy evaluation

Measures of Consumer Welfare Change

The origin of the concept of consumer's surplus is generally credited to Dupuit (4), who in 1844, posited that a buyer can receive a surplus from a transaction His concept was subsequently popularized by Marshall (11) and reinterpreted as the surplus utility a consumer derives from being able to buy a commodity at a particular price An economic measure of this utility surplus, he argued, is given by the Dupuit triangle (that is, the triangular area below the demand curve and above the price line)

Following the development of ordinal utility analysis, Hicks redefined more rigorously measures of the change in consumer welfare resulting from an actual or proposed price change In response to an extension suggested by Henderson to his earlier analysis (9), Hicks defined four measures of consumer welfare change (6, 7, 8) Two of these measures, CV and EV, are relevant to the analysis presented here [4]

Hicks defined CV as the amount of compensation, paid or received, that will leave consumers in their initial welfare position following a price change if they are free to buy any quantity of the commodity at the new price In contrast, he defined EV as the amount of compensation, paid or received, that will leave consumers in their subsequent welfare position in the absence of a price change if they are free to buy any quantity of the commodity at the old price

Generally, CV and EV will not be equal In practice, only one of the two measures will be appropriate as dictated by the existing distribution of property rights and compensation criteria The selection of one of these measures is not a matter of theoretical debate, as argued by some analysts (for example, Boadway (1)), as either measure constitutes a conceptually

valid foundation for analyzing consumer welfare change The selection depends on a social judgment as to whether the initial or the subsequent welfare position is taken as a basis

Two alternative measures of consumer welfare change are LV and PV LV is the change in income required to purchase the original quantities of all goods after prices have changed In contrast, PV is the change in income required to purchase the subsequent set of goods when consumers face the initial price situation LV, like CV, measures the variation required to maintain the initial welfare position PV and EV measure the variation required to maintain the subsequent welfare position

These changes can be expressed by the following equations [5]

$$EV = -\sum_r X_r \Delta P_r - \frac{1}{2}\sum_{rs} \frac{\partial X_r}{\partial P_s} \Delta P_r \Delta P_s \tag{1}$$

$$+\frac{1}{2}\sum_r X_r \Delta P_r \cdot \sum_r \frac{\partial X_r}{\partial M}\Delta P_r$$

$$CV = -\sum_r X_r \Delta P_r - \frac{1}{2}\sum_{rs}\frac{\partial X_r}{\partial P_s}\Delta P_r \Delta P_s \tag{2}$$

$$-\frac{1}{2}\sum_r X_r \Delta P_r \cdot \sum_r \frac{\partial X_r}{\partial M}\Delta P_r$$

$$LV = -\sum_r X_r \Delta P_r \tag{3}$$

$$PV = -\sum_r X_r \Delta P_r - \sum_{rs}\frac{\partial X_r}{\partial P_s}\Delta P_r \Delta P_s \tag{4}$$

$$CS = \frac{LV + PV}{2} \tag{5}$$

where

X_1 X_n are commodities,

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[4] For a thorough review of the historical development of the concept of economic surplus and a discussion of the remaining two measures, compensating surplus and equiv alent surplus, see (3)

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[5] Equations (1) to (5) are from Hicks (6) and ignore all terms of higher order than the second

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$P_1 \quad P_n$ are prices,

$$1 \leqslant r, s \leqslant n$$

and

$$M = \overset{n}{\underset{1}{\Sigma}} P_r X_r$$

and a negative sign indicates a loss of welfare

If $\underset{r}{\Sigma}(\partial X_r / \partial M) \Delta P_r$ is negative for a welfare increase, or is positive for a welfare decrease, then

$$LV < CV < CS < EV < PV \tag{6}$$

Hicks (6) argues that the conditions where equation (6) does not hold are not likely to occur in any realistic policy evaluation Therefore, for most empirical work, it can be considered that LV and PV are the upper and lower bounds for all the relevant measures of consumer welfare change

In addition, the midpoint between LV and CV, which equals CS, is an upper bound for CV and a lower bound for EV Inasmuch as LV, CS, and PV are far easier to calculate than the theoretically more defensible CV and EV, the empirical question raised by Hicks is whether these bounds are sufficiently close approximations to serve as a basis for policy analysis

Additional Theoretical Considerations

The estimation of consumer's surplus as the average of PV and LV is based on the assumption of linear adjustment paths, thus avoiding the indeterminacy problems created by path dependency (2, 17) Furthermore, an estimate of the potential change in aggregate welfare is implied by changes in market prices and quantities as a result of the ban on insecticides [6] Interpersonal utility comparisons are needed to evaluate the distribution of the aggregate potential gain among households, Failing that, a value judgment must be made that society is willing to make certain groups less well off to increase the welfare of others In addition, we do not estimate related changes, such as capital gains and losses

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[6] Winch presents a comprehensive analysis of interpreting alternative measures of welfare change in the absence of complete compensation, and he presents the case for preferring CS over either CV or EV in situations where desired compensation criteria will not be met (22)

to landowners or health implications for applicators For these reasons, the estimates of welfare change presented here are but one input into a comprehensive policy evaluation Our purpose is to examine the convenience-accuracy tradeoff described by Hicks

Finally, we used the Taylor series approximations for CV and EV estimates developed by Hicks (6) and elaborated upon by McKenzie and Pearce (equations (1-4) and (6)) Willig (21) has expressed reservations about these estimates, as their accuracy depends on the remainder term in the Taylor series approximation Our position, like that advocated by McKenzie (12), is that by incorporating additional terms in the Taylor expansion, analysts can make estimates for CV and EV that are as accurate as necessary for evaluating policy The additional terms were not necessary in our application because the remainder is zero, given the structure of the empirical model we used

Estimates for Alternative Measures of Consumer Welfare Change

Pimental and Shoemaker (16) have estimated that implementing a ban on the use of insecticides in corn production would result in a 3-percent reduction in annual U S corn yield This loss in production would have consequences in prices and quantities throughout the agricultural sector Products directly affected by such a change would include pork, beef, veal, chicken, turkey, eggs, ice cream, evaporated milk, fluid milk, cheese, and butter This food group constitutes approximately 50 percent of all food consumed at home and accounts for 10 percent of total consumer expenditures (19, p 68)

We estimated the five measures of consumer welfare change for the 1980-85 period in each of these 11 markets These markets provide a comparatively comprehensive accounting, although a complete general equilibrium analysis would require extending the evaluation to remaining food and nonfood (for example, housing, apparel, transportation, health, and recreation) groups

The Econometric Model

We used the Cross Commodity Forecasting System (CCFS), a multimarket econometric model of the 11 commodities of interest plus feed grains, soybeans, and wheat to assess the impact on consumers of a ban on insecticides in corn production The CCFS was developed by commodity specialists in USDA to "reflect, in an annual aggregate sense, the under-

lying direct and cross economic effects of the crop and live-stock sector" (18, p 1) The model includes 158 endogenous and 136 exogenous variables, with each commodity sub-model containing retail, farm, and investment demand equations, supply equations for live animals and carcasses, product stocks, conversion relationships, and supply and demand identities

Solution values generated by the CCFS, as well as the structural relationships within each commodity submodel, were used to provide the information necessary to calculate measures of consumer welfare change [7] We derived these welfare measures by comparing results *with* the ban to those *without* the ban for a standard set of exogenous variables

Estimation Results

Table 1 summarizes the projected total loss in Marshallian consumer's surplus experienced by the Nation's consumers [8] Losses, by commodity, range from $7 million for ice cream consumption to approximately $29 billion for beef con-sumption Losses were $500 million in the first year and increased to $16 billion in the last year recorded For all commodities, losses total approximately $50 billion for the 6-year period.

Table 2 gives estimates, by year, for all five consumer welfare measures and shows the change in CS as a percentage of disposable income and the deviation of PV and LV from CS PV and LV bound the range of values that EV, CV, and the change in CS can assume

Estimates of the total loss in consumer welfare from 1980 through 1985 range from $49 billion to almost $51 billion Total change in CS is 0 39 percent of total disposable in-come PV and LV differ from this change by 1 7 percent That is, all five estimates for the cumulative impact on consumer welfare lie within a $1 67 billion range that cor-responds to the CS estimate, plus or minus 1 7 percent

We also estimated the five welfare measures by commodity, over the 6-year study period Again, the variations are small Except for veal, estimates lie within a small range of the CS value, varying from as little as 0 1 percent for eggs to 2 4 percent for chicken Variability among estimates of con-

sumer impacts in the market for veal is larger than for other commodities because of the sensitivity of veal production to price changes among substitute commodities Because of price increases in substitutes, veal demand and production increase in the short run Then, as its own price increases, substitution shifts back toward beef and turkey, virtually eliminating veal production

By 1985, production of livestock commodities, except veal, will become stabilized A new equilibrium level appears to be reached A significant change in veal production might occur after 1985, however, because of its relative unimpor-tance as a livestock product, the effect on welfare measures would be small

Conclusions

The results of this empirical analysis demonstrate that the posited 3-percent decrease in corn yields would have a negative impact on consumers Expressed as a percentage of disposable income, losses in CS averaged 0 39 percent over the 6-year period However, this welfare cost, coupled with estimates of the impact on agricultural producers, would have to be weighed against the potential health and environmental benefits of discontinuing the use of insecticides in corn production before a judgment on the overall merit of banning the use of insecticides in corn production could be made In addition, a complete policy analysis would require further verification of the estimate of yield reduction

The estimated differences between alternative measures of consumer welfare change are small The PV and LV provide a narrow range in which the value of the welfare change will lie For the commodities analyzed, PV and LV differ from CS by about 1 percent in most cases, with the deviation be tween these measures increasing over time from 0 4 percent in the first year to 2 2 percent in the last, and averaging 1 7 percent over the 6-year period Of course, the differences between CS and CV or EV are even smaller Our results are consistent with Willig's contention (20) that in most cases, CV and EV will be within 2 percent of CS [9]

Implications for Applied Research

Although the empirical results of this investigation depend on the CCFS model and Pimental's estimate of yield impacts,

[7] See (18) for additional documentation of the CCFS

[8] In a vertically structured sector of the economy like agriculture, CS measures in final output markets have welfare significance for associated input markets as well For a discussion, see (10)

[9] The accord among these measures can be expected whenever (1) the change in consumer surplus is less than 5 percent of consumer income, and (2) income elasticities of demand lie between ±1 (20)

*For most policy evaluations, this simple two-step procedure
will provide information which is detailed and rigorous
enough for decisionmaking*

Table 1—Projected total losses in Marshallian consumer's surplus due to decreased corn yields, 1980-85

Commodity	1980	1981	1982	1983	1984	1985	Total by commodity
	Billion dollars						
Pork	0 2577	0 5810	1 4250	2 5036	2 7346	2 6700	10 1719
Beef	0401	1052	1 8146	6 6052	9 9773	10 2400	28 7824
Veal	0020	0092	0987	2856	2482	0617	7054
Chicken	1057	2328	5831	1 2034	1 5623	1 6467	5 3340
Turkey	[1](0002)	0158	1539	3434	4560	4724	1 4413
Eggs	0651	1453	3315	6625	8685	9602	3 0331
Ice cream	0013	0013	0013	0012	0011	0011	0073
Evaporated milk	0013	0028	0024	0018	0015	0013	0111
Fluid milk	0282	0563	0534	0512	0236	0234	2361
Cheese	0147	0293	0255	0200	0143	0122	1160
Butter	0034	0076	0064	0046	0042	0035	0297
Total	5193	1 1866	4 4958	11 6825	15 8916	16 0925	49 8683

[1] Gain in welfare, for 1980 only

Table 2—Alternative estimates of total consumer welfare losses, 1980-85

Year	Population	Total disposable income	Paasche variation (PV)	Equivalent variation (EV)	Consumer's surplus (CS)	Compensating variation (CV)	Laspeyres variation (LV)	Percentage deviation of PV and LV from CS	CS as a percentage of disposable income
	Millions				*Billion dollars*				*Percent*
1980	221 9	1,725	0 5177	0 5191	0 5192	0 5193	0 5207	0 3	0 03
1981	223 0	1,875	1 1804	1 1861	1 1865	1 1868	1 1925	5	06
1982	225 0	2,033	4 4677	4 4909	4 4958	4 5007	4 5240	6	22
1983	226 7	2,205	11 5331	11 6505	11 6824	11 7143	11 8317	1 3	53
1984	228 4	2,388	15 5895	15 8357	15 8917	15 9477	16 1940	1 9	67
1985	230 1	2,592	15 7438	16 0378	16 0915	16 1452	16 4391	2 2	62
Total	1,355 1	12,818	49 0322	49 7201	49 8671	50 0140	50 7020	1 7	39

they do suggest that, regardless of the specific models used for estimation purposes, adjusting CS estimates to obtain the theoretically preferable measures of CV and EV may indeed be a "fiddling business "

These results generally suggest a pragmatic procedure for analysts to follow in applied policy evaluations First, the analysts should calculate the PV and LV measures of welfare change Next, assuming direct linear paths of price adjustment, as Burns (2) suggests, they can use the midpoint of this range as a practical and reasonable estimate for the change in CS

For most policy evaluations, this simple two-step procedure will provide information which is detailed and rigorous enough for decisionmaking

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M Truman Fossum
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