MODEL VALIDATION AND THE PHILIPPINE PROGRAMMING MODEL

By Gil R. Rodriguez, Jr., and David E. Kunkel*

The use of programming models to analyze the economic implications of supply and demand shifts for the agricultural sector of developing countries has increased significantly Notable models are those by Duloy and Norton (*4*), Pomareda (*16*), Cappi, Fletcher, and others (*3*), Miller and others (*14*), and Heady (*9*) [1] These models use an objective function that incorporates supply and demand functions to simulate competitive market equilibrium

Despite the substantial investment in technical skills and data-processing inputs, validation of sector programming models is rarely discussed explicitly Nugent was the first analyst to test the reliability of programming models (*15*) His work can be summarized in two propositions

1 If a market in the real world approximates a competitive condition closely, any deviation—for that market—of the results of a programming model from an existing observable empirical data base represents model specification errors
2 If the programming model simulates a competitive market solution, but the real world situation being modeled has market imperfections, then these imperfection are likely responsible for some deficiencies in the predictive ability of the programming framework

Recent works by Duloy and Norton (*4*) and Kutcher (*13*) have employed both propositions to validate the Mexican agricultural sector (CHAC) model Later Shumway and Talpez (*17*) concentrated on the first proposition when examining the output of a model of major crops in California

This research demonstrates the need and the procedure for testing sector programming models It compares the model estimates of endogenous variables to carefully selected base period parameters It uses an operational, static, deterministic, and highly aggregate programming model of Philippine agriculture as the framework Alternative formulations of the Philippine model are also examined for possible errors in the consumption, production, and objective function data sets

Keywords

Mathematical programming
Model validation
Philippines
Agricultural sector analysis
Development planning

In this article, we use Nugent's first proposition to validate the optimal levels of production, exports, imports, and the shadow prices of commodities and resources It is our principal objective to illustrate the validity tests conducted on a programming model of the Philippine agricultural sector known as MAAGAP [2]

THE STRUCTURE OF THE PHILIPPINE MODEL

MAAGAP, a highly aggregate, static, and deterministic model, includes rice, corn, sugar, coconuts, vegetables, and livestock products that collectively accounted for about 90 percent of the total value of Philippine agricultural commodities in 1976 Detailed discussion of the data set appears in Gonzalez and Kunkel (*9*) MAAGAP was developed in Project ADAM (Agricultural Diversification and Markets) with the assistance of both Filipino and U S agricultural economists

The MAAGAP model forms an important part of the agricultural policy analysis system within the Philippine Bureau of Agricultural Economics The model has been used for several policy analyses, such as the fertilizer subsidy analysis and the evaluation of supply and demand

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[1] Italicized numbers in parentheses refer to items in References at the end of this article

[2] MAAGAP, a Filipino word which means alert, stands for Model Analysis of Agricultural Adjustments in the Philippines

projections estimated by the National Economic Development Authority (NEDA) The most important papers published during the model development were by Atkinson and Kunkel (*1*) Kunkel, Gonzalez, and Hiwatig (*11*), Gonzalez, Kunkel, and Alix (*8*), Ferrer (*6*), Atkinson and Kunkel (*2*), Foote (*7*), and Encarnacion (*5*) The objective function for the MAAGAP model is

$$\max(Z) = \sum_j \left[\int_0^{C_j^l} P_j dC_j \right] + \sum_j v_j E_j - \sum_j u_j I_j - \sum_n c_n X_n - \sum_k w_k R_k - \sum_t f_t F_t - \sum_j g_j O_j - \sum_m b_m M_m \qquad (1)$$

where the variables for this model are defined in table 1

Equation (1) simply sums the areas under the demand curves and contains the value of exports minus the costs of imports, incidental production items, input supply, and feed-mixing processing The objective function simulates a competitive market by using stepped demand functions

The step demand functions are formed by grid linearization of

$$C_j = \int_0^{C_j^l} P_j dC_j,$$

Thus, the *i*th step is the sum of the area under the demand curve up to the quantity C_j^l The convex combination constraint allows only the corresponding quantity (C_j^l) to be sold In the optimum solution the shadow price is

$$P^* = \frac{\Delta \int P_j^* dC_j}{\Delta C_j},$$

Table 1 — Variables in the Philippine Programming Models

Endogenous		
P_j	=	$f(C_j, Y)$ is the inverse demand function for the *j*th final product
C_j	=	domestic consumption of the *j*th product
E_j	=	quantity of the *j*th product exported
I_j	=	amount of the *j*th commodity imported
X_n	=	production levels of the *n*th production activity
R_k	=	amount supplied of the *k*th input
F_t	=	amount of the *t*th feed ration supplied
O_j	=	activity level of the *j*th final product transferred
M_m	=	activity level of the *m*th processing activity
$\Pi_{\ell j}$	=	shadow prices of the ℓth absolute land class used in production of the *j*th product
*	=	indicates equilibrium value
Exogenous		
Y	=	income level
v_j	=	export price of the *j*th product
u_j	=	import price of the *j*th commodity
w_k	=	input cost of the *k*th input supplying activity
f_t	=	unit cost of the *t*th feed-mixing activity
g_j	=	unit marketing margin of the *j*th final product
b_m	=	unit processing cost for the *m*th processing activity
c_n	=	miscellaneous cost of the *n*th production activity

All input-output coefficients are positive

Thus, only one point on each demand, supply, or transformation function is validated

where $\Delta \quad P_j dC_j$ is the change in the value of the objective function between steps, and ΔC_j is the change in the quantity demanded Thus, by this formulation of the programming problem, *marginal price (shadow' price) of output is equal to the average price*, or the intersection of the supply and the demand curve in a competitive market solution Kunkel, Gonzalez, and Hiwatig (*11*) and Norton (*4*) provide illustrative examples

Such an objective function implies the following individual behavioral assumptions

1 Farmers are technically efficient and governed by profit-maximizing behavior
2 Farmers are pricetakers in the input and commodity markets The income variable appears in the demand function (P_j), income shifts are considered exogenous [3] Because the Philippines is generally a price-taker in international markets, export (v_j) and import (u_j) prices are taken as given

The product price function (P_j) does not contain any cross price elasticity terms They can easily be included through aggregation of commodities into composite groups The formulation used allows substitution possibilities within a group but not across groups Solutions which allowed substitution in the consumption set were found in computer runs not to be significantly different from ones that did not

[3]The model does not capture the income impacts on the farmers' and the other sectors' expenditure patterns within a finite time period

The objective function is maximized subject to a set of constraints defining production, processing, and marketing These constraints are reported by Kunkel and others (*3*) [4] The model structure is shown in figure 1 Programming models for policy analysis are sensitive (particularly the shadow prices of fixed resources) to specification and measurement errors The use of a programming framework imposes strong conditions on variables in the optimum solution If the actual model specification used differs from the theoretical specifications of a perfectly competitive model, equilibrium shadow prices may be biased throughout As shown by Kunkel, Gonzalez, and Hiwatig, the marginal revenue product of all resources used in each production process is equal to resource cost (*11*, p 6) Mathematically, this can be expressed as

$$\lambda_{ijk} \, Q'_{ijk} = \gamma_j \qquad (2)$$

where λ_{ik} is the marginal cost of the *i*th product from the *k*th production process, Q'_{ijk} is the first derivative for the *k*th production process of the *i*th product and *j*th resource, and γ_j is the input cost of the *j*th resource

Whenever price or quantities of inputs supplied or products demanded are fixed *a priori*, then neither $\lambda_{ik} = P_i^*$ nor $\gamma_j = w_j^*$ will hold [5] For example, when resource levels are fixed, as with land in MAAGAP, the shadow price of land may differ from the actual competitive market price To help detect any biases due to measurement and specification errors, the analyst must validate the model against a base period

[4]This report is available from the authors on request Overall, the MAAGAP model contained 158 rows and 504 columns (activities) for the 1976 base

VALIDATING THE PHILIPPINE MODEL[6]

The validation procedure compared the MAAGAP results to actual base period levels for the set of endogenous variables (table 1) The base period, 1976, was chosen as being most representative of recent years We are considering cross-section data and are not validating the ability of the model to capture turning points over a time path

Thus, only one point on each demand, supply, or transformation function is validated

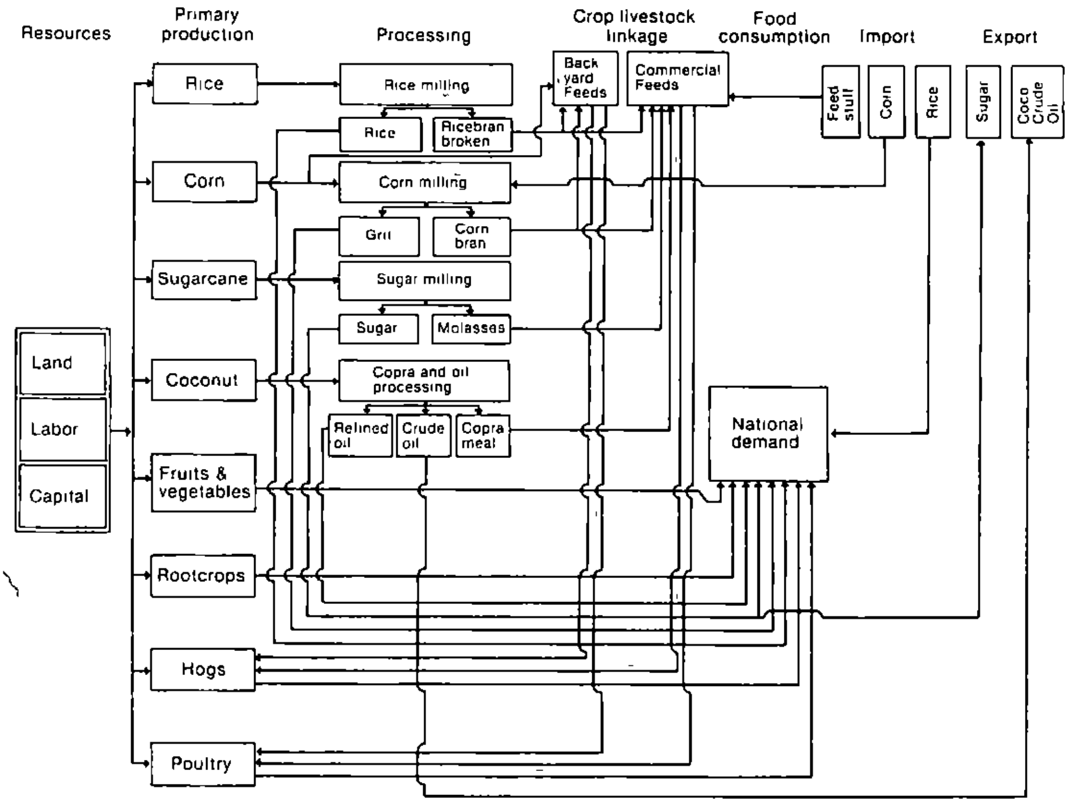
Data limitations made it difficult to determine some of the base period resource levels and prices ($R_k F_t$ and Π_{lj} in table 1) For example, land prices are acquisition costs, and the corresponding shadow prices are rates of return To evade taxes, landowners usually undervalue prices on property not being sold The major input which can be validated is the level of fertilizer use

[5]Note that in this case P_j^* and w_j^* are market equilibrium prices on demand and input supply curves See (*5*) for more detail

[6]The validation tests performed in the Philippine model were influenced partly by the earlier work of Kutcher (*13*) on the consistency tests of the Mexico — Pacific Northwest Regional Model

Figure 1

Schematic Diagram of the Adam National Model



The endogenous variables subjected to a close scrutiny were agricultural market price (P^*), imports (I_j), and production levels (X_m)

The first test involves a check on the production capacity (implicitly involving also the input-output coefficients) of the model This is accomplished by treating final domestic commodity demand as perfectly elastic at fixed price levels If a given commodity is partially or totally imported by the model (when, in fact, it is not imported in the base period), this implies an underestimate by the model of actual capacity That is, the production vector may be too "expensive " The reverse holds true for "excessive" exports of a given commodity

The second validation test entails redefining the model's objective function as the minimization of the costs of producing domestic output levels in the base period This is accomplished by fixing the level of domestic and foreign demand for all products at base period levels The shadow prices generated in the commodity balance equation are then marginal costs To validate the model's assumption of a competitive structure, one then compares these with the base period market prices

The third, and final test, compares the full model results with data for the base period The models corresponding to these tests are called the

We found a high correlation between the crop prices estimated from the full model and the actual prices for 1976

fixed demand, cost minimization, and full model

In these validation tests, the following measures are used to judge how closely the model approximates the base period

1 The correlation between the model derived commodity outputs and prices and those observed for 1976
2 A simple regression of the form,[7]

$$Y_o = a + b\, Y_m \qquad (3)$$

where Y_o is the observed value, and Y_m is the model-estimated value

For this test, the model results and real world data on the various agricultural commodity outputs and prices will have a correlation of one (or, equivalently, the expected value $E(a) = 0$ and $E(b) = 1$ in equation (*10*) if the objective function, production, consumption, and constraints sets of the model are ideal)

3 The Information Inaccuracy Index, that is

$$I(Y_o, Y_m) = \sum_{i=1}^{n} Y_{io} \left[h(Y_{io}) - h(Y_{im})\right]$$

where Y_{io} =

$$Y_{io} / \sum_{i=1}^{n} Y_{io} \qquad Y_{io} > 0$$

Y_{im} =

$$Y_{im} / \sum_{i=1}^{n} Y_{im} \qquad Y_{im} > 0$$

$$h(Y_{io}) = \ell n_e(Y_{io})$$

$$h(Y_{im}) = \ell n_e(Y_{im})$$

The Information Inaccuracy Index was developed by Tilanus and Theil to evaluate the estimation errors of the endogenous variables in an input output model Their rationale in developing the index was that " errors in less important variables are weighed less heavily than the same relative errors in the more important ones " A high value for the Information Inaccuracy Index (which does not have an upper and lower bound) indicates a deterioration of the estimation capability of the model A perfect model would result in $E(Y_o, Y_m) = 0$

However, the critical value that separates "pass" from "fail" depends largely on the utility function of the researcher A logical criterion which the researcher can use in selecting a critical value may depend on an awareness of the marginal returns from the model's improvement and the value of the marginal effort

Crop production and price estimates from various alternative model formulations are compared with observed data in table 2 Table 3 gives the regression and correlation parameters used as indicators of goodness of fit for the linear model[8] The linear regression results indicate two types of directional biases, as *a* and *b* are either less than or greater than 1 0 The first type (T_1 in figure 2) is that used for small values of the relevant base period data (Y_o), the model's estimations are biased upwards The reverse is true for larger values of Y_o The second type of bias (T_2 in figure 2) is one in which all the model's estimates are biased upward if the constant term is positive As indicated by table 3, the full and fixed demand model's estimates of crop prices belong to the second bias type This is not the case for the cost minimization model

The full model's estimate of crop production is also T_2 However, the crop area and production estimates of the fixed demand model are of the first bias type The latter type is also present in the full model's determined crop prices and in the cost minimization model's generated crop

[7] The regression form $\ell n Y_o = \ell n a + b \ell n Y_m$ was also estimated to determine nonlinear biases, but results were not significantly different from the linear case A serious limitation arising from using equation (*10*) (or its log transform) is that formal statistical tests of significance cannot be applied to the regression parameters because the model estimates are not independent Such parameters should merely be interpreted as informal measures of goodness of fit and model biases

[8] Standard errors are given for informational purposes only and should not be used for formal statistical testing See footnote 7

Table 2 — Crop area, production, and prices of alternative model formulations

Crop	Area: Actual base period	Area: Full model	Area: Fixed demand	Production: Actual base period	Production: Full model	Production: Fixed demand	Prices: Actual base period	Prices: Full model	Prices: Fixed demand	Prices: Cost minimization
	--- 1,000 hectares		-	--- 1,000 million tons ---				-- Pesos/kg		---
Palay (rough rice)	3 579 3	4 198 0	4 173 2	6,159	6 705	6 710	0 94	1 01	1 09	1 18
Corn	3,257 0	3,169 3	3 144 0	2,767	3,119	2 960	94	52	33	50
Sugarcane	533 0	538 6	538 6	2,514	2,455	2,455	1 94	1 89	1 89	2 64
Coconut	2,521 2	2 387 0	2,387 0	10 662	8,619	1,730	1 85	[3]1 63	[3]1 63	[3]3 35
Banana	298 7	244 7	224 5	3 068	954	875	41	38	38	50
Cabbage	8 1	15 1	18 0	54	64	69	1 53	1 46	1 44	1 51
Pechay[1]	4 5	6 7	7 3	37	25	27	1 40	1 35	1 20	1 49
Tomatoes	21 0	21 9	26 6	153	79	96	2 04	1 53	1 56	1 63
Eggplant	16 2	26 5	32 1	82	99	120	97	1 13	1 09	1 16
Camote[2]	192 3	196 0	243 3	781	687	745	42	64	61	61
Cassava[1]	118 0	150 5	163 2	621	464	503	38	33	33	38

[1]Leafy vegetable
[2]Root vegetable
[3]Copra equivalent price

Note 7 30 = $1 00

Source Philippine Sugar Commission (PHILSUCOM), Bureau of Agricultural Economics (BAEcon), Philippine Coconut Authority (PCA)

prices Judging from the standard errors for b and the correlation coefficient (r) given in table 3, the full model seems to perform better than the other model formulations for area and price but not for production

The log linear regression results indicate the full and fixed demand model's estimates of crop areas have a nonlinear bias downward for small values of $\ell n Y_m$ and a nonlinear bias upward for large values

We found a high correlation between the crop prices estimated from the full model and the actual prices for 1976 This supports the plausibility of assuming the competitive market structure of the Philippine model However, for coconuts a large gap occurred in the cost minimization model (table 2) This gap can be attributed to data errors which had the following causes

1 The conversion rate was overestimated, a rate of 4 5 per kilograms (kg) of copra (coco nut meat) was used
2 The domestic coconut oil demand was overestimated by 65 percent due to a data error
3 The coconut hectarage constraint was underestimated by 5 3 percent
4 The export levels set up for coconut oil and copra may have been too high due to the absence of any stock adjustments

The coconut data misspecification will likely affect the shadow prices of other major agricultural commodities, particularly sugarcane

A general reason for the prices of the cost minimization model deviating from the actual prices is that, by dropping the first and second terms which allow market pricing of output from equation (1), we are utilizing the total model structure information less efficiently Graphically, this means that if we disregard D_1 in figure 3, the probability of estimating the "true" market price (P_1) is low If S_2 is the implicit supply function generated by the cost minimization model, the error in price estimation is the area abP_1P_2 (fig 3)

The low model price for corn (0 50 pesos per kg) compared with the observed price (0 94 pesos per kg) can be attributed to a possible downward bias in the model's estimate of the cost of producing corn The problem is partly caused by the

The full model predicts the crop production proportions well in comparison with the fixed demand framework

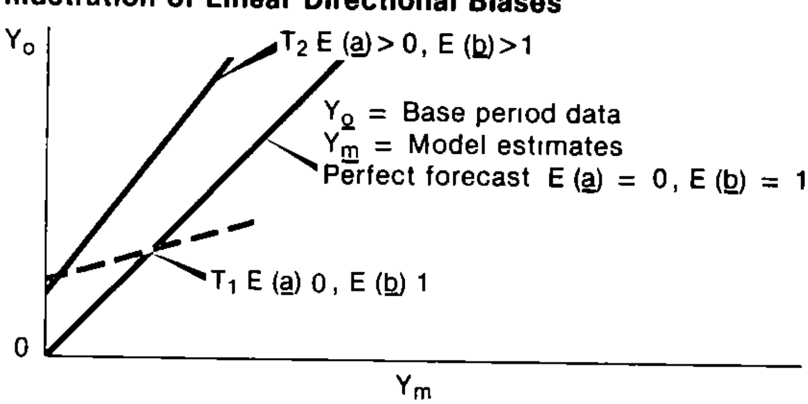
Table 3 — Regression results for actual versus model levels[1]

Result and model	a	b	r
Area			
Full model	32 49	0 930 [2](0373	0 9928
Fixed demand	1051 69	941 (45)	5688
Production			
Full model	148 80	1 086 (0961)	9665
Fixed demand	24 93	938 (109)	9927
Prices			
Full model	00138	1 079 (134)	9370
Fixed demand	0965	1 018 (149)	9157
Cost minimization	4032	56 (120)	8413

[1]Based on table 2
[2]Numbers in parentheses are standard errors

Figure 2

Illustration of Linear Directional Biases



difficulty of determining the appropriate spatial aspects of corn production vectors

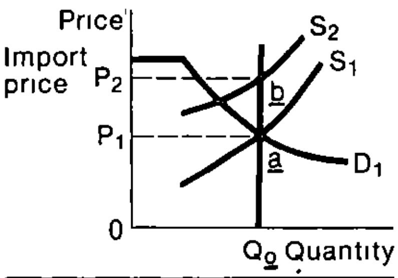
For the production capacity test, the fixed demand model solution registered 40,800 metric tons of commercial broiler imports However, as no broilers were imported by the Philippines in 1976, the domestic commercial broiler production activities incorporated in the Philippine model may be too expensive, that implies an upward bias in the pricing of such activities Comparing the export levels of coconut and sugar products with the base levels indicates an "over-capacity" for centrifugal sugar (1 720 million metric tons (mmt) versus 1 455 mmt) whereas the reverse holds for molasses (0 657 mmt versus 0 792 mmt) and copra meal (0 170 mmt versus 0 497 mmt)

Table 4 gives usage levels obtained from the model formulations All three models overestimated the levels of fertilizer use, probably as a result of aggregation error because production vectors are based on farm survey data However, the full model did well in predicting directions of change in fertilizer prices The full and the fixed demand models performed better than the cost minimization model in predicting nitrogen and potash consumption in 1976 [9]

[9]Nitrogen is considered the most important fertilizer nutrient in the Philippines Experiments conducted by the Bureau of Soils (BS), the Philippine Sugar Commission (PHILSUCOM), and the International Rice Research Institute (IRRI) show that most crops responded favorably compared with their reponse to potash and potassium

Figure 3

Market Equilibrium in the Cost Minimization Model



Although no formal level of significance can be attached to the information parameters provided in table 5, the full model predicts the crop production proportions well in comparison with the fixed demand framework The full model incurs a relative information loss of -3 75 percent, compared with the fixed demand model's loss of -20 74 percent All model types perform well in estimating the crop prices The Information Inaccuracy Index (in absolute terms) ranges from 2 02 percent to 4 92 percent

CONCLUSIONS

Programming models are rarely subjected to validation tests In this article, we have shown that consistency checks on the shadow prices of programming solutions are useful because any misspecifications in resource constraints or prices will tend to affect shadow prices

Tests of a programming model of Philippine agriculture revealed biases in the production, consumption, constraint, and objective function sets Each model compared represented a unique theoretical structure from which to test for inconsistencies The fixed demand model appeared to have significant biases in the crop production vectors for commercial broilers, corn, and copra meal The wide disparity between the cost minimization model's coconut shadow prices and the actual price in 1976 helped to identify measurement errors

The numerical measures used to judge how well a specific programming model approximate the Philippine agricultural conditions in 1976 were simple correlation coefficients, regression of actual versus model results, and the Information Inaccuracy Index Based on these indices, the full model outperformed the others

Validating resource usage and price levels of the MAAGAP model was limited to fertilizer use, a limitation dictated by the availability of the basic data The three models' comparisons of the estimated fertilizer nutrients with actual 1976 levels indicate that all these models overestimated use Nevertheless, the full and fixed model's yields of nitrogen and potash consumption levels were more accurate than those determined by the cost minimization model

Table 4 — Fertilizer usage levels under alternative model assumption

Fertilizer	Model formulation			Actual 1976[1]
	Full model	Fixed demand	Cost minimization	
	1,000 metric tons			
Nitrogen (N)	178	180	195	152
Phosphorous (P_2O_5)	92	84	92	38
Potash (K_2O)	68	68	103	55

[1] Fertilizer and Pesticides Authority

Table 5 — Information indices for evaluating the relative magnitude of estimation errors in the model types

Endogenous variables	Model type	Information Inaccuracy Index	Expected information content[1]	Relative information content[2]
Crop area	Full model	-0 01219	1 4811	0 82
Crop area	Fixed demand	- 0103	1 6654	3 75
Crop production	Full model	- 0624	1 6654	20 74
Crop production	Fixed demand	- 3454	1 6654	20 74
Crop prices	Full model	- 0202	2 2569	0 89
Crop prices	Fixed demand	- 0377	2 2569	1 67
Crop prices	Cost minimization	- 0492	2 2569	2 18

[1] Computed as

$$H(Y_{io}) = \sum_{i=1}^{n} Y_{io} h(Y_{io})$$

[2] Computed as

$$[I(Y_o Y_{mi}) H(Y_{io})] \times 100$$

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