AN ALLOCATION MODEL FOR CONSUMER EXPENDITURES

By Jitendar S Mann*

The Rotterdam model, a complete consumer demand system, was fitted to personal consumption expenditure data for 1949-77 to study the interaction of consumer expenditures A full matrix of direct and cross price elasticities and income elasticities was estimated The 12 categories of expenditures were food at home, food away from home, alcohol and tobacco, clothing, housing, utilities, transportation, medical, durables, other nondurables, services, and miscellaneous

Keywords

Consumer expenditures
Consumer demand
Rotterdam model
Price elasticities
Income elasticities

INTRODUCTION

The consumer's basic problem, as defined by economists, is how to allocate expenditures among different commodities, given their prices and the consumer's income When income and prices change, the consumer changes the income shares spent on different commodities For example, U S food expenditures as a percentage of personal consumption expenditure declined from 21 percent in 1960 to about 18 percent in 1977 The share of expenditure on food purchased for use at home also fell, from 17 to about 13 percent

The objective here is to describe, analyze, and explain the behavior of budget shares (amount spent) for major commodity groups, with emphasis on food expenditures A complete system of demand equations for consumer expenditures is estimated, and a full matrix of direct and cross price elasticities and income elasticities is presented

In studying expenditure allocation, the analyst must specify a complete system, which should allocate consumer expenditures among all categories The Rotterdam model used here (developed by Theil and his associates—(*1-3, 11-13*)) explains the quantity component of the variation in budget shares [1]

BUDGET SHARES

The budget shares are defined as

$$w_i = p_i q_i / m,$$

where w_i is the budget share of the i^{th} commodity, p_i, its price, q_i, the quantity purchased, and m, the total expenditure The shares are non-negative and add up to one for all commodities The consumer expenditure data analyzed are Personal Consumption Expenditures (PCE), published by the U S Department of Commerce The data are combined into 12 major commodity groups food at home, food away from home, alcohol and tobacco, clothing, housing, utilities, transportation, medical services, durables, other nondurables, services, and miscellaneous The details of expenditure items included in each category appear in an appendix

In this article, total expenditure is per capita personal expenditure Saving is assumed exogenous and the terms "total expenditure" and "income" are used synonymously Quantities are represented by per capita constant dollar PCE Because these data are in constant dollars, variation in the time series is due to variation in quantities purchased only Prices are the implicit prices obtained by dividing current dollar expenditure by constant dollar expenditure The use of the implicit price deflator (instead of the Consumer Price Index) assures that price times quantity equals expenditure

The budget shares of the 12 expenditure categories for 1949-77 appear in table 1 The share of food consumed at home declined from about 19 percent in 1949 to about 13 percent in 1977 The share of food consumed away from home has remained almost unchanged The share of alcohol and tobacco used has fallen steadily Clothing expenditure went from about 13 percent in 1949 to about 8 percent in 1977 While the share spent on housing increased, that for utilities remained steady Transportation increased slightly The share spent on medical services more than doubled The shares of durables and other nondurables did not change Services rose a bit during the period The miscellaneous category includes items which do not pass through the marketing system but are included in PCE to account for the output of certain sectors

The partial elasticities of budget share with respect to price, quantity,

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[1] Italicized numbers in parentheses refer to items in References at the end of this article

Table 1—Budget shares of personal consumption expenditures

Year	Food at home	Food away from home	Alcohol and tobacco	Clothing	Housing	Utilities	Transportation	Medical	Durables	Other nondurables	Other services	Miscellaneous
						Percent						
1949	18 86	4 57	6 63	12 98	9 93	3 35	6 98	4 49	12 39	4 93	9 22	5 67
1950	18 07	4 33	6 32	12 21	10 24	3 49	6 90	4 49	14 28	5 02	9 20	5 46
1951	18 82	4 52	6 17	12 11	10 65	3 49	6 94	4 49	12 72	5 11	9 08	5 90
1952	18 89	4 52	6 29	12 05	11 30	3 48	7 15	4 63	11 71	4 90	9 05	6 03
1953	18 25	4 36	6 10	11 54	11 89	3 46	7 27	4 80	12 57	4 80	9 13	5 83
1954	18 38	4 25	5 86	11 32	12 59	3 61	7 24	5 05	12 00	4 70	9 37	5 64
1955	17 67	4 09	5 58	10 01	12 50	3 66	7 23	4 97	13 68	4 69	9 59	5 32
1956	17 62	4 10	5 55	10 96	12 77	3 74	7 42	5 14	12 66	4 74	9 94	5 35
1957	17 80	4 06	5 47	10 51	12 98	3 77	7 55	5 36	12 44	4 77	10 00	5 30
1958	18 13	3 97	5 43	10 30	13 47	3 88	7 59	5 64	11 13	4 79	10 25	5 41
1959	17 39	3 94	5 45	10 13	13 44	3 75	7 64	5 73	12 02	4 80	10 33	5 36
1960	16 96	4 01	5 39	9 90	13 77	3 74	7 76	5 92	11 68	4 88	10 67	5 31
1961	16 80	4 11	5 38	9 81	14 21	3 75	7 68	6 14	10 85	5 01	10 96	5 31
1962	16 01	4 15	5 28	9 67	14 35	3 71	7 64	6 32	11 52	5 20	10 84	5 31
1963	15 46	4 17	5 22	9 43	14 41	3 70	7 50	6 46	12 09	5 27	10 96	5 30
1964	15 33	4 14	5 02	9 54	14 28	3 63	7 35	6 77	12 43	5 31	10 94	5 24
1965	15 35	4 07	4 93	9 35	14 17	3 55	7 35	6 72	12 96	5 35	10 94	5 25
1966	15 41	3 97	4 84	9 45	13 93	3 48	7 39	6 73	12 87	5 56	11 06	5 32
1967	14 94	3 89	4 81	9 37	14 05	3 47	7 50	6 91	12 54	5 54	11 45	5 53
1968	14 70	3 96	4 68	9 34	13 87	3 37	7 42	6 99	13 26	5 58	11 40	5 43
1969	14 49	3 93	4 53	9 29	13 92	3 34	7 51	7 47	13 00	5 59	11 44	5 41
1970	14 72	3 99	4 61	8 96	14 14	3 37	7 71	7 85	12 11	5 61	11 46	5 47
1971	14 00	3 86	4 48	8 90	14 40	3 38	7 82	8 00	12 94	5 55	11 38	5 29
1972	13 61	3 83	4 36	8 83	14 33	3 43	7 72	8 17	13 58	5 58	11 29	5 29
1973	13 80	3 92	4 25	8 86	14 20	3 49	7 66	8 25	13 64	5 67	11 13	5 11
1974	14 29	4 03	4 14	8 57	14 30	3 78	8 32	8 46	12 10	5 72	11 17	5 11
1975	14 27	4 17	4 05	8 36	14 26	4 03	8 13	8 93	11 96	5 49	11 25	5 11
1976	13 60	4 24	3 93	8 13	14 15	4 15	8 15	9 40	12 79	5 35	11 12	4 99
1977	13 28	4 29	3 71	7 91	14 16	4 26	8 30	9 61	13 20	5 24	11 05	4 99
Average	16 10	4 12	5 12	9 96	13 33	3 63	7 55	6 55	12 53	5 20	10 54	5 38

The share of food consumed at home declined from about 19 percent in 1949 to about 13 percent in 1977 The share of food consumed away from home has remained almost unchanged

and income are 1, 1, and −1, respectively To see this, take the total differential of the definition of w_i

$$dw_i = \frac{q_i}{m} dp_i + \frac{p_i}{m} dq_i - \frac{p_i q_i}{m^2} dm$$

$$= w_i d \log p_i + w_i d \log q_i$$

$$- w_i d \log m$$

This equation states that the change in the i^{th} budget share is a weighted sum of logarithmic (relative) changes in price, quantity, and income, the weights being the budget share of the i^{th} commodity Dividing by w_i, we obtain these elasticities

$$\frac{\partial \log w_i}{\partial \log p_i} = 1$$

$$\frac{\partial \log w_i}{\partial \log q_i} = 1$$

$$\frac{\partial \log w_i}{\partial \log m} = -1$$

The relative importance of the varia tion in prices and quantities gives us an idea of the variation of relative shares These changes in prices and quantities appear in tables 2 and 3

Prices of these categories increased throughout 1949-77 food away from home, housing, transportation, medical services, and other services Demand for housing and medical services also rose steadily during the period The largest average annual price increase was for medical services—4 45 percent

The average price increase for food at home was 3 32 percent, for food away from home, 4 13 percent Large price increases in utilities (22 56 percent) and transportation services (16 95 percent) during 1973-74 should be noted

The share spent on housing increased the most annually—3 87 percent During 1949-77, food at home rose 0 93 percent and food away from home, 1 16 percent

The components of change in the share of food consumed at home appear in table 4 Income (total PCE) went up each year from 1949, to 197 percent The price of food at home increased at an average annual rate of 3 32 percent, the quantities consumed increased 0 93 percent, and per capita income rose 5 49 percent Expenditures on food during this period averaged 16 1 percent and declined about 0 2 percentage point annually

We want to know the relationship between income and price elasticity and the change in budget shares during the period First, assume that p_i is constant, and write the above differential as

$$dw_i = w_i d \log q_i - w_i d \log m$$

$$= w_i \left[\frac{d \log q_i}{d \log m} - 1\right] d \log m$$

From this we get the following expression

$$\frac{d \log w_i}{d \log m} \quad p_i = \text{constant} = E_i - 1$$

where E_i is the income elasticity

For it to be positive, for w_i to go up when m increases, we need

$$E_i > 1$$

Now assume that m is unchanged, and

$$\frac{d \log w_i}{d \log p_i} \quad m = \text{constant}$$

$$= \epsilon_u + 1,$$

where ϵ_u is the price elasticity

For the share w_i to go down when p_i increases, we need

$$\epsilon_u > -1$$

A luxury is defined as a commodity with an income elasticity greater than 1 If a good is a luxury, its budget share goes up as income goes up with the price assumed constant This occurs because when $E_i > 1$, a given proportionate increase in income has a larger proportionate effect on $p_i q_i$, the numerator of w_i

THE MODEL

I now present a derivation of the absolute prices version of the allocation model for consumer expenditures A more detailed derivation of the general model appears in (*12*)

The demand function for a commodity can be formulated in income and prices

$$q_i = q_i (m, p_1, \quad , p_n)$$

$$i = 1, 2, \quad , n,$$

Table 2—Relative changes in implicit price deflators, 1949 50 through 1976 77*

Year	Food at home	Food away from home	Alcohol and tobacco	Cloth ing	Hous ing	Util ities	Trans- porta- tion	Medical	Dur- ables	Other nondur- ables	Ser vices	Miscel laneous
1949 50	1 48	2 29	−0 54	−0 63	3 44	1 66	4 68	1 11	2 89	0 63	1 51	0 93
1950 51	10 52	7 16	2 70	8 46	3 98	2 21	5 38	2 63	4 86	7 56	6 30	10 22
1951-52	1 75	57	6 19	− 73	3 98	1 59	4 76	4 64	0	−1 91	6 37	33
1952-53	−1 79	0	83	15	5 32	2 55	5 20	4 83	1 35	44	5 36	− 67
1953-54	0	1 89	1 80	29	3 35	70	3 37	3 29	−3 26	− 44	2 47	−2 04
1954-55	−1 93	93	0	− 29	1 42	1 79	1 57	2 63	1 37	1 76	3 80	− 86
1955-56	75	1 84	1 29	2 16	1 82	1 76	1 85	2 02	2 82	1 88	5 16	1 89
1956 57	3 22	3 75	2 06	1 56	1 79	2 52	4 04	3 92	4 53	3 51	5 08	2 68
1957 58	4 23	2 94	94	56	1 89	1 30	2 75	3 43	25	2 59	3 48	2 61
1958 59	−1 39	2 85	3 51	97	1 33	1 92	2 81	2 83	3 60	1 07	3 03	3 17
1959-60	97	2 45	2 67	1 51	1 57	1 76	2 47	3 07	49	1 59	4 38	2 01
1960-61	96	2 24	87	81	1 29	1 85	94	2 51	97	1 43	79	1 37
1961-62	68	2 65	1 30	40	1 15	0	93	2 00	1 55	77	2 95	1 94
1962-63	1 35	2 28	1 70	1 07	1 01	61	13	1 36	94	1 53	3 46	2 19
1963-64	1 73	1 64	1 54	1 19	1 00	− 85	13	2 08	1 05	1 75	2 05	1 86
1964-65	2 35	2 34	2 06	92	.99	49	3 51	2 61	− 46	49	2 43	2 66
1965-66	5 16	4 67	2 81	2 57	1 34	97	2 77	4 20	23	98	4 15	3 53
1966-67	− 49	4 98	4 01	4 11	1 80	1 32	2 70	5 73	1 84	2 53	3 20	3 15
1967-68	3 51	4 99	4 95	5 11	2 36	1 42	2 39	4 93	3 69	3 86	5 99	4 06
1968-69	4 76	5 94	4 94	5 52	3 21	1 98	4 28	6 21	2 39	3 71	5 76	5 56
1969 70	5 19	7 15	6 12	3 98	4 09	3 84	5 51	4 86	2 54	3 90	5 12	4 59
1970 71	1 92	5 02	4 64	3 11	4 55	6 44	5 02	5 16	3 49	3 96	5 61	5 67
1971-72	5 45	4 08	3 05	2 12	3 36	3 87	1 92	4 71	1 01	2 12	4 71	5 87
1972-73	14 41	8 16	40	3 44	4 21	7 23	4 40	4 97	1 78	2 37	4 40	8 71
1973-74	16 04	11 56	6 18	6 73	4 96	22 56	16 95	8 55	6 56	11 34	8 00	13 52
1974-75	7 39	8 12	7 31	3 81	4 98	11.95	6 34	12 92	8 21	12 37	6 99	9 32
1975-76	95	6 45	4 68	3 42	5 32	9 12	7 80	9 08	5 60	5 62	4 04	5 74
1976-77	3 79	6 59	3 67	3 79	5 77	9 84	7 56	8 20	4 16	5 25	4 56	8 51
Average	3 32	4 13	2 92	2 36	2 90	3 66	4 01	4 45	2 30	2 95	4 33	3 88

*Changes in logs multiplied by 100

Table 3—Relative changes in quantities, 1949-50 through 1976-77*

Year	Food at home	Food away from home	Alcohol and tobacco	Clothing	Housing	Utilities	Transportation	Medical	Durables	Other nondurables	Services	Miscellaneous
1949 50	−0 07	−1 80	1 79	0 40	5 40	8 12	−0 08	4 66	16 99	6 80	3 97	0.95
1950 51	− 24	3 30	83	−3 21	6 13	4 15	1 46	3 71	−10 19	37	−1 31	3 95
1951-52	1 87	2 68	−.91	3 54	5 02	1 31	1 29	1 62	−5 25	1 01	−3 52	5 04
1952-53	2 32	31	− 02	− 53	3 91	67	47	2 68	9 93	1 60	− 50	1 32
1953-54	1 46	−3 51	−4.97	−1 36	3 07	4 39	−2 77	2 63	− 65	−1 02	81	− 65
1954-55	3 34	64	58	2 77	3 25	4 94	3 52	1.24	17 14	3 50	3 99	31
1955-56	1 97	1 04	1 11	39	3 25	3 40	3 70	4 26	−7 65	2 08	1 29	1 79
1956-57	1 17	−1 17	− 15	−2 42	3 27	1 59	1 13	3 54	−2 85	60	−.91	− 28
1957-58	− 85	−3 67	− 17	− 98	3 34	3 07	− 72	3 34	−9.92	− 75	45	90
1958 59	2 69	1 86	2 17	2 76	3 78	22	3 18	4 03	9 50	4 46	3 15	1 22
1959-60	− 69	1 98	− 97	−1 01	3 75	82	1.91	3 11	− 43	2 92	1 74	14
1960-61	− 54	1 66	34	− 28	3 28	− 19	− 43	2 45	−6 93	2 55	3 22	− 14
1961 62	−1 19	2 78	1 37	2 47	4 14	3 23	2 76	5 26	8 71	7 39	27	2 44
1962-63	− 94	2 02	97	27	3 22	3 08	1.96	4 57	7 71	3 71	1 72	1 55
1963-64	2 68	2 98	− 12	5 32	3 37	3.96	3 09	8 08	6 98	4 38	2 86	2 33
1964-65	3 70	1 75	1 88	2 88	4 20	3 43	2 45	2 49	10 60	6 03	3 52	3 45
1965-66	1 91	− 48	2 17	5 23	3 56	3 50	4 44	2 52	5 77	9 62	3 61	4 40
1966-67	1 76	−2 70	− 31	− 51	3 49	2 95	3 14	1 42	− 05	1 45	4 65	5 17
1967-68	2 80	4 78	14	2 47	4 30	3 62	4 45	4 06	9 81	4 80	1 54	1·97
1968-69	73	15	−1 27	69	3 96	3 91	3 81	7 33	3 16	3 28	1 44	93
1969-70	1 73	− 09	93	−2 15	2 89	2 34	2 39	5 52	−4 97	1 84	36	1 73
1970-71	− 49	−1 94	−1 03	2 05	3 75	43	3 03	3 16	9 56	1 39	18	−2 39
1971-72	02	3 34	2 72	5 31	4 42	5 81	4 97	5 67	12 19	6 72	2 75	2 25
1972-73	−3 78	3 56	6 17	6 05	4 10	3 90	4 14	5 26	7 84	8 54	3 45	−2 90
1973-74	−3 90	− 32	− 32	−1 34	4 44	−6 00	− 13	2 53	−9.82	−1 91	94	−4 72
1974-75	1 22	4 03	− 68	2 48	3 48	3 30	12	1 19	− 61	−7 60	2 55	− 73
1975-76	4 24	5 30	2 51	3 74	3 98	3 67	2 52	6 13	11 10	1 72	4 79	1 90
1976-77	3 16	4 06	− 25	2 94	3 57	2 22	3 53	3 35	8 38	2 08	4 21	85
Average	93	1 16	52	1 38	3 87	2 71	2 12	3 78	3 43	2 77	1 83	1 17

*Changes in logs multiplied by 100

Table 4—Components of change in share of food consumed at home 1949-50 through 1976 77

Year	Change in share	Relative change (percent)		
		Price	Quantity	Income
	Number		*Percent*	
1949 50	−0 7936	1 48	−0 07	5 77
1950 51	7564	10 52	− 24	5 89
1951 52	0713	1 75	1 87	3 18
1952-53	− 6462	−1 75	2 32	4 00
1953 54	1361	0	1 46	83
1954 55	− 7090	−1 93	3 34	5 34
1955-56	− 0569	75	1 97	2 93
1956-57	1797	3 22	1 17	3 45
1957 58	3358	4 23	− 85	1 46
1958 59	− 7393	−1 39	2 69	5 40
1959-60	− 4316	96	− 69	2 86
1960-61	− 1631	96	− 54	1 40
1961-62	− 7858	68	−1 19	4 35
1962-63	− 5505	1 35	− 94	3 86
1963-64	− 1307	1 73	2 68	5 26
1964-65	0156	2 35	3 70	5 93
1965-66	0581	5 16	1 91	6 69
1966-67	− 0467	− 49	1 76	4 40
1967-68	− 0241	3 51	2 80	7 81
1968 69	− 0203	4 76	73	6 85
1969-70	0231	5 19	1 73	5 36
1970 71	− 7247	1 92	− 49	6 50
1971 72	− 3908	5 45	02	8 28
1972-73	1941	14 41	−3 78	9 19
1973-74	4941	16 04	−3 90	8 66
1974-75	− 0275	7 39	1 22	8 77
1975 76	− 6722	95	4 24	10 03
1976-77	− 3201	3 79	3 16	9 36
Average	− 1993	3 32	93	5 49

where q_i is the quantity of the i^{th} commodity demanded, p_i, its price, and m, consumer income (equal to total expenditure) Taking the differential of the logarithm of the above demand function, we obtain

$$d(\log q_i) = \frac{\partial(\log q_i)}{\partial(\log m)}\, d(\log m) + \sum_{j=1}^{n} \frac{\partial(\log q_i)}{\partial(\log p_j)}\, d(\log p_j)$$

$$i = 1, 2, \ldots, n$$

Derivations on the right-hand side of this equation are the elasticities The price elasticities are not symmetric To obtain symmetry of the coefficients, multiply both sides of the equation by the budget share of the i^{th} commodity

$$w_i = \frac{p_i q_i}{m}$$

$$w_i\, d(\log q_i) = \left(\frac{p_i q_i}{m}\,\frac{\partial q_i}{\partial m}\,\frac{m}{q_i}\right) d(\log m),$$

$$+ \sum_{j=1}^{n} \left(\frac{p_i q_i}{m}\,\frac{\partial q_i}{\partial p_j}\,\frac{p_j}{q_i}\right) d(\log p_j)$$

$$= p_i \frac{\partial q_i}{\partial m}\, d(\log m)$$

$$+ \sum_{j=1}^{n} \frac{p_j q_j}{m}\,\frac{\partial q_i}{\partial p_j}\, d(\log p_j)$$

The coefficients of $d(\log p_j)$ are now symmetric The left-hand side of this equation is the quantity component (endogenous) of a change in the consumer's budget shares In the microeconomic theory of consumer behavior, prices and income are considered given and the quantities are the endogenous variables Therefore, $w_i d(\log q_i)$ is the endogenous component of variations in budget shares

Let us define

$$\mu_i = p_i \frac{\partial q_i}{\partial m}$$

$$\pi_{ij} = \frac{p_j q_j}{m}\,\frac{\partial q_i}{\partial p_j}$$

so that

$$w_i d(\log q_i) = \mu_i d(\log m)$$

$$+ \sum_{j=1}^{n} \pi_{ij} d(\log p_j) \quad i = 1, 2, \quad , n$$

The coefficient μ_i is called the marginal budget share of the i^{th} commodity It represents the additional amount spent on the commodity when income (total expenditure) increases by 1 dollar It is also called the marginal propensity to spend, and it is the income elasticity weighted by the value share These coefficients satisfy the restriction

$$\sum_{i=1}^{n} \mu_i = 1$$

This is the adding up property of the demand system The μ's do not have to be positive For an inferior commodity, the marginal propensity to spend is negative However, for broad commodity groups, the μ's are expected to be positive Multiplying both sides of the definition of μ_i by $\frac{m}{q_i}$, we obtain

$$\frac{m}{q_i}\mu_i = p_i \frac{\partial q_i}{\partial m} \frac{m}{q_i},$$

or

$$\frac{m\mu_i}{p_i q_i} = \frac{\partial q_i}{\partial m} \frac{m}{q_i},$$

or

$$E_i = \frac{\mu_i}{w_i},$$

where E_i is the income elasticity of demand for the i^{th} commodity

The parameters π_{ij} are the compensated price elasticity (Slutsky) weighted by the budget shares Dividing both sides of the definition of π_{ij} by q_i, we get

$$\frac{\pi_{ij}}{q_i} = \frac{p_i p_j}{m} \frac{\partial q_i}{\partial p_j} \frac{1}{q_i}$$

or

$$\frac{\partial q_i}{\partial p_j} \frac{p_j}{q_i} = \frac{\pi_{ij}}{q_i p_i} m$$

or

$$\eta_{ij} = \frac{\pi_{ij}}{w_i},$$

where η_{ij} is the price elasticity of demand of commodity i for the j^{th} price The coefficients π_{ij} are called the Slutsky coefficients, and the elasticities η_{ij} are the pure substitution elasticities under a compensating income change to keep utility constant

The price coefficients, π_{ij}, form a symmetric, negative, semidefinite matrix of order n Also

$$\sum_{j=1}^{n} \pi_{ij} = 0 \quad i = 1, 2, \quad , n$$

The sum of these coefficients for each commodity is zero This equation represents the homogeneity condition for the demand equations

Substitutes and complements can be defined simply in terms of the sign of π_{ij} (6) If π_{ij} is positive, goods i and j are substitutes, if π_{ij} is negative, they are complements

The Slutsky coefficients are defined as

$$\pi_{ij} = \frac{p_i p_j}{m} \frac{\partial q_i}{\partial p_j}$$

where $\frac{\partial q_i}{\partial p_j}$ are the quantity price slopes with utility unchanging

$$\left(\frac{\partial q_i}{\partial p_j}\right) u = \text{constant}$$

The traditional formulation of the Slutsky equation is

$$\frac{\partial q_i}{\partial p_j} = \left(\frac{\partial q_i}{\partial p_j}\right) u = \text{constant}$$

$$- q_j \frac{\partial q_i}{\partial m}$$

from which we get

$$\left(\frac{\partial q_i}{\partial p_j}\right) u = \text{constant}$$

$$= \frac{\partial q_i}{\partial p_j} + q_j \frac{\partial q_i}{\partial m}$$

Substitute this in the above definition,

$$\pi_{ij} = \frac{p_i p_j}{m} \left[\frac{\partial q_i}{\partial p_j} + q_j \frac{\partial q_i}{\partial m}\right]$$

The commonly estimated elasticities are generally uncompensated The uncompensated cross price elasticities, however, do not tell us whether the goods are substitutes or complements

Change to elasticities

$$\eta_{ij} = \frac{\pi_{ij}}{w_i} = \frac{m}{p_i q_i}\frac{p_i p_j}{m}\left[\frac{\partial q_i}{\partial p_j} + q_j\frac{\partial q_i}{\partial m}\right] = \left[\frac{p_j}{q_i}\frac{\partial q_i}{\partial p_j} + \frac{p_j q_j}{m}\frac{\partial q_i}{\partial m}\frac{m}{q_i}\right] = [\epsilon_{ij} + w_j E_i],$$

where ϵ_{ij} are the uncompensated price elasticities This equation gives the relationship between the uncompensated and compensated elasticities The commonly estimated elasticities are generally uncompensated (*2, 3, 5, 10*) The uncompensated cross price elasticities, however, do not tell us whether the goods are substitutes or complements

ESTIMATION

To apply the model, we take changes in logs, and use the symbol (D) as the log-change operation

$$D p_t = \log p_t - \log p_{t-1}$$

The demand model is thus

$$w_{it}^* Dq_{it} = \mu_i Dq_t + \sum_{j=1}^{n} \pi_{ij} Dp_{jt} + U_{it} \quad i=1, \ldots, n$$

where

$$w_{it}^* = \frac{w_{it-1} + w_{it}}{2}, \quad Dq_t = \sum_{i=1}^{n} w_{it}^* Dq_{it}$$

and U_{it} is a random error term with the following properties

$$E(U_{it}) = 0$$

$$E(U_{it}U_{js}) = \begin{cases} \omega & \text{if } s = t \\ o & \text{if } s \neq t \end{cases}$$

The random errors do not correlate over time but do correlate across demand equations for each observation It can be shown that the sum of n disturbances U_{it} equals zero for each time period, and that the matrix ω_{ij} is of rank n 1

The variable Dq_t is a weighted sum of the logarithm of quantities demanded It is the sum of the left-hand side of all the demand equations Dq_t measures relative change in total consumption and can be used to measure the relative change in real income Formally, we have 12 demand equations

$$w_{it}^* Dq_{it} = \mu_i Dq_t + \sum_{j=1}^{12} \pi_{ij} Dp_{jt} + U_{it} \quad i=1, 2, \ldots, 12$$

However, it can be shown that only 11 equations are independent Summing the first 11, we get

$$\sum_{i=1}^{11} w_{it}^* Dq_{it} = (\sum_{i=1}^{11} \mu_i)Dq_t + \sum_{j=1}^{12}(\sum_{i=1}^{11} \pi_{ij})Dp_{jt} + \sum_{i=1}^{11} U_{it}$$

The left hand side is $Dq_t - w_{12}^* Dq_{12t}$ because

$$Dq_t = \sum_{i=1}^{12} w_{ij}^* Dq_{it}$$

The first term on the right-hand side is $(1 - \mu_{12})Dq_t$, because

$$\sum_{i=1}^{12} \mu_i = 1$$

From

$$\sum_{i=1}^{12} \pi_{ij} = \sum_{j=1}^{12} \pi_{jt} = 0$$

and symmetry, we have

$$\sum_{j=1}^{11} \pi_{ij} = -\pi_{j12} = -\pi_{12j}$$

Because the sum of U_{it} is zero for each t,

$$\sum_{i=1}^{11} U_{it} = -U_{12}$$

Using these values we obtain

$$Dq_t - w_{12}^* Dq_{11t} = (1 - \mu_{12})$$

$$Dq_t - \sum_{j=1}^{12} \pi_{12j} Dp_{jt} - U_{12t}$$

which is the 12th equation In other words, we can leave out the 12th equation because all the information is contained in the other 11 Also, Barten has shown that it makes no difference which equation is left out, the estimates of the coefficients will be the same (*3*)

Here, the equation for the miscellaneous category was omitted, being of little interest because the

As utility prices rise, food consumption goes up, while that of alcohol and tobacco, clothing, other nondurables, and services declines Low income elasticity of food consumed at home explains the fall in the budget share of this category from 1949 to 1977

items are included in the PCE for accounting purposes only

We can impose homogeneity on the model by using the miscellaneous price as a deflator for the other 11 prices Estimates of the coefficients were obtained in several stages First, the model was fitted without symmetry restrictions and with and without the intercepts The model without the intercepts gave positive price elasticities for the clothing, medical services, and durables categories The model with intercepts also gave a positive price elasticity for clothing Estimates of price elasticities for medical and durables were negative, however, while the income elasticity estimate for durables was high

Second, the model was fitted with the symmetry constraint, estimates of price coefficients were required to be symmetric Again, positive estimates were obtained for price elasticities for the medical services and durables categories So this problem could be overcome, the negative price elasticities from an unconstrained system with intercepts were included as prior estimates in a symmetric system The model also includes the theoretical restriction implied by the homogeneity, adding-up, and symmetry conditions

RESULTS

As discussed above, the demand system was fitted by including prior own price coefficients for medical services (−0 01176) and durables (−0 10426) (table 5) Because the Slutsky matrix is symmetric, only the upper triangle of the price coefficient matrix appears Values in parentheses under the coefficients in table 5 are the t values All the income coefficients have high t values, as do the own price coefficients (except for *a priori* medical services and durables) Estimates of all the price and income elasticities appear in table 6

All the income elasticities are positive in table 6, which means that all goods are normal One expects this behavior at this level of aggregation The following categories have income elasticity estimates greater than one, which shows they are luxuries food away from home (1 16), medical services (1 289), durables (2 459), non durables (1 275), and services (1 009)

The positive sign of cross elasticity means substitutability, the negative sign indicates complementarity between two goods Food consumed at home substitutes for food away from home, alcohol and tobacco, clothing, housing, utilities, medical, other nondurables, and services Food at home shows complementarity with transportation and durables Food away from home substitutes for food at home, alcohol and tobacco, clothing, housing, utilities, and other nondurables Food away from home shows complementarity with the transportation, medical, and services categories The estimates of income and price elasticities for food at home are 0 364 and −0 463, respectively The elasticity estimates for food consumed away from home are 1 16 (income) and −0 917 (price) The income elasticity of demand for food consumed at home is, not surprisingly, the smallest of all the expenditure categories

The cross elasticities in table 6 show that utilities substitute for food consumed at home and food away from home As utility prices rise, food consumption goes up, while that of alcohol and tobacco, clothing, other nondurables, and services declines Higher priced transportation services, a major part of which is oil and gasoline, are associated with lower use of food at home, food away from home, housing, medical, durables, and other nondurables A rise in the prices of medical services is associated with a decline in consumption of food away from home, alcohol and tobacco, transportation, and nondurables

Low income elasticity of food consumed at home explains the fall in the budget share of this category from 1949 to 1977 The income elasticity of food consumed away from home is close to one, which agrees with the almost constant budget share

LIMITATIONS

The analysis and results presented here have three drawbacks limitations of the general approach, the specific model, and the data

The general approach, based on classical consumer demand theory, explains variations in consumption in terms of consumer income and prices Although the model is a "complete system," it does not account for all the possible variables Thus, specification error could occur in, for example, the equation for durables The demand for durables, which is complex, depends on many

Table 5—Estimates of the coefficients of a demand system for consumer expenditures

Item	Marginal shares	Slutsky coefficients										
		Food at home	Food away from home	Alcohol and tobacco	Clothing	Housing	Utilities	Trans-portation	Medical	Durables	Other non-durables	Services
Food at home	0 05863 (5 91508)	−0 07456 (6 62960)	0 01013 (1 31471)	0 00848 (1 67941)	0 01304 (1 47319)	0 01820 (1 94207)	0 01175 (2 72088)	−0 00890 (1 95838)	0 01052 (1 07927)	−0 01106 (66704)	0 02019 (3 14762)	0 00543 (62554)
Food away from home	0 4780 (5 50558)		− 03777 (2 35581)	00650 (86211)	00824 (63824)	00161 (14739)	01856 (3 04852)	− 01523 (2 05569)	− 00839 (75708)	01599 (1 50829)	01439 (1 55846)	− 00487 (49808)
Alcohol and tobacco	01911 (3 56263)			− 01594 (2 24856)	− 01215 (1 44359)	00005 (00713)	− 00622 (1 52223)	00526 (1 13153)	− 00674 (96656)	03769 (5 10675)	− 01799 (2 88649)	− 00807 (1 22929)
Clothing	06616 (6 52404)				− 03933 (2 15044)	− 01373 (1 10681)	− 00688 (1 03315)	00122 (16839)	00826 (66061)	03721 (2 74923)	− 00083 (08521)	00300 (26591)
Housing	12540 (10 83640)					− 03169 (1 98882)	01014 (1 75852)	− 00873 (1 28231)	00741 (60499)	− 02250 (1 64094)	00878 (99544)	03809 (3 48907)
Utilities	03496 (7 41158)						− 01843 (4 04832)	00965 (2 30511)	00976 (1 65818)	00853 (1 34284)	− 00031 (05964)	− 00985 (1 77126)
Transportation	05615 (11 16300)							− 02075 (3 38544)	− 01383 (2 20837)	− 01226 (1 84286)	− 00389 (67674)	03135 (5 29064)
Medical	08444 (6 69910)								− 01176 (a priori)	01569 (97996)	− 02901 (3 28437)	01346 (1 31675)
Durables	30808 (9 72334)									− 10426 (a priori)	01963 (1 96979)	− 00073 (05275)
Nondurables	06626 (9 51261)										− 03552 (3 32586)	02858 (3 55683)
Services	10626 (10 23920)											− 08789 (5 94478)

Table 6—Estimates of income and price elasticities

Item	Income elasticities	Price elasticities										
		Food at home	Food away from home	Alcohol and tobacco	Clothing	Housing	Utilities	Trans portation	Medical	Durables	Other non-durables	Services
Food at home	0 364	−0 463	0 063	0 053	0 081	0 113	0 073	−0 055	093	−0 069	0 125	0 034
Food away from home	1 160	246	− 917	158	200	391	450	− 370	− 204	388	349	− 118
Alcohol and tobacco	373	166	127	− 311	− 237	001	− 121	103	− 132	736	− 351	− 158
Clothing	665	131	083	− 122	− 395	− 138	− 069	013	083	374	− 008	030
Housing	941	137	012	000	− 103	− 283	076	− 065	056	− 169	066	286
Utilities	964	324	511	− 171	− 190	279	− 507	266	269	235	− 009	− 271
Transportation	743	− 118	− 202	070	016	− 116	128	− 275	−183	− 162	− 052	415
Medical	1 289	161	− 128	− 103	126	113	149	− 211	− 180	240	− 443	205
Durables	2 459	− 088	128	301	297	− 180	068	− 098	125	− 832	157	− 006
Nondurables	1 275	388	277	− 346	− 016	169	− 006	− 075	− 558	378	− 683	550
Services	1 009	052	− 046	− 077	028	361	− 093	288	128	− 007	271	− 834

Earlier studies have usually treated food demand in isolation from consumers' other allocation decisions

other variables besides prices and income An elaborate model for durables would include credit availability, interest rate, average life of the equipment, and so on

The specific model used here, the Rotterdam Model, is based on the principle of maximization of utility without restriction on the functional form Therefore, it is more realistic and general than other complete systems, such as the linear expenditure system or the indirect addilog model However, the Rotterdam Model assumes that marginal budget shares (μ_{ie}) and the Slutsky coefficients (π_{ij}) are constant, that is, they are independent of prices and income The rapid rise in prices since 1972 makes this a restrictive assumption A model encompassing variable parameters for these two elements must await further developments in the theory of consumer demand

The third category of limitations is inherent in the Personal Consumption Expenditure data used here The PCE represents the most comprehensive series available on consumer expenditure, but it has many limitations when considered for use in demand analysis Developed as a part of the national income accounts, the PCE must fit into these accounts' requirements and definitions

For example, the PCE on durables is obtained by multiplying the number of pieces of equipment sold by an average price and allocating the expenditure between personal consumption and producer durable equipment What consumers actually pay during any given year is the installment payment Any difference between the PCE and the amount of equipment sold is a source of error

PCE expenditure on medical service measures the expenditure by the private sector In recent years, the proportion of health expenditure financed by the Government has increased considerably According to U S Department of Health, Education and Welfare estimates, public expenditure on health services increased from 20 percent in 1950 to 42 percent in 1976 (*6*) See (*8*) for a critique of personal consumption expenditure data for food

The study reported on here represents, despite these limitations, a major step in studying food demand as part of an interrelated system of consumer demand equations Earlier studies have usually treated food demand in isolation from consumers' other allocation decisions Hassan and others made the only other application of the Rotterdam model to U S data known to this author (*7*) They fitted the relative prices versions of the model to PCE data for 1929-65, and, to estimate the coefficients, they incorporated the separability hypothesis Recent revisions of the U S national income accounts (*15*) provided additional motivation for the present work

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APPENDIX: THE DATA

Data used here are per capita U S personal consumption expenditures for 1949-77, divided into 12 commodity groups

(1) Food at home includes food purchased for off-premise consumption excluding alcohol
(2) Food away from home includes purchased meals and beverages
(3) Alcohol and tobacco.
(4) Clothing includes shoes, and other footwear, shoe cleaning and repair, clothing and accessories except footwear, cleaning, laundering, dyeing, pressing, alteration, storage, and repair of garments, and jewelry and watches
(5) Housing includes owner-occupied nonfarm dwellings and tenant-occupied nonfarm dwellings
(6) Utility includes electricity, gas, fuel oil, and coal
(7) Transportation includes tires, tubes, accessories, and other parts, repair, greasing, washing, parking, storage and rental, gasoline and oil, bridge, tunnel, ferry, and toll roads, insurance premiums less claims paid, purchased local transportation, and purchased intercity transportation
(8) Medical care expenses include drug preparations and sundries, physician, dentist, and other professional services, and privately controlled hospitals and sanitariums, medical care and hospitalization insurance, income loss insurance, and workmen's compensation insurance
(9) Durable goods include furniture, mattresses, and bedsprings, kitchen and other household appliances, china, glassware, tableware, and utensils, other durable house furnishings, books and maps, wheel goods, durable toys, sports equipment, boats, and pleasure aircraft, radio and television receivers, new autos, net purchases of used autos, and other motor vehicles
(*10*) Other nondurable goods include toilet articles and preparations, semidurable household furnishings, cleaning and polishing preparations, miscellaneous household supplies and paper products, stationery and writing supplies, magazines, newspapers, and sheet music, nondurable toys and sport supplies, and flowers, seeds, and potted plants
(11) Other services include personal business expenditures, barber shops, beauty shops, and baths, water and other sanitary services, telephone and telegraph, domestic service, other household operations, radio and television repair, admissions to spectator amusements, clubs and fraternal organizations, parimutual net receipts, other recreation, and commercial participant amusements
(12) Miscellaneous includes private education and research, religious and welfare activities, net foreign travel, food furnished employees, food produced and consumed on farms, clothing furnished military, rental value of farm dwellings, other housing, and ophthalmic products and orthopedic appliances

Detailed expenditures from the Commerce Department public use tapes were aggregated into these 12 categories Dividing the current dollar expenditure by the constant dollar expenditure produced implicit price deflators.