THE RECURSIVE ADAPTIVE PROGRAMMING HYBRID MODEL: A TOOL FOR ANALYZING AGRICULTURAL POLICIES

By Wen-yuan Huang, Reuben N. Weisz, Kenneth H. Baum, Earl O. Heady, and Lloyd Teigen*

A comprehensive model of U.S. agriculture incorporates the spatial pattern of supply, resource use, and the technical structure of agricultural production that is generated by a linear programming component, and it utilizes detailed information on market structure, processes, and prices that is provided by an econometric component. The methodology for the hybrid model is explained, and a summary of lessons learned from a recent test of this model is presented.

Keywords

Policy
Simulation
Forecasting
Models
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INTRODUCTION

Many agricultural policies have been region specific or have had different regional impacts. Policies for cotton, public irrigation programs, and others have been region specific. In the supply control programs of the fifties, for example, the Southeast could shift land from cotton to feedgrains and wheat. Thus, the region partially escaped the rigors of supply control in a way that other regions could not.

Because of the different impacts of agricultural policies among regions, we need models that reflect price, income, resource use, and related items over space and time. Econometric models and mathematical programming models can be used independently, or in combination with each other, depending on the needs of the analysis.

Econometric models can be *positive* or predictive, forecasting the response that farmers and regions will take (*13, 14, 24, 25*).[1] These models predict future response based on past experience as reflected in time series data.

Programming models can be *normative*, suggesting the response that farmers and regions ought to take. Such models indicate, for example, whether natural resources or environmental possibilities are sufficient (*9, 11, 12, 17, 21, 38*).

In some instances, we want to examine production potential or resource capability and to learn the market outcome if these potentials were realized. Here, we need to link a normative model with a positive model. Sonka and Heady's study (*32*) is typical.

In some instances, we combine an econometric component and a programming component to generate predictive estimates, for example, Day's (*6*), Schaller and Dean's (*29*), and Sahi's (*28*) recursive linear programming tableaus linked yearly by econometrical flexibility constraints. Positive and normative aspects have been combined in quadratic programming models (*19, 23, 35*). Although these solutions are simultaneous, they utilize econometrical demand functions in the objective functions and conventional linear programming constraints. Generally, the recursive linear programming models are used for shortrun analyses, while the simultaneous models are used for longrun analyses.

We present a Recursive Adaptive Programming (RAP) hybrid model in this article which combines a large-scale econometric model with a large-scale programming model. Ideally, such a hybrid model would provide the best features of both types of models, while eliminating problems associated with each. The ideal hybrid would incorporate information on the spatial pattern of supply, resource use, and the technical structure of production generated by the programming model. And it would use detailed information on market structure, processes, and prices provided by the econometric model. Such a hybrid can simulate a dynamic sequence of interrelated events over space and through time and provide a consistent set of economic performance indicators. Our model achieves these objectives.

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[1] Italicized numbers in parentheses refer to items in References at the end of this article.

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FOUR METHODS OF COMBINING AN ECONOMETRIC MODEL AND A PROGRAMMING MODEL

Before presenting our hybrid model, we discuss four alternative hybrid approaches for linking econometric and mathematical programming models

The One-Way Communication Model

In the One-Way Communication Model, output from one type of model becomes input for the other For example, information can flow from the econometric model to the programming model This hybrid is best characterized by a single-period and interregional programming model with fixed demands determined by a set of econometric equations

The National Water Assessment conducted by the U S Water Resources Council used a One-Way Communication Model to analyze alternative future potentials for U S agriculture (*18*) The quantities of agricultural products demanded were projected by an econometric model for 1985 and 2000 (*24*) These demand projections became constraints in a linear interregional programming model (*18*) That model then projected the least-cost (competitive equilibrium) spatial pattern of agricultural production and resource use subject to these minimum fixed demands

This One-Way Communication Model has worked well for long range analysis However, its ability to simulate the shortrun behavior of the agricultural sector (not its original purpose) is limited by the lack of feedback from the programming to the econometric model within or between time periods This model obtains nonfeasible solutions when the econometrically estimated values of the linkage variables fall outside the feasible region defined by restraints in the programming model

The Simultaneous Solution Model

The Simultaneous Solution Model uses equations from an econometric model as identities (rather than inequality constraints) within the programming model (*19*, *23*, *35*) Its conceptual appeal is that the solution will simultaneously satisfy the assumptions of both parent models

Penn and others (*22*) used this approach to evaluate the shortrun impacts of energy shortages on the U S economy Their Simultaneous Solution Model incorporated input output data developed by the U S Department of Commerce for 85 sectors into a linear programming model that contained two energy constraint equations (*37*)

Problems will arise in applications of a Simultaneous Solution Model when any of the following three conditions occur

1 The feasibility region defined by the equations derived from the positive model is smaller than the computational errors inherent in the linear programming software package
2 A static equilibrium solution is imposed on a dynamic disequilibrium system (*2*)
3 Nonlinear equations derived from the econometric component result in prohibitive computational costs or cannot be solved when cast within a mathematical programming framework

A Simultaneous Solution Model constructed from large scale ESCS econometric and programming models would contain thousands of equations and tens of thousands of variables A Simultaneous Solution Model of this size would be computationally impossible and/or prohibitively costly (particularly if bounding procedures were used)

Recursive Interactive Programming (RIP) Models

Unlike the static hybrid models described earlier, RIP models (*1*, *30*, *31*) simulate the evolutionary structure of the economy over space and through time A RIP model can be characterized as an intertemporal sequence of One-Way Communication Models that has the following basic features

- Within each stage of time, the individual (econometric and programming) components are solved once in a prespecified sequence
- Within each stage of time, the state of the model is defined by historical information derived from preceding stages of simulation and by exogenous events (that is, input data) not brought about by the previous history of the simulation

The RIP Models have many advantages over those described earlier For example

The RAP model uses an econometric model (26) as the first component of the hybrid and a linear programming model (15) as the second component

1 They allow for a flow of communication within each stage and between stages
2 They present fewer computational problems than the Simultaneous Solution Models because the feasibility set is not restricted to equality solutions of the econometric model
3 They dynamically simulate a sequence of events over space and through time in a nonsimultaneous, or cobweb, frame work
4 They allow evaluation of potential supply capacities for the future in contrast to being based on time series data

The RIP approach also has limitations For example

1 If the first component within each stage of time is a linear programming model, the RIP hybrid tends to overestimate total production and, therefore, to underestimate prices because the linear programming component produces an economically efficient use of resources
2 The RIP hybrid begun by being run with an econometric model may encounter an infeasible solution The econometric component may give an estimated production that exceeds capacity
3 If either of the components has been specified incorrectly, the model's recursive nature may result in propagation of errors over time

The first problem has been ameliorated by introducing pseudo-behavioral constraints into the programming component The RIP models cited earlier had a procedure for adjusting upper and lower bounds on regional acreage limitations to respond to the price impacts produced by the econometric component, this is appropriate

The second and third problems presented by the RIP model can be partly addressed by incorporating a two-way flow of communication between the econometric and programming components within each stage of the analysis This feedback concept resembles a self-adaptive control system (7) It is a model able to change values of variables that link components through an internal process of estimation, evaluation, and adjustment according to a predetermined rule It forms the basis of our RAP model which is described below

THE HYBRID MODEL

Recursive Adaptive Programming (RAP) Model

The RAP model uses an econometric model (*26*) as the first component of the hybrid and a linear programming model (*15*) as the second component It is constructed from the RIP model by including a feedback structure in each stage

The Cross Commodity (CED-CC) Model

The econometric component (*36*) descends from the Commodity Economics Division (CED) Commodity Forecast System (*4*) Commonly referred to as the CED Cross Commodity (CED-CC) Model, it includes both crop and livestock sectors

The model has 127 exogenous variables and 164 endogenous variables represented by 164 regression and identity equations These equations are divided into 10 groups retail demand, retail product supply relations in the dairy sector, farm demand for the livestock sector, capital stocks, livestock supply, crop demand, product stocks, planted acreage relations, supply and utilization identity, and index definitions The crop sector includes corn, sorghum, barley, oats, wheat, and soybeans The CED-CC model can be expressed as

$$Y_{it} = a_{it} + \sum_{n=1}^{164} (b_{0in} Y_{nt}) + \sum_{n=1}^{164} \sum_{k=1}^{5} (b_{kin} Y_{n,t-k}) + \sum_{m=1}^{127} (c_{im} z_{mt}) + e_{it} \qquad (1)$$

where i=1, , 164, Y_{nt} and z_{mt} denote endogenous and exogenous variables, respectively, the diagonals of b_0 are zero, the b_k matrices are increasingly sparse, and e_{it} is an error term

The Linear Programming Model

The linear programming component (*15*) updates the National Water Assessment Model (*18*) described earlier To reduce the cost of

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testing this hybrid model, the programming component has only one land class and it uses only land as the resource restraint There are 13 commodities ($i = 1 \ldots 13$) in the model For computational purposes, these are divided into two groups
$i = 1 \ldots 6$ includes corn, sorghum, oats, barley, wheat, and oilmeals,
$i = 7 \ldots 13$ includes corn silage, sorghum silage, nonlegume hay, legume hay, cotton, summer fallow, and sugar beets

The programming component can be expressed as

Maximize

$$\sum_{i=1}^{6}\sum_{j=1}^{105}\Big[\sum_{k=1}^{k_j}(XD_{ijkt} + XI_{ijkt})P_{ijt} - \sum_{k=1}^{k_j} XD_{ijkt}CD_{ijkt} - \sum_{k=1}^{k_j} XI_{ijkt}CI_{ijkt}\Big] - M\sum_{i=1}^{6}(V_i^+ + V_i^-) \qquad (2)$$

Subject to

National production balance restraints

$$\sum_{j=1}^{105}\sum_{k=1}^{k_j}(XD_{ijkt} + XI_{ijkt}) + V_i^+ - V_i^- = Q_{it} \qquad (3)$$

$i=1, \ldots, 6$, k_j varies from region to region, and

regional production response restraints

$$\sum_{k=1}^{k_j}(XD_{ijkt} + XI_{ijkt}) \leqslant [\bar{\beta}_{ijt}]\Big[\sum_{k=1}^{k_j}(XD_{ijk\,t-1} + XI_{ijk\,t-1})\Big]$$

$$\sum_{k=1}^{k_j}(XD_{ijkt} + XI_{ijkt}) \geqslant [\beta_{ijt}]\Big[\sum_{k=1}^{k_j}(XD_{ijk\,t-1} + XI_{ijk\,t-1})\Big], \qquad (4)$$

land restraints,

$$\sum_{i=1}^{13}\sum_{k=1}^{k_j} VD_{ijkt}\,XD_{ijkt} \leqslant LD_{jt}$$

$$\sum_{i=1}^{13}\sum_{k=1}^{k_j} VI_{ijkt}\,XI_{ijkt} \leqslant LI_{jt}$$

$$j=1, \ldots, 105, \qquad (5)$$

where XD_{ijkt} (or XI_{ijkt}) is defined as the quantity of production of crop i using rotation and tillage practice k on dry (or irrigated) land in producing area j in time period t CD_{ijkt} or CI_{ijkt} is the cost of producing one unit of XD_{ijkt} or XI_{ijkt}, respectively VD_{ijkt} or VI_{ijkt} is acres of land used to produce one unit of XD_{ijkt} or XI_{ijkt}, respectively LD_{jt} or LI_{jt} is total dry or irrigation land available in producing area j in time period t P_{ijt} is the farm level price for crop i in producing region j in the time period t M is an arbitrarily large penalty cost that is associated with the deviational variables, V_i^+ and V_i^- $\bar{\beta}_{ijt}$ and $\underline{\beta}_{ijt}$ are, respectively, the maximum and minimum proportionate increases or decreases of production of crop i in producing area j from year $t-1$ to year t, the price elasticities are used to determine their values

Linkages Between Components

To evaluate shortrun impacts of agricultural policies, one would use the econometric model for the principal component in the hybrid model and the linear programming (LP) model for the following subordinate and complimentary roles

- Three sets of endogenous variables are selected as linkage variables to transfer information from the econometric component to the programming component These three sets (expressed as Y_{it} in the econometric component) are regional crop price P_{ijt}, cost of production CD_{ijkt} (and CI_{ijkt}), and national aggregate crop production Q_{it} At time period t, the values of P_{ijkt} and CD_{ijkt} are used to revise the coefficients in the objective function, the values of P_{ijt} are used in the regional production response restraints, the value of Q_{it} is used as the value of the right-hand side of the national aggregate production balance restraints

- For each commodity, the LP model contains an accounting row that measures the deviation (V_i) between aggregate produc tion as forecast by the econometric component and the aggregate contained in the LP solution Large penalty costs are assigned to the deviational variables in the profit maximizing objective function to force the LP solution to approach the econometric solution as nearly as possible

- If all the deviational (production) variables in the LP solution vector are equal to zero, the solutions produced by the two components are assumed consistent the econometric estimates are within the feasible region In this case, the RAP model initiates computations for the next stage in time
- However, if any deviational variables in the LP solution vector are not equal to zero, the production forecast by the econometric component lies outside the production possibilities region defined by the feasibility constraints in the LP component Here, the predetermined adaptive feedback mechanism is invoked The production variables in the econometric component become linkage variables from the programming component to the econometric component, they are set equal to the LP solution values The econometric component is resolved producing a new set of endogenous variables such as prices These newly adjusted values are used subsequently in the simulation, they comprise the historical information that defines the state of the model in the next stage of time

Test Methods

The hybrid model's performance in estimating agricultural production, prices, and levels of other agricultural activities was tested with static and dynamic simulation Both test methods were applied to the hybrid model and to the CED-CC Model Estimated values from these two models are compared with actual observations

In the static simulation, actual observed data are used for all predetermined variables (including lagged and exogenous) for each time period In the dynamic simulation, the lagged endogenous variables are estimated recursively and used as input in the next time period

Results from the static test provide information on how well the model can perform when errors from input data are removed or kept at a minimum Results from the dynamic test provide information on how well the model can be used for multiperiod simulation—for example, how seriously the error accumulated in previous time periods will affect the performance of the model in later time periods

The years 1969 and 1972 were selected arbitrarily for the static simulation of the hybrid model [2] The years 1969 through 1973 and the years 1972 through 1976 were selected for the dynamic test However, only results for 1969-73 are presented here The regression coefficients of the econometric component (CED-CC model) were established in 1977 from historical data for 1950-77 Endogenous and exogenous data for 1960-77 were updated

The data set in the programming component was derived from the 1975 LP data base at the Center for Agricultural and Rural Development (CARD) at Iowa State University Initial data (1968 and 1971) were derived from this data base The production costs were adjusted according to cost indices for production, interest, taxes, and wage rates Projected production costs were adjusted by a constant rate from test periods 1969 and 1971 Constant yield was assumed during the test period The derived regional to national price ratio (1972-74) was assumed unchanged The values of elasticities are from Richardson and Ray (27)

Test Results

Each year's simulation of the econometric component determines 164 values for endogenous variables—livestock and crop production, utilization, and marketing activities The programming component gives spatial distributions of thousands of crop production activities and land use patterns in 105 producing areas Empirical results are available from the authors for the 48 contiguous States Key data from a selected State are presented in the following table (Iowa was selected because two of the authors are currently working there)

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[2] In conducting a static simulation, one must use actual values for all predetermined variables as input data Although this requirement poses no difficulty in the econometric component, it does pose difficulty in the programming component The LP component uses extensively synthesized data that do not have observed values, therefore, it only approximates a static simulation

> *The static simulation results indicate that the hybrid model does well estimating production of major crops (that is, corn and soybeans) at both State or national levels but performs poorly in estimating output of minor crops (for example, oats)*

Static simulation results of hybrid model, 1969 and 1972

Area and crop	Results		Error
	Actual	Estimated	
1969 national production			
	Million bushels		*Percent*
Corn	4,687	4,487	0 27
Soybeans	1,133	1,116	1 50
Oats	965	959	62
Wheat	1,442	1,453	76
1969 Iowa production			
	Thousand bushels		
Corn	1,012,563	1 001,146	1 13
Soybeans	179,850	182,530	1 49
Oats	93,840	108,720	13 69
Wheat	1,320	1,755	32 95
1972 national production			
	Million bushels		
Corn	5,570	5,444	0 24
Soybeans	1,270	1,312	3 31
Oats	690	784	13 62
Wheat	1,546	1,601	3 56
1972 Iowa production			
	Thousand bushels		
Corn	1,212,200	1,154,493	4 76
Soybeans	217,800	215,161	2 92
Oats	70,000	81,362	16 23
Wheat	1,238	1,360	9 90

The static simulation results indicate that the hybrid model does well estimating production of major crops (that is, corn and soybeans) at both State or national levels but performs poorly in estimating output of minor crops (for example, oats) At the State and national levels for both years (1969 and 1972), there was less than a 5-percent error in estimation for the major crops However, at the State and national level for a minor crop (oats), there was a more than 13 percent error in estimation

In the dynamic simulation test, most of the national crop production generated by the econometric component was adjusted by the programming component This caused a significant discrepancy between the hybrid model and the CED-CC model in their estimates of national crop production and prices

Figures 1 through 4, grouped at the end of this article, illustrate the following significant features of the hybrid model

- When using regional restraints, the hybrid model does not yield better estimates for aggregate national production and price than those generated by the CED-CC model alone This failure occurs because the restraints caused by using national price elasticities do not represent the regional responses adequately
- The adjustment mechanism in the hybrid model assumes that national aggregate production can be estimated better by summing the individual regional production estimates than by using the national aggregate figure from the CED-CC econometric model This assumption is true only if a set of accurate regional response functions can be formulated To improve the performance of the hybrid model, we should estimate and use region specific elasticities of production with respect to price instead of the national (β) elasticities that were available for use in this study
- The time recursive structure used by the hybrid model will accumulate error and pass it on to the next time period (The estimates illustrate this point) This error might be reduced by formulating regional restraints as a function of the endogenous

The Recursive Adaptive Programming (RAP) hybrid model is the most sophisticated method of linking econometric and programming components

variable in the econometric component rather than depending heavily on the previous year's production, as in (*4*)

Therefore, we suggest that whenever accurate regional response restraints are not available, the One-Way Communication Model will probably perform better between time periods than will any model with a recursive structure

In a second dynamic simulation run, we did not include the previously described regional restraints but used instead four regression equations representing corn, soybean, oat, and wheat production responses to generate the right hand side values of the regional restraints for Iowa The hybrid model gave the same estimation of national production as the CED-CC model Furthermore, we made significant improvement in simulating regional (Iowa) crop production, as judged by the values of the root mean square error (RMSE) This outcome demonstrates that if we use a regional response function that is better estimated econometrically, the hybrid model will yield better estimates of national and regional production and price

CONCLUSION

The need for a policy model with space and time characteristics of price, production, and resource use has led to the development of hybrid models combining econometric and programming components The Recursive Adaptive Programming (RAP) hybrid model is the most sophisticated method of linking econometric and programming components It uses a programming component to validate the estimates by the econometric component and adjusts the estimates when they fall outside the feasible production region

The static simulation tests of RAP show it performs well in estimating corn and soybean production at both national and regional (Iowa) levels, but they show inconsistencies in estimating production of oats and wheat The dynamic simulation tests show that both national and regional (Iowa) estimates follow the general movement of the observations but have cumulative error The model could be used as a national model if the bounds of regional restraints were relaxed The regional restraints need to be improved considerably, before a high degree of confidence can be attached to the region specific results of the RAP model

Figure 1

Corn: Performance of Hybrid and CED-CC Models Compared with Actual Observations

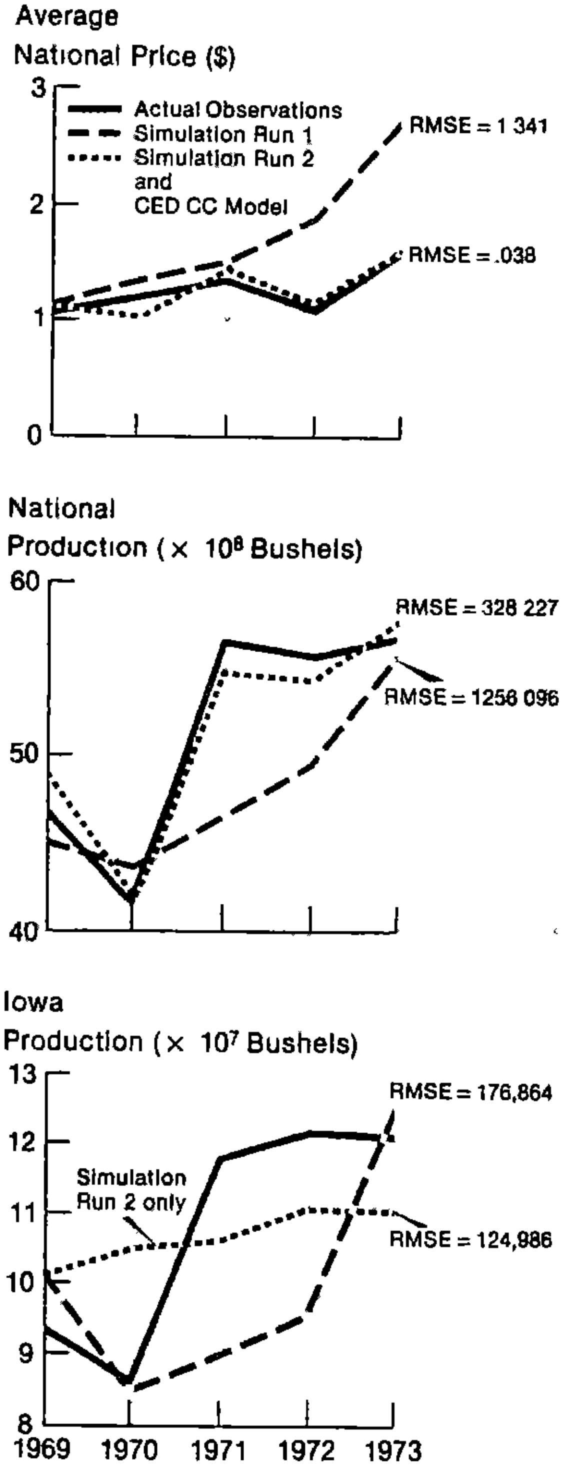


Figure 2

Soybeans: Performance of Hybrid and CED-CC Models Compared with Actual Observations

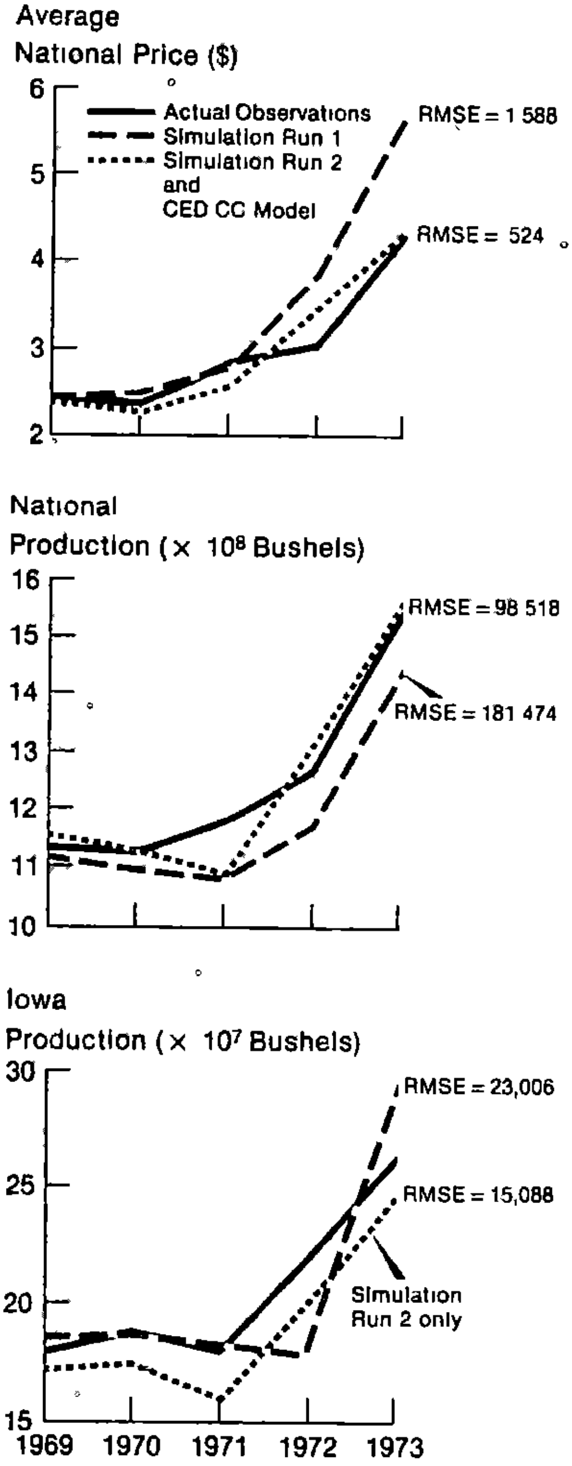


Figure 3

Oats: Performance of Hybrid and CED-CC Models Compared with Actual Observations

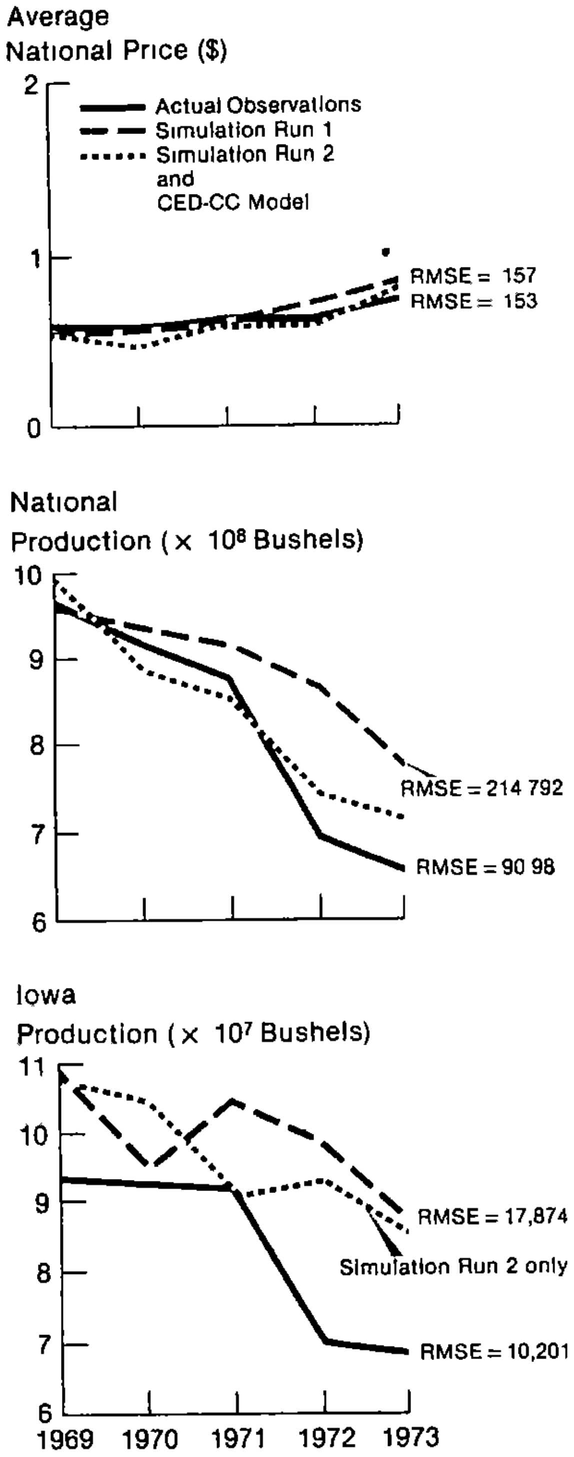
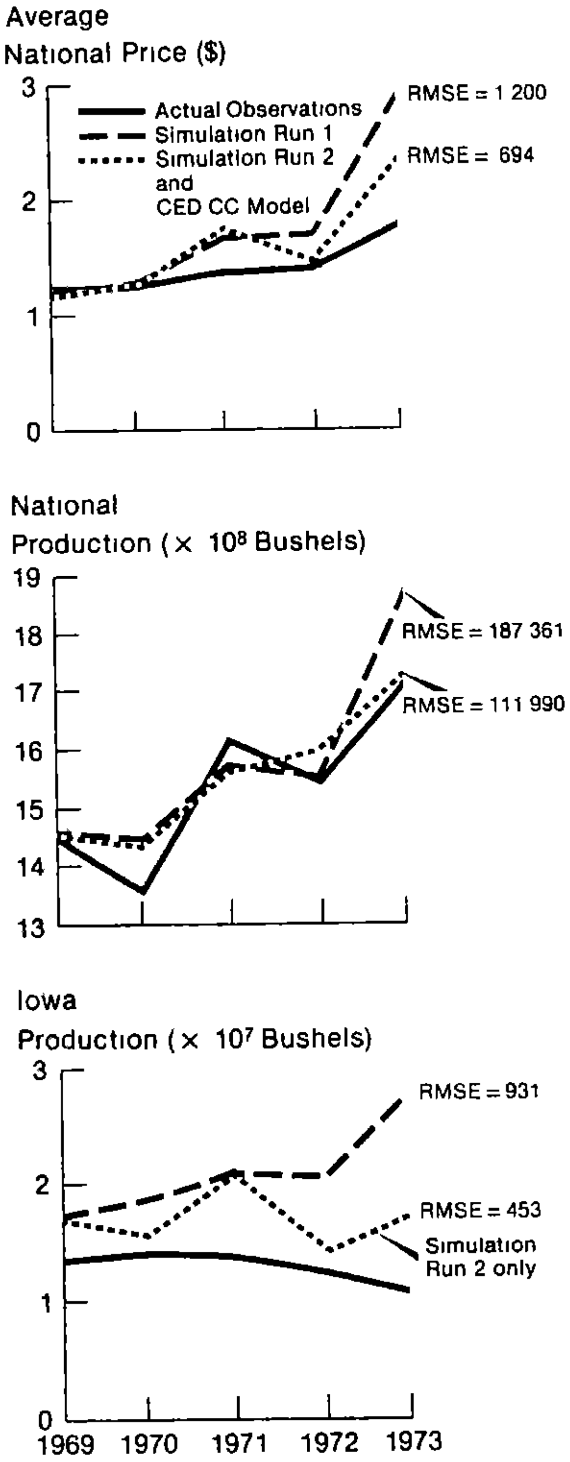


Figure 4

Wheat: Performance of Hybrid and CED-CC Models Compared with Actual Observations



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