Structural Stability and Recursive Residuals: Quarterly Demand for Meat

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Methods have been developed recently for continual evaluation of structural stability for model specification. CUSUM and CUSUMSQ tests suggested by Brown, Durbin, and Evans employ recursively calculated residuals to permit the examination of structural stability. Easily developed plots can be used to generate the inferences which can be supported by the tests. This article reports on an application of these tests for five quarterly meat demand equations for Canada.

Keywords

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Structural change
Demand for meat
CUSUM
CUSUMSQ

Government and private sector decisionmakers and analysts commonly use quarterly estimates of demand for meat and other agricultural commodities in policymaking and forecasting. Models from which these estimates are developed are necessarily highly simplified as to demand theory and the institutional specifics of the industry. Accordingly, careful analysts and forecast users want to be assured of the accuracy with which these models approximate the true structure of the situation studied. They often make re-estimations based on different data periods and respecifications of the models to evaluate the approximations originally provided from the models.

More formal procedures for continual evaluation of structural stability for model specifications have recently begun to be developed. Various methods, beginning with the Chow (*3*) and F tests on various sample partitions and nested specifications, can now be used to assess structural change statistically (*7, 4*).[1] Two of these with considerable intuitive and computational appeal have been suggested by Brown, Durbin and Evans (*2*). These so-called CUSUM and CUSUMSQ tests employ recursively calculated residuals to permit the examination of structural stability against quite complex alternatives. Easily developed plots can be used to generate the inferences which can be supported by the tests.

We report in this article on an application of the CUSUM and CUSUMSQ tests to five quarterly meat demand equations for Canada. The application demonstrates the feasibility of applying these tests in routine forecasting and policy contexts. Results indicate the importance of misspecification errors implicit in these simple models and the sample periods over which the approximation can be used with relative confidence.

THE RECURSIVE RESIDUALS

Consider the linear model of the form

$$y_t = \underline{X}'\beta_t + u_t \;, \qquad t = 1, \ldots, T, \tag{1}$$

where y_t is the observation on the dependent variable, $\underline{X}_t$ is the column vector of observations on the k independent variables, $\underline{\beta}$ is a corresponding vector of coefficients (the subscript, t, implying that the β's may not be constant over time) and u_t is an additive disturbance term. The first variable, X_{1t}, takes the value of unity for all T observations. The remaining regressors are assumed nonstochastic. Thus, lagged dependent variables and autoregressive schemes are excluded from the specification. The error terms are also assumed independently and normally distributed with mean zero and constant variance σ^2. The hypothesis to be tested is $\underline{\beta}_1 = \underline{\beta}_2, \ldots, \underline{\beta}_T = \underline{\beta}$.

The ordinary least squares (OLS) estimates based on T observations are given by

$$\underline{\hat{b}} = (X'X)^{-1}X'\underline{y} \tag{2}$$

where X is a T by k matrix of observations on the regressors and y is a similarly defined T by 1 vector for the dependent variable. Now suppose that only r observations are used to estimate β. Then for $r \geq k$,

$$\underline{\hat{b}}_r = (X_r'X_r)^{-1}X_r'y_r, \qquad r = k+1, \ldots, T, \tag{3}$$

where $X_r' = [\underline{X}_1, \ldots, \underline{X}_r]$ and $\underline{y}_r' = [y_1, \ldots, y_r]$. By introducing successive (for example, new sets) observations, one can obtain T$-k+1$ estimates of $\underline{\beta}$ denoted by $\underline{\hat{b}}_k, \underline{\hat{b}}_{k+1}, \ldots, \underline{\hat{b}}_T$. The $\underline{\hat{b}}_r$'s may be obtained recursively (without repeated matrix inversion) from the expression

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[1] Italicized numbers in parentheses refer to items in References at the end of this article. Research support for this project was provided in part by a cooperative agreement with the former Economic Research Service, U.S. Department of Agriculture.

We fit five demand equations which link per capita disappearance of beef, pork, veal, chicken, and turkey to prices and consumer income

$$\hat{\underline{b}}_r = \hat{\underline{b}}_{r-1} + (X_r'X_r)^{-1}X_r(\underline{y}_r' - X_r'\underline{b}_{r-1}) \quad (4)$$

where

$$(X_r'X_r)^{-1} = (X_{r-1}'X_{r-1})^{-1} - \frac{(X_{r-1}'X_{r-1})'\underline{X}_r\underline{X}_r'(X_{r-1}'X_{r-1})^{-1}}{1 + X_r'(X_{r-1}'X_{r-1})^{-1}X_r}$$

(See (*6*) and (*1*))

Now consider the T-k quantities defined as

$$w_r = \frac{y_r - \underline{X}_r'\underline{b}_{r-1}}{[1 + X_r'(X_{r-1}'X_{r-1})^{-1}X_r]^{1/2}}, \quad r = k+1, \quad , T \quad (5)$$

These recursive residuals can be obtained if $\hat{\underline{b}}$ is computed recursively by equation (4) Note that the w_r is the standardized prediction error of $\underline{y}_r$ when predicted from $\underline{X}_1'$, , $\underline{X}_{r-1}'$ The recursive residuals can be shown to be independent, given the aforementioned error assumptions They are normally distributed with mean zero and constant variance σ^2 (*2*)

If the coefficient $\underline{\beta}_t$ is constant up to time $t = t_o$ and different from then on, the recursive residuals, w_r, will have zero mean up to t_o and nonzero means thereafter These residuals therefore give information about the temporal stability of the estimated coefficients Brown, Durbin, and Evans suggested two tests based on the recursive residuals the plot of the cumulative sum of recursive residuals (CUSUM) and the plot of the cumulative sum of squares recursive residuals (CUSUMSQ)

The CUSUM Test

The CUSUM test is based on plotting the following variable W_r against time

$$W_r = \frac{1}{\hat{\sigma}} \sum_{j=1}^{r} w_j \qquad r = k+1, \quad , T, \quad (6)$$

where $\hat{\sigma}$ is the estimated standard deviation based on all T observations The expectation of W_r is $E(W_r)$ equal to 0 The plot of W_r should be distributed about this mean value, if we assume that the β's are constant An intuitive basis for a test for the departure of the sample path of W_r from its mean value of zero would then be to find a pair of lines lying symmetrically above and below the line W_r equals 0, so that the probability of crossing one or both lines is α, the required significance level [2]

When W_r departs sufficiently from the mean under the null hypothesis, it would cross one of these lines, which would indicate the presence of a structural change Pairs of lines satisfying the intuitive basis criteria are those through the points defined by $[k \pm a(T-k)^{1/2}]$, $[T \pm 3a(T-k)^{1/2}]$ where "a" is a parameter, whose value depends on the level of significance α At the 5-percent level of significance, a equals 0 943 (*2*, p 154) We may reject the hypothesis of constancy for the coefficients β_t at the selected significance level, if the sample path of W_r falls outside the pair of reference lines

[2] The variances used are calculated from the ordinary least squares residuals Alternative BLUS residuals could be employed (*8*) The BLUS residuals have the same distribution as the structural disturbance under the null hypothesis but are more difficult to compute

The CUSUMSQ Test

The CUSUMSQ test is based on the plot of the values for

$$S_r = \frac{\sum_{j=k+1}^{r} w_j^2}{\sum_{j=k+1}^{T} w_j^2} \qquad r = k+1, \quad , T \quad (7)$$

Note that the quantity S_r lies between zero and one ($S_r = 0$ if $r < k+1$ and $S_r = 1$ if $r = T$), and the expectation of S_r is $E(S_r) = (r-k)/(T-k)$ Significance tests again can be performed by drawing a pair of lines parallel to the mean value line The reference lines take the form $(r-k)/(T-k) \pm C$ The required values for C, corresponding to specific values for α (the significance level), appear in (*2*, p 4, table 1) For a given value of α, we find the value for C by entering the table at $n = 1/2(T-k) - 1$ and $1/2\alpha$

MODELS

We fit five demand equations which link per capita disappearance of beef, pork, veal, chicken, and turkey to prices and consumer income (table 1) Per capita disappearance of each commodity is expressed as a linear function of the own price, price(s) of selected other commodities (meats), and per capita disposable income Thus, the five demand equations typify those used to study consumption behavior in applied contexts at

Table 1—Demand equations for meat and poultry, ordinary least squares, first quarter 1965—fourth quarter 1976

Commodity	Explanatory variables[1]						
	Constant	SUD2	SUD3	SUD4	RPBF	RPRK	RPVL
Beef	19 93 (0 6786)	–0 4077 (0,2813)	–0 5200 (0 2869)	–0 1477 (0 3151)	–0 0889 (0 0246)	0 0086 (0 0091)	0 0464 (0 0222)
Pork	16 07 (0 7170)	–0 1028 (0 2968)	–0 9172 (0 2993)	–1.6179 (0 3085)	0 0373 (0 0095)	–0 0939 (0 0095)	(0 0075)
Veal	2 61 (0.1492)	–0 1709 (0 0687)	–0 1710 (0 0680)	–0 2921 (0 0697)	0 0218 (0 0050)		–0 0402 (0 0042)
Chicken	5 35 (0 2917)	–0 4552 (0 1634)	–0 1970 (0 1617)	0 8288 (0 1719)			
Turkey	5 01 (0 1251)	–3.4800 (0 0731)	–3 1400 (0 0727)	–2 730 (0 079)			
	RPCK	RPTK	RPHB	PCDY	von Neumann Ratio	F-test for hetero skedasticity	R^2
Beef				0 0083 (0 0014)	2 44	0 48	0 92
Pork				0 0091 (0 0012)	[2]0 73	[3]2 19	0 82
Veal				0 0010 (0 0001)	[2]0 97	[3]2 12	0 87
Chicken	–0 0437 (0 0053)		0 0260 (0 0032)	0 0058 (0 0006)	1.89	1 45	0 90
Turkey	0 0050 (0 0033)	–0 0065 (0 0036)		0 0001 (0 0003)	1 99	1 08	0 99

[1] Equation specifications are indicated by the table SUD, (, = 2, 3, 4) are seasonal dummies for the second, third, and fourth quarters, respectively, the retail price indices per pound are, beef (RPBF), pork (RPPL), veal (RPVL), chicken (RPCK), turkey (RPTK), and hamburger (RPHB), finally, PCDY is per capita personal disposable income in current dollars Standard errors are in parentheses [2] The von Neumann statistic indicates positive serial correlation at the 5-percent significance level [3] The F test for heteroskedasticity is significant at the 5-percent significance level

The CUSUM and CUSUMSQ tests suggest some structural stability in the quarterly meat demand functions

disaggregated commodity levels Their relationship to demand theory is limited and they have likely evolved through a trial and error process with the available sample data

THE DATA

To estimate the parameters, we used quarterly observations for Canada on per capita meat and poultry disappearance, consumer price indexes (1971 = 100), and per capita personal disposable income for the period, first quarter 1965 to fourth quarter 1976 The data sources were *Prices and Price Indexes*, Statistics Canada (Catalogue No 62-002), *National Income and Expenditure Accounts*, Statistics Canada (Catalogue No 13-201), and files of the Livestock Division, Statistics Canada Newly available quarterly Canadian data were used There was a question as to whether the specifications evolved in the annual Canadian data would prove appropriate and stable in the quarterly time frame

RESULTS

Estimates of the five demand equations for the full sample period are presented in table 1, and plots of the forward and backward CUSUM and CUSUMSQ tests are shown in figures 1-5 [3] The backward tests are conducted using the same procedure as the forward test described in the section on recursive residuals The difference is that the observation index is reversed, the first being the last, and so on In this case the base observations used to begin the test procedure are from the most recent *k* periods The confidence lines shown in the figures are for α level 0 05

The estimated relationships in table 1 conform generally with results of previous applied work All the estimated coefficients have the anticipated signs Most of the estimates are more than twice the corresponding standard errors The pork and veal equations show some evidence of positive autocorrelation and of increasing variance over time at the 5-percent significance level [4]

Figure 1 (A-D) shows the CUSUM and CUSUMSQ plots for beef Observe that the forward CUSUM plot gives the appearance of structural stability The forward CUSUMSQ statistic deviates from the mean value line, and tends to underpredict in the early part of the sample (up to observation 24) and then overpredict The backward CUSUM plot also shows a tendency for overprediction (fig 1 (C)), and the backward CUSUMSQ plot indicates a structural change at observation 29 (or 20 forward)

A corresponding series of F and Chow tests suggest that the demand equation for beef underwent structural change around observation 20 [5] The subsample regressions in table 2 show the variation in the estimated coefficients between the two periods The estimated coefficients for the full sample period are more similar to those in the second subsample (observation 22-48), except for the dummy variables Coefficients of determination did not differ markedly from that for the combined sample

The forward and backward CUSUM plots for pork (fig 2 (A-D))

[3] These figures appear at the end of this article, just before the References

[4] This means that the hypothesis of serial independence of the structural disturbance terms may be violated, which makes the interpretation of the recursive residuals tests somewhat difficult Johnson and Bradshaw have shown that the CUSUM and CUSUMSQ tests are not robust in this situation (*5*)

[5] The process for making Chow and F tests is familiar and so it will not be discussed The Chow tests applied were based on the one additional new observation The F statistics were calculated (where possible) by partitioning the sample at the point at which the recursive residual was being calculated

Table 2—Estimated demand equations for beef during two sample subperiods

Variable[1]	Observations (1 21)[2]	Observations (22-48)[2]
Constant	23 76 (9 97)	19 5400 (19 99)
SUD2	−1 33 (3 15)	0 2545 (0 67)
SUD3	−1 2400 (3 45)	0 0271 (0 07)
SUD4	−0 4500 (0 50)	0 1873 (0 46)
RPBF	−0 2223 (3 85)	−0 0767 (2 86)
RPPK	0 0284 (1 70)	0 0043 (0 38)
RPVL	0 1234 (1 63)	0 0356 (1 40)
PCDY	0 0078 (0 68)	0 0089 (5 11)

[1] See table 1 for definitions of variables [2] "t" statistics are in parentheses

Along with the plots, these evaluations indicate sample periods over which the structure can be taken as stable

deviate from the mean value lines, although these and the CUSUMSQ statistics lie within the confidence bounds The backward CUSUM tends to overpredict up to observation 28 and then underpredict The forward and backward CUSUMSQ plots have the tendency of being below the mean value lines We expected this for forward CUSUMSQ (when heteroskedasticity is present) but not for the backward plot (7) Computed sequential F tests confirmed the instability of the structure in the early part of the sample period The Chow test for backward recursion indicated a structural change at observation 29 (or 20 forward)

The veal equation in table 1 showed both autocorrelation and heteroskedasticity The CUSUM and CUSUMSQ plots for veal appear in figure 3 (A-D) The forward CUSUM statistic begins to deviate (overpredict) from the mean value around observation 29 The backward CUSUM also diverges from the mean value as early as the second observation These deviations in the forward and backward plots, however, are not significant at the 5-percent level Both the forward and backward CUSUMSQ plots tend to be below the mean values but, again, not at statistically significant levels The F and Chow tests computed at each iteration indicate structural instability at observation 29 Users should interpret these conclusions with caution because of the autocorrelation and heteroskedasticity present

The forward and backward CUSUMSQ plots (fig 4 (B and D)) for chicken appear more structurally stable than the forward and backward CUSUM plots (fig 4 (A and C)) The CUSUM plots tend to diverge from the mean value After observation 27, the forward CUSUM underpredicts while the backward CUSUM overpredicts None of these plots, however, crosses the confidence bounds The series of F and Chow tests are not consistent in indicating a change in structure

Finally, the forward and backward CUSUM plots for turkey (fig 5 (A and C)) show strong tendencies to underpredict early in the sample period and continue to do so throughout The deviations from the mean values, however, are not significant The forward and backward CUSUMSQ plots (fig 5 (B and D)) give the appearance of structural stability, as confirmed by the sequence of F and Chow tests

CONCLUSIONS

The CUSUM and CUSUMSQ tests suggest some structural stability in the quarterly meat demand functions Applied as a part of routine estimation procedures, these recursive residual analyses can be used to select appropriate sample periods and model specifications The major limitation is that the power of these tests is erratic and may be low against alternative hypotheses (*4*) In addition, these tests are sensitive to possible errors in the specification for the distribution of the disturbances, specifically, serial correlation and heteroskedasticity

From a practical viewpoint, these tests simply add structure to a procedure widely used in applied work for examining the appropriateness of estimated equations, that of comparing calculated residuals The difference is that, by using the CUSUM and CUSUMSQ methods, statistical evaluations of these differences can be obtained Along with the plots, these evaluations indicate sample periods over which the structure can be taken as stable The recursive estimation procedure makes the tests possible with only minor additional computational burden Major sources of predictive errors for simple models of the type presented are associated with changes in structure Thus, the tests provide a valuable addition to the stock of diagnostic techniques available to the careful analyst

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Figure 1A Beef, CUSUM of Recursive Residuals, Forward Recursion

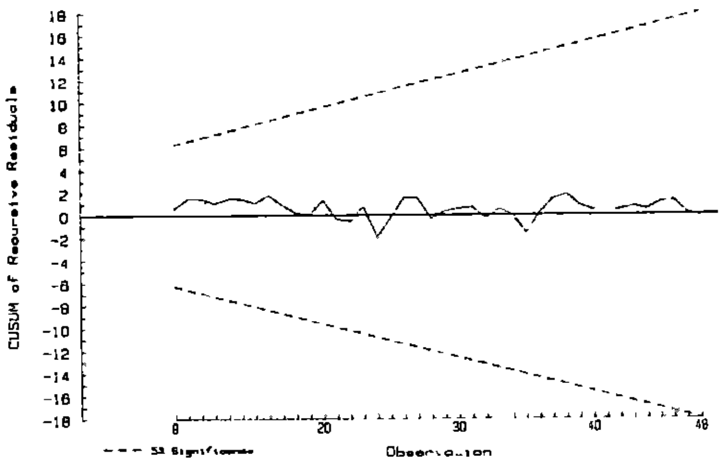

Figure 1B Beef, CUSUMSQ of Recursive Residuals, Forward Recursion

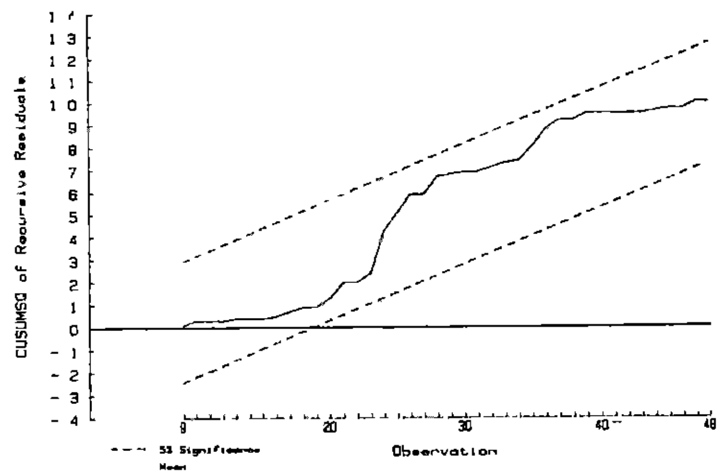

Figure 1C Beef, CUSUM of Recursive Residuals, Backward Recursion

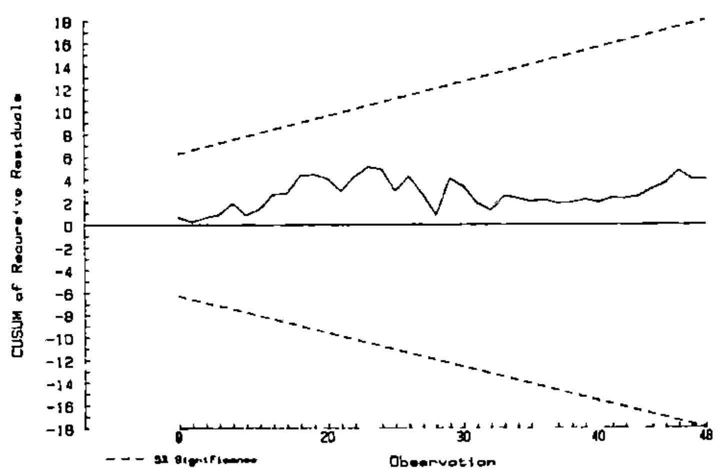

Figure 1D Beef, CUSUMSQ of Recursive Residuals, Backward Recursion

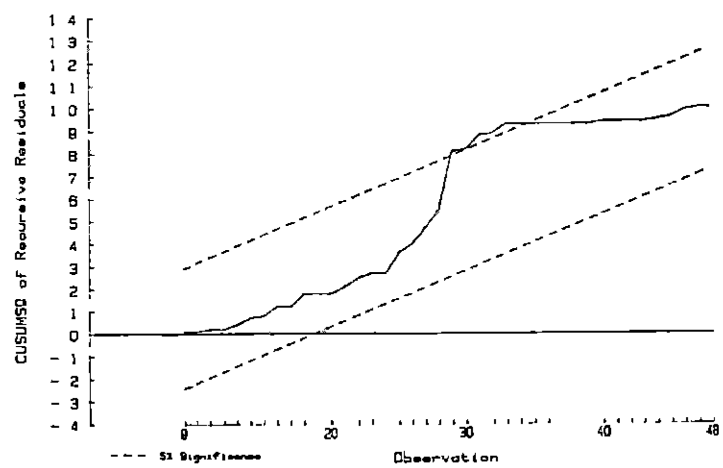

Figure 2A Pork, CUSUM of Recursive Residuals, Forward Recursion

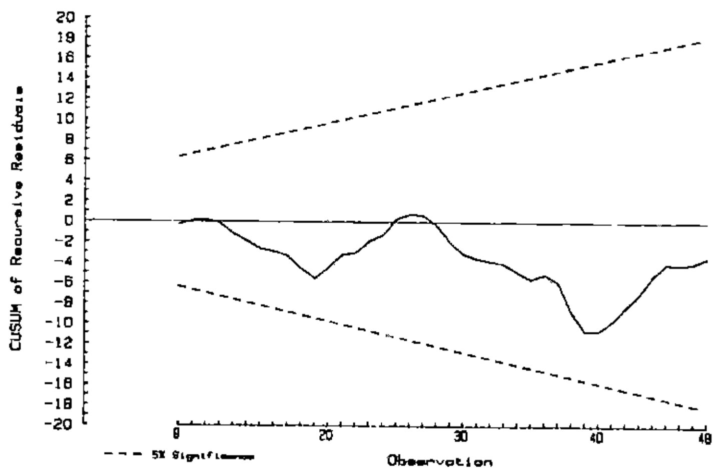

Figure 2B Pork, CUSUMSQ of Recursive Residuals, Forward Recursion

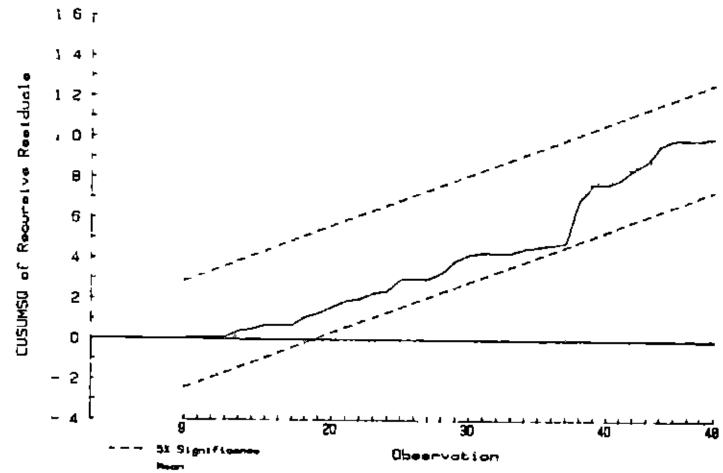

Figure 2C Pork, CUSUM of Recursive Residuals, Backward Recursion

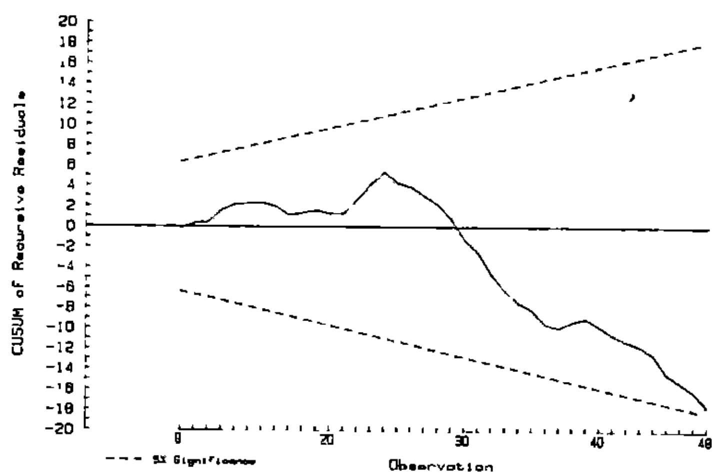

Figure 2D Pork, CUSUMSQ of Recursive Residuals, Backward Recursion

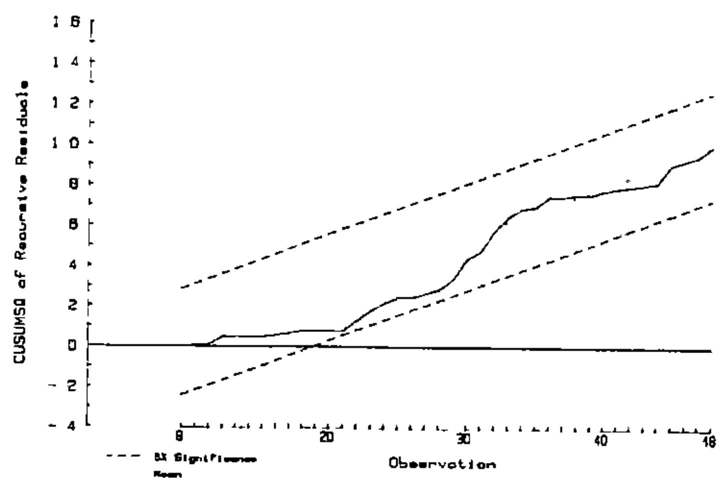

Figure 3A Veal, CUSUM of Recursive Residuals, Forward Recursion

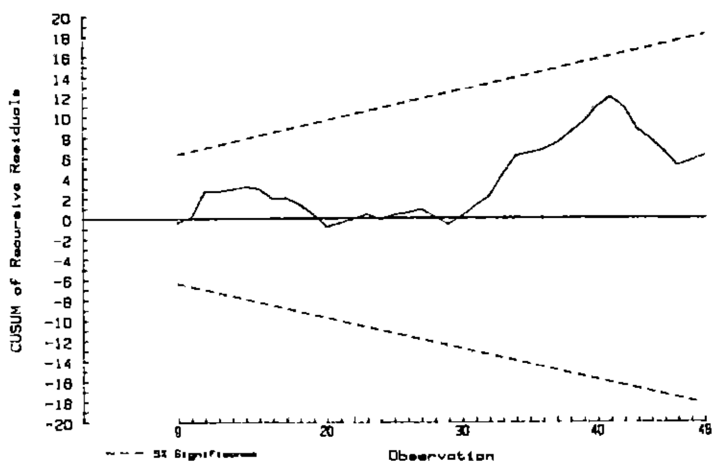

Figure 3B Veal, CUSUMSQ of Recursive Residuals, Forward Recursion

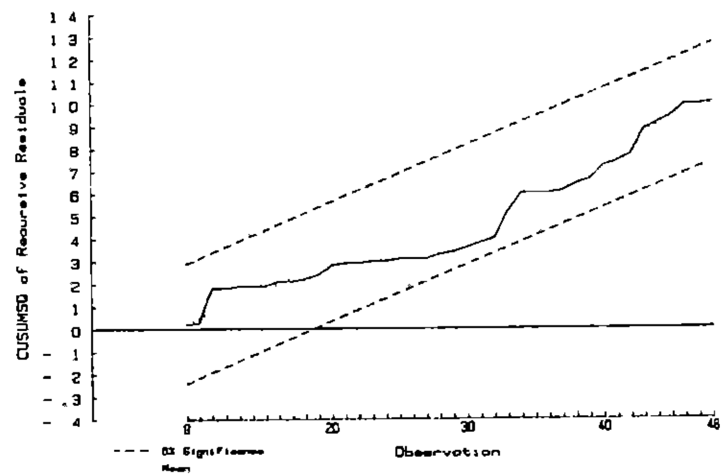

Figure 3C Veal, CUSUM of Recursive Residuals, Backward Recursion

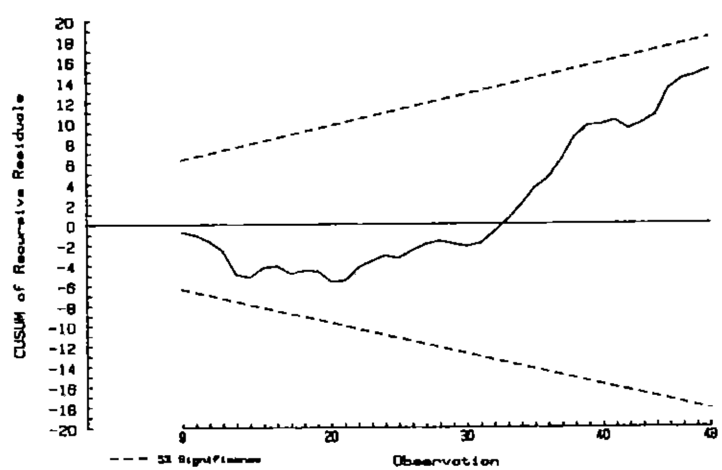

Figure 3D Veal, CUSUMSQ of Recursive Residuals, Backward Recursion

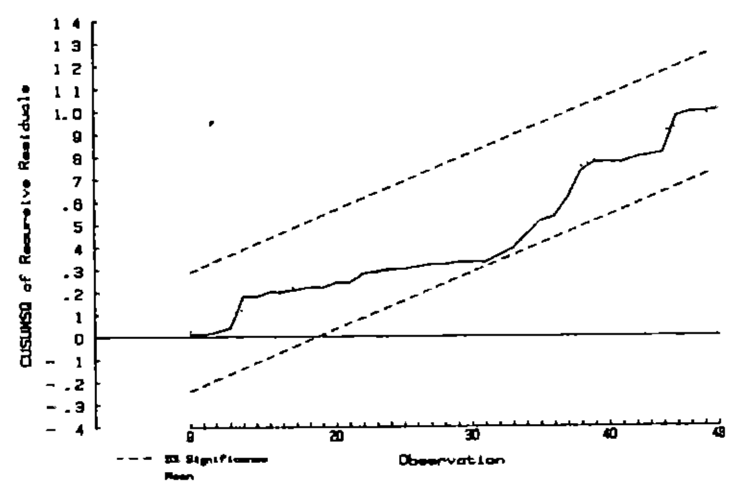

Figure 4A Chicken, CUSUM of Recursive Residuals, Forward Recursion

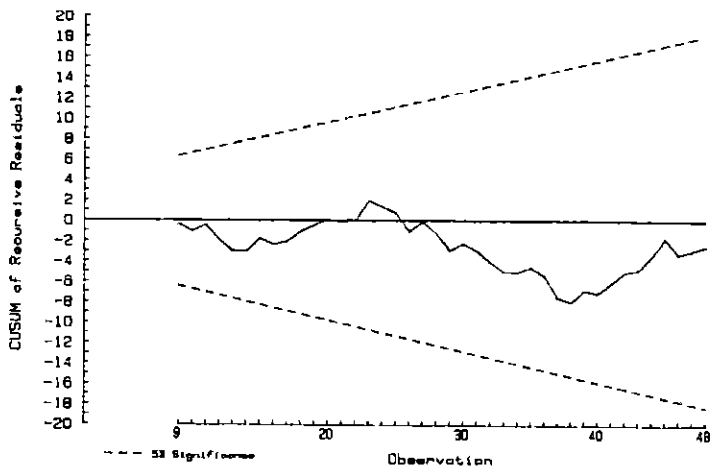

Figure 4B Chicken, CUSUM of Recursive Residuals, Forward Recursion

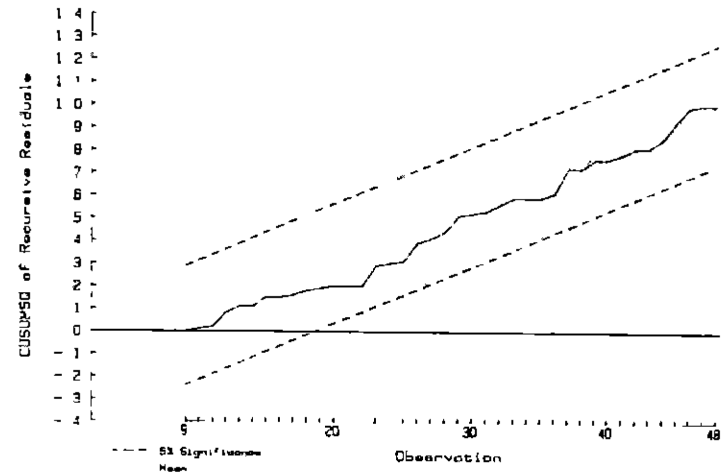

Figure 4C Chicken, CUSUM of Recursive Residuals, Backward Recursion

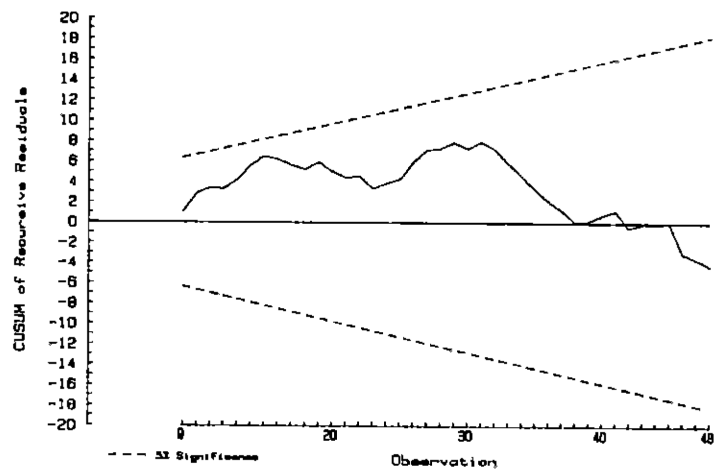

Figure 4D Chicken, CUSUMSQ of Recursive Residuals, Backward Recursion

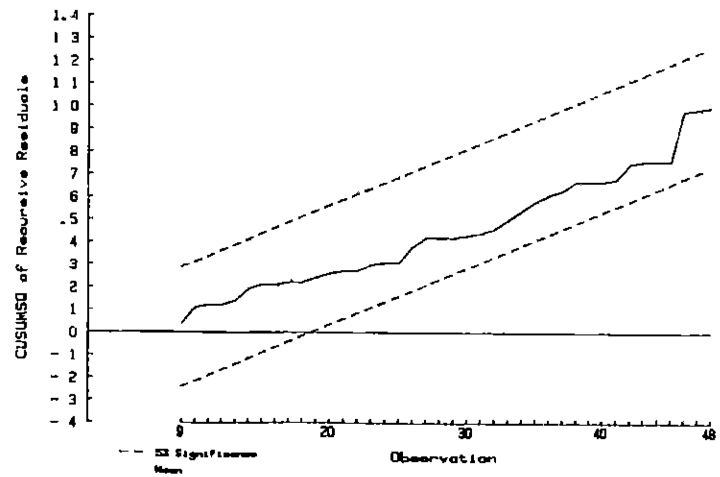

Figure 5A Turkey, CUSUM of Recursive Residuals, Forward Recursion

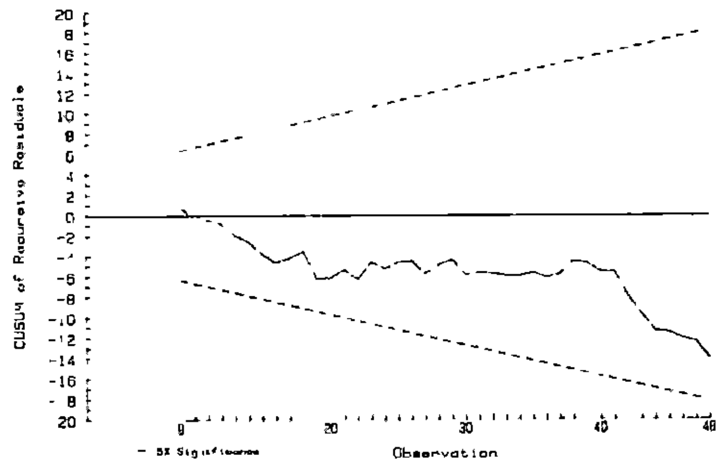

Figure 5B Turkey, CUSUMSQ of Recursive Residuals, Forward Recursion

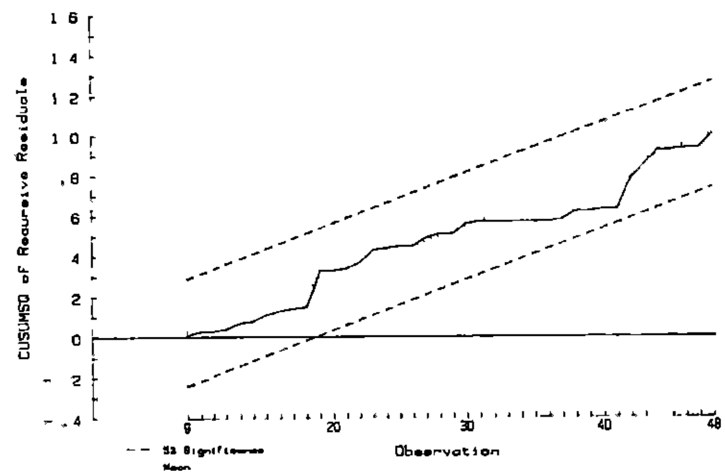

Figure 5C Turkey, CUSUM of Recursive Residuals, Backward Recursion

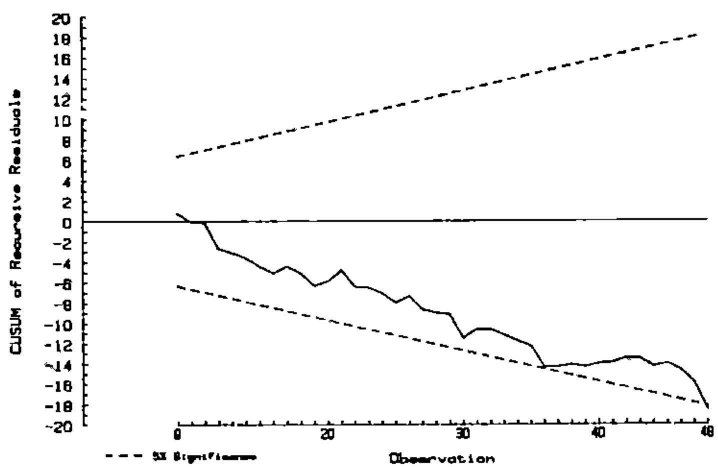

Figure 5D Turkey, CUSUMSQ of Recursive Residuals, Backward Recursion