Advertising Effectiveness and Coefficient Variation Over Time

By Ronald W. Ward and Lester H. Myers*

Changes over time in consumer demand, influenced by advertising, may not be accounted for in traditional, fixed parameter explanatory and forecasting models. A distributed lagged advertising model was developed and tested with coefficients that had random and systematic adjustments. A variable coefficient model clearly captured the dynamic effects of advertising on consumer demand.

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Models used to forecast or explain behavior are normally based on a regression function:

$$\hat{Y}_t = \sum_{j=0}^{k} \hat{\beta}_j X_{jt}.$$

In such regression models, the $\hat{\beta}_j$ represent OLS estimates of structural parameters; OLS estimates of reduced-form coefficients; or reduced-form estimates derived from statistically estimated structural relationships. When the $\hat{\beta}_j$'s coefficients are estimated using time-series data for observation periods 1 through n, a common problem in forecasting develops. For some period, $n+s$, $\hat{Y}_{n+s}$ tends to deviate from the actual or observed value of Y during $n+s$.

Within the regression model, three particularly disturbing problems frequently occur. First, the observed value (Y_{n+s}) often lies outside an acceptable confidence interval for $\hat{Y}_{n+s}$. Second, the magnitude of the absolute deviation between predicted and actual values tends to increase as s increases. Third, these two problems often occur regardless of the degree of fit obtained in the original regression.

These problems may be caused by several factors including model misspecification, errors in the projected values for the independent variables (X_j's), and parameter change between period n and the forecast period $n+s$. In this article, we focus on the problems by studying the effectiveness of advertising frozen concentrated orange juice (FCOJ) when demand is suspected to have changed over time. Specifically, our objective is to illustrate the usefulness of varying demand parameter procedures.

VARYING PARAMETER REGRESSION MODELS

The sources of parameter variation can be grouped into three broad categories: (a) structural changes in the economic phenomenon being studied; (b) model misspecification; and (c) aggregation. Technological and institutional changes always lead to structural changes in economic phenomena. Sources of misspecification are varied but include omission of independent variables, wrong functional form, and use of proxy variables. If the more influential independent variables are excluded, the resulting estimates of the coefficients of the variables included will be biased. The extent and the direction of their bias depends on the strength and direction of correlation between the included and excluded variables. If the strength varies over time, the estimated coefficients of the included variables will not be stable over alternative observation periods (*19*).[1]

Aggregation Can Cause Parameters to Vary

In macromodels, aggregation can be a source of parameter variation as the process generating macrovariables cannot be stationary unless the process generating the underlying microvariables is stationary. Over time, the relative importance of microeconomic agents will change, which may not be reflected in aggregation weights. Another common difficulty is that the aggregated (macro) variables are discrete (indexed as discrete time points) whereas the underlying microvariables may be continuous. This situation often causes parameter variation in macromodels. Lastly, structural changes in the economy give rise to discrete shifts in parameters.

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[1] Italicized numbers in parentheses refer to items in Literature Cited at the end of this article.

Methods for Handling Parameter Variance

There are two courses of action to handle varying parameters. First, we can eliminate the parameter variation itself by respecifying the model to account for the above three sources of variation explicitly and we can apply existing econometric methods of estimating constant parameters. Second, we can develop new econometric methods to deal with each pattern of parameter variation. Model re-specification may not be possible because the economic data are nonexperimental, and, even if re-specification could be done, data, time, and money may be too limited.

Types of Parameter Variance

Recently, the hypothesis of parameter variation has been accepted as a real-world situation. Attempts to incorporate explicitly this hypothesis into the estimation procedure have resulted in modified econometric methods that have proven useful. Parameter variation can be classified into three broad categories: (a) nonstochastic, caused by structural change in the economic phenomenon being studied; (b) stochastic and stationary; and (c) stochastic but nonstationary.

Nonstochastic parameter variation is divided further into: (1) discrete variation or switching regression and (2) systematic variation. The former permits only a finite (usually small) number of parameter values; the latter, an infinite number. Various approaches have been tried for switching regression under a variety of assumptions. A widely used procedure has been the incorporation of zero-one shifter variables and/or the use of continuous "time" variables.

More recently, stochastic and stationary parameter variation models (random coefficient models) have been used increasingly. Regression parameter vectors are assumed to be random drawings from a common multivariable distribution with mean vector μ and covariance matrix Σ_{μ}. If only the intercept is assumed to vary, the model reduces to an analysis of covariance with random effects. The random coefficient models have been analyzed with a single sample of cross-section data or time-series data or with use of panel data collected over time.

For variations wherein only time or household parameter variation is permitted, see (*33, 13, 9, 21, 8, 26, 28, 29, 30, 14, 15, 16, 19*).

Stochastic and nonstationary parameter variation models are termed sequential (Markovian) models. Extensive research work on sequential parameter variation is embodied in control theory and the applied physical sciences wherein optimal estimation methods usually known as the Weiner-Kalman-Bucy filters are used (*17, 18*). For variations to suit economic applications, see (*22, 23, 31, 7, 2*). An adaptive regression model has been developed wherein only the intercept varies sequentially (*3, 4, 5*). Models have also been developed for situations in which all the coefficients vary (*6, 27, 25*).

SEQUENTIAL PARAMETER VARIATION MODELS

Since many sources can cause parameter variation, any explanation (hypothesized pattern) of parameter variation must be sufficiently general to incorporate several possibilities. The pattern assumed by Cooley and Prescott (*4-6*) is sufficiently general to accommodate many realistic situations. Those assumed by Rosenberg and Swamy (*22-26, 28-30*) emerge as special cases of that assumed by Cooley and Prescott.

The Cooley-Prescott Explanatory Model

The Cooley-Prescott model assumes that the parameters of the model $Y_t = x_t \beta_t$ are adaptive and subjected to both permanent and transitory changes, wherein x_t is a 1 times (K+1) vector. The transitory changes are temporary shocks whose effects do not persist over time. Permanent changes, as they reflect changes in behavioral, technological, and institutional aspects of the economic phenomenon being studied, are more likely to persist

$$R = \begin{bmatrix} x_1' \Sigma_\mu x_1 & 0 & 0 & 0 \\ 0 & x_2' \Sigma_\mu x_2 & 0 & 0 \\ \cdot & & \cdot & \cdot \\ \cdot & & \cdot & \cdot \\ \cdot & & \cdot & \cdot \\ 0 & 0 & \ldots & x_T' \Sigma_\mu x_T \end{bmatrix}$$

over time and be generated in a systematic pattern. An important feature is that the model picks up structural "drifts" as opposed to uniformly constrained shifts.

The parameter vector β_t is assumed subject to:

(a) transitory changes $\beta_t = \beta_t^p + \mu_t$ and

(b) permanent changes $\beta_t^p = \beta_{t-1}^p + v_t$

wherein μ_t and v_t are identically and independently distributed multivariate normal vector variables with zero mean vectors and covariance matrices Σ_μ and Σ_v.

A particular covariance structure used by Cooley and Prescott is $\text{Cov}(\mu_t) = (1-\gamma)\,\sigma^2\,\Sigma_\mu$ and $\text{Cov}(v_t) = \gamma\sigma^2\,\Sigma_v$, wherein Σ_μ and Σ_v are assumed known up to scale factors. Let first elements be one ($\sigma_{11\mu} = \sigma_{11v} = 1$) when the intercept is subjected to the above pattern of variation. Again, Cooley and Prescott's pattern is sufficiently general to accommodate a wide variety of causes. The proportions γ and $(1-\gamma)$ of the total parameter variation can be attributed to permanent and transitory changes, respectively. The parameter γ represents the speed of parameter adaptation to structural changes in the phenomenon being studied. The larger (smaller) value of γ implies that the sources of parameter variation are more (less) of a permanent nature. Changing elements of Σ_μ and Σ_v imply varying rates of changes for the parameters and different degrees of permanency of changes. Parameters are estimated using maximum likelihood procedures. For properties of the estimators, see (*4, 5, 6, 3*).

The Variable Coefficient Models

Given the sample of T values over time, the variable coefficient (VC) model is written as:

$$y = x\beta + w \tag{1}$$

wherein y is a T times 1 vector, x is a T times (K+1) matrix of explanatory variables, and β is a (K+1) times 1 vector of permanent components for the parameters estimated for the base period T+1. Define

$$\beta_{T+1}^p = \beta_T^p + v_{T+1},$$

then from (a) and (b) earlier on this page, it follows that

$$\beta_{T+1}^p = \beta_t^p + \sum_{s=t+1}^{T+1} v_s \text{ and } \beta_t = \beta_{T+1}^p + \mu_t - \sum_{s=t+1}^{T+1} v_s.$$

The vector of disturbances (w) is defined with the t^{th} element being

$$w_t = x_t\mu_t - x_t \sum_{s=t+1}^{T+1} v_s$$

with the variance-covariance matrix.

$$\text{Cov}\,(w) = \sigma^2[(1-\gamma)\,R + \gamma C] = \sigma^2\,\Omega\,(\gamma) \tag{2}$$

wherein:

$$R = \begin{bmatrix} x_1' \Sigma_\mu x_1 & 0 & 0 & 0 \\ 0 & x_2' \Sigma_\mu x_2 & 0 & 0 \\ \cdot & & \cdot & \cdot \\ \cdot & & \cdot & \cdot \\ \cdot & & \cdot & \cdot \\ 0 & 0 & \ldots & x_T' \Sigma_\mu x_T \end{bmatrix}$$

and $C = (c_{ij})$ and $c_{ij} = \min(T-i+1, T-j+1)x_i'\Sigma_v x_j$. Conditional maximum likelihood estimators of β and σ^2 for a given γ, respectively, are:

$$\hat\beta(\gamma) = [x'\Omega(\gamma)^{-1}x]^{-1}\, x'\Omega(\gamma)^{-1}y \tag{3}$$

and

$$s^2(\gamma) = \frac{1}{T}\,[(y-x\hat\beta(\gamma))'\Omega(\gamma)^{-1}\,(y-x\hat\beta(\gamma))]. \tag{4}$$

$$\Sigma_u = \Sigma_v = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}$$

The concentrated likelihood function, obtained by substituting $\hat{\beta}(\gamma)$ and $s^2(\gamma)$ for β and σ^2, respectively, will be a function of $0 \leqslant \gamma \leqslant 1$. The optimum value, $\hat{\gamma}$, of γ is the one which maximizes the likelihood function. The ML estimators of β and σ^2 are $\hat{\beta}(\hat{\gamma})$ and $s^2(\hat{\gamma})$.

The procedures assume Σ_μ and Σ_v to be known. Since the relative importance of permanent and transitory changes is determined empirically by $\hat{\gamma}$ and no *a priori* basis exists to assume otherwise, we assume $\Sigma_\mu = \Sigma_v$. Similarly, with no *a priori* basis to believe that random changes in parameters are correlated, we can assume both Σ_μ and Σ_v are diagonal:

$$\Sigma_\mu = \Sigma_v = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & \sigma_{22} & 0 & & \vdots \\ 0 & 0 & \sigma_{33} & & \vdots \\ \vdots & & & \ddots & \vdots \\ 0 & \cdots & \cdots & \cdots & \sigma_{KK} \end{bmatrix} \qquad (5)$$

after normalizing on σ_{11}. To assign numerical values to the σ_{jj}, one alternative frequently used is to calculate the ordinary least squares estimates of the VC model, substitute the covariance matrix of estimates in the matrices Σ_μ and Σ_v, and normalize on $\hat{\sigma}_{11}$. The resulting matrix is Σ_μ or Σ_v.

Various assumptions can be evaluated by incorporating different structures for Σ_μ and Σ_v. For example, if one believes that only the constant term is subject to permanent and transitory variation, the proper model would be:

$$\Sigma_\mu = \Sigma_v = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix} \qquad (6)$$

If one suspects transitory but not permanent variation:

$$\Sigma_\mu = \begin{bmatrix} 1 & 0 & & 0 \\ \vdots & \sigma_{22} & & \\ \vdots & & \ddots & \\ 0 & & & \sigma_{KK} \end{bmatrix}, \quad \Sigma_v = \begin{bmatrix} 0 & \cdots & \cdots & 0 \\ \vdots & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & \cdots & 0 \end{bmatrix} \qquad (7)$$

If the variation among the parameters is not independent, the off-diagonal terms should not be ignored.

The above model has received much attention for its forecasting merits in that β is not calculated for the entire sample period but instead for period T+1, which suggests the most recent parameter values should lead to better forecasts. While forecasting is important, this class of VC models is equally important to modeling in general. Frequently, one may suspect changes in the economic responsiveness to a set of exogenous variables but theory provides little clue. The VC models can be used to identify and trace out structural drifts. Finally, statistical problems such as serial correlation, multicollinearity, and heteroskedasticity can sometimes be remedied with the use of VC models.

Given this introduction, we now present a specific case study that illustrates many of the procedure's merits.

ADVERTISING MODEL

Ward's advertising model showed that current consumption of frozen concentrated orange juice is related to price, seasonality, and a distributed lag specification of advertising effectiveness (*34*). The final model was specified where the dependent variable is expressed in first differences to compensate for serial correlation problems:

$$\dot{q}_t = \alpha_0 + \alpha_1 \dot{p}_t + \alpha_2 s + \sum_{j=0}^{N} \lambda_{j+1} \dot{a}_{t-j} + \epsilon_t \qquad (8)$$

q_t = per capita consumption of frozen concentrated orange juice (gallons of single-strength equivalent)

p_t = average quarterly price of FCOJ (dollars/gallon deflated by the Consumer Price Index: 1967=100)

a_t = advertising expenditures (million dollars)

s = quarterly seasonal dummy

t = a series of consecutive numbers beginning with t=2 in 3rd quarter 1967 through t=35 in 4th quarter 1975

$\dot{q}_t = q_t - q_{t-1}, \dot{p}_t = p_t - p_{t-1}, \dot{a}_t = a_t - a_{t-1}.$

This model obviously allows systematic adjustments in the model, as is evident with α_2. The lagged effect was estimated using a polynomial approximation of different degrees (*34*). The empirical results indicated that a first-degree polynomial with up to four lags is an acceptable specification of the model.

Initially, we estimated the model with advertising data up to 1973, and the specification restricted the parameters to remain fixed over the sample period. In contrast, advertising is designed to influence consumers' preferences toward the product advertised. Thus, the model specification may be unduly restrictive; λ_j may change over time as consumers are exposed to advertising for the product. Similar arguments may be made for other variables in the model. An alternative would be to allow the parameters to vary with both the systematic and random components.

While Ward's previous work consistently showed a first-degree polynomial model to be satisfactory, we adopt a slight variation in the polynomial model here. We use the apparent lag structure λ_{j+1}, and specify the lagged structure as:

$$\lambda_{(j+1)t} = \beta_{0t} + \beta_{1t} \sqrt[3]{j}$$

This lag structure has properties of the geometric decay function:

$$\left(\frac{\partial \lambda_{j+1}}{\partial_j} < 0 \text{ and } \frac{\partial^2 \lambda_{j+1}}{\partial_j^2} > 0 \right.$$

assuming $\beta_{0t} > 0$ and $\beta_{1t} < 0$) but can be estimated using the Almon lag procedures.[2] Similarly, if both the immediate and decay effects from advertising are changing, they can be estimated easily with this definition of λ_{j+1}. The parameters λ_{j+1} may have systematic and random components. Hence, β_0 and β_1 may change systematically with an added random component. Equation (9) now represents an alternative specification of the original equation (8):

$$q_t = \alpha_{0t} + \alpha_{1t} p_t + \sum_{j=0}^{4} \lambda_{(j+1)t} a_{t-j} + \epsilon_t \qquad (9)$$

The revised model from (9) now follows, with the variation in the Almon lagged structure:

$$q_t = \alpha_{0t} + \alpha_{1t} p_t + \beta_{0t} Z_{1t} + \beta_{1t} Z_{2t} + \epsilon_t \qquad (10)$$

The Z_{it}'s follow from the Almon procedure where:

$$Z_{1t} = a_t + a_{t-1} + a_{t-2} + a_{t-3} + a_{t-4}$$

$$Z_{2t} = a_{t-1} + 1.257\, a_{t-2} + 1.437\, a_{t-3} + 1.580\, a_{t-4}.$$

Applying the random coefficient model to (10) shoud show any path of parameter adjustments over the time period analyzed. If consumption is changing as well as varying seasonally, this would appear in equation (10) in α_{0t}. Similarly, changes in advertising effectiveness and decay become evident with the time adjustments in β_{0t} and β_{1t}. These parameters are especially important for analyzing advertising policies.

We now analyze assumptions using the Cooley and Prescott's random coefficient model.

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[2] Many alternative lag structures were hypothesized and estimated with the OLS model. The polynomial specification adopted proved consistently to give superior statistical results relative to other polynomial specifications. This specification of λ_j decays in a theoretically plausible way. It can be adapted readily to the VC model as only two parameters (β_0 and β_1) completely determine the lagged structure.

$$\hat{\Sigma}_\mu = \begin{bmatrix} 1.000 & & & \\ -0.6787 & 0.5959 & & \\ -0.0311 & 0.0163 & 0.0044 & \\ 0.0079 & -0.0109 & -0.0031 & 0.0032 \end{bmatrix}$$

APPLYING RANDOM COEFFICIENTS

Equation (10) represents the distributed lag structure wherein the parameters can be estimated without transforming the error terms. This preservation of the error structure is, of course, a major advantage of the Almon lag procedure. The parameters in (10) may also be random, as suggested previously. Results of re-estimating Ward's model and including 12 additional quarters of data suggest that the advertising parameters may have changed. Serious questions emerge as to the extended validity of the initial parameters. Also, the re-estimation with ordinary least squares does not clearly define any structural changes that may have occurred. Thus, it seems appropriate, recognizing that change over time occurs, to use random coefficients, which allow for systematic and nonsystematic change.

OLS Base Model

As indicated by Cooley and Prescott, all or some parameters may be fixed or the intercept only may be random. Also, the empirical values for Σ_μ and Σ_v follow from an OLS estimate of (10). Further, equation (10) is altered slightly by including a time variable to permit the intercept to shift in fixed increments (that is, time is a proxy for income and other shifters not explicitly included in the model). Equation (11) provides the inputs for Σ_μ; the standard errors are in parentheses.

$$\begin{aligned} \hat{q}_t = {} & 0.5340 - 0.2346\, p_t + 0.0163\, Z_{1t} \\ & (0.1083) \quad (0.0836) \quad (0.0072) \\ & - 0.0015\, Z_{2t} + 0.0076\, t \\ & (0.0061) \quad (0.0009) \end{aligned} \tag{11}$$

$R^2 = 0.9465$ D.W. = 1.7023

The normalized values for the lower triangle of $\hat{\Sigma}_\mu$ are:

$$\hat{\Sigma}_\mu = \begin{bmatrix} 1.000 & & & \\ -0.6787 & 0.5959 & & \\ -0.0311 & 0.0163 & 0.0044 & \\ 0.0079 & -0.0109 & -0.0031 & 0.0032 \end{bmatrix}$$

Σ_μ from equation (11) has been used in the random model primarily because equation (11) captures at least part of the intercept adjustment known to be necessary from prior research. Further, preliminary estimates of Σ_μ using OLS of equation (10) did not change the VC estimates appreciably. Similarly, the varying coefficient estimates are robust for slight deviations from the OLS estimates of Σ_μ.

The subsequent analyses assume the matrices Σ_μ and Σ_v are diagonal and equal. Relaxing this assumption in several estimates indicated only slight differences in the parameter values across models, again supporting the robustness of the estimates with different configurations of Σ_μ and Σ_v.

Random Models

The variable coefficient (VC) model was estimated with quarterly observations for the 4th quarter of 1968 (t=7) through the 4th quarter of 1975 (t=35). Data for 1976 and 1977 were omitted and used later for validation. Parameters were allowed to change for each observation period and γ, showing the weighting of permanent and transitory effects, indicated that 98 percent (γ=0.98) of the parameter change was permanent. The proxy trend variable initially included in the fixed model is now dropped as the permanent and transitory adjustments appear in the changing intercept estimates. Equation (12) represents the parameter estimates for period t=35 with the standard error in parentheses. This equation corresponds to β_{t+1} initially shown in equation (1).

$$\hat{q}_t = 0.7162 - 0.4139\, p_t + 0.0191\, Z_{1t} - 0.0057\, Z_{2t} \quad (12)$$

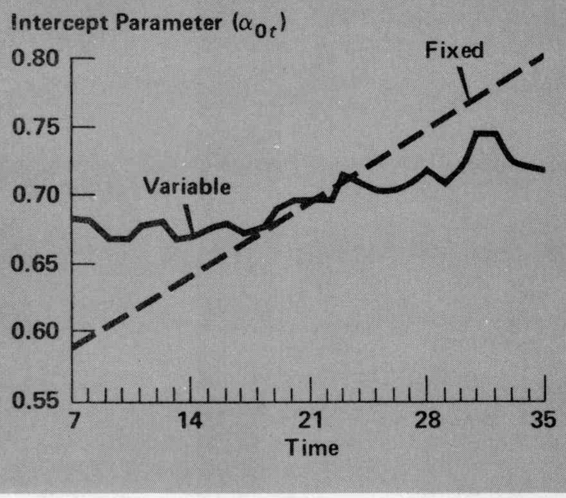
$$(0.1410) \quad (0.1693) \quad (0.0075) \quad (0.0064)$$

We now illustrate the differences and dynamics of the VC estimates compared with the fixed OLS estimates.

Intercepts

The pattern of intercept adjustments for the random and fixed models over the estimation period appears in figure 1. Intercept α_{0t} moves up somewhat seasonally in the variable model and intercept $\alpha_0 + \alpha_4 t$ is constrained to increase linearly in the fixed model. A more detailed analysis might indicate the type of fixed variables to include to account explicitly for the change. However, a simple time trend adjustment is unduly restrictive. In particular, the fixed model generated lower initial estimated values of the intercept compared with the VC model. For later periods, the fixed model suggested much stronger growth patterns than may have actually occurred. Applying the fixed model over extended time periods could continually overforecast consumption trends.

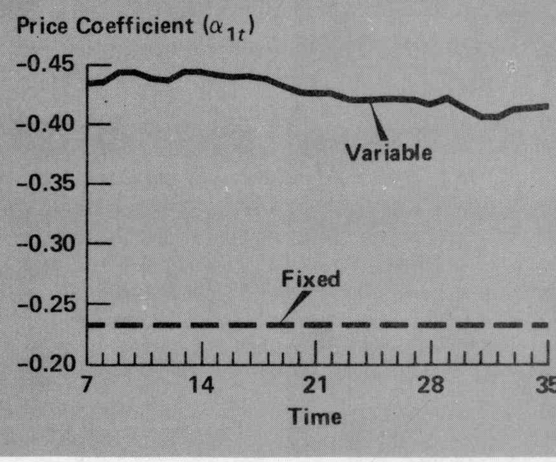
FIGURE 1
Intercept Parameter (α_{0t}) Adjustment for Fixed and Variable Models



Price Coefficients

The VC model estimated only slight downward adjustments in the price coefficient over the time periods analyzed. Current statistical procedures are limited when one attempts to test the significance of these parameter changes over time (*11*, p. 54; *10*, p. 329). However, a more important result is the difference between the fixed and varying price coefficients. The fixed price coefficient was estimated as -0.2346 while the closest value of the random parameter was -0.4044 (fig. 2).

FIGURE 2
Adjustments in Price Coefficient (α_{1t}) in the Variable Model and Comparison with the Fixed Model

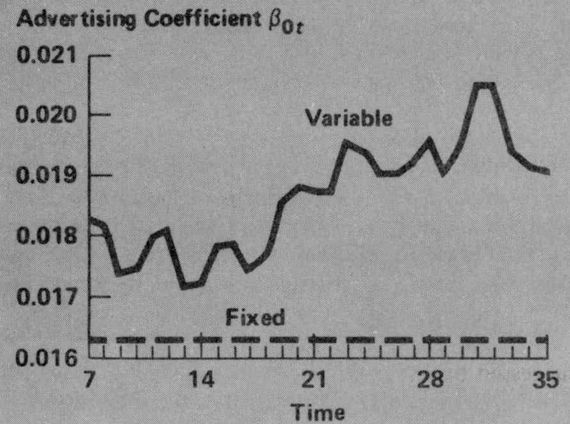


In all fixed models in which time was used as a proxy for income and growth trends, the price coefficient had small absolute values relative to alternative OLS models which included per capita income instead of time. Prices and time were negatively correlated over the period of the analysis, which may affect estimated coefficient values directly. Deleting the time variable in the random model reduced the multicollinearity and produced a larger absolute price effect.[3]

The difference in OLS and VC model estimates can be illustrated further by comparing price elasticity estimates for the end of the sample period used for estimation. With OLS model and price and quantity values for the 4th quarter 1975, the price elasticity is -0.268. With the VC model, it is -0.489.

Although most analyses of VC models emphasize their forecasting merits, the above results suggest that the models are equally useful in identifying specific problems of parameter values throughout the entire sample period. Here, the price parameter is expected to have been underestimated because of multicollinearity. If the trend variable is deleted from the VC model, the systematic adjustment in the intercept reflects the effects initially measured with the time variable in the fixed model and the price coefficient, freed of the multicollinearity problem, can be estimated. Obviously, the usefulness of the VC model is specific to the particular equation.

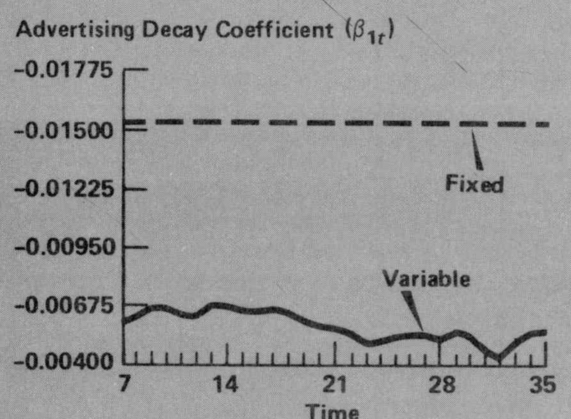
FIGURE 3
Adjustments in Advertising Coefficient (β_{0t}) in the Variable Model and Comparison with the Fixed Model



Random Advertising Effects

Advertising components of the models were calculated wherein β_0 is the immediate effect and β_1 shows the decay. Figures 3 and 4 compare these parameters for the fixed and variable models. The path of adjustment in β_0 clearly shows a positive trend, which indicates that advertising effect has increased during the latter time periods. The systematic adjustments in β_0 suggest some seasonal variation in response to advertising plus the increased advertising effectiveness over time. Also, a comparison of the fixed and variable

FIGURE 4
Adjustments in Advertising Decay Coefficient (β_{1t}) in the Variable Model and Comparison with the Fixed Model



[3] A similar model to equation (11) was estimated with real per capita shown income rather than time; however, the statistics shown in (11) suggest that its structure is preferable. Price was correlated with both time (t) and income (i) ($\rho(pt) = -0.83$ and $\rho(pi) = +0.80$)). The price parameter was estimated to be -0.2346 with the time equation and -0.5305 for the income equation. In contrast, the VC model with the multicollinearity problem removed produced price parameters in the midrange of the fixed models.

advertising parameters indicates a substantial numerical difference in the effects of advertising expenditures, one obviously accentuated in the more recent periods.

The decay parameter β_1 also differs considerably from that of the fixed model (fig. 4). This difference is important in that the fixed model suggests a rapid advertising decay while the random model shows that advertising's effect extends over several quarters. In fact, calculating the lagged parameters from

$$\beta_0 + \beta_1 \sqrt[3]{j}$$

shows that the rate of advertising decay has declined over the sample period. That is, not only has advertising become more effective as an immediate demand stimulator but it has also become more effective because its impacts last longer.

The magnitude of impact of an advertising policy is suggested by the estimated effect on orange concentrate sales per capita from a $1 million advertising expenditure during quarter t over a five-quarter time horizon. The OLS estimates suggest increased per capita FCOJ sales of 0.017 gallons per capita; for the variable coefficient model, 0.065 gallons.[4] Adjusted for a population of 220 million, the OLS estimates indicated added sales of 3.74 million single-strength gallons versus 14.3 million for the variable coefficient estimates.

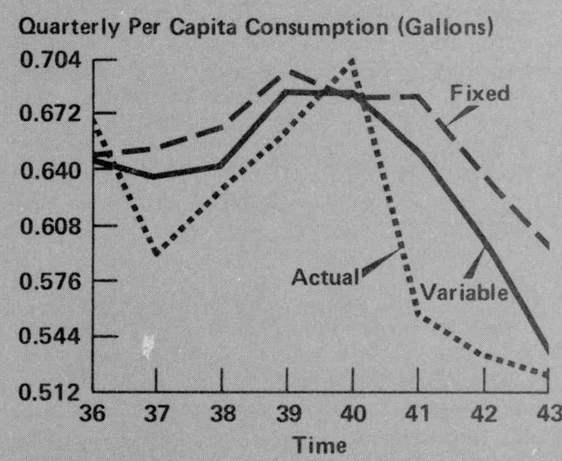
While our abilities to test these parameter differences statistically are limited, the numerical values suggest that considerable error can occur when the fixed model is used. The variable coefficient procedure is extremely useful for modeling when structural change is suspected but the systematic component cannot be hypothesized *a priori*.

Model Validation

We now briefly compare the accuracy of the two models. As indicated earlier, they were estimated with data through the fourth quarter of 1975; complete data are available through 1977. We used data from these eight quarters to evaluate the predictive ability of the models.

Generally, the parameter variation model predicted values nearer to actual levels of per capita consumption and it predicted turning points better (that is, the Theil u statistics were $u_{ols} = 0.197$ and $u_{vc} = 0.132$) (fig. 5). Also, the average absolute error for the OLS model was 54 percent greater than for the VC model (0.0608 versus 0.0395). As the use of the model is extended beyond period 35, the nonstochastic model consistently generated larger errors than did the random model.

FIGURE 5
Forecasting with the Variable and Fixed Models



CONCLUSION

The VC variable model used assumes the parameters estimated for period T+1 provide the best equation for forecasting in period T+k. This procedure, frequently employed, ignores the systematic parameter patterns that may have occurred over periods 1 to T. Such

[4] The OLS model actually indicated negative effects after the second quarter. The VC model showed an extended carryover of advertising consistent with the initial first-difference model shown in equation (8). Hence, the comparison above is calculated using two quarters for the OLS model.

patterns can often be captured in re-specified fixed models which in turn would reflect the parameter adjustments in the forecasting periods beyond T+1. Thus, we judge the primary usefulness of the VC variable procedures to be for model specification, which, in turn, should lead to improved forecasting. Again, the need for re-specification depends on the observed parameter patterns. For example, if the parameters tended to plateau after n periods ($1 < n < T+1$), the initial model may not need to be re-specified. In some situations re-specification may recreate the statistical problems initially remedied with the VC model.

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In Earlier Issues

Economists have recognized and accepted the representation of all economic variables of an economy or major segment in terms of a mutually interdependent set defined by a system of simultaneous equations—following the concepts of Walras and Pareto. Most economists and statisticians would be surprised to learn that their procedures in separately fitting individual economic equations to observational data are frequently inconsistent with the postulate of mutual economic interdependence.

Trygve Haavelmo pointed out that the simultaneous character of a system of economic equations and the mutual and simultaneous determination of a set of interdependent economic variables imposed logical restrictions upon the estimating procedure used to calculate statistical constants for these equations. To Ragnar Frisch is credited the first suggestions leading to development of this new line of analysis.

Review of: *Statistical Inference in Dynamic Economic Models*
Richard O. Been
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