Decoupling of Commodity Programs and Other Modern Fairy Tales

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Abstract

Many countries have policies such as quotas, tariffs, support prices and target prices to raise the prices which their farmers receive for commodities. Higher prices cause farmers to produce more. The extra production is dumped on world markets. Decoupling is proposed as a way for a country to subsidise its farmers without distorting markets. Direct income transfers would replace policies which manipulate prices. However, income transfers will also distort markets because farmers' supply decisions under risk depend upon income. This study shows that income transfers can distort markets by more than manipulating prices.

Keywords: government programs, decoupling, risk, stochastic optimal control

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As the old saying goes, farmers want to earn their subsidies, they don't want charity. This is one reason countries manipulate the prices of agricultural commodities instead of giving money directly to farmers. Quotas, tariffs, support prices and target prices are politically acceptable, direct income transfers are not. The provisions for agriculture in the General Agreement on Tariffs and Trade (GATT) seek to replace quotas, support prices and target prices with tariffs, making price distortions obvious. Then, the tariffs are to be reduced and, perhaps, replaced by income transfers. This process is called decoupling. Commodity markets are to be "decoupled" from subsidies to farmers.

In an informal survey of agricultural economists, all believed that the manipulation of prices distorts world commodity markets and is inefficient. The world would be more prosperous with free-trade. At the same time, all believed that income transfers do not distort markets and do not hinder free trade. Countries have a right to give money to farmers for reasons of justice and equity. Therefore, agricultural economists support the GATT.

In a second informal survey of agricultural economists, all believed that farmers respond to risk. Farmers attitudes toward risk depend upon their incomes. A rich farmer facing risks makes different decisions than a poor farmer. In other words, agricultural economists have two hands. On the one hand, they believe that farmers supplying commodities for international trade will make the same decisions regardless of their income. The elasticity of supply with respect to income is zero. On the other hand, they believe that farmers respond to risk according to their income. The elasticity of supply with respect to income is not zero. Which hand holds the truth?

The GATT flashes a 'red light' at government programs which manipulate prices but gives a 'green light' to income transfers. This study assumes that farmers respond to risk and investigates whether direct income transfers will decouple government programs from commodity markets. It is very likely the GATT gives a green light to distortions of commodity markets.

Household Decisions under Risk

Demand and supply of a commodity will be derived from a model of household decisions under risk. Farmers will make both demand and supply decisions. Consumers will make only demand decisions. The objective of the model is to maximize expected utility of consumption over time, subject to a stochastic budget constraint for the change in household wealth.

$$(1) \qquad J(W_0, t_0) = \underset{F,c}{Max}\, E \int_{t_0}^{\infty} e^{-\rho t} U(c) dt,$$

subject to

$$(2) \qquad dW = \delta(W, F, c)dt + \sigma(F)dZ$$

In the objective (1), expected utility, J, depends upon initial wealth, W_0, and the initial time, t_0. Expected utility is found by choosing the acres of the farm, F, and the quantity of consumption, c, which maximize the expected discounted value of utility, U, during each time period, t. Utility is discounted at the rate of time preference, ρ, and the household's time horizon is infinite. Merton has shown that an infinite horizon can be equivalent to a finite horizon if the rate of time preference is increased to include the risk of dieing unexpectedly. In the budget constraint (2), current wealth, W, is expected to change at rate δ, which is a function of wealth, the acres farmed and consumption. The change in wealth has standard deviation σ, which is a function of the area farmed because the income from farming is risky. The error term dZ, is a Wiener process which makes the change in wealth an Ito stochastic process. Hertzler reviews the optimal control of Ito processes.

Expected utility over the time horizon is found by maximizing the Hamilton-Jacobi-Bellman (HJB) equation is each time period.

$$(3) \qquad 0 = J_t + \underset{F,c}{Max}\left\{ e^{-\rho t} U + J_W \delta + \tfrac{1}{2} J_{WW} \sigma^2 \right\}.$$

The HJB equation (3) is a partial differential equation in time and wealth. It relates the marginal utility with respect to time, J_t, to the certainty equivalent shown in brackets. The certainty equivalent depends upon marginal utility with respect to wealth, J_w, and the change in the marginal utility with respect to wealth, J_{ww}. It is denominated in utils and has three terms. The first term is discounted utility of consumption at time t, the second term the expected change in utility from an expected change in wealth; and the third term is the premium paid by the household to avoid risks having σ^2.

Functional Forms

Once functional forms are specified, the area farmed and consumption can be chosen to maximize the HJB equation. In some cases, the HJB equation can be integrated to find a closed-form solution for expected utility. For example, expected utility has a closed-form if utility of consumption is a member of the Hyperbolic Absolute Risk-Aversion (HARA) family, if the expected change in wealth is linear in the area farmed and in consumption and if the standard deviation is linear in the area farmed (Merton).

A prominent member of the HARA family is the Stone-Geary utility function

(4) $U(c) = \beta(c - \gamma)^{\alpha}$,

Parameter α ($\alpha \neq 1$) defines the curvature of the utility function. Parameter β ($0 \leq \beta$) shifts utility. Parameter γ ($\gamma \leq c$) is sometimes interpreted as the subsistence level of consumption. In this interpretation, utility depends upon consumption above subsistence.

The wealth of the household is expected to change from returns on investments, earned income and expenditures on consumption

(5) $\delta(W,F,c) = \delta_w W + I + (\delta_f - \delta_w)p_f F + p_y YF - p_c c$

The expected change in wealth in (5) has five terms. The first term is the return from investing wealth, W, off the farm at rate δ_w. The second term is income, I, from a job off the farm. The third term is real capital gains from investment in the farm. Each acre in the farm has a price p_f which appreciates at the rate δ_f. Multiplying by the acres in the farm, F, gives capital gains of $\delta_f p_f F$. Wealth invested in the farm cannot be invested off farm, however, and has an opportunity cost of $\delta_w p_f F$. Subtracting this opportunity cost gives real capital gains. The fourth term in the change in wealth is income from production. Multiplying the gross margin per bushel, p_y, by the yield per acre, Y, gives the gross margin per acre. Multiplying by the acres, F, gives income from production. The last term is the expenditure on consumption, c, at price p_c.

Because income from production is risky, the change in wealth has a standard deviation

(6) $\sigma(F) = F\sigma_y$

The gross margin per acre is the source of risk, with standard deviation σ_y. Multiplying by the number of acres gives the standard deviation of the change in wealth.

These functional forms are reasonably flexible. The utility function allows the household to have various types of risk preferences, including decreasing, constant or increasing absolute risk-aversion and decreasing, constant or increasing relative risk-aversion. A single source of risk is considered, but other sources could be added without changing the results to follow. The most restrictive assumption is constant returns to scale. Income from farm production is a linear function of the acres in the farm.

Closed-Form Solution

Analyzing the effects of government policies is an exercise is comparative dynamics. As with comparative statics, a dual solution to the model is required. General results for stochastic dynamic duality are unknown, but a closed-form solution is a special case. The functional forms in equations (4), (5) and (6) allow the Hamilton-Jacobi-Bellman equation in (3) to be integrated into a closed-form solution for expected utility.

$$(7) \qquad J(W,t) = e^{-\rho t}\beta\left[\frac{(1-\alpha)}{r}\right]^{1-\alpha}\left[\frac{\delta_* W + I - p_c\gamma}{p_c\delta_*}\right]^{\alpha},$$

where:

$$(8) \qquad r = \rho - \alpha\left[\delta_* + \frac{1}{2}\frac{\left[(\delta_f - \delta_*)p_f + p_y Y\right]^2}{(1-\alpha)\sigma_y^2}\right]$$

Proof is in Appendix A. Where utility in equation (4) depends upon consumption above subsistence, expected utility in equation (7) depends upon off-farm income above subsistence expenditures. In addition, utility is for a single time period but expected utility is for the whole time horizon. The contribution to expected utility of a single time period is capitalized by a term containing the risk-adjusted real rate of time preference, r. As defined in equation (8), the risk-adjusted real rate of time preference is the rate of time preference less a term for the rate of return to off-farm investments and a premium for risky on-farm investments.

The Arrow-Pratt coefficient of absolute risk-aversion is usually defined for utility in equation (4). A similar coefficient of absolute risk-aversion can be defined for expected utility in equation (7).

$$(9)\qquad -\frac{J_{WW}}{J_W} = \frac{(1-\alpha)\delta_w}{\delta_w W + I - p_c\gamma}$$

A household is risk averse if the coefficient of absolute risk-aversion is positive. The greater is the coefficient, the greater is the degree of risk aversion. The household has decreasing or increasing absolute risk-aversion if α is less than or greater than one. It has constant absolute risk-aversion if income from an off-farm job, I, greatly exceeds subsistence expenditures, $p_c\gamma$.

The coefficient of relative risk aversion equals the coefficient of absolute risk-aversion multiplied by wealth.

$$(10)\qquad -\frac{J_{WW}W}{J_W} = \frac{(1-\alpha)\delta_w W}{\delta_w W + I - p_c\gamma}$$

The coefficient of relative risk-aversion is also the elasticity of expected utility with respect to wealth. The household has decreasing, constant or increasing relative risk-aversion depending upon whether income from an off-farm job is less than, equal to or greater than subsistence expenditures.

The household's consumption and area farmed also have closed-form solutions.

$$(11)\qquad c = \frac{r}{(1-\alpha)p_c\delta_w}\left[\delta_w W + I - p_c\gamma\right] + \gamma.$$

$$(12)\qquad F = \frac{(\delta_f - \delta_w)p_f + p_y Y}{(1-\alpha)\sigma_y^2\delta_w}\left[\delta_w W + I - p_c\gamma\right].$$

Proof is in Appendix A. Consumption is equation (11) can be rearranged to show that expenditures on consumption above subsistence are a proportion of off-farm income above subsistence. The higher is the household's rate of time preference, the more is consumed now rather than saved for the future. The area farmed in equation (12) depends upon the return per acre, the variance per acre and the household's degree of risk aversion. The return per acre equals real capital gains plus the gross margin. This is divided by the variance per acre weighted by the coefficient of absolute risk-aversion. The higher the return, the lower the variance and the less the household's aversion to risk, the greater will be the area farmed.

Market Demand and Supply

Aggregating consumption and the area farmed by all the households in a country gives market demand and supply functions. Some households will be consumers with no income from farming nor risks. Other households will be farmers who both consume and produce. Market demand adds together the household consumption of all consumers and all farmers.

$$(13) \quad D = n_c\left[\frac{r_c}{(1-\alpha_c)p_c\delta_w}[\delta_w W_c + I_c - p_c\gamma_c] + \gamma_c\right] + n_f\left[\frac{r_f}{(1-\alpha_f)p_c\delta_w}[\delta_w W_f + I_f - p_c\gamma_f] + \gamma_f\right]$$

In equation (13), all consumers are the same and all farmers are the same. Multiplying consumption in equation (11) by the number of consumers, n_c, and by the number of farmers, n_f, and adding the results gives market demand, D. The subscripts c and f denote consumers and farmers. Similarly, market supply adds together the production of all farmers.

$$(14) \quad S = n_f\left[Y\frac{(\delta_f - \delta_w)p_f + p_y Y}{(1-\alpha_f)\sigma_y^2\delta_w}[\delta_w W_f + I_f - p_c\gamma_f]\right]$$

An individual farmer's production equals the yield per acre multiplied by the area farmed in equation (12). Then multiplying by the number of farmers gives market supply, S.

Because of risk, supply depends upon farmers' off-farm income above subsistence. Direct transfers of income to farmers will shift the supply curve. The magnitude of the shift can be determined from the elasticity of supply with respect to wealth.

$$(15) \quad S_W\frac{W_f}{S} = \frac{\delta_w W_f}{\delta_w W_f + I_f - p_c\gamma_f}.$$

The elasticity in equation (15) equals the return to off-farm investments, $\delta_w W$, divided by the total off-farm income above subsistence. It will be greater than, equal to, or less than one depending upon whether off-farm income from a job, I, is less than, equal to, or greater than subsistence expenditures, $p_c\gamma$.

The elasticity of supply with respect to wealth is independent of a farmers' degree of risk aversion. Income transfers will shift supply, not because farmers are averse to risk, but simply because farming is risky. However, the elasticity of supply does depend upon changes in the degree of risk aversion as wealth changes. The elasticity of supply equals one if relative risk-aversion is constant, as defined by the derivative of equation (10). It only becomes zero if absolute risk-aversion is constant, as defined by the derivative of equation (9). A study of farmers' decisions by Just and Pope rejects the hypothesis of constant absolute risk-aversion but accepts constant relative risk aversion. The elasticity of supply with respect to wealth is likely to be closer to one than to zero. If so, decoupling of government programs from commodity markets may not be possible.

Market Distortions from Decoupling

A large number of commodity programs could be analyzed. Quotas, tariffs, support prices and target prices are possibly the most important. The results for quotas are very similar, although not identical, to those for tariffs. For these reasons, the market distortions from tariffs, support prices and target prices will be compared with the distortions from direct income transfers. If the distortions from income transfers are significantly less, then subsidies to farmers can be decoupled from the market.

Table 1. Model Parameters.

	Importing Country		Exporting Country	
Parameter	Consumers	Farmers	Consumers	Farmers
t (year)	0	0	0	0
n (people)	80	20	95	5
α	0.2	0.2	0.2	0.2
β	1	1	1	1
γ	-20,000	0[1]	-20,000	0[1]
		-80,000[2]		-600,000[2]
ρ	0.04	0.04	0.04	0.04
δ_f	0	0.01	0	0.01
δ_w	0.03	0.03	0.03	0.03
p_f ($/acre)	0	1,500	0	3,000
$\bar{Y}$ (bu./acre)	0	50	0	100
σ_y ($/acre)	0	175	0	250
$\dot{W}$ ($)	200,000	2,000,000	200,000	15,000,000
I ($)	50,000	0	50,000	0

[1]Value of γ for constant relative risk-aversion and unitary elasticity of supply with respect to wealth.
[2]Value of γ for increasing relative risk-aversion and inelastic supply with respect to wealth.

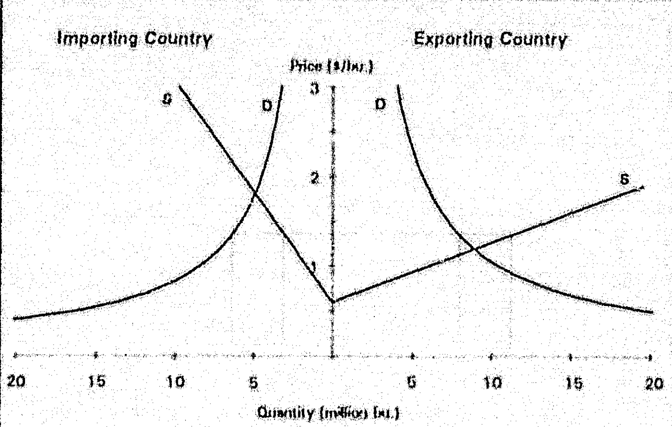


Figure 1: Free-trade Equilibrium when Relative Risk-Aversion is Constant and the Elasticity of Supply with Respect to Wealth Equals One.

Table 1 lists parameters of the model for two countries: an importing country and an exporting country. Consumers are the same in both countries. However, the importing country has relatively more farmers and smaller farms. The exporting country has fewer farmers and larger, more productive farms. Two cases will be considered. In the first case, farmers in both countries have constant relative risk-aversion and, hence, an elasticity of supply with respect to wealth equal to one. In the second case, farmers have increasing relative risk aversion and an elasticity of supply less than one. Market equilibria are calculated using the solver in MicroSoft Excel.

Constant Relative Risk Aversion and Unitary Elastic Supply with Respect to Wealth

As a baseline, suppose there are no government programs in either country. This is the

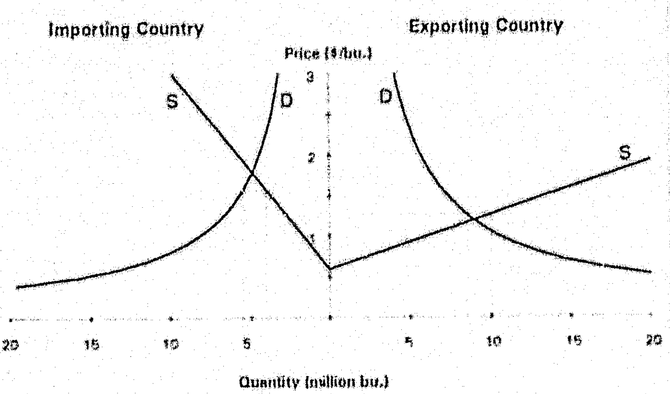
Table 2. Elasticities at the Free-trade Equilibrium when Relative Risk-Aversion is Constant and the Elasticity of Supply with Respect to Wealth Equals One.

Type of Elasticity	Importing Country	Exporting Country
Price Elasticities		
Demand	-0.90	-0.90
Supply	1.80	1.80
Wealth Elasticities		
Demand	0.08	0.08
Supply	1.00	1.00

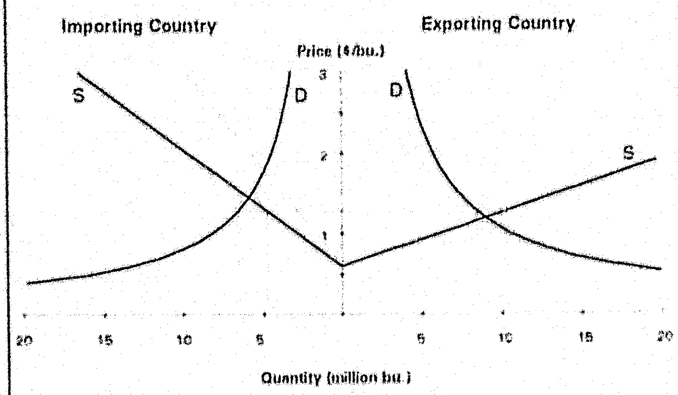
free-trade equilibrium shown in Figure 1. The exporting country sells its excess demand to the importing country. The price, quantities and expected utilities of individual consumers and farmers are detailed in Table A1 of Appendix B.

Demand and supply elasticities at the free-trade equilibrium are listed in Table 2. Because of the different wealth elasticities, an income transfer from consumers to farmers will shift the demand curve less than the supply curve.

Import Tariff. Consider now the decoupling of an import tariff. In panel a) of Figure 2, an import tariff raises the price in the importing country and lowers the price in the exporting country. Compared with free trade, the tariff reduces trade between countries.



Panel a) Import Tariff.



Panel b) Direct Income Transfer.

Figure 2: Decoupling an Import Tariff when Relative Risk-Aversion is Constant and the Elasticity of Supply with Respect to Wealth Equals One.

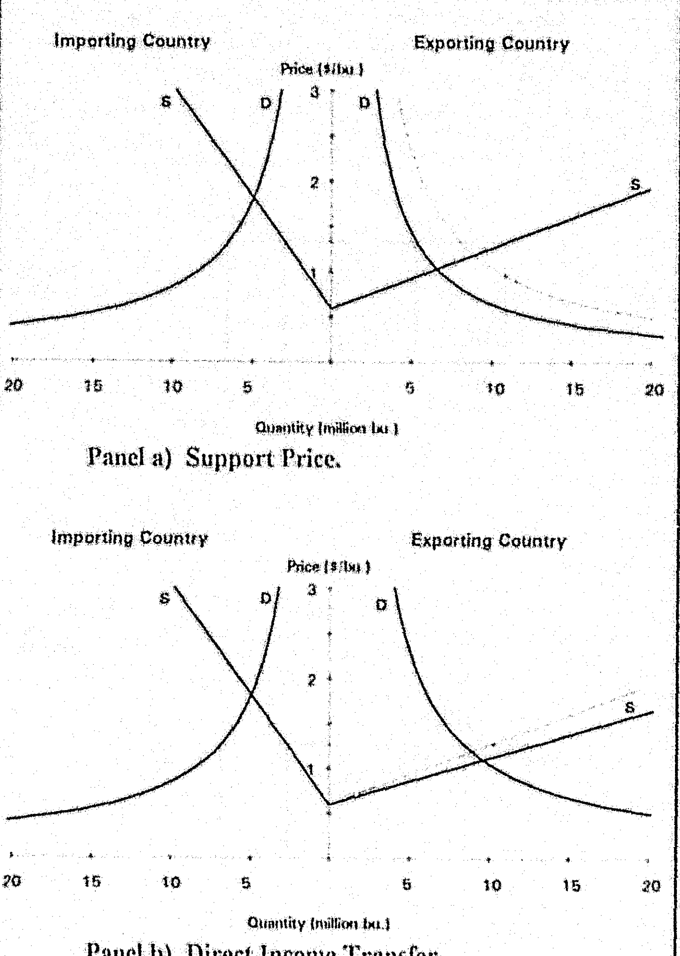
In panel b), the import tariff is replaced by a direct income transfer which maintains the expected utility of farmers in the importing country. Loss of income by consumers tends to shift the demand curve in, but, because farmers also consume, the gain of income by farmers tends to shift the demand curve back out. On balance, the demand curve is almost unchanged. However, the supply curve does shift. Supply in the importing country is more distorted by a direct income transfer, although the world price and trade between countries appear to change very little.

From the graphs alone, it is difficult to assess whether a direct income transfer reduces distortions of the market. A measure of deadweight loss is needed. The deadweight loss is the sum of consumers' and farmers' willingness to pay for free trade. Because a closed-form for expected utility is known, there is no need to calculate consumer and producer surpluses. Instead, willingness to pay is the amount of income a consumer or farmer could forgo under free trade and still have the same expected utility as under an import tariff or a direct income transfer. It is a compensating variation which makes consumers and farmers as well off under free trade as under the government program.

Measures of willingness to pay can help explain the political process as interest groups lobby for against a government program (Just and Rausser). Consumers in the importing country and farmers in the exporting country are harmed by either an import tariff or an income transfer. They are willing to pay a positive amount for free trade. (See Table A2 of Appendix B.) Consumers in the exporting country and farmers in the importing country benefit from a tariff or income transfer and would be willing to pay a negative amount for free trade. In other words, they would require compensation. However, the sum of all consumers' and farmers' willingness to pay will be a positive deadweight loss.

To eliminate the import tariff and obtain free trade, consumers and farmers in the importing country would be willing to pay $659,667 and -$660,070, respectively. In total, the importing country benefits slightly from the import tariff and would be willing to pay -$403. Consumers and farmers in the exporting country would be willing to pay -$329,116 and $620,005. In total, the exporting country would be willing to pay $290,889. The deadweight loss to both countries from the import tariff is $290,486. To eliminate the direct income transfer, consumers and farmers in the importing country would be willing to pay $628,056 and -$660,070 for a country total of -$32,014. Consumers and farmers in the exporting country would be willing to pay -$262,367 and $523,822 for a country total of $262,455. The deadweight loss to both countries from the direct income transfer is $230,441. Although its deadweight loss is slightly less, the direct income transfer does not significantly reduce distortions of the market. In this example, decoupling does not work.

Support Price. Suppose the exporting country has a program to support the price. The government purchases commodities and withdraws them from the market. In panel a) of Figure 3, the excess supply in the exporting country is larger than the excess demand in the importing



Panel a) Support Price.

Panel b) Direct Income Transfer.

Figure 3: Decoupling a Support Price when Relative Risk-Aversion is Constant and the Elasticity of Supply with Respect to Wealth Equals One.

country. The difference is purchased by the government. Purchases financed by taxes may reduce the income of consumers and the off-farm income of farmers. The demand curve shifts in. The irony is that the world price is slightly lower with the support price program than with free trade. This result may not hold if several commodities are considered and the supported commodity is a small share of consumers' expenditures. Nor will the result hold if purchases of commodities do not directly reduce consumers' income. Even so, it is possible for a price support program to lower, not raise, the price.

In panel b), the support price is replaced by a direct income transfer which maintains the expected utility of farmers in the exporting country. The demand curve is back near its original position, but the supply curve is shifted out. An income transfer distorts the market differently from the price support. Whether the distortion is better or worse is difficult to judge. Everyone but consumers in the importing country are willing to pay a positive amount for free trade. (See Table A2 of Appendix B). For the support price, total willingness to pay by consumers and producers in the importing country is -$46,082 and in the exporting country is $2,826,837, giving a total deadweight loss of $2,780,754. For the direct income transfer, total willingness to pay in the importing country is -$187,307 and in the exporting country is $571,410, giving a deadweight loss of $330,103. In this example, decoupling can work, although not perfectly.

Target Price. Perhaps the exporting country has a target price program in which the government pays farmers a price premium. In panel a) of Figure 4, the price to farmers in the exporting country is above the world price. Financing of the price premium reduces consumers' income and shifts the demand curve in. Because of this shift, the target price received by farmers is only slightly above the price they would receive with free trade.

In panel b), the target price is replaced by a direct income transfer. The demand curve resumes its original position but the supply curve shifts dramatically. Trade between countries is wildly distorted. (See Table A4 of Appendix B.) Consumers in the importing country prefer the income transfer but consumers in the exporting country must pay the transfer and are especially harmed. In total, the importing country benefits from both the target price and direct income transfer and would be willing to pay negative amounts of -$328,330 and -$1,182,153 to replace them with free trade. The exporting country is harmed by its own target price and income transfer programs and would be willing to pay \$862,196 and \$3,302,167 for free trade. Total deadweight loss for the target price, \$533,866, is much less than the deadweight loss for the direct income transfer, \$2,120,014. In this example, decoupling is a disaster.

Increasing Relative Risk Aversion and Inelastic Supply with Respect to Wealth

When does decoupling work and when does it not? Figure 5 shows decoupling of a target price with a lower elasticity of supply respect to wealth equal to 0.42. In panel a), the target price shifts the demand curve and distorts the market, as in the previous example. In panel b), a direct income transfer restores the demand curve and shifts the supply curve, but much less than before. The deadweight loss from the target price is \$798,782, which is higher than before. However, the

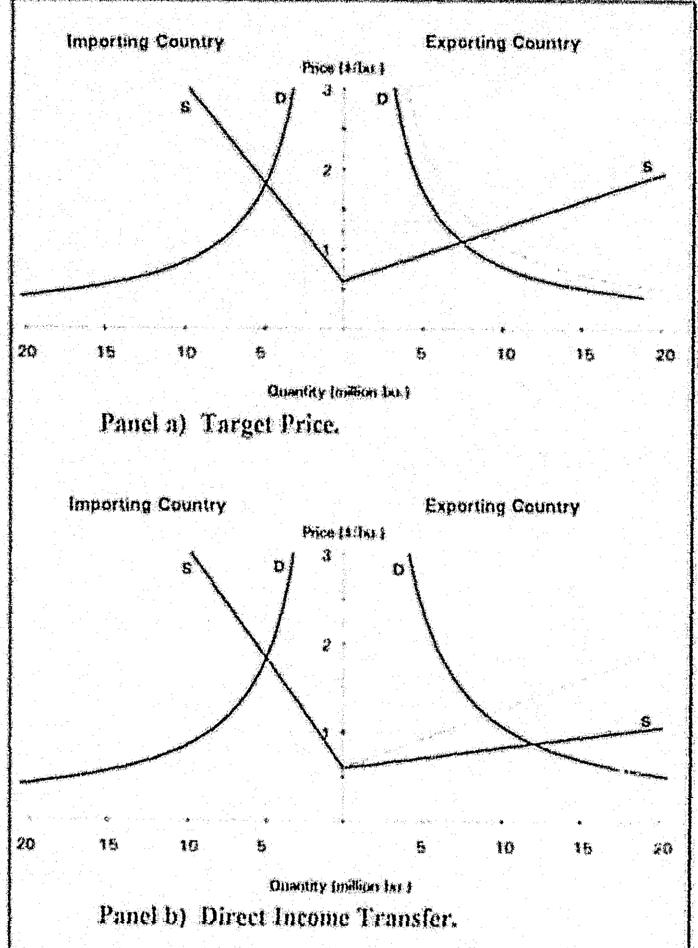


Figure 4: Decoupling a Target Price when Relative Risk-Aversion is Constant and the Elasticity of Supply with Respect to Wealth Equals One.

deadweight loss from the direct income transfer is only $183,996. (See Table A5 in Appendix B.) The exporting country is still harmed by its own policies, but a direct income transfer is a definite improvement and decoupling works.

Direct income transfers will partially decouple other government programs as well, if supply is inelastic with respect to wealth. For supply to be inelastic, relative risk aversion must be increasing. Income transfers will completely decouple government programs if the supply elasticity is zero. Absolute risk-aversion must be constant (Saha). Therefore, decoupling will work only if farmers' risk preferences have a specific structure which empirical evidence rejects (Pope and Just).

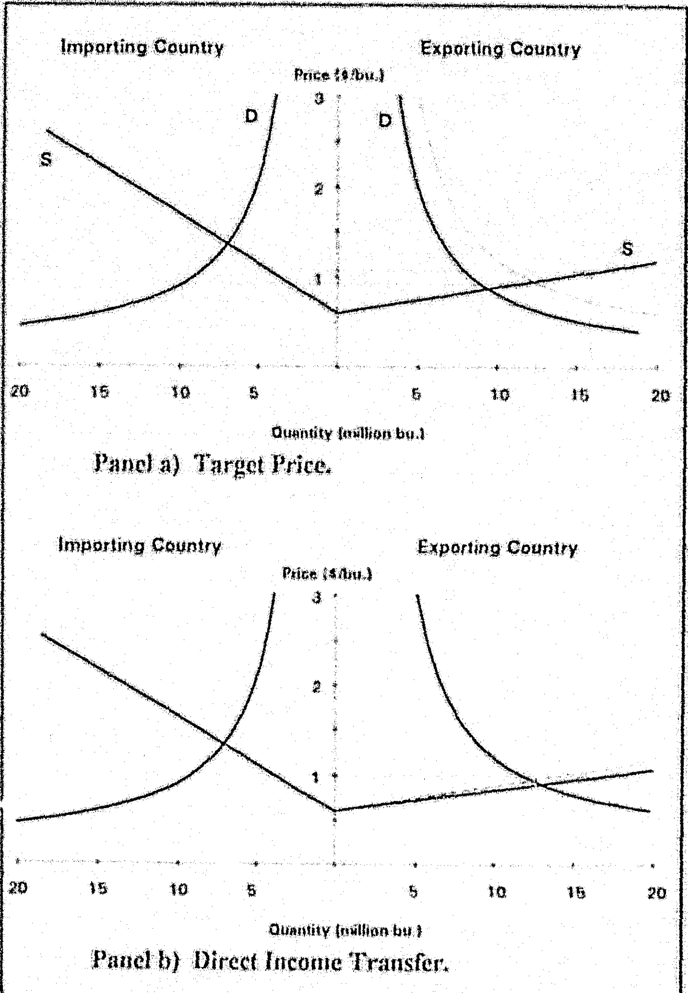


Figure 5: Decoupling a Target Price for Increasing Relative Risk Aversion and Inelastic Supply with Respect to Wealth.

Conclusions

A fairy tale begins with a villain who spreads pestilence and tribulation over the land. Then talented and brave heroes and heroines stop the villain and everyone lives happily ever after. The process of decoupling begins with government programs which distort world trade. Negotiators build the GATT to flash a "red light" at government programs which manipulate prices. In the sake of justice and equity, the GATT gives the "green light" to direct income transfers and other programs which may improve the environment. Unfortunately, our faith in decoupling is based on an heroic assumption. Farmers must have nearly constant absolute risk-aversion. In the real world, farmers may behave differently and we might not live happily ever after.

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Appendix A

It will be shown that indirect utility, J, in equation (7), consumption, c, in equation (11) and farmland investment, F, in equation (12) comprise a closed-form solution. First find the optimality conditions for consumption and area farmed by partially differentiating the Hamilton-Jacobi-Bellman (HJB) equation (3):

$$0 = e^{-rt}\alpha\beta(c-\gamma)^{\alpha-1} - J_W p_c;$$
$$0 = J_W\left[(\delta_f - \delta_w)p_f + p_y Y\right] + J_{WW}\sigma_y^{\,2} F$$

Solve the optimality conditions.

$$c = \left[e^{\rho t} J_W p_c / \alpha\beta\right]^{1/(\alpha-1)} + \gamma;$$

$$F = \frac{(\delta_f - \delta_w)p_f + p_y Y}{[-J_{WW}/J_W]\sigma_y^2}.$$

Substitute the optimality conditions to maximize the HJB equation.

$$\begin{aligned}
0 = {} & J_t + e^{-\rho t}\beta\left[e^{\rho t} J_W p_c / \alpha\beta\right]^{\alpha/(\alpha-1)} \\
& + J_W\left[\delta_w W + I + \frac{\left[(\delta_f - \delta_w)p_f + p_y Y\right]^2}{[-J_{WW}/J_W]\sigma_y^2} - p_c\left[e^{\rho t} J_W p_c / \alpha\beta\right]^{1/(\alpha-1)} - p_c\gamma\right] \\
& + \tfrac{1}{2} J_{WW}\frac{\left[(\delta_f - \delta_w)p_f + p_y Y\right]^2}{[-J_{WW}/J_W]^2\sigma_y^2}
\end{aligned}$$

Simplify the HJB equation and, for convenience, place the partial derivative with respect to time on the left-hand side.

$$\begin{aligned}
J_t = {} & -(1-\alpha)e^{-\rho t}\beta\left[e^{\rho t} J_W p_c / \alpha\beta\right]^{\alpha/(\alpha-1)} \\
& - \tfrac{1}{2}\frac{J_W}{[-J_{WW}/J_W]}\frac{\left[(\delta_f - \delta_w)p_f + p_y Y\right]^2}{\sigma_y^2} - J_W\left[\delta_w W + I - p_c\gamma\right]
\end{aligned}$$

Next, differentiate indirect utility in equation (7 to find its first partial-derivatives with respect to time and wealth and its second partial-derivative with respect to wealth.

$$\begin{aligned}
J_t &= -\rho J; \\
J_W &= \frac{\alpha}{p_c} e^{-\rho t}\beta[(1-\alpha)/r]^{1-\alpha}\left[W/p_c + I/p_c\delta_w - \gamma/\delta_w\right]^{\alpha-1} \\
&= \frac{\alpha}{p_c}\left[W/p_c + I/p_c\delta_w - \gamma/\delta_w\right]^{-1} J, \\
J_{WW} &= \frac{\alpha(\alpha-1)}{p_c^2} e^{-\rho t}\beta[(1-\alpha)/r]^{1-\alpha}\left[W/p_c + I/p_c\delta_w - \gamma/\delta_w\right]^{\alpha-2} \\
&= \frac{\alpha(\alpha-1)}{p_c^2}\left[W/p_c + I/p_c\delta_w - \gamma/\delta_w\right]^{-2} J
\end{aligned}$$

Use these derivatives to simplify various components of the HJB equation

$$
\begin{aligned}
-J_{WW} / J_W &= \frac{(1-\alpha)}{p_c}\left[W / p_c + I / p_c\delta_w - \gamma / \delta_w\right]^{-1} \\
&= \frac{(1-\alpha)}{\left[W + I / \delta_w - p_c\gamma / \delta_w\right]};
\end{aligned}
$$

$$
\frac{J_W}{\left[-J_{WW} / J_W\right]} = \frac{\alpha}{(1-\alpha)} J;
$$

$$
\begin{aligned}
(1-\alpha)e^{-\rho t}\beta\left[e^{\rho t} J_W p_c / \alpha\beta\right]^{\alpha/\alpha-1} &= (1-\alpha)e^{-\rho t}\beta\left[(1-\alpha)/r\right]^{-\alpha}\left[W / p_c + I / p_c\delta_w - \gamma / \delta_w\right]^{\alpha} \\
&= rJ;
\end{aligned}
$$

$$
J_W\left[\delta_w W + I - p_c\gamma\right] = \alpha\delta_w J.
$$

Finally, substitute these components into the HJB equation and use equation (8) which defines the risk adjusted real rate of time preference, r.

$$
\begin{aligned}
-\rho J &= -rJ - \frac{\alpha\left[(\delta_f - \delta_w)p_f + p_y\bar{Y}\right]^2}{2(1-\alpha)\sigma_y^{\,2}} J - \alpha\delta_w J \\
&= -rJ + (r-\rho)J \\
&= -\rho J.
\end{aligned}
$$

Therefore, indirect utility in equation (7) is the solution of the HJB equation and consumption and area farmed in equations (11) and (12) maximize expected utility subject to the stochastic change in wealth.

Appendix B

Table A1. Free-trade Equilibrium when Relative Risk-Aversion is Constant and the Elasticity of Supply with Respect to Wealth Equals One.

Item	Importing Country	Exporting Country
Prices ($/bu.)		
Demand	1.35	1.35
Supply	1.35	1.35
Quantities (bu.)		
Demand	6,405,703	7,940,653
Supply	3,068,739	11,277,617
Expected Utilities (utils)		
Consumer	228.84	228.84
Farmer	248.83	443.37

Table A2. Decoupling an Import Tariff when Relative Risk-Aversion is Constant and the Elasticity of Supply with Respect to Wealth Equals One.

Item	Import Tariff		Direct Income Transfer	
	Importing Country	Exporting Country	Importing Country	Exporting Country
Prices (\$/bu.)				
Demand[1]	1.57	1.27	1.29	1.29
	(+16.26)	(-5.93)	(-4.77)	(-4.77)
Supply[1]	1.57	1.27	1.29	1.29
	(+16.26)	(-5.93)	(-4.77)	(-4.77)
Quantities (bu.)				
Demand[1]	5,480,165	8,599,999	6,601,842	8,433,965
	(-14.45)	(+7.80)	(+3.06)	(+6.21)
Supply[1]	3,965,811	10,074,354	4,725,542	10,310,265
	(+29.23)	(-10.67)	(+53.99)	(-8.58)
Income Transfers (\$)				
Consumers	0	0	-821,252	0
Farmers	0	0	821,252	0
Individual Utilities (utils)				
Consumer	224.35	230.76	224.27	230.37
Farmer	271.62	415.69	271.62	420.43
WTP for Free Trade (\$)				
Consumers	659,667	-329,116	628,056	-262,367
Farmers	-660,070	620,005	-660,070	524,822

[1]The percentage distortion compared to equilibrium with no government intervention is shown in parentheses.

Table A3. Decoupling a Support Price when Relative Risk-Aversion is Constant and the Elasticity of Supply with Respect to Wealth Equals One.

	Support Price		Direct Income Transfer	
Item	Importing Country	Exporting Country	Importing Country	Exporting Country
Prices ($/bu.)				
Demand[1]	1.33	1.33	1.26	1.26
	(-1.96)	(-1.96)	(-6.68)	(-6.68)
Supply[1]	1.33	1.33	1.26	1.26
	(-1.96)	(-1.96)	(-6.68)	(-6.68)
Quantities (bu.)				
Demand[1]	6,535,449	5,305,529	6,868,019	8,464,359
	(+2.03)	(-33.19)	(+7.22)	(+6.56)
Supply[1]	2,960,637	10,880,342	2,700,240	12,632,137
	(-3.52)	(-3.52)	(-12.01)	(+12.01)
Income Transfers ($)				
Consumers	0	-2,650,712	0	-614,175
Farmers	0	0	0	614,175
Individual Utilities (utils)				
Consumer	229.45	211.27	231.01	227.21
Farmer	247.02	433.22	243.35	433.22
WTP for Free Trade ($)				
Consumers	-88,954	2,580,802	-313,635	271,325
Farmers	42,872	246,065	126,328	246,035

[1]The percentage distortion compared to equilibrium with no government intervention is shown in parentheses

Table A4. Decoupling a Target Price when Relative Risk-Aversion is Constant and the Elasticity of Supply with Respect to Wealth Equals One.

Item	Target Price		Direct Income Transfer	
	Importing Country	Exporting Country	Importing Country	Exporting Country
Prices ($/bu.)				
Demand[1]	1.21	1.21	1.00	1.00
	(-10.43)	(-10.43)	(-26.03)	(-26.03)
Supply[1]	1.21	1.36	1.00	1.00
	(-10.43)	(+0.66)	(-26.03)	(-26.03)
Quantities (bu.)				
Demand[1]	7,155,252	6,749,655	8,632,455	10,635,152
	(+11.70)	(-15.00)	(+34.76)	(+33.93)
Supply[1]	2,493,028	11,411,879	1,632,286	17,635,321
	(-18.76)	(+1.19)	(-46.81)	(+56.37)
Income Transfers ($)				
Consumers	0	-1,711,782	0	-4,364,734
Farmers	0	0	0	4,364,734
Individual Utilities (utils)				
Consumer	232.33	220.80	238.80	198.36
Farmer	241.07	457.62	236.91	457.62
WTP for Free Trade ($)				
Consumers	-504,180	1,247,769	-1,443,235	3,687,450
Farmers	175,850	-385,573	261,082	-385,283

[1]The percentage distortion compared to equilibrium with no government intervention is shown in parentheses.

Table A5. Decoupling a Target Price for Increasing Relative Risk Aversion and Inelastic Supply with Respect to Wealth.

	Target Price		Direct Income Transfer	
Item	Importing Country	Exporting Country	Importing Country	Exporting Country
Prices ($/bu.)				
Demand[1]	0.92	0.92	1.00	1.00
	(-11.43)	(-11.43)	(-4.15)	(-4.15)
Supply[1]	0.92	1.07	1.00	1.00
	(-11.43)	(+2.94)	(-4.15)	(-4.15)
Quantities (bu.)				
Demand[1]	9,904,011	8,962,089	9,182,345	11,893,324
	(+12.43)	(-21.83)	(+4.24)	(+3.74)
Supply[1]	2,960,770	15,905,330	3,818,953	17,256,716
	(-31.73)	(-0.20)	(-11.94)	(+8.28)
Income Transfers ($)				
Consumers	0	-2,385,799	0	-1,205,846
Farmers	0	0	0	1,205,846
Individual Utilities (utils)				
Consumer	241.59	222.52	238.77	230.20
Farmer	278.77	455.52	280.69	455.52
WTP for Free Trade ($)				
Consumers	-560,044	1,944,944	-191,839	1,016,077
Farmers	163,290	-749,407	78,764	-719,006

[1]The percentage distortion compared to equilibrium with no government intervention is shown in parentheses.