Linking Farm Risk to Institutional Credit Risk

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Contributed paper to be presented at the 38th Annual Conference of the Australian Agricultural Economics Society, Victoria University, Wellington, 8–10 February 1994.

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Abstract

This paper describes a stochastic simulation model of a financial institution's portfolio of rural credit accounts. In this model, the market, production and financial risk exposure attributes facing farm business borrowers and the financial institution's loan assets are linked using portfolio theory. The model generates a distribution around the time path of returns on the financial institution's loan assets and a similar distribution of returns to farmer borrowers.

With further development, the model should provide a framework for the assessment of optimal lending and pricing policies to the rural sector, given the capacity of financial institutions to augment the terms and conditions associated with debt facilities in a deregulated financial system.

Introduction

A key objective in the deregulation of the financial system following the release of the Campbell Report (1982) and the Martin Report (1984) was to raise the level of competition between various institutions in the financial sector. The establishment of a more competitive banking industry has focussed attention on the pricing policies and credit (or default) risk management strategies employed by financial institutions.

Australian banks that have exposures to the rural sector have directed significant attention to credit risk management systems appropriate to this sector. The past decade has seen marked changes in the nature of market based risk faced by farmers. For example, the removal of interest rate controls has seen farmers become exposed to interest rate fluctuations, while reforms to agricultural pricing mean that prices received by farmers are now largely market driven (Piggott 1990). The greater variability in prices received and interest rates paid by farmers has seen Australian banks faced with significantly increased credit risk on portfolios of rural loans.

While frameworks for the management of operational risk, liquidity risk, interest rate and exchange rate risk appear to be reasonably well documented, models for credit risk management from a portfolio perspective remain primitive (Davis and Harper 1991, p. 1, 75). Significant research effort is required in developing portfolio models of credit risk given the range of

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* The views expressed in this paper are those of the authors and do not neccessarily reflect those of the Commonwealth Development Bank. Thanks are due to Andrew Saunders and Bruce Brown for their assistance and support in the development of this paper.

specific factors that determine the risk profile of loan exposures of different sectors of the economy. For example, Australian agriculture is primarily subject to production risk due to weather variability and price risk due to the inelastic nature of aggregate supply and demand functions (Piggott 1990). In addition, there is inadequate time series data on risk rated portfolios given the recency of credit risk rating systems in use by banks in Australia. This lack of data does not allow the use of econometric techniques to estimate the impact of these specific factors on portfolio credit risk. In the interim, a normative approach must be utilised where the production, market and financial risk factors facing farmers are linked to the returns on a bank's portfolio of rural credits.

The aim of this paper is to develop a simulation model of a bank's portfolio of rural credit accounts. In the next section, contemporary investment and portfolio theory is discussed. This is done to define institutional credit risk and to show how portfolio selection principles can be used in loan asset pricing. In the third section, the stochastic properties of market, production and financial risk attributes of rural loans are linked to institutional credit risk using portfolio theory. A stochastic simulation model is developed, and farm risk is linked to the probability of default and then related to portfolio credit risk. Some of the key strategies for the management of institutional credit risk and loan asset selection from a portfolio perspective are then discussed in the fourth section, and some concluding remarks made in the final section.

Principles of institutional credit risk

Measurement of risk

The most widely used method of risk analysis uses the expected (mean) return as an indicator of an investment's anticipated profitability and the variance (or standard deviation) as an indicator of risk (Harrington 1987, p. 5). In general terms, the expected value of an investment is simply the net present value of the possible return outcomes weighted by their probability of occurrence. However, in the case where a financial institution or a bank invests funds in the form of a loan the upside of bank returns on a loan contract are known to a bank and are limited to the promised interest rate, fee income and the loan value. On the down side, the bank's return is limited to the extent that it retains collateral in the event of loan default. This may be expressed algebraically as follows:

(1) $$E(BR_i) = (1 - d).(r+f+L) + d.(C - L)$$

where

$E(BR_i)$	=	expected bank returns;
r	=	promised interest rate;
f	=	bank fees;
C	=	collateral;
L	=	loan value; and
d	=	probability of default.

While the terms of a loan contract are known to a bank, the probability of default, and any loan loss in the event of default, are uncertain. The last component in equation (1), $d.(C-L)$, measures the average rate of possible loss of assets on a loan account expected over time and thus may be termed as expected credit risk.

In practice, there can be varying degrees of loan default that depends on a range of circumstances. In the event of default, a borrower may choose to liquidate some assets, obtain new credit or seek other income to meet debt servicing obligations. The bank also faces several options including allowing the borrower to miss several payments, renegotiating the terms of the debt contract (lowering interest rates, extension of loan maturity or overdraft credit limit) or the use of legal remedies to repossess collateral (Webb 1982). In some cases, a bank may not be able or wish to renegotiate on a loan contract, particularly where a borrower is a corporate entity because of the ramifications of corporate law due to aiding and abetting provisions. In the rural sector, a bank may allow significant level of indulgence when a farm borrower fails to meet repayments on a due date. Banks behave in this way due to the extensive fixed asset base of Australian farms. This offers significant security on loan assets. Banks are also aware of the inherent volatility of farm income conditions in Australia. As a result, a model of institutional credit risk associated with a rural loan portfolio exposure should reflect to some extent varying degrees of default.

The variance measures the dispersion of returns around the mean (expected) value of returns. It provides information on the extent of the possible deviations of the actual return from the expected return. The variance of the distribution ($Var(BR_i)$) is given by the expression:

$$(2) \qquad Var(BR_i) \quad = \quad E\{BR_i - E(BR_i)\}^2$$

Thus unexpected credit risk may be defined as the volatility of likely credit losses versus the average or expected loss of income and loan assets in any given period. In other words, unexpected risk is associated with actual outcomes of the probability of default and losses on loan accounts deviating from their expected levels through time.

A key element of defining a bank asset is the loan contract. For example, in the case of a term loan, an immediate cash outlay in the form of loan principal is provided to a farm. The bank expects positive net income and loan principal repayments in the following years during the agreed term of the contract. To account for the time value of money, these returns should be discounted back to a net present value (Levy and Sarnat 1990, p. 30). For example, assuming a loan account has a two year term, the expected net present value of a loan asset is defined as the sum of the present values of the expected cash flows in the first year and the second year, less the initial loan principal outlay:

(3) $\quad NPV(BR_i) = \emptyset_{i1}.BR_{i1} + \emptyset_{i2}^2.BR_{i2} - LP_{i0}$

(3') $\quad NPV\{E(BR_i)\} = \emptyset_1.E(BR_{i1}) + \emptyset_2^2.E(BR_{i2}) - LP_{i0}$

where $NPV(BR_i)$ = net present value of bank returns on loan asset *i*.
$NPV\{E(BR_i)\}$ = expected net present value of bank returns on loan asset *i*.
$E(BR_{i1})$ = expected value of bank returns on loan asset *i* in the first year;
$E(BR_{i2})$ = expected value of bank returns on loan asset in the second year;
LP_{i0} = initial loan principal outlay on loan asset *i*; and
$\emptyset$ = $1/(1+k)$; a coefficient for capitalising bank returns over time where *k* denotes the discount rate.

The variance of bank returns on a loan asset may be defined by the expression below given that bank returns BR_{i1} and BR_{i2} are statistically independent over time:

(4) $$Var\{NPV(BR_i)\} = \emptyset^2 \sigma^2_{i1} + \emptyset^4 \sigma^2_{i2} = \frac{\sigma^2_{i1}}{(1+k)^2} + \frac{\sigma^2_{i2}}{(1+k)^4}$$

where σ^2_{i1} = variance of the bank returns distribution in year 1;
σ^2_{i2} = variance of the bank returns distribution in year 2; and
$Var\{NPV(BR_i)\}$ = variance of the NPV.

Given that a bank's loan portfolio consists of many individual loan accounts, the return on a portfolio is simply the summation of returns from each loan asset in the portfolio. Similarly, the expected return on a portfolio is simply the summation of expected returns from each asset in the portfolio (Markowitz 1959, 1992).

Portfolio selection and asset pricing

The primary focus of the application of portfolio theory has been to portfolios of actively traded securities within a static framework. For marketable securities, their returns may be measured by the holding period rate of return. The holding period is specified (for example, one year) and all the benefits received during the year are theoretically reinvested. For multi-period investments, the holding period rate of return can readily be converted to an equivalent return per period by compounding the holding period rate of return for each period. In this model, the expressions for the mean and the variance of returns to an individual security held by an investor are measured as a rate of return on assets in percentage points terms. The weight of each security is equal to it's percentage value in the portfolio.

In contrast, the distribution of returns to an individual loan asset held by a bank are described by the parameters NPV{E(BR_i)} and Var{NPV(BR_i)}, which are measured in dollar terms. The bank is assumed to classify its loan assets according to various characteristics that are similar among these loan assets. The mean and variance of returns on a loan asset class are equal to the summation of the NPV{E(BR_i)} and Var{NPV(BR_i)} of each loan account for the particular loan class. In addition, the weight of the loan asset class in a portfolio will be the value of gross loan balances in the loan asset class. For exposition purposes, the mean and variance of returns on each respective loan asset class are expressed in terms of E(R_i) and σ^2 respectively below.

If an investor (or bank) holds n securities (loan assets) with weights of w_i in the portfolio, the expected return on such a portfolio is given by:

(5) $$E(R_p) = \sum_{i=1}^{n} w_i . E(R_i)$$

with a variance of

(6) $$\sigma_p^2 = \sum_{i=1}^{n} w_i . \sigma_i^2 + 2 \sum_{\substack{i=1}}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{m} w_i w_j . Cov(R_i, R_j)$$

where E(R_p) = expected portfolio return;
σ_i^2 = variance of portfolio returns (per dollar invested) on the ith security;
Cov(R_i,R_j) = covariance of returns on security i and security j; and
w_i = proportion of a portfolio in the ith security.

Based on the measures of risk and returns provided in equations (5) and (6), portfolio theory focuses upon the selection of alternative groups of securities on their risk-return characteristics as reflected in the portfolio's expected return and its variance. The risk averse investor will seek a portfolio management strategy that will stabilise returns, that is, minimise the variance for a given level of expected return. Algebraically, the mean-variance criterion for selecting securities where x and y are the returns on two different securities may be specified as follows:

(7) $$E(x) \geq E(y), \quad \sigma^2(x) \leq \sigma^2(y)$$

Portfolio theory leads to the notion of efficient portfolios. An efficient portfolio is one that provides the maximum return for a given level of risk (in other words, the variance) or the minimum risk for a given level of return. By varying the given level of return, and minimising at each of these levels, the volatility of returns on the portfolio, a locus of optimal combinations of individual investments into portfolios may be constructed. This analytical tool is called the efficient frontier. It is the locus of those portfolios that minimise risk for each level of expected return. To derive the efficient frontier, quadratic programming may be used to minimise portfolio variance for differing levels of return. The objective function and the constraints for the quadratic programming problem may be specified as follows:

$$(8) \quad \text{Minimise} \quad \sigma_p^2 = \sum_{i=1}^{n} w_i.\sigma_i^2 + 2\sum_{\substack{i=1 \\ i\neq j}}^{n}\sum_{j=1}^{m} w_i w_j.Cov(R_i, R_j)$$

subject to

$$(9) \qquad \sum_{i=1}^{n} w_i = 1$$

$$(10) \qquad \sum_{i=1}^{n} w_i.E(R_i) \geq E(R_p)$$

$$(11) \qquad w_i \geq 0, \; i = 1,\ldots,n$$

Equation (8) specifies the objective function of the quadratic programming problem as minimise the portfolio variance. The first constraint (equation (9)) assumes all the weights of each security sum to one. The second constraint (equation (10)) states that the expected return on the portfolio is a weighted sum of the expected returns on the securities that must at the least equal $E(R_p)$. The final restriction (equation 11) precludes the possibility of negative holdings in securities *i*.

The primary conclusion of portfolio theory is that investment risk can be reduced by diversification; that is, by spreading the portfolio across different classes and types of assets (Levy and Sarnat 1990, p. 268). The risk associated with holding any given portfolio of assets consists of two distinct types of risk: the first component of equation (6), $\sum_{i=1}^{n} w_i.\sigma_i^2$, represents unsystematic risk; and the second component, $2\sum_{i=1}^{n}\sum_{j=1}^{m} w_i w_j.Cov(R_i, R_j)$, represents systematic risk. Unsystematic risk measures the portion of portfolio risk that is associated with individual asset returns behaving independently of each other. As diversification increases, meaning that as progressively more assets are added to the portfolio with each weighted equally in the portfolio, the unsystematic risk converges to zero. Thus, unsystematic risk is easily diversified away.

Systematic risk involves a different concept. If the covariance term is redefined as $\sigma_i \sigma_j \rho_{ij}$ where ρ_{ij} represents the correlation coefficient between the returns on assets *i* and *j*, then clearly systematic risk measures the portion of portfolio risk that is a consequence of returns on different assets being correlated with one another. When returns are positively correlated, their return variabilities do not cancel one another out completely. As diversification increases and the portfolio grows in value, the systematic portion of the risk gradually converges to the average covariance of the rates of returns on all assets included in the portfolio.

Thus the bank's decision making process on loan selection may be modelled as a problem of choosing an optimal combination of loan accounts (portfolio) out of the subset of efficient combinations (portfolios). Because of possible covariance between the returns from loan proposals and those generated by existing loan asset holdings, the combinations should include existing loan assets as well as newly proposed loan assets. These efficient loan asset holdings facing a bank can be described by the envelope curve in the $E(R_p)$ and σ_p space as illustrated in Figure 1. All of the interior combinations should not be chosen since they represent inefficient options, in the sense that a bank can always improve its position (increase return with no increase in risk, or reduce risk without sacrifice of return) by choosing a different combination on the efficient frontier.

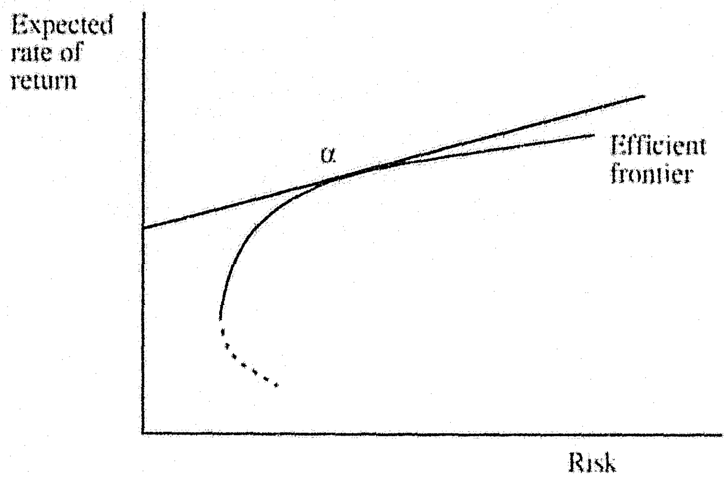

FIGURE 1—The efficient frontier

Markowitz's analysis provided a convenient framework for measuring risk and return. The Capital Asset Pricing Model (CAPM) takes this framework a step further by examining at its implications for pricing risky assets (Levy and Sarnat 1992, p. 291). In particular, the CAPM provides a method for measuring risk that cannot be eliminated through diversification subject to some additional fairly restrictive assumptions and concludes that only those risks that cannot be diversified away should be rewarded with higher expected returns.

Under CAPM, an index of risk is measured by Beta:

$$\beta_i = \frac{Cov(R_i R_m)}{\sigma_m^2} = \frac{\rho_{i,m}\sigma_i\sigma_m}{\sigma_m^2} \qquad (12)$$

where β_i = beta coefficient on the *i*th security;

$Cov(R_i R_m)$ = covariance between the return on the *i*th security and the returns on the market portfolio;

$\rho_{i,m}$ = correlation coefficient between the return on the *i*th security and the portfolio; and

σ_m, σ_m^2 = standard deviation and the variance of the market portfolio.

The basic CAPM equation for estimating required rate of return on securities is:

$$\text{(13)} \qquad E(R_i) = rl + \{E(R_m) - rl\}\, \beta_i$$

where rl = riskless rate of return and

$E(R_{mt})$ = expected return on the market portfolio.

Thus a security that moves exactly with the market portfolio has a ß = 1 and a risk premium equal to the market premium (as shown by the point α in Figure 1). For stocks that are more volatile than the market portfolio, ß > 1, the risk premium is greater than the market premium. For stocks that are less volatile than the market portfolio, ß < 1, the risk premium is less than the market portfolio.

Given these CAPM results, each individual investor in a perfect market should be responsible for extracting all potential gains from diversification. The only remaining risk after diversification is systematic risk and is due to the common exogenous factors that affect all assets in the market. Since all assets will not be affected identically, differential risk premiums across assets should simply reflect the extent of the relationship of each asset with these factors. The extension of the CAPM into the Arbitrage Pricing Model allows for some decomposition of the overall risk premium on an asset into its responsiveness to common exogenous factors (Levy and Sarnat 1990, p. 306).

Application of portfolio theory to a bank loan asset portfolio

There are several factors that preclude the use of the holding period return on bank loan assets. First, the maturity structure of a bank's portfolio cannot be ignored. Once a term loan has been granted to a customer, a bank can not readily disinvest in this loan without reneging on the terms of the original loan contract. Second, the size of debt facilities held by farm borrowers differ widely. Hence differences in scale need to be accounted for in a simulation model of a rural loan portfolio. Finally, there is no established secondary market for customer loans. This reduces the marketability of securities based on customer loans (Juttner 1986). As a result, the use of the holding period rate of return sits uncomfortably as a measure of bank returns on term loan assets (Carmichael and Davis 1990). A model of return-risk relationships of a bank portfolio must use the net present value of bank returns arising over the term of the loan that may be derived through use of the simulation approach (Carmichael and Davis 1990).

A further criticism of the portfolio theory approach is the use of the mean-variance criterion for describing the distribution of returns on securities. Bank returns on individual loan accounts are non-normally distributed. However, when a sufficiently large number of non-normally distributions are included into a loan asset class, the Central Limit Theorem may allow the use of normality as an approximation to the true distribution. Further, the results of the CAPM assume that an investor holds a portfolio of assets that is equivalent to portfolios held by all participants in the securities market. Notwithstanding this particularly restrictive assumption, the key results of CAPM may be used to guide specialist lenders in their choice of credit risk management strategies.

A model of return-risk relationships of a bank's loan asset portfolio must also consider both the expected and unexpected components of institutional credit risk as described in equations (1) and (2). The price outcomes on actively traded securities are primarily a result of exogenous market forces. However, banks may pro-actively manage their expected returns on different loan asset classes through the use of risk pricing mechanisms. This indicates significant endogeniety in the level of expected risk that a bank may be willing to be exposed to. Thus banks can use certain types of loan pricing policies to augment its expected credit risk exposure.

Banks also for a variety of reasons may choose, or are constrained by government legislation, to specialise in certain loanable funds market segments. Consequently, the diversification option for risk management as suggested by Markowitz (1959) may be precluded.

A simulation model

The aim of this section is to develop a simulation model that will enable the generation of an efficient frontier for a bank's rural loan portfolio. The bank is assumed to offer two collaterised debt instruments, an overdraft and a term loan facility. These are the most common debt instruments used by the Australian farm sector. Farm income is stochastic in nature and therefore has implications for the probability of default, the level of write-offs and the covariance of bank returns over time. The proposed framework introduces notions of partial default, resource costs associated with bank returns, and differences in loan size and maturity structure across a loan portfolio. As suggested by Carmichael and Davis (1992), the principles of portfolio theory are used to link farm risk to institutional credit risk.

Measuring bank returns and bank capital

The treatment of returns and capital losses in the standard accounting frameworks utilised by banks depend on their performing and default status and, in particular, whether a credit account has been identified as a bad and doubtful debt in a previous period in the form of a specific provision. Bank returns may be measured using the following equations:

(14) $BR_t = II_t + BFC_t - PD_t - C_t$

where II = interest income;
BFC = fees and charges;
PD = provisions for bad and doubtful debts;
C = cost of funds and other associated costs; and
t = 1, ..., n time periods.

(15) $PD_t = SPN_t - RSP_t + WNSP_t$

where SPN = new specific provisions for bad and doubtful debts;
RSP = recovery of specific provisions previously specifically provided for; and
WNSP = write-offs of assets not previously provided for as a bad and doubtful debt.

The change in gross bank assets in year t is measured by:

(16) $BAG_t - BAG_{t-1} = IC_t + LD_t - RP_t - WO_t$

where BAG = gross bank asset balances;
IC = interest capitalised or compoundable on accrual accounts;
LD = loan drawings;
RP = repayments; and
WO = write-offs of assets.

(17) $WO_t = WNSP_t + WSP_t$

where WSP = write-offs of assets previously specifically provided for as a bad and doubtful debt.

The precise measurement of returns and capital losses is complex, especially if differing degrees of default by bank customers are considered. For example, both interest and fee income not yet received in cash form in a particular year (but likely to be fully recouped in future years on outstanding loan repayments, including any compounded interest) may be accounted for as income, and compounded for balance sheet reporting purposes. To simplify the analysis, the returns and capital losses to the bank are defined below in terms of the originally agreed terms of debt contracts for the overdraft and term loan facilities. However, in this model it is assumed that all bank fees and charges must be met by customers when due.

It is also assumed that the bank forgives repayments on the term loan facility for a designated time period in event of partial default, but does not offer any restructuring options on this facility. Forgiveness in this case is defined to be where a bank is prepared to lose some principal and interest repayments on the term loan facility during the designated period.

It is further assumed that the farm customer is permitted to draw further on the overdraft facility up to the originally agreed overdraft credit limit in order to remain in operation. In this event, the bank expects that during the period the farm is unable to meet its fixed debt commitments on the term loan facility, the farm is required to honour all its commitments on the overdraft facility provided its total liabilities do not exceed its credit limit.

If farm incomes rise in later periods to the extent that term loan repayments can be resumed at the originally agreed repayment rate, then the bank restores the farm customer to performing status. In this event, the bank receives all commitments in terms of the originally agreed principal, interest and fee payments on both the term loan and the overdraft facility.

On the other hand, if farm incomes fall to the extent that the farm reaches its credit limit, then the farm attains full default status. In this case, the bank realises on its security.

Given these assumptions, the following conditions in which a farm account would reach partial or full default status may be specified as follows:

(18) *Partial default* If $PI \neq PI_t$ but $FL_t < \cap.FA_t$

(19) *Full default* If $FL_t \geq \cap.FA_t$ then realisation on bank security

where PI = principal and interest repayments on an amortised term loan facility in year t
FL = total farm liabilities in year t
$\cap$ = designated credit limit for the farm customer (expressed as a percentage of total farm assets); and
FA = total farm assets in year t.

Clearly bank returns and changes in its asset base are influenced by whether or not a farm customer is in performing or partial default status. Further, expected or actual capital losses can only occur if the salvage value of assets does not match the value of liabilities for a particular farm credit account at the time of full default.

If a farm customer falls into partial default status, the bank must make a provision for a bad and doubtful debt if capital losses may occur. If capital losses are likely, the bank can not accrue interest income. On the other hand, if the bank does not expect to incur capital losses then the bank may accrue any earnings on the farm credit account. Thus, as described in equations (14) to (17) above, the treatment of capital losses in bank returns varies depending on whether a farm enters full default status from a performing status or partial default status. In the former case, write-offs occurs immediately following full default while in the latter, write-offs may or may not occur depending on the extent to which farm customers are required to draw on their credit reserves given the time path of farm incomes. In any event, actual capital losses can only arise when full default by a credit account occurs.

In practice, a farm may hold credit accounts with more than one financial institution. Each financial institution may impose different credit limits or lending policies on the same customer. Further, varying accounting standards are used by different financial institutions. The model may be augmented to account for these variations as required.

Linking farm risk to probability of default

Each farm credit account held by the bank is assumed to have fixed production plans and exhibits constant farm costs through time. In addition, the farm is assumed to have no off-farm assets or off-farm income, and thus its only source of income is derived only from its farming operations.

The farm is assumed to have farm liabilities at time t, FL_t, with a bank overdraft balance of BB_t and an amortised term loan balance, LP_t. The farm begins with an initial loan principal amount of LP_0 in year $t = 0$ with fixed annual repayments of PI_t. The bank is assumed to limit the farm to a certain level of BB, equal to BB^*, such that if total farm liabilities rise above a certain critical level of the value of farm assets, $\cap.FA$ in year t, then the bank realises on its collateral. The farm offers the bank all its farm assets as collateral that has a constant value of FA.

Farm income after debt servicing for consumption purposes, FIAD, in year t is defined as:

$$(20)\quad FIAD_t = GFI_t - FC_t - T_t - D_t$$

$$(21)\quad GFI_t = p_t.y_t.Q_t$$

$$(22)\quad T_t = t_t\,.\,FIT_t$$

where GFI = gross farm income;
FC = farm costs;
T = tax payments;
D = debt servicing obligations less credit drawings;
p = vector of commodity prices;
y = vector of yields,
Q = vector of enterprise size;
t = average tax rate; and
FIT = taxable farm income.

The annual tax payments of a farm will depend on the average tax rate and the level of farm income for taxation purposes. The measurement of these two variables will depend on farm tax management strategies. In particular, assumptions would have to be made regarding the legal entity structure of the farm business and the utilisation of taxation provisions such as income splitting and tax averaging.

The debt servicing component is defined as:

(23) $D_t \quad = \quad ro_t.BB_{t-1} - PI_t - BFC_t + \Delta BB_t$

(24) $PI_t \quad = \quad LP_0 \{rt_t(1+rt_t)^{nt}\} / \{(1+rt_t)^{nt} - 1\}$

(25) $BB^*_t \quad = \quad \cap.FA - LP_{t-1}$

where

$\cap.FA$	=	credit limit defined as a proportion of farm assets;
ro	=	rate of interest on the bank overdraft balance;
rt	=	rate of interest on the term loan facility;
BFC	=	bank fees and associated charges;
ΔBB	=	change in the bank overdraft balance; and
nt	=	duration term of term loan facility.

In this model, farm income after debt servicing is assumed to vary according to commodity prices, yields and interest rates, each of which are normally distributed with the following mean-variance properties:

(26) $p \sim N(E(p), Var(p))$

(27) $y \sim N(E(y), Var(y))$

(28) $r \sim N(E(r), Var(r))$ (where $r = ro, rt$).

According to Gabriel and Baker (1981), total farm risk may be defined as the probability, d, that the farm will be unable to generate a minimum level of funds needed for consumption, PE as well as business requirements after having serviced debt in a particular time period after fully drawing on its available credit reserve $BB^*_t - BB_{t-1}$. Thus, the probability of full default, d may be defined using equations (20) to (28) as:

(29) $P\{ (GFI_t - FC_t + BB^*_t - BB_{t-1} - ro.BB_{t-1} - PI_t - BFC_t) \leq PE \} \leq d$

where $P\{ . \}$ = probability density function.

A farm may achieve partial default status when the farm has not fully drawn on its available credit reserve on its overdraft facility but has insufficient farm income to cover its repayment obligations on its term loan facility. This situation may be modelled by assuming that the bank forgives any amortised term loan repayments for a certain designated time period. However, the farm must meet any fee commitments and allows the farm to resume amortised loan repayments when farm income rises. The farm draws on its credit reserve, the overdraft facility, on which both interest costs and fees must be met.

Given these assumptions, partial default risk may be defined as the probability, *pd*, that a farm is unable to generate a minimum level of funds needed for home consumption as well as the repayment commitments on its term loan facility during a particular designated time period.

$$(30) \quad P\{(GFI_t - FC + BB_t - BB_{t-1} - ro.BB_{t-1} - BFC_t) \leq PE + PI_t\} \leq pd$$

$$\text{with} \quad pd > d,\ 0 \leq pd,\ d \leq 1 \ \text{ and } BB_t < \cap.FA - LP_{t-1}.$$

The distribution of farm cash flows is shown in diagrammatic terms in Figure 2:

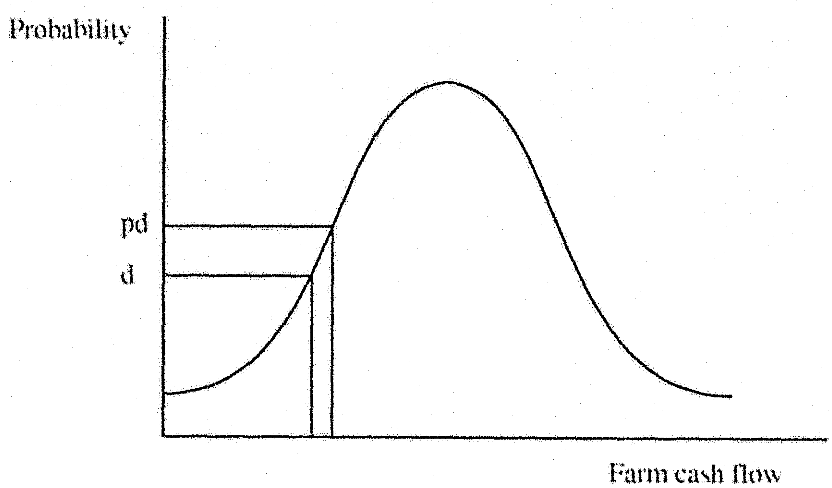

FIGURE 2 The distribution of farm cash flow after debt servicing

The implications for full loan repayment performance, partial default and full default by the farm model described in equations (20) to (30) on the bank returns and the change in the bank asset base as described in equations (14) to (17) are illustrated in the Appendix.

Linking probability of default to portfolio credit risk

Portfolio classification

The bank is assumed to conduct risk classification of its portfolio of rural credit accounts according to three key dimensions: the probability of full default, the probability of partial default and the expected capital loss. In practice, the precise effect of these variables on individual credit accounts is difficult to measure (Juttner 1986). However, financial institutions may classify rural customers for risk on the basis of similar characteristics using a credit scoring model (Barry and Ellinger 1989). In addition, by selecting a limited number of risk classes with each class reflecting a minimum and maximum range for each of these three factors, a bank may arrive at reasonably accurate predictors of the portfolio credit risk associated with each particular loan asset class.

Using portfolio theory, the bank asset base may be aggregated by summing the average size of liabilities of farm customers in each particular asset class, c. Thus, in time period t the bank's total asset base may be defined as:

$$(31) \quad BAG_t = \sum_{j=1}^{m} X_{c,t} . \overline{FL}_{c,t}$$

where BAG_t = gross bank asset balances in time period t
$X_{c,t}$ = number of credit accounts in asset class c in time period t and
$\overline{FL}_{c,t}$ = average level of farm liabilities in asset class c in time period t.

The bank is also assumed to classify its rural loan portfolio according to three further dimensions in order to standardise its credit accounts for different sources of risk. These dimensions are financial product type, agricultural region and farm enterprise mix. First, a bank may offer both fixed and variable interest rate loan products. The nature of farm financial risk will differ given the utilisation of either product other things held equal. Second, farm production risk may vary across agricultural regions due to climatic variations (such as the incidence of drought). Third, farm market risk will vary according to farm enterprise mix given that specialist farms face inherently higher market risk from commodity price fluctuations than farms with a diversified production base.

Given the farm model described earlier in the paper, the remaining sources of credit risk on a risk rated portfolio standardised for the three dimensions described above will stem from differing debt to equity ratios and the management ability of farmers across different risk classes.

For the purpose of the simulation model, it is proposed to provide a farm model with liabilities equal to the average level in the particular asset class. Each farm model will exhibit the average business and financial risk characteristics associated with a particular loan asset class.

Accounting for credit account flows

The above model assumes that the bank holds an existing portfolio of credit accounts. In any particular year, a bank may gain new credit accounts through its loan approval process or lose credit accounts as a result of term loans maturing and through write-offs. Equation (32) specifies ending period balances of the credit accounts in a particular class of assets.

(32) $X_{c,t} = X_{c,t-1} + NX_{c,t} - MX_{c,t} - \pi_{c,t} \cdot X_{c,t-1}$

Rearranging equation (32) and collecting for like terms gives:

(32') $X_{c,t} = (1 - \pi_{c,t}) \cdot X_{c,t-1} + NX_{c,t} - MX_{c,t}$

where $X_{c,t}$ = number of credit accounts in asset class c at the end of time period t
$NX_{c,t}$ = number of new accounts in asset class c approved during time period t
$MX_{c,t}$ = number of accounts maturing in risk class c during time period t and
$\pi_{c,t}$ = proportion of credit accounts written off in asset class c during time period t.

Given that the bank's risk classification system accurately groups credit accounts with similar rates of probability of default into a particular asset class, the probability of full default derived from each farm model is equivalent to the proportion of credit accounts written off during time period t.

This may be modelled as:

(33) $\pi_{c,t} = 0$ if $WO_{c,t} = 0$ and

(33') $\pi_{c,t} = d_{c,t}$ if $WO_{c,t} > 0$

where $d_{c,t}$ = probability of full default for asset class c in time period t *and* $0 \leq d_{c,t} \leq 1$.

The flow of new accounts ($NX_{c,t}$) may be modelled in the form of a loan offer function following Juttner (1986) such that:

(34) $NX_{c,t} = f(r'_c, Z_c)$

where r'_c = risk adjusted interest rate on loan asset class c and
Z_c = vector of exogenous factors determining loan demand for the loan asset c from the bank.

The number of credit accounts maturing, $MX_{c,t}$, will simply be a function of the number of credit accounts surviving the average term of loans in an asset class during time t:

(35) $MX_{c,t} = f(X_{c,at})$

where $X_{c,at}$ = number of credit accounts surviving the average term of loan (at) in asset class c.

For simplicity, the simulation model could set $NX_{c,t} = MX_{c,t}$, such that the flow of new loan accounts is simply equal to the flow of loan accounts maturing. This assumption would enable the simulation to proceed given any difficulties in developing a set of behavioural equations as specified in equation (34).

Expected bank portfolio returns

Given equations (14) to (35) and by specifying the initial conditions on the bank holdings of assets, $X_{c,t0}$, bank returns in time period t will equal the weighted average of returns achieved on m asset classes:

(36) $$R_{pt} = \sum_{i=1}^{m} X'_{c,t} . \overline{BR}_{c,t}$$

where R_{pt} = bank returns on portfolio for time period t and
$\overline{BR}_{c,t}$ = average bank returns on assets in class c for time period t.

By inserting the expected levels of commodity prices, yields and interest rates to derive expected levels of farm income after debt servicing for each respective asset class, expected loan asset portfolio returns in each time period may be calculated by using equation (36')

(36') $$E(R_{pt}) = \sum_{j=1}^{m} X'_{c,t} . E(\overline{BR}_{c,t})$$

and after discounting for the time value of money

(37) $$E(R_{pt}) = \sum_{t=1}^{n} \left\{ \sum_{j=1}^{m} \frac{X'_{c,t} E(\overline{BR}_{c,t})}{(1+k_t)^t} - X_{c,t0} . \overline{FL}_{c,t0} \right\}$$

where k_t = discount rate for time period t.

Variance of bank portfolio returns

As indicated in Section 2, the variance of (a bank's) portfolio returns may be differentiated into two separate components: unsystematic risk and systematic risk. Using a two stage process, these sources of portfolio risk may be distinguished from each other. First, the combined distribution for bank returns from a portfolio of loan assets is generated by simulating each loan asset's payoff function by initially assuming zero covariances of bank returns across different asset classes and summing these into a portfolio value. If this process is repeated many times, an empirical distribution for the portfolio (assuming no systematic risk) may be constructed as illustrated in Figure 3.

In order to include the impacts of systematic risk, a covariance matrix must be constructed for bank returns across different loan asset classes. The source of covariability of returns on a bank's rural loan portfolio will stem from the covariability of commodity prices, yields and interest rates between loan asset class i and loan asset class j. Clearly, in many cases, these covariance measures will indicate a considerable element of positive covariability of bank returns on different asset classes over time.

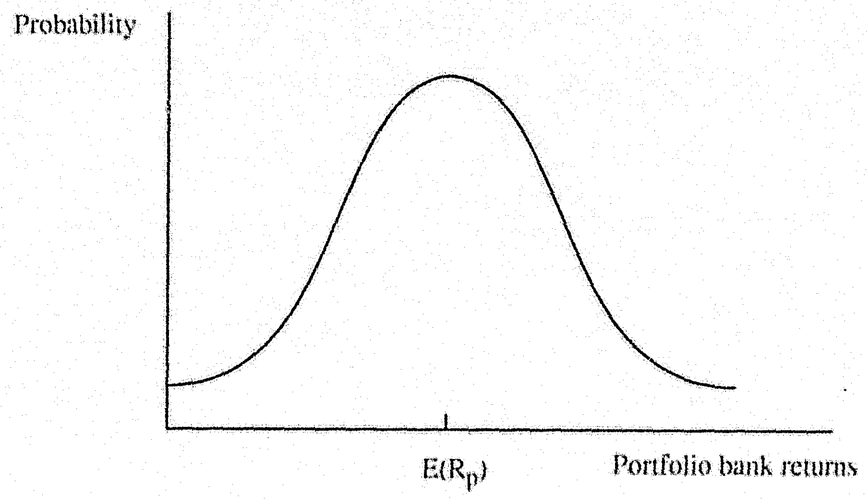

FIGURE 3 —The distribution of the net present value of bank returns

By using the variance-covariance matrix for bank returns by asset class, a second stage for generating the distribution of a bank's portfolio of rural loans may be established. The loan payoff functions are then simulated and summed into a portfolio value. Again, if this process is repeated many times, a distribution for the portfolio value may be constructed in which the portfolio variance includes both the unsystematic risk and the systematic risk components. The level of systematic risk can be estimated by subtracting the portfolio variance generated in the simulation from the portfolio variance estimated in the first stage. This result holds since the portfolio variance is simply the addition of the unsystematic risk and systematic risk measures as illustrated in equation (6).

The efficient frontier for a bank's rural loan portfolio may be derived from the quadratic programming model described in equations (8) to (11). By constraining the quadratic program to different levels of portfolio returns, a set of optimal portfolios that minimise the portfolio variance at each given level of portfolio return may be determined.

Some implications for institutional credit risk management

Credit risk management from a portfolio perspective may be modelled as involving two key objectives: first, to ensure that the bank is operating on, or as close as is feasible to, its efficient frontier, and second, to optimise the particular mix of the expected return on its portfolio and the volatility of bank returns subject to the risk preference function of the bank. There are three key strategies that a bank may pursue in order to achieve these objectives. These include a loan risk pricing model, the setting of portfolio exposure limits and the determination of appropriate bank capitalisation levels (Davis and Harper 1991, Wymann 1991).

A bank may manage expected credit risk by using an efficient loan pricing model in a similar fashion to a self insurance policy. For example, the promised rate of interest on the bank's lowest risk class would determine its prime rate of interest. A risk premium is added to the prime rate of interest for greater levels of expected credit risk (Juttner 1986, Sinkey 1986, p. 396). The key elements of expected risk that may be included in a loan pricing model are default risk, capital loss risk, portfolio risk and term maturity risk (Sinkey 1991, p. 400). The extent to which a bank penalises each of these factors through risk pricing depends on the risk preference function and the particular set of portfolio management strategies chosen by the bank.

The determination of the size of risk premiums for expected credit risk is by no means clear cut given the coincidence of financial risk for customers and default risk exposure for the bank. A trade-off exists between achieving greater returns on higher risk customers through a risk premium and the financial risk faced by these borrowers (Sinkey 1986, p. 400). This is due to the associated rise in the volatility of borrower incomes resulting from the imposition of a higher interest rate structure on these borrowers. The higher the risk premium, the greater the degree of default of a borrower other things being held equal. This trade-off would equally apply to a bank fees charging policy. For example, on term loan facilities, an up-front establishment fee is generally charged at a time during which a farm's debt-equity ratio is generally at its highest level thereby increasing default risk. Thus, a similar set of optimal fees charging policy rules could also be developed which account for the trade-off between promised bank returns and the expected credit risk.

In portfolio theory, the application of the mean-variance rules to loan asset selection from a portfolio perspective suggests that if a bank does not price for expected credit risk, then loan assets with the lowest credit risk will earn the highest expected return to the bank. Such a pricing policy is essentially risk preferring behaviour as the efficient frontier drawn in $E(R_p)$ and σ_p^2 space (as illustrated in Figure 1) is negatively sloped. In addition, the application of the mean-variance rules for loan asset selection as described in equation (7) implies that a risk averse bank not using risk pricing would only select loan proposals with the lowest default risk in a loanable funds market.

A bank may adopt an expected risk pricing policy on a risk neutral basis. Under this risk preference regime, the expected returns across all loan asset classes would be equalised regardless of the level of unexpected credit risk. A risk premium would be determined for each risk class on the basis of their respective expected capital losses expressed in percentage points. A key result of this policy is that the unsystematic risk component of loan asset portfolio is effectively forced to zero. This occurs because any variability in bank returns on a loan asset class that behave independently of those on other loan assets is effectively nullified. Any remaining volatility in bank returns will solely be due to systematic risk exposure.

Under an expected risk averse pricing policy, much higher expected returns would be required on credit accounts associated with greater levels of credit risk than compared to an expected risk neutral pricing policy. However, a utility function of the nature of risk aversion by the bank would need to be specified to gauge the size of the risk premium required. The differentiation of the volatility of bank returns into its respective unsystematic risk and systematic risk components under risk aversion is also more complex.

There are a range of options open to bank decision makers to manage systematic risk exposure. The primary portfolio based strategies are three fold: pricing, selective credit rationing and bank capitalisation. First, Sinkey (1986, p. 399) proposes that a specialist lender, for example a rural bank, may place an additional risk premium into its interest rate structure on the basis on their Beta coefficient. In this case, a loan asset that generates returns that are negatively correlated to the returns on the portfolio is worth more to a specialist lender than a loan asset that adds proportionately more to the volatility of bank portfolio returns. Second, a bank may effectively practise credit rationing through the centralised setting of exposure limits. Under this strategy, a bank actively diversifies into loan assets associated with negative or low levels of covariability of returns and places exposure limits on loan assets with high covariability of returns to the returns on its existing portfolio. Finally, a bank may augment its own financial structure to manage its systematic risk exposure (Davis 1990, Wymann 1991). Under this strategy, banks direct sufficient amounts of capital towards each risk class based on their relative volatility of returns.

Concluding remarks

This paper describes a stochastic simulation model of a bank's portfolio of rural credit accounts. The model is sufficiently general to incorporate various alternative lending policy rules regarding credit limits imposed on farm borrowers. It also explicitly includes treatment of partial default exposures.

The model will enable analysis of the trade-off between bank returns and the expected risk given the use of risk premiums in interest rate structures and the use of various types of fees charging policies. In addition, bank returns and the credit risk profile associated with a range of lending policies may be established using the model. With further development, the model should provide a framework for deriving optimal lending and pricing policies to the farm sector which reflect any given risk preference function held by decision makers in banks.

References

Barry, P.J. and Ellinger, P.N. (1989), 'Credit scoring, loan pricing and farm business performance', *Western Journal of Agricultural Economics* 14(1), 45-55.

Campbell, J.K. (Chairman), Coates, A.W., Halkerston, K.W., McCrossin, R.G., Mallyan, J.S. and Argy, F. (1981), *Australian Financial System: Final Report of the Committee of Inquiry*, AGPS, Canberra.

Carmichael, J. and Davis, P. (1991), 'The relevance of portfolio theory to risk management in financial institutions', (in Davis, K. and Harper, I. (ed.)), *Risk Management in Financial Institutions*, Allen and Unwin, Sydney.

Davis, K. T. (1990), 'Bank pricing and risk-adjusted capital requirements', *Australian Journal of Management* 15(2), 243-59.

Davis, K. and Harper, I. (1991), 'Risk management in financial institutions: an overview', (in Davis, K. and Harper, I. (ed.)), *Risk Management in Financial Institutions*, Allen and Unwin, Sydney.

Gabriel, S. and Baker, C. B. (1980), 'Concepts of business and financial risk', *American Journal of Agricultural Economics* 62(3), 560-4.

Harrington, D.R. (1987), *Modern Portfolio Theory, The Capital Asset Pricing Model and Arbitrage Theory - A User's Guide*, Second Edition, Prentice Hall, New Jersey.

Juttner, J. (1986), *The risk structure of interest rates*, Working Paper 8604-A, Centre for Studies in Money, Banking and Finance, Macquarie University, Sydney, November.

Levy, H. and Sarnat, M. (1990), *Capital Investment and Financial Decisions*, Fourth Edition, Prentice Hall, London.

Markowitz, H.M. (1959), *Portfolio Selection: Efficient Diversification of Investments*, Wiley, New York.

Markowitz, H.M. (1992), *Portfolio Selection - Efficient Diversification of Investments*, Blackwell, Oxford, Allen and Unwin.

Martin, V.E., (Chairman), Beetham, R.M., Cleary, D.J. and Hancock, K.J. (1983), *Australian Financial System: A Report of the Review Group*, AGPS, Canberra.

Piggott, R. (1990), 'Agricultural marketing', (in D.B Williams (ed.)), *Agriculture in the Australian Economy*, 3rd Edition, Sydney University Press.

Sinkey, J.F. (1986), *Commercial Bank Management In the Financial Services Industry*, MacMillan, New York.

Webb, B G. (1982), 'Borrower risk under alternative mortgage instruments', *Journal of Finance* 37, 169-83.

Appendix

The implications for performing, partial default (equation (18)) and full default (equation (19)) by the farm model (equations (20) to (30)) on the bank returns and the change in the bank asset base (equations (14) to (17)) are shown below. In this model, the bank's cost of funds and operating costs, C_t, are assumed to be given. These costs are determined by a variety of factors including fund raising strategies and bank operating structures.

A.1 Performing

In the case where a rural credit account is servicing fully its debt commitments, the bank earns interest and fee income. The farm makes repayments of an amount of principal, P_t, on its term loan facility. If in the previous year, the bank overdraft balance was positive or zero and the farm has sufficient liquidity to meet its personal and business commitments, the farm consumes this excess income and does not draw on its credit reserve. In the circumstance that the bank overdraft balance was negative in the previous year, the farm makes a repayment, RP_t, on this facility of an amount, $BB_t - BB_{t-1}$. Thus, depending on the level of farm income, the following profit and asset changes to the bank will occur:

$$BR_t = II_t + BFC_t - C_t \tag{A.1}$$

$$BAG_t - BAG_{t-1} = IC_t - RP_t + LD_t \tag{A.2}$$

where
$$\begin{aligned} II_t &= ro_t.BB_t + rt_t.LP_{t-1} \\ IC_t &= ro_t.BB_t + rt_t.LP_{t-1} \\ LD_t &= (BB_t - BB_{t-1}) \text{ and/or} \\ RP_t &= (BB_t - BB_{t-1}) + P_t. \end{aligned}$$

A.2 Partial default

When a farm enters into partial default, bank returns and the change in the bank asset base depend on whether or not the bank is required to make a provision for a bad and doubtful debt. If a loan is fully secured and thus no capital losses are expected then no provision for a bad and doubtful debt is required. However, if capital losses are expected then a provision must be made.

Performing—partial default but fully secured

In this circumstance, the bank earns no interest on the term loan account. Bank returns are limited to any interest received on the overdraft account and any fee income from both the overdraft and term loan facilities. Since no capital losses are expected, then both the bank overdraft balance and the term loan principal outstanding can be capitalised for balance sheet reporting purposes. No principal is repaid on the term loan facility, and the level of loan drawings increases as the farm increases its bank overdraft balance.

$$\text{(A.3)} \quad BR_t = II_t + BFC_t - C_t$$

$$\text{(A.4)} \quad BAG_t - BAG_{t-1} = IC_t - RP_t$$

where $II_t = ro_t.BB_t$

$IC_t = ro_t.BB_t + rt_t.LP_{t-1}$ and

$LD_t = (BB_t - BB_{t-1})$.

Performing —partial default but not fully secured

If there is potential for a farm to enter full default status and the loan is not fully secured, the bank is required to also make a reservation for the full extent of the possible loss by increasing its specific provisions for bad and doubtful debts, SPN.

The expected size of the capital loss to the bank will equal to the salvage of the farm assets less any debt outstanding at time period t.. Upon sale of the farm assets, the salvage value of the farm assets is defined to be equal to $s.FA$ where s is the proportion of the value of farm assets realised at its sale. The amount of farm debt outstanding at this juncture will equal to the term loan principal outstanding, LP_{t-1}, plus the amount of its credit reserve on the overdraft, BB^*_t. Thus the value of expected capital losses will equal to $s.FA - LP_{t-1} - BB^*_t$ if farm liabilities outstanding at time t are expected to exceed $s.FA$. The farm may also increase its liquidity by drawing on its overdraft facility. The impact on bank returns will equal:

$$\text{(A.5)} \quad BR_t = II_t + BFC_t - PD_t - C_t$$

where $II_t = ro_t.BB_t$ and

$PD_t = s.FA - LP_{t-1} - BB^*_t = SPN_t$.

If a provision for bad and doubtful debt has been made, the bank can not compound any interest outstanding. The farm may draw further on its overdraft facility and thus loan drawings, LD, may rise. Thus the change in bank assets is measured as:

$$\text{(A.6)} \quad BAG_t - BAG_{t-1} = LD_t - SPN_t$$

where $LD_t = (BB_t - BB_{t-1})$ and

$SPN_t = s.FA - LP_{t-1} - BB^*_t$.

Partial default—performing if was fully secured

In this case, the farm resumes all debt servicing commitments and the bank earns all interest and fee income agreed under the original contract. The repayment of P_t is met. However, depending on the level of farm income, a repayment or an additional loan drawing on the bank overdraft facility could occur. Bank returns and the change in bank assets are:

(A.7) $BR_t \quad = \quad II_t + BFC_t - C_t$

(A.8) $BAG_t - BAG_{t-1} \quad = \quad IC_t - RP_t + LD_t$

where $II_t = ro_t.BB_t + rt_t.LP_{t-1}$
$IC_t = ro_t.BB_t + rt_t.LP_{t-1}$ and
$RP_t = P_t + (BB_t - BB_{t-1})$ or
$LD_t = (BB_t - BB_{t-1})$.

Partial default—performing if was not fully secured

In addition to earning interest and fees, the bank recovers the specific provision made in the previous year by decreasing its specific provisions for bad and doubtful debts.

(A.9) $BR_t \quad = \quad II_t + BFC_t + RSP_t - C_t$

where $RSP_t = s.FA - LP_{t-1} - BB^*_{t}$.

With respect to the change in bank assets, interest is compounded on both the overdraft and term loan facilities, a repayment occurs on term loan principal. Again, depending on the level of farm income, a repayment or an additional loan drawing could occur on the bank overdraft balance.

(A.10) $BAG_t - BAG_{t-1} \quad = \quad IC_t + RSP_t - RP_t + LD_t$

where $IC_t = ro_t.BB_t + rt_t.LP_{t-1}$
$RP_t = P_t + (BB_t - BB_{t-1})$ or
$LD_t = (BB_t - BB_{t-1})$ and
$RSP_t = s.FA - LP_{t-1} - BB^*_{t}$.

If bank credit policy is to offer forgiveness on interest and principal repayments to farms for only one year, and the farm does not meet its amortised term loan commitments in the second year in a succession, the bank realises on its security and the farm enters into full default.

Clearly the model could encompass a range of different credit policy options regarding treatment of customers in partial default.

A.3 Full default

In the circumstance of full default, the bank receives no return and may suffer capital losses. A farm enters a full default position when its farm liabilities, FL, exceed a specified proportion, $\cap$, of the value of farm assets, FA. This condition may be expressed as:

(A.11) Full default if $FL_t \geq \cap.FA$

The impact on bank returns for an account in full default that is well secured, in other words if $s.FA$ exceeds FL_t, is indifferent to whether a credit account enters full default from a performing or partial default status. No returns are earned by the bank in either case. In practice, unpaid interest may well be recouped in the event that $s.FA > FL_t$.

(A.12) $$BR_t = 0$$

However, bank returns in either case are affected if $s.FA$ exceeds FL_t upon the credit account entering full default

Performing—full default but fully secured

If a farm enters into full default and if $s.FA$ exceeds FL_t, then clearly the credit account is well secured and the bank recovers all outstanding overdraft balance and term loan principal. In which case, the following value would be imputed into the balance sheet:

(A.13) $$BAG_t - BAG_{t-1} = -RP_t = FL_t \qquad \text{if } s.FA \geq FL_t$$

Performing—full default but not fully secured

If the value of $s.FA$ is less than FL_t, in other words the credit account is not well secured, the bank must write-off the difference between these two items. However, the maximum possible level of write-off is limited only to the value of FL_t. In addition, the treatment of write-offs on a bank balance sheet depends on whether the farm account has entered full default from a performing status in the previous year or partial default status. In either case, the impact on the bank balance sheet is the same. However, the impact on bank returns depends on whether the account has been previously provided for as a bad and doubtful debt.

(A.14) $$BAG_t - BAG_{t-1} = -WNSP_t$$

$$= -s.FA + FL_t$$

where $s.FA - FL_t < WNSP_t \leq FL_t$

$$\text{(A.15)} \qquad BR_t = -WNSP_t$$

$$= -s.FA + FL_t$$

where $s.FA - FL_t < WNSP_t \leq FL_t$

Partial default – full default but not fully secured

$$\text{(A.16)} \qquad BAG_t - BAG_{t-1} = -WSP_t$$

$$= -s.FA + FL_t$$

where $s.FA - FL_t < WSP_t \leq FL_t$

$$\text{(A.17)} \qquad BR_t = +WSP_t$$

$$= s.FA - FL_t$$

where $s.FA - FL_t < WSP_t \leq FL_t$

The term WSP_t is an additive term since the account has been specifically provided for in the profit and loss statement in the previous year as a bad and doubtful debt.