CRITERIA FOR CRITIQUING SOCIAL SCIENCE COMPUTER MODELS WITH APPLICATION TO BARR-GALE MODEL FOR CONSUMER PRICE INDEX OF FOOD

By Howard Morland, Narasimhan P. Kannan, and Dennis L. Meadows*

The authors propose six questions relevant to the evaluation of all social system models, and apply them to the Barr-Gale model for forecasting the Consumer Price Index for Food. While analysts invest much effort in developing new economic models, relatively little attention is paid to critiquing existing models. For models to become scientifically acceptable and widely used tools for policy design in the social sciences, they must be accompanied by complete documentation and data. Then independent investigators can confirm published results and determine a model's sensitivity to changes in assumptions. Greater emphasis on critical analysis and standard procedures for evaluation will help potential users evaluate models for accuracy and suitability for their purposes.

Keywords: Food prices, farm prices, forecasts, model testing, model critique.

INTRODUCTION

Much attention has been given to the task of formulating social system models that forecast prices. Much less effort has been devoted to analyzing such models and putting them to practical use. The lack of interest in critiquing social system models is illustrated by the fact that numerous econometric models are published in journals such as *Agricultural Economics Research*, yet no critical evaluations of such models by independent researchers are published. Decisionmakers who could profitably employ accurate forecasting models often lack the statistical skills necessary to evaluate them, to choose the most useful model from among those available, and to interpret the outputs with confidence. In the absence of any generally accepted system for evaluating social system models, good and bad models alike are frequently ignored.

In the physical sciences, models are expected to pass tests of independent verification and critical analysis. Premiums are placed on simplicity, accuracy, and usefulness. If the art of social system modeling is to advance toward the level of its physical science counterpart, ways must be developed for independent researchers to verify conclusions and test the predictive accuracy of the models of other social scientists.

There is no one procedure that will completely analyze all models, but the questions proposed below cover the areas of greatest concern in most social science models. The critical process must be flexible enough to accommodate a wide variety of subjects and modeling techniques, and yet reveal any flaws of either application or method.

- Is the model's output unambiguous? Does it consist of variables that can be identified and measured in the real system?
- How accurate is the model? Does it closely reproduce historical data; were any computational errors involved in its construction?
- Can a potential user understand the way the model works and hence evaluate its structure in relation to reality?
- Is documentation complete enough so that a potential user can independently confirm the published results and test their sensitivity to reasonable changes in the model's assumptions?
- Is the theoretical basis of the model sound?
- Can a simpler model with equivalent or better performance be constructed?

We applied these questions to an econometric model constructed to predict the Consumer Price Index for Food on a quarterly basis. This model was developed by T. W. Barr and H. F. Gale and published in *Agricultural Economics Research* in January 1973 (*1*, pp. 1-14).[1] Their model was chosen because the description was unusually complete, suggesting that independent analysis should be feasible based only on information in the article; the goals of the model are specific, so that interpretation of its output is unambiguous; and recent drastic increases in food prices make the model of current interest.

ANALYSIS OF THE MODEL

Description

The model consists of six equations. Five are simultaneous linear equations with five unknowns which can be solved to yield five independent formulae, one for each unknown. The sixth equation depends on the other

*Howard Morland is a graduate student working toward a masters degree; Narasimhan P. Kannan is a Ph.D. candidate; and Dennis L. Meadows is an associate professor of engineering and business. All three are associated with the Systems Dynamics Group, Thayer School of Engineering, Dartmouth College, Hanover, New Hampshire.

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five. The five equations are published in two forms:

- Unsolved, simultaneous equations, called the structural equations, with coefficients obtained through two-stage least squares regression analyses of the base period data;
- Solved, independent equations, called the reduced-form equations, with coefficients produced by inversion and multiplication of the coefficient matrices of the structural equations.

All the equations appear in the appendix.

Users of the model are advised by Barr and Gale to perform calculations with the reduced-form equations as the two forms are mathematically equivalent and the reduced-form equations do not require the use of matrix algebra nor access to a high-speed computer.

The model requires quarterly inputs of seven variables, six dummy variables, numerical values for five model parameters in the previous quarter, and a time trend variable. With this information, the model produces values for six output variables, five of which serve as inputs for the next quarter's calculations. For the first quarter's calculations, the model must have historical values of these five variables.

The seven variables of input data are forecasts obtained in the form of expert opinions. They include forecasts of the wage rate of workers in the food marketing industry and of various food prices. Output forecasts are not generated beyond quarters for which the model receives input forecasts. Therefore the modeling process is simply a weighting scheme by which predictions of seven variables are converted into predictions of more useful quantities.

Documentation

Input Information. Six of the inputs are averages of agricultural prices—prices received by farmers for meat, dairy products, poultry, oil crops, fruits, and vegetables. Historical values of these variables are published monthly in *Agricultural Prices* by USDA's Crop Reporting Board. Forecasts are available from commodity specialists. The seventh input is an estimate of wages paid in the food marketing industry, historical values of which appear quarterly in the ERS publication *Marketing and Transportation Situation*. Wage forecasts are, presumably, available from the U.S. Department of Labor, although Barr and Gale do not suggest where the model user may obtain such forecasts.

Output Information. The two primary outputs, the Consumer Price Index for Food at Home and the Consumer Price Index for All Food, are published monthly by the Labor Department's Bureau of Labor Statistics in *Monthly Labor Review*. The other four outputs—the farm values and the farm-retail price spreads for both livestock and crop-food products—are components of USDA's market basket price, published in *Agricultural Outlook*. Lagged values of five of the six outputs (all but the farm-retail price spread for livestock), re-enter the model as inputs; consequently, historical values of the five quantities must be supplied to the model for its first iteration.

We were not always able to tell from the text exactly which values should be extracted from the three publications and entered into the model. Moreover, all inputs, except the dummy and time-trend variables, take the form of indices (1967=100). As many of the inputs are published in current dollars only, a readily available compilation of the index values used in the model would save users and evaluators time and effort. Where such information seems too bulky to publish, it should appear in a separate appendix or user's manual, which could be available from the authors on request. Despite the data gaps, the Barr-Gale documentation is unusually complete: all the equations were published.

Errors

When we attempted to run the model, we discovered some miscellaneous errors.

The Sixth Equation. The equation for the Consumer Price Index for All Food, TCPIF, which did not belong to the set of five simultaneous equations, was, itself, presented as five equations. Unable to interpret the equations unambiguously, we contacted one of the authors by telephone and were instructed to ignore the first of the TCPIF equations, combine the other four, and solve for TCPIF. Such an interpretation of the equations presented was not apparent from the text.

Typographical and Editing Errors. In attempting to reproduce the results of the published prediction-interval test for the first quarter of 1972, we obtained a value of 128.2 for CPIF, the Consumer Price Index for Food at Home, compared with the Barr-Gale figure of 118.2. Predictions for subsequent quarters showed similar deviations from the Barr-Gale values. After eliminating the possibility of error in the input data, we resolved the set of structural equations on the assumption (which later proved correct) that the reduced-form equations contained a typographical error.

Our calculations reproduced the coefficients of the published reduced-form equations to six-digit accuracy. According to our results, the 19th term in the equation for CPIF should read $-0.03836\ FVC_{t-2}$ rather than $+0.03836\ FVC_{t-2}$ as the term appears in the published equation. After making the indicated sign change, we obtained a prediction of 118.7 for CPIF for the first quarter of 1972. Our figure is acceptably close to the 118.2 cited by Barr and Gale as their prediction for the same quarter.

A second typographical error in the reduced-form equation for FVL placed the wrong time subscript (2) on the variable FVC_{t-3}. This mistake does not significantly influence the model predictions.

From our new set of reduced-form equations, we recorded the following differences between the constant terms resulting from our calculations and the published terms:

Variable	Barr-Gale	Our recalculations
Farm value of livestock	21.35997	23.45924
Farm-retail price spread for livestock	6.64355	5.87997
Consumer Price Index for Food	29.25527	36.93179

These three discrepancies in the reduced-form equations are resolved if the constant term in the structural equation for CPIF is assumed to be -4.08410 rather than +3.40227, as published.

The text provides no explanation for the failure of the two forms of the model, structural and reduced, to be mathematically equivalent. The explanation, which Barr helped us locate through telephone contact, is that the structural equations contained the constant term error and the reduced-form equations contained the numerical sign and time subscript errors.

One way to avoid such errors is to print computer output directly rather than to retype equations and computer-generated results. The appendix contains a complete listing of the Barr-Gale equations in the form of output generated by a short BASIC computer program. [illegible]e three errors in the equations have been corrected to [illegible]ke the two forms of the model equivalent. Such programs can take computer-generated numbers and print them in a publishable format.

Forecasting Ability

The model was developed using data from 1960 through the third quarter of 1971. Its performance was demonstrated by Barr and Gale using data from the next three quarters. Sufficient time has now elapsed to permit a more complete test of the model's predictive powers.

Rather than attempt to evaluate performance with forecasted input estimates, we have chosen to supply historical values. Thus, we can determine the maximum predictive power of the model without any errors introduced by forecasts of the input data. This approach finesses a real difficulty: the model requires inputs based on subjective opinions.

A regression model's coefficients are most accurate near the mean of the data from which the model is constructed (*3*, pp. 21-24). The normalized independent variables for the base period range from values of around 90 in 1960 to around 120 in 1971. Given the usual assumption about variance of the coefficients, the expected deviation between forecast and actual prices will increase as values of the inputs move away from their base period averages of approximately 100. Since this model's forecasts begin at the high end of the data range and move beyond, the model operated outside its range of greatest accuracy in our tests.

The figure shows how the model would have forecast each of the six outputs during 1972-74 if perfect predictions had been available for each of the input variables, if the model were updated quarterly, and if projections were made for only one quarter in advance. The graphs supply visible evidence that the Barr-Gale model, like many others, has performed poorly since the end of 1973. The average deviation between forecast and actual values for 1974 ranges from 58.8 percent for the farm value of crops (FVC), to 2.8 percent for the farm-retail price spread for livestock (FRSL). The FVC error clearly reduces the utility of the model.

Reasons for Poor Performance

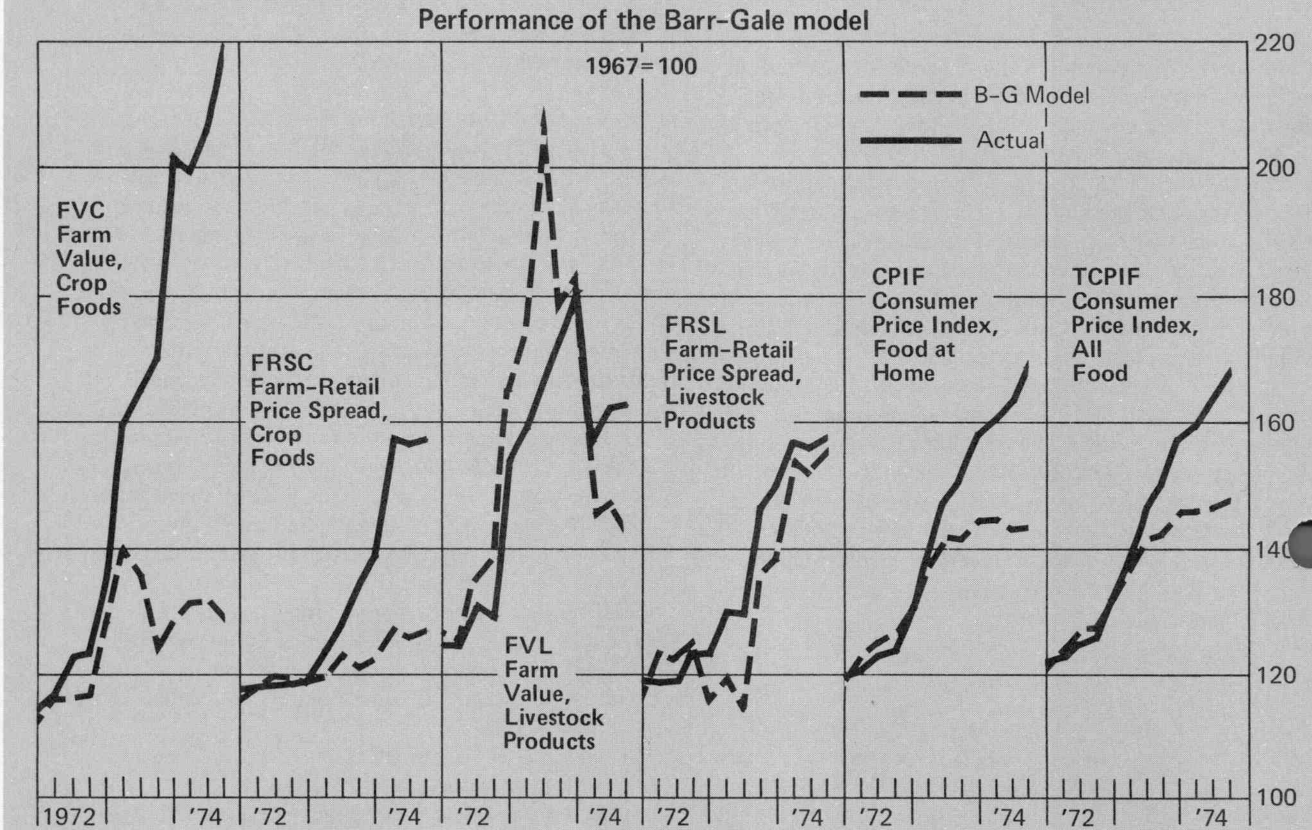
Sensitivity analysis reveals that the wage input and certain constants are by far the most important driving forces in the model and in large part they are responsible for the model's poor predictive performance. For instance, the equation for the farm value of crops, FVC, has a constant term of 74.5 index points (an index point is 1 percent of the total in 1967, the index base year). The large constant term, which has no real-world analog, holds the forecast values as many as 90 index points below the actual values by the end of the test period. Significantly, the equation for the farm-retail price spread for livestock, FRSL, has the smallest constant term, 6.6 index points, and it exhibits by far the best performance.

The six-digit coefficients and the large number of terms in each equation can obscure the strong sensitivity of the model to wages, and its lack of sensitivity to prices. The sensitivity of simple models, such as the Barr-Gale model, can be analyzed directly. By inspecting the coefficients, the user can obtain the necessary information about sensitivity of the equations to changes in independent variables.

In the table we list the 23 terms of the equation for CPIF, with their average values for the index base year of 1967. Prices received for meat, PRM, contributed 6.652 index points to the CPIF predictions in that year. Therefore, a 50-percent increase over the 1967 values of prices received for meat would increase the predicted Consumer Price Index for Food at Home by 3.326 index points, or about 3 percent. The equation is more sensitive to PRM than to any other price input.

On the other hand, the importance of wages in the CPIF equation is overwhelming. Half the value of CPIF, 49.428 index points, is attributable to the current and lagged values of WFMI, wages of food marketing workers. Since the constant term is almost 30 percent of the value of CPIF, 29.25527 index points, the remaining 20 terms of the equation, including all inputs of food prices and distributor markups, contribute just over 20 percent of the total prediction.

The negative signs on the coefficients for the time trend, the farm value, and the farm-retail price spread



NOTE: For each of the six equations in the model, forecasts are made for the 12 quarters of 1972–74 using actual values of all current and lagged input variables. Actual values of the output variables are shown for comparison.

Sensitivity analysis of reduced-form equation for CPIF

Coefficient	Input	Product	Term name
29.25527	1	29.25527	(constant)
+0.06652	100	6.652	Prices received for meat (t)
+0.02552	100	2.552	Prices received for dairy (t)
+0.01662	100	1.662	Prices received for poultry (t)
+0.01054	100	1.054	Prices received for oil (t)
+0.01262	100	1.262	Prices received for fruit (t)
+0.01947	100	2.947	Prices received for vegetables (t)
−0.09859	0.25	−0.02465	Dummy, first quarter (t)
+0.43540	0.25	0.1089	Dummy, second quarter (t)
+0.76243	0.25	0.1906	Dummy, third quarter (t)
−0.80680	0	0	Dummy, 1960-64 (t)
−0.54244	1	−0.54244	Dummy, 1967-68 (t)
+0.18579	100	18.579	Food marketing wages (t)
+0.65825	0.25	0.16456	Dummy, fourth quarter (t)
−0.09970	30.5	−3.04085	Time trend (t)
+0.07819	100	7.819	Farm value of livestock (t−1)
+0.06774	100	6.774	Farm value of livestock (t−2)
+0.02395	100	2.395	Farm value of crops (t−1)
−0.03836	100	−3.836	Farm value of crops (t−2)
−0.00347	100	−0.347	Farm value of crops (t−3)
−0.00291	100	−0.291	Farm-retail spread, crops (t−1)
−0.02930	100	−3.930	Farm-retail spread, crops (t−2)
+0.30849	100	30.849	Food marketing wages (t−1)
Total		100.2524	

Note: Because all data are fed into the model with the index 1967 equal to 100, the contribution of each data variable to the total is roughly equal to 100 times the coefficient, exactly so in 1967 when all data inputs are equal to 100. Quarterly dummy variables average 0.25 for each year. The time trend variable averages 30.5 for 1967; its value for each quarter equals the number of elapsed quarters since the first quarter of 1960. Dummy variables for the wheat subsidy and the wheat allotment programs have values of zero and 1.0, respectively, in 1967. Prices received are prices received by farmers.

apparently reflect relationships that were valid during the sixties but no longer hold today. A regression analysis on more recent data would certainly be expected to yield positive coefficients for such terms.

Because regression analysis captures the statistical properties of coincidental variation of quantities rather than their causal interrelationships, the terms of the model's equations may not, and in this case do not, reflect the real-world contributions of the input quantities to the totals. Recent dramatic increases in consumer prices for food have not been accompanied by similar increases in wages for food workers, despite the close correlation of the two quantities during the sixties. Consequently, the model could not have predicted recent events, and, as we have seen, it does not, even with perfect-input forecasts. The minor importance (and sometimes negative influence) of food producer prices and distributor markups in the CPIF equation are responsible for the model's poor performance since the recent increase in food prices began.

To answer whether there is a simpler model for CPIF which will perform adequately, we developed an exponential smoothing model which is considerably less complicated than regression analysis.[2] It followed the actual trends more closely than the Barr-Gale model, but it too had weaknesses. Since the last quarter of 1972 marks the end of one linear trend and the beginning of another, the smoothing model did not provide accurate four-quarter projections for 1973.[3]

CONCLUSIONS

Referring, then, to our original test questions, which we believe all models should be subjected to, we draw six conclusions.

The model predicted the CPIF acceptably well for its published test period and for the first three quarters

[2] For an explanation of exponential smoothing, see (*2*, pp. 128-135).

[3] Detailed documentation of this model is available from the System Dynamics Program Office, Box 8000, Dartmouth College, Hanover, N.H. 03755. Request "A Critique of the Barr-Gale Econometric Model for Forecasting the Consumer Price Index for Food." DSD #45, $1.30.

after its publication. But, as with many other CPIF models, the radical changes in the influences on the index in 1973 caused great errors in the model's forecasts thereafter. While it would be possible (though difficult) to revise the model to reflect these changes, any later similar changes would cause another breakdown. A quantity which is increasing steadily would not be expected to return to the range of its historical values. Yet only within the historical range can a least squares regression model be expected to have reasonable accuracy. While a base period of almost 12 years may seem desirable, in the Barr-Gale model such a long period actually serves to increase the difference between the mean value of the base period data and the values which are being forecast.

The model's output, being a published number, is measurable and unambiguous; hence, the model's performance is easily rated.

With the help of the sensitivity analysis published here, potential users should find the model comprehensible, and should be able to evaluate its strengths and limitations for their own purposes.

The Barr-Gale model is more completely presented and documented than most similar models in the literature, but improvements are still necessary. Because the model's inputs are indexed to 1967=100, it is easy to understand the equations, though developing indexed data is a nuisance.

We have demonstrated that a simpler model with comparable predictive ability is possible.

The underlying difficulty with both the Barr-Gale model and the exponential smoothing model is that they are mathematically derived projections based on statistical coincidence: when the nature of the marketplace changes radically, as it did in 1973, such models no longer reflect real-world behavior.

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(3) Draper, R. N. and H. Smith, *Applied Regression Analysis*. John Wiley and Sons, 1966.

APPENDIX

UNKNOWN TERMS OF THE STRUCTURAL EQUATIONS

EQUATION NUMBER

ONE (FVC)	TWO (FRSC)	THREE (FVL)	FOUR (FRSL)	FIVE (CPIF)
1.0000 FVC	0.1442 FVC	0.0000 FVC	0.0000 FVC	-0.1535 FVC
-0.4644 FRSC	1.0000 FRSC	0.0000 FRSC	0.0000 FRSC	-0.2758 FRSC
0.0000 FVL	0.0000 FVL	1.0000 FVL	0.3685 FVL	-0.2253 FVL
0.0000 FRSL	0.0000 FRSL	0.3966 FRSL	1.0000 FRSL	-0.3656 FRSL
0.0000 CPIF	0.0000 CPIF	-0.2335 CPIF	0.0000 CPIF	1.0000 CPIF

KNOWN TERMS OF THE STRUCTURAL EQUATIONS

EQUATION NUMBER

ONE (FVC)	TWO (FRSC)	THREE (FVL)	FOUR (FRSL)	FIVE (CPIF)[1]
49.79600	63.91080	17.16370	14.51470	-4.08410
0.00000 PRM	0.00000 PRM	0.61150 PRM	0.00000 PRM	0.00000 PRM
0.00000 PRD	0.00000 PRD	0.23460 PRD	0.00000 PRD	0.00000 PRD
0.00000 PRP	0.00000 PRP	0.15280 PRP	0.00000 PRP	0.00000 PRP
0.09640 PRO	0.00000 PRO	0.00000 PRO	0.00000 PRO	0.00000 PRO
0.11550 PRF	0.00000 PRF	0.00000 PRF	0.00000 PRF	0.00000 PRF
0.26960 PRV	0.00000 PRV	0.00000 PRV	0.00000 PRV	0.00000 PRV
0.00000 DFQ	0.00000 DFQ	-0.90640 DFQ	0.00000 DFQ	0.00000 DFQ
0.00000 DSQ	1.30530 DSQ	0.00000 DSQ	0.00000 DSQ	0.00000 DSQ
0.00000 DTQ	2.28570 DTQ	0.00000 DTQ	0.00000 DTQ	0.00000 DTQ
-7.38160 DWS	0.00000 DWS	0.00000 DWS	0.00000 DWS	0.00000 DWS
-4.96290 DWA	0.00000 DWA	0.00000 DWA	0.00000 DWA	0.00000 DWA
0.00000 WFMI	0.55700 WFMI	0.00000 WFMI	0.00000 WFMI	0.00000 WFMI
0.00000 D4Q	0.00000 D4Q	0.00000 D4Q	1.98420 D4Q	0.00000 D4Q
0.00000 T	0.00000 T	0.00000 T	-0.46900 T	0.05450 T
0.00000 FVL-1	0.00000 FVL-1	0.00000 FVL-1	0.23570 FVL-1	0.00000 FVL-1
0.00000 FVL-2	0.00000 FVL-2	0.00000 FVL-2	0.20420 FVL-2	0.00000 FVL-2
0.00000 FVC-1	0.07180 FVC-1	0.00000 FVC-1	0.00000 FVC-1	0.00000 FVC-1
0.00000 FVC-2	-0.11500 FVC-2	0.00000 FVC-2	0.00000 FVC-2	0.00000 FVC-2
0.00000 FVC-3	-0.01040 FVC-3	0.00000 FVC-3	0.00000 FVC-3	0.00000 FVC-3
-0.02660 FRSC-1	0.00000 FRSC-1	0.00000 FRSC-1	0.00000 FRSC-1	0.00000 FRSC-1
-0.35960 FRSC-2	0.00000 FRSC-2	0.00000 FRSC-2	0.00000 FRSC-2	0.00000 FRSC-2
0.00000 WFMI-1	0.00000 WFMI-1	0.00000 WFMI-1	0.92990 WFMI-1	0.00000 WFMI-1

Note: Each of the five simultaneous equations is listed in a separate column. Terms with dependent, or unknown, variables are grouped at the top of each column; constant terms and terms with independent, or known, variables are grouped below. Sign changes have been made in the top section, so that each group of unknown terms is equal to the corresponding group of known terms. Numbers following term names are time subscripts. Terms with no subscripts are current to the quarter being forecasted; others are lagged by the number of quarters indicated by the subscript.

[1] The constant term in the fifth equation has been changed to make the constants of the structural equations algebraically equivalent to those of the published reduced-form equations.

MATRIX ALGEBRA SOLUTION TO THE STRUCTURAL EQUATIONS

EQUATION NUMBER

ONE (FVC)	TWO (FRSC)	THREE (FVL)	FOUR (FRSL)	FIVE (CPIF)
74.48798*	53.16963	21.35997	6.64355	29.25527
0.00000 PRM	0.00000 PRM	0.73436 PRM	-0.27061 PRM	0.06652 PRM
0.00000 PRD	0.00000 PRD	0.28173 PRD	-0.10382 PRD	0.02552 PRD
0.00000 PRP	0.00000 PRP	0.18350 PRP	-0.06762 PRP	0.01662 PRP
0.09035 PRO	-0.01303 PRO	0.00288 PRO	-0.00106 PRO	0.01054 PRO
0.10825 PRF	-0.01561 PRF	0.00345 PRF	-0.00127 PRF	0.01262 PRF
0.25268 PRV	-0.03644 PRV	0.00806 PRV	-0.00297 PRV	0.02947 PRV
0.00000 DFQ	0.00000 DFQ	-1.08850 DFQ	0.40111 DFQ	-0.09859 DFQ
0.56814 DSQ	1.22337 DSQ	0.11907 DSQ	-0.04388 DSQ	0.43540 DSQ
0.99486 DTQ	2.14224 DTQ	0.20850 DTQ	-0.07683 DTQ	0.76243 DTQ
-6.91831 DWS	0.99762 DWS	-0.22063 DWS	0.08130 DWS	-0.80680 DWS
-4.65141 DWA	0.67073 DWA	-0.14834 DWA	0.05466 DWA	-0.54244 DWA
0.24244 WFMI	0.52204 WFMI	0.05081 WFMI	-0.01872 WFMI	0.18579 WFMI
0.00000 D4Q	0.00000 D4Q	-0.74162 D4Q	2.25749 D4Q	0.65825 D4Q
0.00000 T	0.00000 T	0.19058 T	-0.53923 T	-0.09970 T
0.00000 FVL-1	0.00000 FVL-1	-0.08810 FVL-1	0.26816 FVL-1	0.07819 FVL-1
0.00000 FVL-2	0.00000 FVL-2	-0.07632 FVL-2	0.23232 FVL-2	0.06774 FVL-2
0.03125 FVC-1	0.06729 FVC-1	0.00655 FVC-1	-0.00241 FVC-1	0.02395 FVC-1
-0.05005 FVC-2	-0.10778 FVC-2	-0.01049 FVC-2	0.00387 FVC-2	*-0.03836 FVC-2
-0.00453 FVC-3	-0.00975 FVC-3	*-0.00095 FVC-3	0.00035 FVC-3	-0.00347 FVC-3
-0.02493 FRSC-1	0.00359 FRSC-1	-0.00080 FRSC-1	0.00029 FRSC-1	-0.00291 FRSC-1
-0.33703 FRSC-2	0.04860 FRSC-2	-0.01075 FRSC-2	0.00396 FRSC-2	-0.03930 FRSC-2
0.00000 WFMI-1	0.00000 WFMI-1	-0.34756 WFMI-1	1.05798 WFMI-1	0.30849 WFMI-1

EQUATION NUMBER SIX (TCPIF)

TCPIF = 0.2913 + 0.9592 TCPIF-1 + 0.7804 (CPIF - 0.9592 CPIF-1) + 0.4047 (T - 0.9592 T-1)

*Note: These equations differ from the published reduced-form equations in the numerical sign of the 19th term (FVC-2) of the 5th equation and in the time subscript of the 20th term (FVC-3) of the 3rd equation.