Towards Ecological Reality in Economic Models

by

Nick Milham[1]
CSIRO Wildlife and Ecology
Canberra

Abstract

In this paper, the increasing importance being placed on improving the interface between
economic and ecological models is noted and commended. However, several serious
deficiencies in the current state of economic models of farm-level resource management still
exist. In particular, overlooking the ecological phenomenon of threshold effects and multiple
stable states is highlighted as a potential problem in models of this nature. Special forms of the
objective function and Hamiltonian function which account for ecological thresholds, are
proposed and developed. The intent, rather than to develop a new technique, is to suggest a
simple modification to the optimal control framework to accommodate these types of
ecological processes. Practical problems in the application of the model, including data
requirements, are noted.

[1]Current location: NSW Agriculture, AGCOST Unit, Parkes.

Introduction

In recent years, increasing efforts have been made to accommodate aspects of the natural environment and ecological relationships in theoretical and applied economic models. The importance placed on these developments is reflected in the publication of several new professional journals and the establishment of new research organisations that aim to promote research in the field of environmental and natural resource economics.[1]

One issue that has received considerable attention is that of land degradation in farming and pastoral lands. Attempts to model and explain the optimal utilisation of natural resources at the farm level have proliferated over the last decade (e.g., Barrett, 1989; Hertzler, 1990; McConnell, 1983; Milham 1992; Passmore and Brown, 1992; Saliba, 1985; Segarra and Taylor, 1987). However, these models have tended to overlook ecological relationships or account for them in only very superficial ways.

Without exception, these models all assume that the ecosystem in which the agricultural production activity is undertaken is either in or will approach a single, stable equilibrium. Reflecting this is the usually implicit assumption of a linear, reversible response in the status of environmental resources. That is, that if utilisation of the resource base is slowed below some critical level, or ceases entirely, then the system will linearly recover to what is was before.

The model developed in Milham (1992) implicitly assumed equal, linear responses in soil structure or fertility following investment in these productive aspects of the soil in any period. The sophisticated grazing management model described in Barrett (1989), which accounts for dynamic interactions between stocking rates and the carrying capacity of the range, also assumes that range resources will recover linearly in any period in which grazing pressure is reduced. These assumptions have strong potential to conflict with the ecological phenomena of threshold effects and multiple stable states.

Ecological Thresholds and Multiple Stable States

Only a very cursory review of ecology literature is required to find discussion of the concepts of stability and resilience in ecosystems. In this context, stability refers to the ability of the system to recover from small shocks and return to its previous state. Resilience refers to the ability of the system to recover from large shocks. That is, it is a measure of the size of the

---

[1]Examples include the Journals of Ecological Economics and Environmental and Resource Economics, and the London Environmental Economics Centre.

domain of attraction around the original state. Authors such as Buffington and Herbel (1965), Noy-Meir (1975) and Walker *et al.* (1981) have reported scientific evidence of the existence of at least two stable ecological states in semi-arid rangelands, while Holling (1973) provided a more general treatise on stability and resilience in ecological systems.

There is some debate over whether the observed ecological 'states' are in fact different. For example, Noy-Meir and Walker (1986) suggest that rather than an irreversible change having occurred, an ecosystem may just require a very long time to recover. This argument raises the question as to how long marked changes in an ecosystem have to persist for the new conditions to be regarded as a stable alternative state.

In a review of literature on woody shrub invasion of semi-arid grasslands, Walker *et al.* (1981) reported a general conclusion that even if the large herbivores were removed, the shrub-dominated state would persist over a period of at least 20 years. In an agricultural production context, 20 years is a very long planning horizon and ecological conditions that extend over a period of that magnitude can be justifiably regarded as an unique, stable state.

If the change in state involves a decline in productive potential, the issue of management-induced, discontinuous environmental change, with thresholds between alternative states (Friedel, 1991; Laycock, 1991), becomes an important aspect of resource management at the farm level. Why? Because, for a given technology, the rate of consumption of environmental resources, and the associated marginal user costs, could be expected to differ in each state. That is, the optimal trajectory of utilisation of the set of environmental resources and, hence, the path of dynamic profit, will change if an ecological threshold is crossed.

In this paper, a more general form of the optimal control model developed in Milham (1992) is presented and a means of incorporating the concept of ecological thresholds in this type of analytical framework is proposed.

A General Economic Model of Farm-Level Resource Use

In Milham (1992) the following dynamic economic model of soil use at the farm level is developed:[2]

$$(1) \quad \max J \ = \ \Pi_{pv} + R_{pv}$$

$$= \int_0^T e^{-\rho t} \{p_t.q_t - c_t.VI_t - P_{Pt}.PI_t - P_{St}.SI_t - P_{Ct}.CI_t\} dt +$$

$$e^{-\rho T} R(SD, SP, SS, CK) \quad ,$$

s.t.   constraints, control equations and boundary conditions.

In this expression, J represents the aggregate, present-value profit function derived from a single-commodity production function with a given farm technology. Profit comprises two components Π_{pv}, being the discounted stream of net income from utilising the soil resource over the planning horizon (T), and, R_{pv}, being the terminal resale value of the land.

In the expanded form, ρ is the farmer's time preference (discount) rate; p_t is the price received per unit of the commodity in time t; q_t is the vector of output of the commodity; c_t is an index of prices paid for the volume of farm inputs (VI_t); and P_{Pt}, P_{St} and P_{Ct} are the prices per unit of investment in soil fertility (PI), soil structure (SI) and soil conservation (CI), respectively.[3] SD, SP, SS and CK represent the stocks of the soil characteristics depth, fertility and structure, and the stock of soil conservation capital (equipment and infrastructure), respectively.

It is noted that it is not generally the case in the real world that a single production technique is selected and applied unchanged throughout the entire farm planning horizon. To accommodate this observation, a discrete choice version of expression (1), after Hertzler (1990) and Kim *et al.* (1989), is formulated. This model,

---

[2]The development of this model relied heavily on the work of McConnell (1983), Saliba (1985) and Segarra and Taylor (1987).

[3]Depth, fertility and structure are recognised as the three most important aspects of soil condition in terms of pasture and crop production (Milham 1992).

$$(2) \quad \max J^Y = \int_0^T e^{-\rho t} \sum_y \{p_t.q_{yt} - c_t.VI_{yt} - P_{Pt}.PI_{yt} -$$

$$P_{St}.SI_{yt} - P_{Ct}.CI_{yt}\}dt +$$

$$e^{-\rho T}\sum_y R(SD_{yt}, SP_{yt}, SS_{yt}, CK_{yt}) \quad , \quad y = 1, 2, ..., Y \quad ,$$

allows selection of any particular technology y from a set of available technologies Y. Maximising profit over time is then shown to require maximising the current value Hamiltonian:

$$(3) \quad H^Y = e^{-\rho t}\sum_y \{p_t.q_{yt} - c_t.VI_{yt} - P_{Pt} PI_{yt} - P_{St}.SI_{yt} - P_{Ct}.CI_{yt}\}$$

$$+ \sum_y \{- \beta_{t+1}SL_{yt} + \mu_{t+1}(PI_{yt} - PL_{yt}) + \nu_{t+1}(SI_{yt} - SC_{yt})$$

$$+ \phi_{t+1}(CI_{yt} - \alpha.CK_{yt}\} \quad , \quad y = 1, 2, ..., Y \quad ,$$

where β_{t+1}, μ_{t+1}, ν_{t+1} and ϕ_{t+1} are the discount d marginal user costs of soil depth, soil fertility, soil structure and the stock of conservation capital, respectively. That is, they represent the discounted values of one more unit of soil depth, productivity, structure and conservation capital in some future period beyond T. SI_{yt}, PL_{yt} and SC_{yt} represent decrements to the stocks of soil depth, soil productivity and soil structure, respectively. The coefficient α, is the rate of depreciation of the stock of conservation capital. The level of the stock of conservation capital and the stocks of the three environmental factors, soil depth (which is non-renewable in a farm management sense), fertility and structure (which are renewable), are the state variables in this optimisation problem.

From expression (3) it can be seen that each technology has a unique dynamic profitability associated with it, and the management problem is to determine which is optimal at each point in time, and then to choose it. The maximum value of H^Y is obtained by summing across the outcomes of the set or sequence of optimum techniques selected throughout the planning horizon.[4]

---

[4]Switching from production of one commodity to another without changing the production technology, such as changing from wheat to barley or from sheep to cattle, may also have this effect.

Now, while this model was developed in the context of the soil resource, there is no reason why other management-sensitive, environmental resources (such as vegetation composition, soil surface condition, proximity of the water table to the soil surface etc) could not also be included. All that would be required would be to insert the relevant variables and control relationships into the model in a similar fashion to the way soil depth etc were accommodated. Of course, this would have the effect of increasing the size and complexity of the model and would not necessarily provide any further enlightenment in a theoretical sense.

A simplified and hence more amenable, but equally complete, form of the model can be obtained by separating the potential gamut of environmental state variables into just two categories, renewable and exhaustible resources. Renewable resources will be distinguished by the fact that they will have an associated control variable representing growth (perhaps associated with investment) in the stock of that resource. When developing an applied model, the relationships for all exhaustible and renewable resources will have the general forms described below

Denoting renewable resources as RR and exhaustible resources as ER and making equivalent changes to subscripts, equations (2) and (3) can be re-expressed as

$$(4) \quad \max J^i \;=\; \int_0^T e^{-\rho t} \sum_y \{p_t \cdot q_{yt} - c_t \, VI_{yt} - P_{Rt} . RI_{yt} - P_{Ct} \, CI_{yt}\} \;+$$
$$e^{-\rho T} \sum_v R(ER_{yt}, RR_{yt}, CK_{yt}) \;,\quad y = 1, 2, \dots, Y \;,$$

where RI is investment in formation of the renewable resource(s), and

$$(5) \quad H^Y \;=\; e^{-\rho t} \sum_y \{p_t . q_{yt} - c_t . VI_{yt} - P_{Rt} . RI_{yt} - P_{Ct} . CI_{yt}\} \;+$$
$$\sum_y \{- \beta_{t+1} EL_{yt} + \mu_{t+1}(RI_{yt} - RL_{yt}) + \phi_{t+1}(CI_{yt} - \alpha . CK_{yt})\} \;,$$
$$y = 1, 2, \dots, Y \;,$$

where EL and RL represent consumption of the two categories of environmental resources.[5]

The optimising conditions for this model can be derived as:

---

[5] If the stock of conservation capital is not subject to depreciation, then it will have similar characteristics to a renewable environmental resource and could thus be included in that category.

(6) $\delta H/\delta EL = 0 = e^{-\rho t}p.\delta q/\delta EL - \beta_{t+1}$;

(7) $\delta H/\delta RL = 0 = e^{-\rho t}p.\delta q/\delta RL - \mu_{t+1}$;

(8) $\delta H/\delta VI = 0 = e^{-\rho t}p.\delta q/\delta VI - e^{-\rho t}c_t$;

(9) $\delta H/\delta RI = 0 = -e^{-\rho t}P_{Rt} + \mu_{t+1}$;

(10) $\delta H/\delta CI = 0 = -e^{-\rho t}P_{Ct} + \phi_{t+1}$;

and,

(11) $\delta H/\delta CK = 0 = e^{-\rho t}p\,\delta q/\delta CK - \alpha.\phi_{t+1}$

The terminal stock conditions are:

(12) $\delta R/\delta ER_{T+1} = \beta_{T+1}$,

(13) $\delta R/\delta RR_{T+1} = \mu_{T+1}$.

and,

(14) $\delta R/\delta CK_{T+1} = \phi_{T+1}$.

How can the above expressions provide an avenue for incorporation of ecological thresholds into the optimal control framework? The answer lies in defining a further parameter Z, of a similar form to Y, being the set of alternative ecological states that the system can be

forced into by crossing certain thresholds, and in recognising that the dynamic profit associated with a given production technology will be different in each state.[6]

Investment (CI) and depreciation (α) of the stock of conservation capital are not likely to depend on the ecological state (z). Nor is the rate of investment (growth) in the renewable environmental resource(s) (RI). However, for a given technology, it could be expected that the required volume of variable inputs (VI), the rate of consumption of both the renewable (RL) and non-renewable (EL) resources and their marginal user costs (β and μ respectively), will all vary with z. The required adjustments to the model are thus to simply insert additional summation signs and subscripts in expressions (4) and (5) such that summation is occurring across technology/state pairings. That is,

$$(15) \quad \mathcal{N} = \int_0^T e^{-\rho t} \sum_z \sum_y \{p_t.q_{yt} - c_t.VI_{yzt} - P_{Rt}.RI_{yt} - P_{Ct}.CI_{yt}\} +$$
$$e^{-\rho T} \sum_z \sum_y R(ER_{yzt}, RR_{yzt}, CK_{yt}) , \quad z = 1, 2, ..., Z \text{ and } y = 1, 2, ..., Y ,$$

and

$$(16) \quad H^p = e^{-\rho t} \sum_z \sum_y \{p_t q_{yt} - c_t VI_{yzt} - P_{Rt}.RI_{yt} - P_{Ct} CI_{yt}\} +$$
$$\sum_z \sum_y \{- \beta_{zt+1} EL_{yzt} + \mu_{zt+1}(RI_{yt} - RL_{yzt}) + \phi_{t+1}(CI_{yt} - \alpha.CK_{yt})\} ,$$
$$z = 1, 2, ..., Z \text{ and } y = 1, 2, ..., Y.$$

In practice, the primary difference between the z and y parameters will be that while the production technology is under the manager's control, the ecological state is not. It may however be possible to recognise that a threshold is being approached and take action to prevent this occurring. For a given commodity, the terminal ecological state (and sequence of any interim states) will be a function of the initial ecological state, the sequence of technologies selected, and the sequence of environmental events (e.g., rainfall, fire etc) experienced over the planning horizon. The terminal ecological state will also reflect the combination of the terminal states of the set of environmental resources (e.g., land condition, water quality, vegetation

---

[6] If there is no difference in the dynamic profit achieved with the same commodity/technology pairing across different ecological states then, for farm management purposes, the states are the same.

composition etc). That is, the ecological state could be described as a kind of higher level or *super-state* variable.

In any applied modelling exercise it would be necessary to incorporate expressions representing the ecological responses to each alternative production technology, in each possible ecological state, in the objective function and as constraints in the optimisation process. Furthermore, it could be expected that for a given technology, commodity output would also vary with changes in the ecological state. It is immediately apparent that the optimising conditions for the problem will proliferate very rapidly as the number of alternative technologies and ecological states increase.

An additional practical point is that the set of possible ecological states and the nature of the 'triggers' that will cause an ecological threshold to be crossed can be expected to be unique to individual ecosystems. There are also likely to be unique restrictions on the sequence of transition between alternative states. That is, it may be possible to move to one particular state from only one or a subset of the total number of other possible states. (See, for example, Westoby *et al.* 1989.) Once these critical ecological state-and-transition relationships are adequately captured in mathematical functions, which will be a non-trivial task, derivation of the Hamiltonian and the optimising conditions will be a mechanical process.

To illustrate the potential bias in analytical results if the phenomenon of ecological thresholds is ignored, a simple example is now posed and the associated optimising conditions are developed

Optimising Conditions for a Simple Example

Let us assume that there is only one technology, one commodity, two ecological states, and that the threshold between the two states is crossed at the end of the planning horizon, i.e., at time T. The two states will be denoted by z_1 and z_2.

Now, since the threshold is crossed at time T, the change in ecological state will only affect the second part of the right hand side of the profit and Hamiltonian functions. That is, z will equal only 1 in the first part of these functions (which represents the period up to time T) and only 2 in the second part (which represents the period from time $T+1$). Thus, dropping out the summation signs but leaving y to denote the production technology, the Hamiltonian for this example becomes

(17) $\quad H^y = \quad e^{-\rho t}\{p_t.q_{yt} - c_t.VI_{yz_1t} - P_{Rt}.RI_{yt} - P_{Ct}.CI_{yt}\} -$

$$\beta_{z_2t+1}EL_{yz_2t} + \mu_{z_2t+1}(RI_{yt} - RL_{yz_2t}) + \phi_{t+1}(CI_{yt} - \alpha.CK_{yt}) \quad,$$

for which the optimising and terminal stock conditions are:

(18) $\quad \delta H/\delta EL = 0 = e^{-\rho t}p.\delta q/\delta EL_{z_1} - \beta_{z_2t+1} \quad;$

(19) $\quad \delta H/\delta RL = 0 = e^{-\rho t}p.\delta q/\delta RL_{z_1} - \mu_{z_2t+1} \quad;$

(20) $\quad \delta H/\delta VI = 0 = e^{-\rho t}p.\delta q/\delta VI_{z_1} - e^{-\rho t}c_t \quad,$

(21) $\quad \delta H/\delta RI = 0 = -e^{-\rho t}P_{Rt} + \mu_{z_2t+1} \quad;$

(22) $\quad \delta H/\delta CI = 0 = -e^{-\rho t}P_{Ct} + \phi_{t+1} \quad,$

(23) $\quad \delta H/\delta CK = 0 = e^{-\rho t}p.\delta q/\delta CK - \alpha.\phi_{t+1} = \dot{\phi} - \alpha.\phi_{t+1} \quad;$

(24) $\quad \delta R/\delta ER_{T+1} = \beta_{z_2T+1} \quad,$

(25) $\quad \delta R/\delta RR_{T+1} = \mu_{z_2T+1} \quad,$

and,

(26) $\quad \delta R/\delta CK_{T+1} = \phi_{T+1} \quad.$

In comparing these conditions with those derived earlier an important change can be noted. For example, from expressions (18) and (19) it can be seen that resource depletion over the planning horizon will be now tolerated until the discounted foregone profits from utilising those resource stocks over the planning period, i.e., under state z_1, equals the discounted marginal unit value of the resource over the remaining time horizon, i.e., under state z_2. Similarly, the terminal stock conditions now indicate that it is profitability under the ecological

states prevailing after time T that is critical for measuring the true marginal user costs of resources and for determining when it becomes uneconomic to continue consuming them. If crossing the ecological threshold causes a marked decline in future profitability and the terminal value of the property then optimal resource consumption over the planning horizon will involve strategies that push the change in state further into the future.

Data Requirements

While it is relatively simple to identify the nature of the functional relationships required to adequately represent ecological thresholds and multiple stable states in a dynamic model of resource management, to determine these relationships empirically will not be a trivial matter. Comprehensive and detailed data sets on a large range of ecological, agronomic and economic variables will be required.

As a general rule, at least in developed countries, economic and market related data and data on the productivity of specific commodities under different production technologies and environmental conditions are readily available (e.g., Jones and Sandland, 1974; Murphy *et al.*, 1992, O'Sullivan, 1987, Wilson, 1991). The major problem areas appear to lie in obtaining sufficient scientific data to

(i)   identify the set of alternative states for an ecosystem. This requires being able to identify when an irreversible (in a management sense) change has occurred, i.e., when an ecological threshold has been crossed;

(ii)   determine the network of possible transitions between the identified alternative ecological states, and,

(iii)   achieve a predictive understanding of the events and relationships that will trigger a transition from one ecological state to another.

Sufficient understanding of the relevant processes, together with data adequate for these purposes, may not be currently available for any ecosystem on the globe. Problems of sparse or even non-existent ecological data should not, however, daunt the prospective model-builder. The development of models of this nature can, and should, play an important role in identifying key areas for further scientific research. As an interim measure, expert judgement combined with sensitivity testing is a valid approach for approximating responses. As further data become available, parameters can be adjusted and relationships modified to more precisely reflect real world conditions.

Conclusion

The foregoing represents a further attempt to improve the interface between economic models of resource management and ecological reality. However, only one of the current shortcomings of economic models, that of overlooking the existence of multiple stable states, has been addressed. And considerable development of the proposed approach is still required to get it to an operable stage. Many other issues, such as spatial patchiness (non-homogeneity), and protracted instability (systems that are neither in nor appear to be approaching an equilibrium), are yet to receive attention.

References

Barrett, S. (1989). *On the Overgrazing Problem*. LEEC Paper 89-07. London: London Environmental Economics Centre, University College London.

Buffington, L.C. and Herbel, C.H. (1965). Vegetational changes on a semidesert grassland range. *Ecological Monographs* 35(1), 139-64.

Friedel, M.H. (1991). Range condition assessment and the concept of thresholds: a viewpoint. *Journal of Range Management* 44(5), 422-6.

Hertzler, Greg (1990). Dynamically optimal adoption of farming practices which degrade or renew the land. Paper presented to the 34th Annual Conference of the Australian Agricultural Economics Society, 13-15 February. St. Lucia: University of Queensland.

Holling, C.S. (1973). Resilience and stability of ecological systems. *Annual Review of Ecology and Systematics* 4, 1-23.

Jones, R.J. and Sandland, R.L. (1974). The relation between animal gain and stocking rate: derivation of the relation from the results of grazing trials. *Journal of Agricultural Science (Camb.)*, 83, 335-42.

Kim, C.F., Moore, M.R., Hanchar, J.J. and Nieswiadomy, M. (1989). A dynamic model of adaptation to resource depletion: theory and an application to groundwater mining. *Journal of Environmental Economics and Management* 17(1), 66-82.

Laycock, W.A. (1991). Stable states and thresholds of range condition on North American rangelands: a viewpoint. *Journal of Range Management* 44(5), 427-33.

McConnell, Kenneth E. (1983). An economic model of soil conservation. *American Journal of Agricultural Economics* 65(1), 83-89.

Milham, N.P. (1992). An analysis of farmers' incentives to conserve or degrade the land. unpublished working paper.

Murphy, J., Barclay, R. and Hertel, K. (1992). *1992 Crop and Livestock Budgets: Cobar, Bourke, Nyngan Shires & Far West*, Farm Business Notes No.47. Dubbo: NSW Agriculture.

Noy-Meir, I. (1975). Stability of grazing ecosystems: an application of predator-prey graphs. *Journal of Ecology* 63, 459-81.

Noy-Meir, I. and Walker, B.H. (1986). Stability and resilience in rangelands. In *Rangelands: A resource Under Siege*, Proceedings of the Second International Rangeland Congress (P.J. Joss, P.W. Lynch and O.B. Williams, eds). Canberra: Australian Academy of Science.

O'Sullivan, S. (1987) *Enterprise Budgets for the North West of N.S.W.*, COMPLAN Handbook No.7. Armidale: Agricultural Business Research Institute.

Passmore, G. and Brown, C. (1992). Analysis of rangeland degradation using stochastic dynamic programming. Contributed paper to the 36th Annual Conference of the Australian Agricultural Economics Society, 10-12 February. Canberra, Australian National University

Saliba, B.C. (1985). Soil productivity and farmers' erosion control incentives - a dynamic modelling approach. *Western Journal of Agricultural Economics* 10(2), 354-64.

Segarra, E. and Taylor, D.B. (1987). Farm level dynamic analysis of soil conservation: an application to the Piedmont area of Virginia. *Southern Journal of Agricultural Economics* 19(2), 61-73.

Walker, B.H., Ludwig, D , Holling, C.S. and Peterman, R.M. (1981). Stability of semi-arid savanna grazing systems. *Journal of Ecology* 69, 473-98.

Westoby, M., Walker, B. and Noy-Meir, I. (1989). Opportunistic management for rangelands not at equilibrium. *Journal of Range Management*, 42(4), 266-74.

Wilson, A.D. (1991). The influence of kangaroos and forage supply on sheep productivity in the semi-arid woodlands. *Rangelands Journal*, 13(2), 69-80.