37th Annual Conference of the
Australian Agricultural Economics Society
University of Sydney, 9-11 February 1993
Sydney, Australia

Determining optimal farm input levels using stochastic experimental response data

R.J. Farquharson *

NSW Agriculture
Agricultural Research and Advisory Station
Private Mail Bag
Grafton NSW 2460

The interpretation of response data from biological experiments and the resulting provision of recommendations for input usage to farm decision makers is an important part of the research and development process. In this paper an analysis of a set of experimental results is undertaken. Particular attention is focused on the variability of these results over the period of experimentation and on dealing with the fact that the data are relatively scarce. Two methods of analysis are undertaken to indicate the impact of different manipulations of the raw data to specifically allow for the sparseness of the data. Recommended levels of input usage are determined for a production-maximising, a profit-maximising and a utility-maximising decision maker.

* Acknowledgment is made to Drs K Helyar and J Ayres for providing the experimental data

Determining optimal farm input levels using stochastic experimental response data

1. Introduction

The interpretation of response data from biological experiments and the provision of recommendations for input usage to farmers based on that interpretation is an important part of the research and development (R & D) process. The proper formulation of recommendations from analyses of experimental data is vital in avoiding unrealistic advice which may cause failure by farmers to accept a new technology and embarrassment for the adviser.

In this paper an analysis of a set of experimental results is undertaken. Particular attention is focused on the variability of these results over the period of experimentation and in dealing with the fact that the data are relatively 'scarce'. The recommended level of input usage is adjusted for the variation in experimental output and accounts for a typically risk averse farmer. A comparison is made of recommendations with two alternative methods of data analysis.

2. Experimental results

The instability of the legume component has been a major problem of pasture development in the sub-tropical region of north-eastern NSW, Australia. Temperate legumes, such as white clover, will persist when the growth of associated warm climate grasses is controlled by high grazing pressure. However, under farm conditions it is not always possible to control grass growth in summer over large areas with the result that white clover fails to persist.

Kenya white clover (*Trifolium semipilosum*) is persistent in association with kikuyu grass because of its ability to produce vertical stems from runners. To test persistence and productivity, grazing areas of two cultivars of Kenya white clover and two cultivars of white clover were sown. The experiment reported here was conducted in a region of high rainfall on a red soil plateau near Wollongbar in the North Coast region of NSW. It consisted of a cattle grazing trial where Hereford weaner steers were grazed for 12 months on pastures of Kikuyu and the legume *Trifolium semipilosum* (Kenya clover, cv. Safari). The experiment investigated steer liveweight gain (LWG) from pastures treated with four levels of superphosphate (P) stocked at 'low' and 'high' stocking rates (SR) over 8 years and the results of the trial are presented in Table 1.

SR differed for each level of P, so it was not included in any factorial or fractional factorial experimental design. In Table 1 the figures on LWG are presented on a per head (/hd) and a per hectare (/ha) basis. The LWG/ha figures have been derived as LWG/hd multiplied by SR (steers/ha).

This experiment was conducted after an initial three-year pilot study in which the pastures were established. Therefore the figures in Table 1 are steady-state equilibria after a pasture development phase. The analysis in this paper is essentially a comparison of these equilibria rather than an assessment of cash flow patterns in moving towards these equilibria.

There is considerable variation in LWG over the eight years of the trial. This is due to climatic patterns but will also have been a result of agronomic or management factors. The effect of these factors will be referred to later, although no analysis could be undertaken in a multi-disciplinary context. It is the variation in LWG over time that is of primary importance in terms of making recommendations to farmers from these data.

From Table 1 an increase in SR reduces mean LWG/hd at each level of P, whereas mean LWG/ha increases with SR for each level of P except the highest. In making recommendations to farmers for whom the land resource is probably most limiting and who must decide on SR (i.e. cattle numbers are a management tool), an analysis on a /ha basis is probably the most worthwhile if each kilogram of live steer weight is valued at the same price. However, if meat quality is important (in determining price received) and it differs according to steer weight then LWG/ha may not be the most important factor. The approach followed in this paper is to assume no price changes for different steer weights and to concentrate on LWG/ha.

The average response and distribution of LWG/ha to increased P is shown in Figures 1 and 2, where LWG/ha is plotted against P for 'low' and 'high' SR. Generally there is a LWG/ha response to increasing levels of P but this is not large compared to some other soil types (Dr P.T. Mears, personal communication). The particular soil type in this experiment has a relatively high capacity to hold extra P and only release it slowly to plants. The levels of LWG/ha at zero P are relatively large compared to those at higher P. This has implications for the optimum input levels derived in this analysis.

The degree of variation in LWG/ha over time can be seen in Table 1 (from the coefficient of variation) and Figures 1 and 2 (from inspection). This variation generally increases with higher levels of P and higher SR within levels of P. For farmers who are typically risk averse such behaviour could be important in considering appropriate levels of inputs. The next section of the paper outlines the theoretical basis for incorporating this variation into recommendations for risk averse farmers, and a later section contains results of two different approaches to estimating the effect of such variation on the optimal level of input for risk-averse farmers.

3. Theoretical approach

In considering the information presented in Table 1, the first step undertaken was to briefly investigate the biological response of LWG to changes in P and SR. Jones and Sandland (1974) examined the relationship between animal gain and stocking rate from the results of a number of grazing trials. They found that a simple linear model best related gain per animal and stocking rate . Using their nomenclature gain per animal (Y_a) was related to stocking rate (x) by.

$$Y_a = a - b\,x$$

Production /ha (Y_h) was related to stocking rate by:

$$Y_h = a\,x - b\,x^2$$

These types of functional forms were tested in this analysis as a first step in gaining an understanding of the biological responses. Then economic concepts were introduced.

The basic approach used by economists in dealing with the question of 'how much' of an input to use in a production process involves assessing the extra or marginal revenues (MR) from increments in the input and comparing them with the marginal costs (MC) of the increment. As long as the MR is greater than the MC, the input level should be raised by that amount. If there are decreasing returns to scale then MR eventually declines with increasing input, and when the point is reached where MR = MC the level of input is optimal. The use of marginal analysis of changes in discrete input levels has been outlined in CIMMYT (1988). In the approach followed here a response surface approach is used where the level of MR is adjusted to account for variability in output for a farmer who is averse to risk.

To account for revenues and costs a profit function is specified after Anderson, Dillon and Hardaker (1977) (ADH) such that profit π is a function of LWG over 12 months valued at P_B (price of beef), less applied P (superphosphate in kg/ha) multiplied by unit price of P (P_P), less other costs (OC). Other costs will include pasture maintenance, animal health, marketing, transport and overhead costs. The profit function is:

$$\pi = LWG.P_B - P.P_P - OC \tag{1}$$

LWG in kg/ha is given by LWG in kg/hd multiplied by SR in hd/ha. SR is an important management factor in the production process which must be decided in conjunction with the decision on the level of P.

The decision-maker is assumed to exhibit risk aversion and his or her preferences for risky profits are expressed in a utility function U which relates in some manner to π. The optimal decision includes the values of P and SR that maximise expected utility where expectations are taken over the distributions of LWG and P_B (ADH p. 161)

This problem of determining maximum U can be approached through specifying expected utility in terms of the probability distribution of π. The moment method (ADH p. 96) assumes that output (LWG) of the individual firm and P_B are stochastically independent. Therefore the mean and variance of profit are given by:

$$\begin{aligned} E(\pi) &= E(LWG.P_B - P.P_P - OC) \\ &= E(LWG).E(P_B) - P.P_P - OC \qquad (2) \\ V(\pi) &= V(LWG.P_B - P.P_P - OC) \\ &= [E(P_B)]^2.V(LWG) + [E(LWG)]^2.V(P_B) + V(P_B).V(LWG) \qquad (3) \end{aligned}$$

In (2) and (3), functions for E(LWG) and V(LWG) need to be specified in terms of the input variables.

The expected utility of the decision-maker for the risky prospect π can be assessed as the utility level at the mean of π plus a series of products of moments of π, corresponding derivatives of the utility function and inverse factorials, other than that involving the first derivative (ADH p.97). A Taylor series approximation which ignores terms beyond the second derivative is:

$$U = U[E(\pi)] + U_2[E(\pi)].V(\pi)/2 \qquad (4)$$

Then, following ADH (p. 163), the first-order condition for maximising utility is:

$$0 = E(P_B).dE(LWG)/dP - P_P - REDQ\{[[E(P_B)]^2 + V(P_B)].dV(LWG)/dP$$
$$+ 2\,V(P_B).E(LWG).dE(LWG)/dP\} \qquad (5)$$

Here REDQ is the risk evaluation differential quotient which measures the rate of utility substitution between the expected value and variance of profits:

$$REDQ = [dE(\pi)/dV(\pi)]_U = -[dU/dV(\pi)]/[dU/dE(\pi)] \qquad (6)$$

Equation (5) can be further simplified by assuming the price of beef (P_B) is fixed, so that $E(P_B) = P_B$ and $V(P_B) = 0$, and (5) can be written as:

$$P_P = P_B.dE(LWG)/dP - REDQ[P_B^2\,dV(LWG)/dP] \qquad (7)$$

Equation (7) indicates that the utility-maximising level of P occurs when the marginal factor cost (ie price of P) equals the value of the marginal expected profit less a marginal risk reduction that is determined by the utility function and the marginal variance of the revenue (ADH p. 163). The corresponding profit-maximising level of input is given by deleting the last term of (7).

This formulation is based on the assumption that the mean and variance of profits provide all the necessary information about the distribution of π. However, it could be that the distribution of LWG is skewed, in which case the third moment ($M_3(\pi)$) becomes important. To incorporate this in the optimising process $M_3(\pi)$ needs to be specified in terms of profits as in (2) and (3). If this is done, ADH (p. 170) provide the first-order condition for maximising utility as:

$$P_P = P_B.dE(LWG)/dP - REDQ[P_B^2 dV(LWG)/dP] - MSQ[P_B^3.dM_3(LWG)/dP] \qquad (8)$$

Here MSQ = -[dU/dM_3(LWG)]/dU/dE(WLG)]. (9)

MSQ is the marginal skewness quotient which measures the rate of utility substitution between expected value and skewness of profits.

Finally, in many cases when output is expressed on a /ha basis it may be necessary to aggregate the process to find the impact on a firm of any given size. If a firm is A ha in size, then the first-order condition (7) becomes (from ADH, p. 167):

$$A.P_p = A.P_B.dE(LWG)/dP - REDQ[(AP_B)^2.dV(LWG)/dP] \qquad (10)$$

4. Utility functions

ADH discuss several non-linear forms of utility function including quadratic, cubic, logarithmic, power and negative exponential. In this analysis the negative exponential function is used. This function is characterised by constant (absolute) risk aversion (coefficient c). This function is of the form:

$$U = 1 - \exp(-c\pi) \qquad (11)$$

The absolute and relative risk aversion coefficients for Australian farmers have been considered by researchers. Bardsley and Harris (1987) estimated some of these coefficients. The absolute risk aversion coefficient is the relative risk aversion coefficient divided by level of wealth. From Bardsley and Harris' (1987) estimates the absolute risk aversion coefficient in the high rainfall zone in Australia was 1 x 10[illegible]. The purpose of this analysis is to illustrate the impact of allowing for risk in deriving recommendations to groups of farmers and the risk coefficient can not be used with great precision. The target audience for the particular experiment considered here is likely to consist of relatively smaller sized beef properties in the NSW north coastal region. It was assumed that those producers may therefore have increased risk aversion and a figure of $c = 0.001$ was used in this analysis. This figure must be considered a ball-park for illustration only.

If it is accepted that gross margins are approximately normal then the expected utility function of (11) is a simple function of $E(\pi)$ and $V(\pi)$ which can be maximised by maximising $E(\pi) - ½.c.V(\pi)$. This means that REDQ is (½)c, which is a constant. The implication of using a function such as this is that risk aversion does not change as wealth changes and hence equation (7) can be used to solve for P rather than equation (10).

5. Alternative analytical approaches

5.1 Accounting for variation in many variables

The approach outlined above investigates the decision on a single risky input. In practice there will be a large number of risky factors influencing the production function. How can analysts assess the influence of other factors which might be observable and influence the decision-maker?

Two broad approaches are identified by ADH. One approach is to relate output to a wider group of variables than just those tested in the experiment. In this 'analytical' approach farmer-controlled input variables can be related to other variables outside the control of the decision-maker. These variables may be stochastic and they may be known or unknown at the time of the decision.

This approach involves describing the effect of the controllable and uncontrollable input variables in the production process and then assessing joint probability distributions associated with these variables. A particular problem with this approach is knowing and specifying all of the stochastic processes that influence production. Analyses of particular factors can be undertaken in a multi-disciplinary context, although no such analysis is attempted here.

An alternative approach is to compound all of the variation into a 'gross' relationship without identifying the effect of individual risk sources and quantifying the composite probability distributions so that they might be functionally related to the decision variables. This type of analysis generally involves using the first two or three moments to undertake a decision analysis. This approach is followed here.

In this case there are two important management decisions to be made. They are the level of P and the level of SR. The analysis here will incorporate decisions on levels of inputs for both variables.

5.2 Working with sparse data

When there are sufficient data, moment-estimation formulas can be used for estimation. When data are sparse (say, $n < 10$) it is more difficult to make good estimates of moments beyond the mean. A sparse-data smoothing rule can be applied to derive cumulative density functions (CDFs) from which moments may be calculated. Once the moments have been estimated for particular combinations of the decision variables, least-squares regressions can be fitted which relate these moments to the decision variables.

Anderson (1973) used a six-step procedure to accomplish this task for a set of sparse experimental data. These steps were:

1. For each set of data on yield Y in relation to inputs X_j ($j = 1, 2, \ldots m$), find a least-squares response function that best describes the relationship, i.e. for each of $i = 1, 2, \ldots n$ data sets (years) estimate,

$$Y_i = f(X_{1i}, \ldots X_{mi})$$

2. Use each of the n selected 'best' response functions to predict output levels for the design points of a convenient and adequate experimental design for each of the controlled inputs;

3. For each design point rank the n predicted outputs, apply the sparse-data rule and hand-smooth a graphical CDF incorporating any additional information available;

4. Use each empirical CDF to estimate the first few moments of the yield distribution at each design point;

5. Use least-squares regression to fit the derived moments and any other pertinent distribution features as functions of the controlled inputs or decision variables, i.e. estimate;

$$\text{i.e.} \quad E(Y) = g(X_{1i}, \ldots X_{mi}))$$
$$V(Y) = h(X_{1i}, .. \; X_{mi})$$

6. The final step consists of obtaining the producer's preference function for risk defined in terms of the first few moments of net profits and using this to determine the input rates that maximise his or her preference or utility.

In this paper two methods of analysing the data in Table 1 will be compared to determine what recommendations arise on input levels. The first method involves estimating means and variances directly from Table 1 (as if the data were not sparse) and deriving functions using ordinary least squares (OLS) for E(LWG) and V(LWG) in terms of the decision variables. In addition the distributions can be tested for skewness by estimating $M_3(\pi)$. If skewness is shown to be significant, a function for $M_3(\pi)$ can be estimated and the problem solved using equation (8). These functions are combined with the utility function (11) to derive the optimal input levels using equation (7).

The second method involves using the six steps of Anderson (1973) to estimate the optimal level of P to recommend given that the data are sparse.

The essential differences between these methods lie in the manipulation of the raw data and the effects of specifically allowing for sparseness of the data. It is of interest to know how much impact these differences have on the recommended levels of P and SR.

6. Results

6.1 Biological relationships

The mean responses of LWG/steer and LWG/ha are shown in Table 1. Because of the levels of P used in the experiment, a transformed variable PL = $P^{0.5}$ was tested in the models as well as P. The natural log of P could not be used because zero P was one treatment. Jones and Sandland (1974) showed that a simple linear model explained LWG/steer in terms of stocking rate. In this dataset, mean LWG/steer is expected to be positively related to P or PL, and negatively related to SR. There also appears to be some interaction between P or PL and SR.

Results from the best two fitted regressions for LWG/steer are presented in Table 2. The first includes all three of the explanators (PL, SR and SR.PL), and the second has the SR variable removed. Both equations explain a substantial amount of the variation in LWG/steer and have the expected signs of coefficients, but the second equation is slightly preferred.

If LWG/ha equals LWG/steer multiplied by SR, then the resulting equation for LWG/ha should include SR, PL.SR and PL.SR² as explanatory variables. The results of that regression are also shown in Table 2. In this case SR and PL.SR have positive coefficients and PL.SR² is negatively related. All these coefficients are highly significant and the overall regression explains a high proportion of the variation in LWG/ha.

From these results the influence of SR is negative on production/steer but positive on production/ha. In both cases there is an interaction between PL and SR - production/ha initially increases with higher levels of superphosphate and stocking rate but at some point production decreases.

The point of maximum production of LWG/ha can be derived from the equation in Table 2 by taking the first derivative of LWG/ha with respect to SR and equating it to zero. As the level of superphosphate application increases the stocking rate associated with maximum production decreases. This is shown in Figure 3. The optimum levels of SR are 8.[illegible], 5.2 and 3.9 steers/ha at P levels of 55, 220 and 660 kg/ha/year respectively. This trend is the result of the negative interaction between these variables at higher levels of input.

6.2 Economic analysis - Method 1

As well as estimates of $E(\pi)$ and $V(\ )$, Table 1 contains an estimate of relative skewness (α_3) for each set of data (treatment). The estimates of α_3 were derived using :

$$\alpha_3 = \frac{n}{(n-1)(n-2)} \sum \left[\frac{x_i - \bar{x}}{\sigma} \right]^3$$

To test whether the α_3 values are significant the variance of the skewness statistic is utilised. The variance of skewness, V_{α_3}, is given by (Kendall and Stuart 1969):

$$V_s = \frac{6n(n-1)}{(n-2)(n+1)(n+3)}$$ where n = number of observations.

Here, n = 8 and V_s = 0.57. The standard deviation of α_3 is 0.75 and to test for significance of each α_3 the ratio of α_3/SD_3 is tested as a normal t statistic. None of the α_3 figures in Table 1 are significant and so the distributions of predicted LWG/ha are assumed to be not skewed. Therefore the analysis using Method 1 will proceed without needing to account for skewness.

OLS regressions were fitted to explain the estimated E(LWG) (mean) and V(LWG) (variance) figures shown in Table 1. Explanatory variables tested were P, PL and SR and the interaction terms of superphosphate applied with SR and SR^2 from the results in Table 2. The results of the preferred equations are presented in the first part of Table 3.

From Table 3 in explaining E(LWG), the levels of SR and the interaction terms were both very significant. SR and PL.SR were positively related to E(LWG) but the other interaction term (PL.SR^2) was negatively related. This implies that in explaining increased E(LWG), SR and P were generally positively related but at higher levels of P and SR the effect was to reduce E(LWG). In explaining V(LWG) the intercept term and SR were significant factors but the interaction terms were less significant.

To estimate equation (7), dE(LWG)/dP was derived as:

$$dE(LWG)/dP = [dE(LWG)/dPL].[dPL/dP]$$

and dV(LWG)/dP was derived as:

$$dV(LWG)/dP = [dV(LWG)/dPL].[dPL/dP].$$

The estimate for REDQ was combined with these other assumptions:

P_B = \$1.15/kg liveweight, and
P_P = \$0.23/kg Superphosphate applied.

From Table 3, equation (7) becomes:

$$0.23 = 1.15\,[2.8\,SR\,P^{-1/2} - 0.7\,SR^2\,P^{-1/2}] - .0005[1.15^2(28.5\,SR\,P^{-1/2} - 10\,SR^2\,P^{-1/2})].$$

This equation is plotted in Figure 4 showing the level of P that maximises utility for each level of SR. For each level of SR, the level of P indicates the point where marginal factor cost equals the marginal utility. As stocking rate increases up to 2 steers/ha the optimal level of P increases to nearly 200 kg/ha but at higher levels of SR the optimal level of P reduces sharply to zero at 5 steers/ha.

The corresponding profit-maximising levels of P for each SR are also shown in Figure 4. At lower levels of SR the profit-maximising optimum level of P is slightly higher than the utility-maximising level, but at higher levels of SR the reverse is the case. This result may be interpreted as implying that there is more variability in output associated with the level of P than with the level of SR. However the differences do not appear to be very large.

6.3 Economic analysis - Method 2

Anderson (1973) used this procedure to analyse sparse response data when conventional procedures of obtaining probability distributions fail. His example was of a fractional factorial trial concerning the response of wheat to Nitrogen and Phosphorus fertiliser at one site in the red-brown earths of the eastern Darling Downs area of Queensland, Australia. He estimated response functions for each data set and used these to predict outputs for the design points of a complete factorial design. He then plotted CDFs and estimated moments of these distributions for fitting further functional relationships.

In this second analysis OLS regressions for each data set (year) in Table 1 were estimated and the results are presented in Table 4 (step 1 from Section 5.2). In most years SR and the interaction terms (PL.SR and PL.SR^2) were significant explanatory factors of LWG/ha. In all cases the first two factors were positively related and the last negatively related to LWG/ha.

These regressions were then used to predict LWG/ha for a convenient experimental design covering the range of input levels for the important variables in the experiment (step 2). This experimental design includes each of the four levels of P and five levels of SR (SR = 1, 2, 3, 4 and 5 steers/ha). These predictions were made for the eight year period of the experiment.

The next step involved (for each design point) ranking the eight predicted outputs in increasing order of magnitude and applying the sparse-data rule. This rule (ADH p. 42) is that if N observations are available on a continuous random variable and are ranked in ascending order of size, the Kth observation is a reasonable estimate of the K/(N + 1) fractile. The fractile estimates so derived were then plotted and a CDF smoothed subjectively through the coordinates. The CDF of a unimodal two-tailed distribution has an S shape (ADH p. 43).

Step 4 of Method 2 involves using the empirical CDFs to estimate the first two moments of the yield distribution at each design point. The 0.05, 0.15, ... , 0.95 fractiles (predicted values of LWG/ha) were read from each of the twenty plotted CDFs and the mean and variance of the distributions were estimated for each design point. These values are shown in Table 5.

The final step in Method 2 involves estimating the E(LWG) and V(LWG) functions in terms of the input decision variables and then determining the optimal input levels as in Method 1. The estimated E(LWG) and V(LWG) functions are shown in Table 3. A comparison of the regression results between Methods 1 and 2 shows that the E(LWG) equations are very similar. However, the V(LWG) equations differ in that the signs of the interaction terms are interposed and the Method 2 equation explains less of the variation in V(LWG). The magnitude of the coefficients are also broadly similar to the results for Method 1.

The utility- and profit-maximising levels of P for each level of SR are plotted for Method 2 in Figure 5. A similar shape and pattern to Figure 4 (Method 1) is seen. However, the effect of the different interactions between P and SR from the second set of regressions meant that the utility-maximising levels of P at each SR were higher than the profit-maximising levels, although only by a small amount. In broad terms the results from both methods of analysis are that the optimum level of P increases to just under 200 kg/ha/year at 2 steers/ha and then declines sharply to zero P at around 5 steers/ha.

7. Discussion and conclusions

The initial examination of the experimental data in this paper involved explaining the biological response (LWG/ha) in terms of P and SR. The maximum level of production with respect to SR was derived as a function of P and a broadly inverse relationship was indicated between these factors. For instance at a P level of 200 kg/ha/year production was maximised at a SR of 5.4 steers/ha. At 100 kg/ha/year the relevant SR was 6.8 steers/ha. These results are due to the negative interaction between P and SR at higher levels of inputs. This outcome is influenced in part by the soil type and the resulting ability to release Phosphorus to plants. Different results might be expected from other soil types.

The focus then shifted to the main point of the paper which was to estimate economic optima for levels of inputs. Two sub-issues were addressed. The first was to account for the increased variability in output observed at higher input levels by calculating profit-maximising and utility-maximising optimal levels of inputs. The second was to test a method which attempted to overcome the relative sparseness of the dataset in terms of numbers of observations (eight years in total).

The optimal economic input levels derived are shown in Table 6. Examination of the figures there confirms firstly, that in terms of making broad recommendations to groups of farmers from these data there is relatively little to be gained in accounting for increased variability in output for the levels of risk aversion assumed in this analysis. Even though increased variability was observed at higher levels of inputs, the negative interaction between inputs at these levels was the most important factor in estimating economic optima.

The second point from Table 6 is that there seems to be only relatively small differences in recommendations in using Anderson's (1973) method of accounting for sparseness of data. Although he states that sparseness may be characterised by $n < 10$, it seems that the eight years of information available here have provided a reasonable picture of the response surface.

Perhaps the major point to come from this analysis is of the importance of accounting for the interaction between inputs in conjunction with an analysis of the marginal costs and benefits in deriving recommendations for farmers. The optimal level of P varies substantially according to the level of SR.

However , these relationships may vary with soil types having different capacities to allow plants to respond to added P and therefore to provide production responses, and hence further analyses of this type may be valuable.

References.

Anderson, Jock R. (1973), 'Sparse data, climatic variability, and yield uncertainty in response analysis', *American Journal of Agricultural Economics* 55(1): 77-82.

________, Dillon, John L. and Hardaker, J. Brian (1977), *Agricultural Decision Analysis*, Ames, Iowa.

Bardsley, P. and Harris, M. (1987), 'An approach to the econometric estimation of attitudes to risk in agriculture', *Australian Journal of Agricultural Economics*, 31(2): 112-126.

CIMMYT (1988), 'From agronomic data to farmer recommendations', Economics Training Manual, Mexico.

Kendall, M.G. and Stuart, A. (1969), *The Advanced Theory of Statistics, Volume 1, Distributional Theory*, Third Edition, Griffin.

Table 1
Experimental data

Superphosphate applied	Stocking rate	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Mean	Variance	Coefficient of variation	Skewness coefficient
kg/ha	strs/ha	kg/yr	kg/yr	kg/yr	kg/yr	kg/yr	kg/yr	kg/yr	kg/yr	kg/yr			
		Liveweight gain per steer											
0	1.7	119	170	176	142	190	129	140	147	152	604	16.2	
0	2.5	110	121	159	127	165	158	135	86	133	749	20.6	
55	2.3	103	145	174	137	202	163	130	127	148	962	21	
55	3.5	104	102	194	134	201	163	135	97	141	1,686	29.1	
220	3	121	162	223	133	188	184	172	115	162	1,387	22.9	
220	4.7	140	110	175	138	142	136	88	55	123	1,395	30.4	
660	3.3	142	164	215	127	182	205	145	102	160	1,507	24.3	
660	5	78	89	126	83	140	133	90	36	97	1,197	35.7	
		Liveweight gain per hectare											
0	1.7	202	289	299	241	323	219	238	250	258	1,746	16.2	0.36
0	2.5	275	303	398	318	413	395	338	215	332	4,684	20.6	-0.43
55	2.3	237	334	400	315	465	375	299	292	340	5,090	21	0.5
55	3.5	364	357	679	469	704	571	473	340	494	20,656	29.1	0.47
220	3	363	486	669	399	564	552	516	345	487	12,486	22.9	0.19
220	4.7	658	517	823	649	667	639	414	259	578	30,819	30.4	-0.73
660	3.3	469	541	710	419	601	677	479	337	529	16,415	24.2	0.07
660	5	390	445	630	415	700	665	450	180	484	29,917	35.7	-0.39

Table 2
Regression results for biological relationships

Dependent variable	Intercept	SR	PL	PL.SR	PL.SR^2	Adjusted R^2
LWG/steer	151	-4.6	4.9	-1.2		0.87
	(10)	(-0.8)	(4.5)	(-3.7)		
	140		5.3	-1.4		0.88
	(31.8)		(6.0)	(-6.9)		
LWH/ha	9.9	134		5.6	-1.4	0.97
	(0.3)	(10.3)		(8.4)	(-9.2)	

t Statistics in brackets, n = 8.

Table 3
Economic analysis: Preferred regression results for estimation of first two momentsof LWG/ha distribution

Dependent variable	Intercept	SR	PL.SR	PL.SR^2	Adjusted R^2
		Method 1			
E(LWG)	9.9	134.3	5.6	-1.4	0.97
	(0.3)	(10.3)	(8.4)	(-9.2)	
V(LWG)	-19280.2	11180.6	57.0	-19.9	0.95
	(-4.4)	(6.3)	(0.6)	(-0.9)	
		Method 2			
E(LWG)	2.6	138.8	5.7	-1.5	0.99
	(0.4)	(74.1)	(27.4)	(-30.2)	
V(LWG)	-8761.6	10857.8	-88.2	9.8	0.82
	(-1.7)	(6.8)	(-0.5)	(0.2)	

t Statistics in brackets.

Table 4
Estimated response functions for LWG/HA in each data set: Method 2

Year	Intercept	SR	PL.SR	PL.SR^2	Adjusted R^2
1	-157.7	175.5	4.6	-1.3	0.64
	(-1.0)	(2.8)	(1.4)	(-1.8)	
2	172.5	53.7	6.4	-1.2	0.78
	(2.0)	(1.6)	(3.7)	(-3.1)	
3	-148.4	233.2	9.0	-2.4	0.94
	(-1.7)	(7.0)	(5.3)	(-6.0)	
4	-121.3	189.4	2.5	-1.1	0.84
	(-1.4)	(5.4)	(1.4)	(-2.6)	
5	57.4	158.8	4.0	-1.1	0.82
	(0.5)	(3.7)	(1.8)	(-2.1)	
6	-8.2	145.8	7.8	-1.7	0.96
	(-0.1)	(6.3)	(6.6)	(-6.0)	
7	102.2	87.1	5.9	-1.4	0.61
	(0.9)	(2.0)	(2.6)	(-2.5)	
8	173.5	34.7	4.9	-1.3	0.74
	(3.0)	(1.5)	(4.2)	(-4.6)	

t Statistics in brackets.

Table 5
Predicted moments of LWG/ha for each experimental design point - Method 2

Level of Superphosphate	Estimated moment	Predicted LWG/ha				
		Stocking rate				
		1	2	3	4	5
0	Mean	146	278.9	419	547	711.5
0	Variance	6,310	5,278	10,962	36,107	55,834
55	Mean	171.9	318.4	442.6	535.4	638.5
55	Variance	6,956	5,296	11,989	32,570	52,188
220	Mean	208.9	347	485.3	550.8	571.2
220	Variance	8,442	7,433	15,452	29,593	44,522
660	Mean	257.9	423.9	516	533.7	481.5
660	Variance	10,238	11,383	19,199	27,577	41,223

Table 6
Comparison of optimal economic input levels

Stocking Rate	Method 1		Method 2	
	Level of Superphosphate to maximise:			
	Profit	Utility	Profit	Utility
steers/ha	kg/ha/year	kg/ha/year	kg/ha/year	kg/ha/year
0	0	0	0	0
1	109	100	111	113
2	194	194	184	190
3	109	138	83	88
4	0	12	8	6
5	-	-	-	-

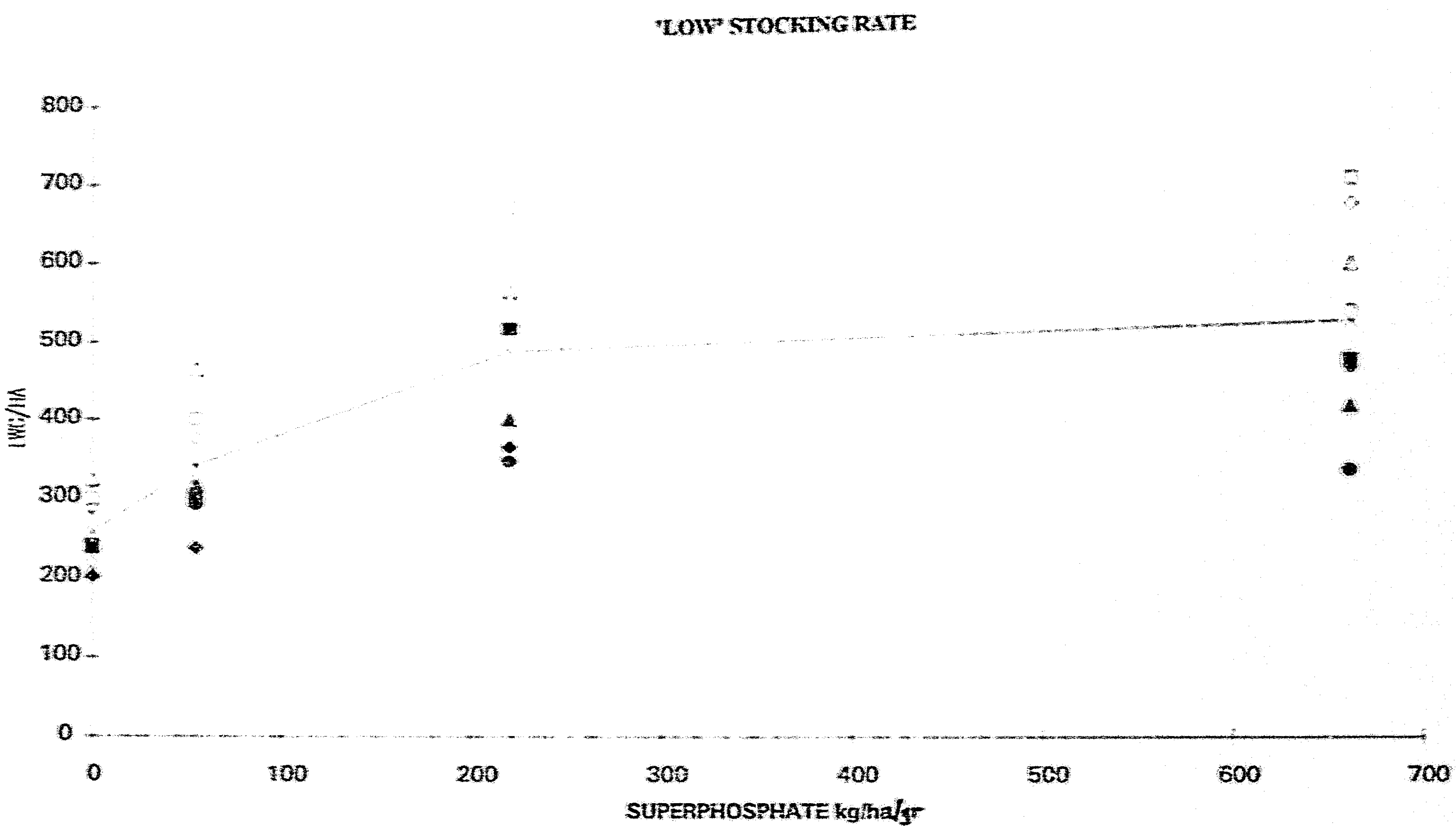
'LOW' STOCKING RATE
800
700
600
500
400
300
200
100
0
LWG/HA
0
100
200
300
400
500
600
700
SUPERPHOSPHATE kg/ha/yr

Figure 1

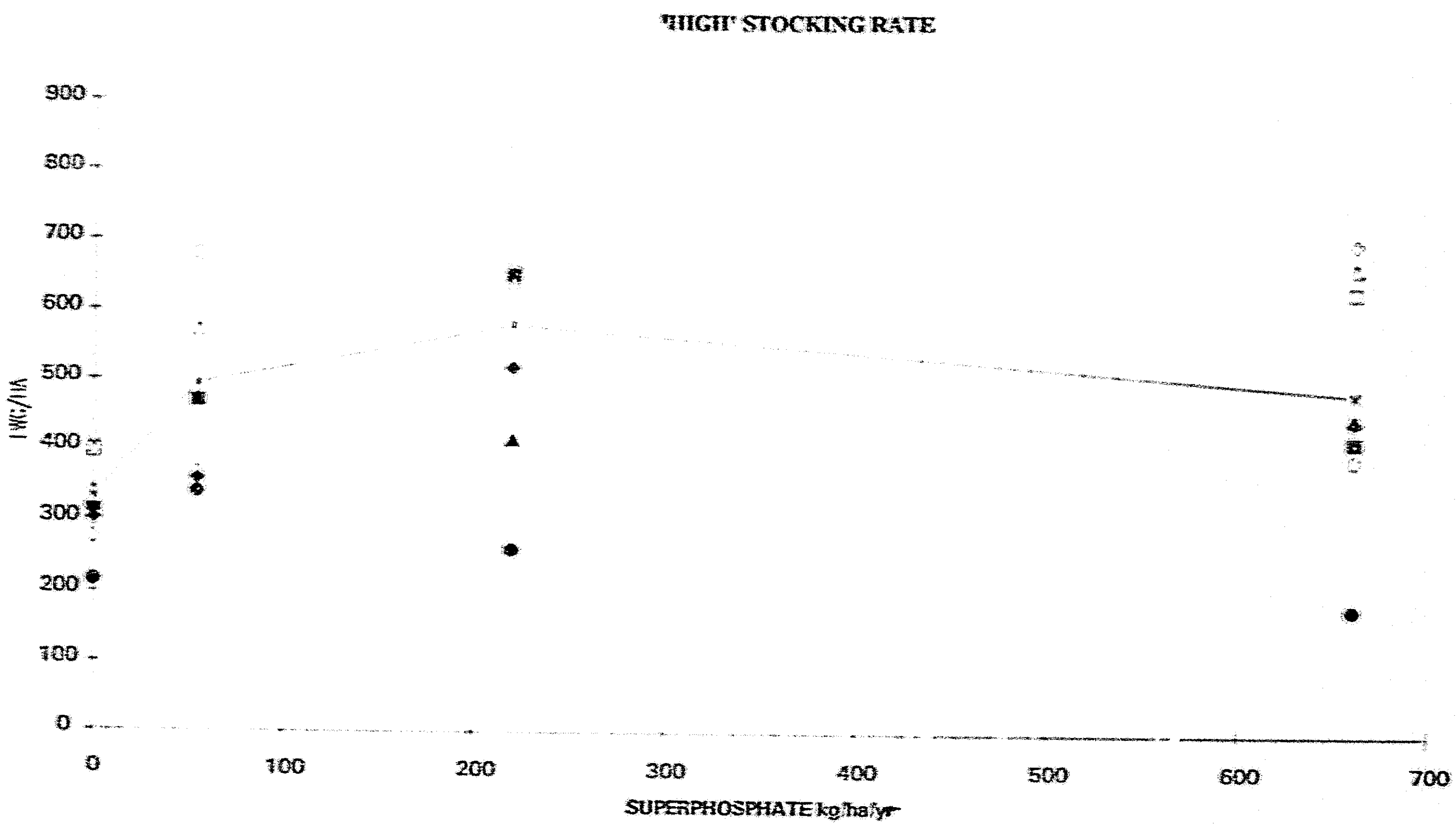
'HIGH' STOCKING RATE
900
800
700
600
500
400
300
200
100
0
0
100
200
300
400
500
600
700
SUPERPHOSPHATE kg/ha/yr

Figure 2

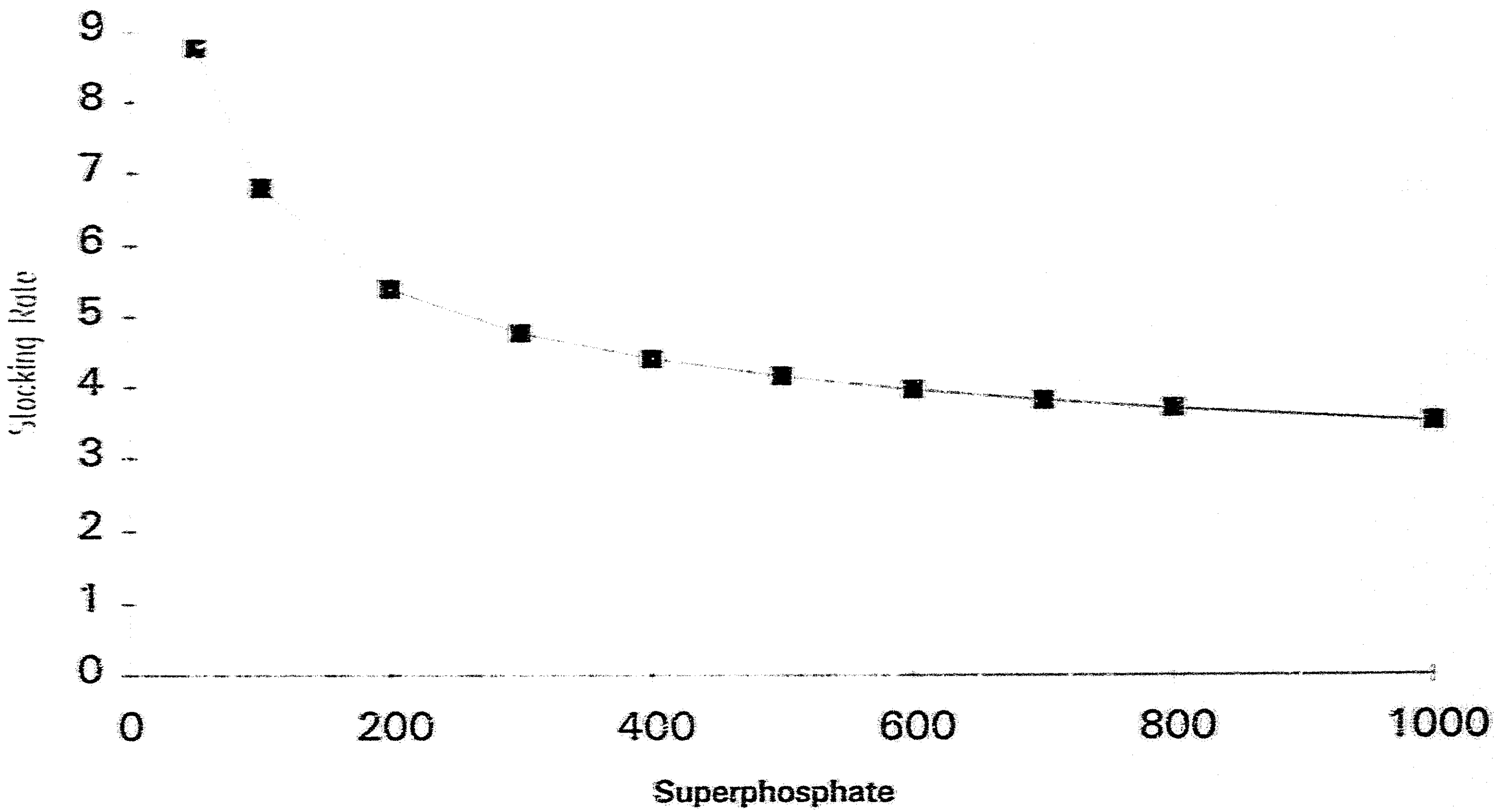

Figure 3
Level of stocking rate maximising LWG/ha
for different levels of P

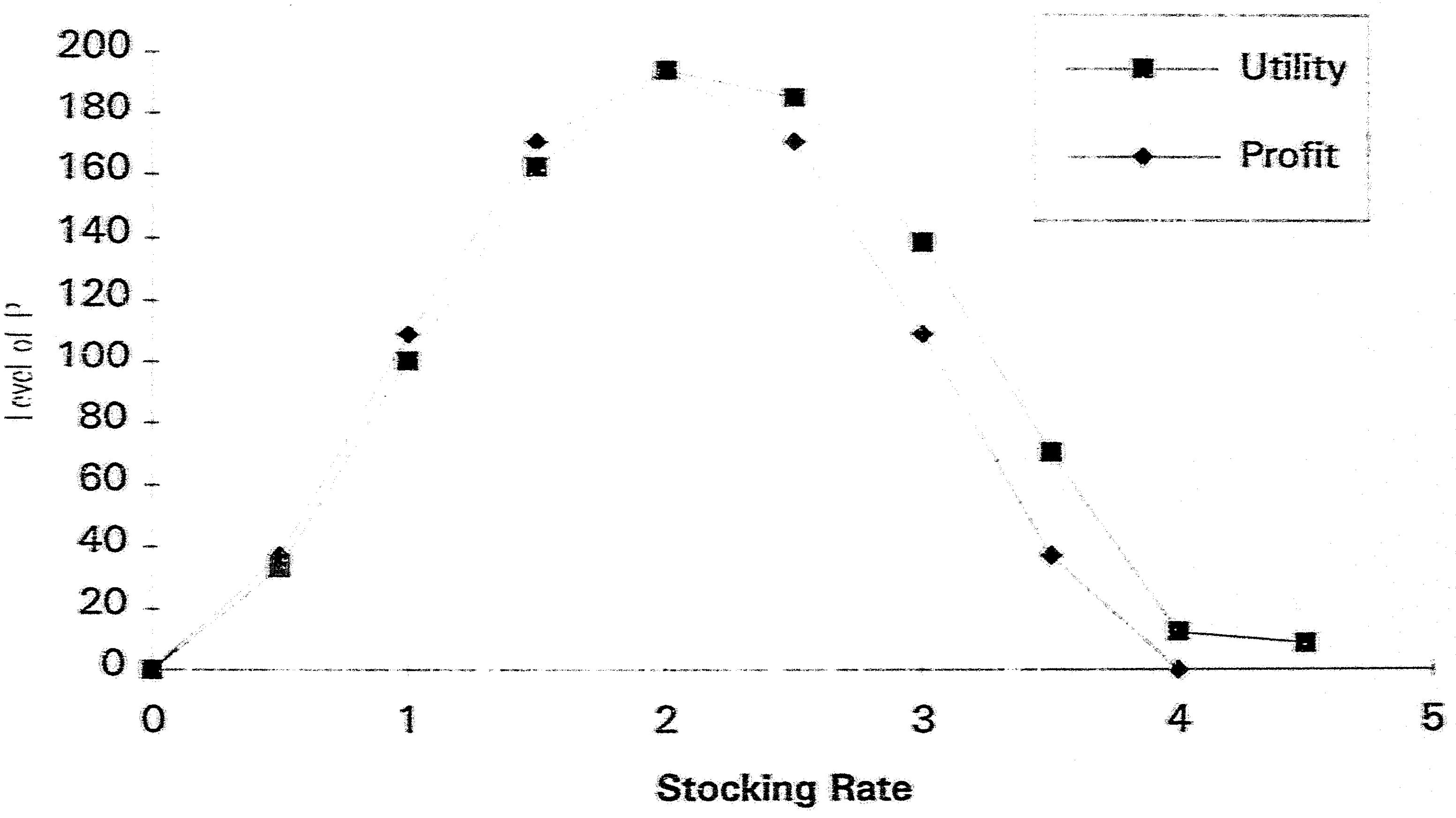

Figure 4
Level of P at each SR to maximise Utility and Profit - Method 1

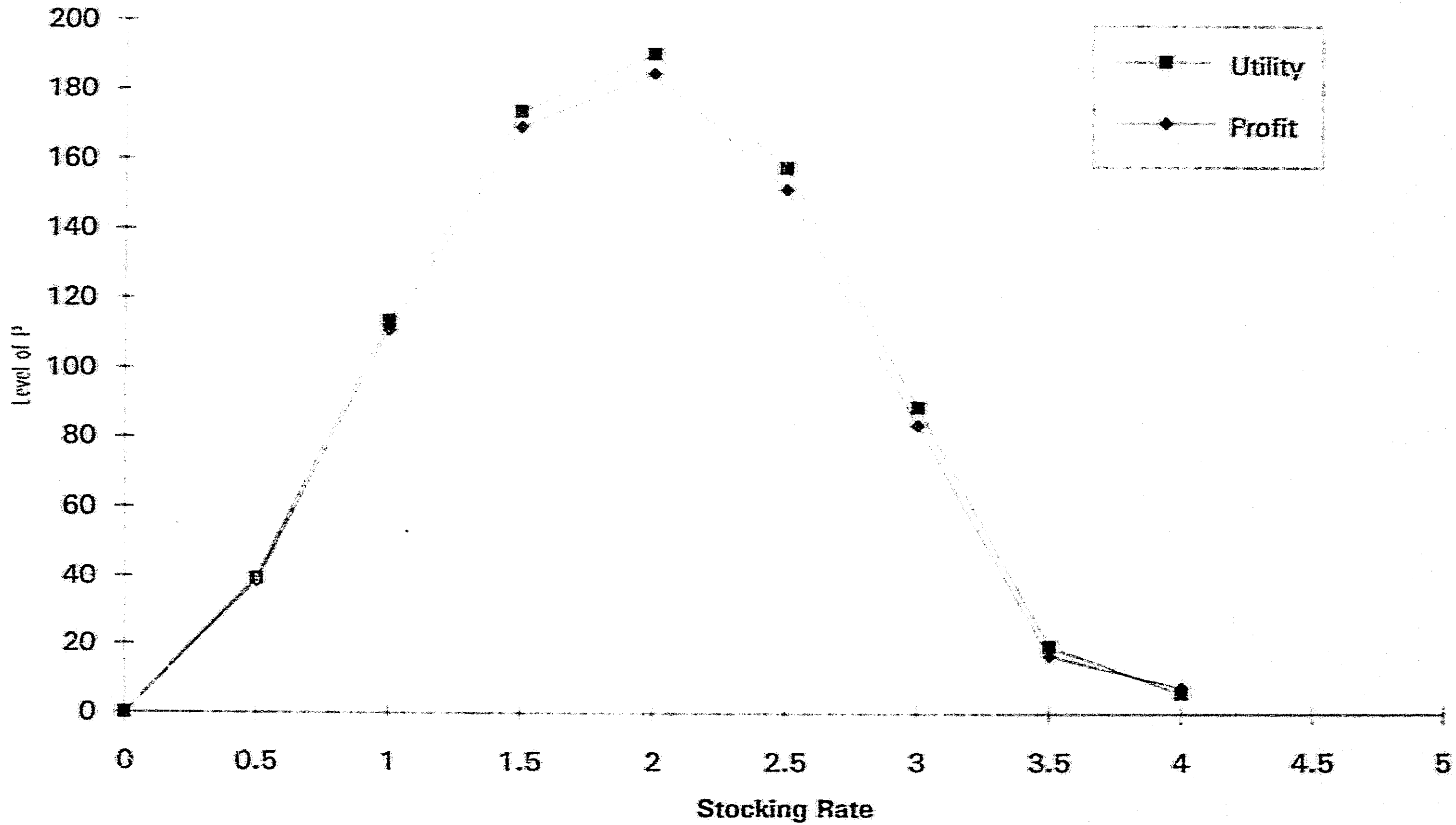

Figure 5
Level of P at each SR to maximise Utility and Profit - Method 2