A Methods Note on the Gauss-Seidel Algorithm for Solving Econometric Models

By Dale Heien, Jim Matthews, and Abner Womack

A particular numerical analytical technique for solving systems of simultaneous equations which offers several advantages to the user over other numerical techniques is discussed. The Gauss-Seidel algorithm is simply an iterative technique which requires no derivatives, matrix inversion, eigenvalue computation, or any other sophisticated numerical methodology. While the technique has been used successfully by a few large scale model builders and by the authors for several commodity models, the experience gained in the use of the technique has not been generally disseminated. Obtaining convergence with such an iterative technique is critically dependent on a number of factors which are taken up in some detail by the authors.

Keywords: Solutions, mathematical analysis, numerical, methodology, nonlinear, equations.

Once an econometric model has been specified and estimated, the next step involves some procedure to solve the model. This is done to obtain the reduced-form multipliers and to make forecasts for some future period or to examine the model's "track record" over the period of fit. The typical textbook approach to this problem is to treat the model as a set of simultaneous linear equations. In the conventional matrix terminology we have

$$\beta y + \Gamma x = 0$$

where β is a $G \times G$ matrix of coefficients of the endogenous variables, y is a $G \times 1$ vector of endogenous variables, Γ is a $G \times K$ matrix of coefficients on the exogenous variables, and x is a $K \times 1$ vector of exogenous variables. The solution, or reduced form, of the model is

$$y = -\beta^{-1} \Gamma x = \pi x$$

where

$$\pi_{ij} = \frac{\partial y_i}{\partial x_j}$$

is the $G \times K$ matrix of reduced-form multipliers. The main problem here is the calculation of β^{-1}, and this can be accomplished by well-known numerical techniques.

Unfortunately, few econometric models can be represented by a set of linear equations. For example, fundamental identities (such as price times quantity equals total revenue) as well as many other basic variables (relative prices, real income, etc.), form ratios that render the model nonlinear. In the past, this nonlinear impasse was handled in three ways. First, the nonlinear relations in the model could be linearized by using the first-order terms of a Taylor series expansion of the function.[1] Second, classical Newtonian numerical analytic techniques could be applied to solve simultaneous nonlinear equations. Third, the model was simply never solved. Unfortunately, the third route was often the one chosen. The advent of large scale models, such as the Brookings-SSRC model with extensive nonlinearities, rendered the first two approaches uneconomical and cumbersome. Attempts were also made to divide the model into linear blocks with nonlinear relations between them. Iterative techniques were then proposed to bridge these blocks.[2]

Recently, large scale model builders have rediscovered an old numerical analytic technique for solving linear and nonlinear simultaneous equations. This technique is the Gauss-Seidel method. The Gauss-Seidel method is simply an iterative technique which requires no derivatives, matrix inversion, eigenvalue computation, or any other sophisticated numerical methodology. Writing the equations of the model in the following form,

$$\begin{aligned} y_1 &= f_1(y_2, y_3, \ldots, y_G, x_1, x_2, \ldots, x_K) \\ y_2 &= f_2(y_1, y_3, \ldots, y_G, x_1, x_2 \ldots, x_K) \\ &\vdots \\ y_G &= f_G(y_1, y_3, \ldots, y_{G-1}, x_1, x_2, \ldots, x_K) \end{aligned}$$

Footnotes are on p. 75.

$$y^0 = (y_2^0, y_2^0, \ldots, y_G)$$

we can compute a first round of y's (y^1) from these initial guesses

$$y_1^1 = f_1\,(y_2^0, y_3^0, \ldots, y_G^0, x_1, x_2, \ldots, x_K)$$

$$y_2^1 = f_2(y_1^1, y_3^0, \ldots, y_G^0, x_1, x_2, \ldots, x_K)$$

$$\begin{array}{cc} \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{array}$$

$$y_G^1 = f_G(y_1^1, y_2^1, y_3^1, \ldots, y_{G-1}^0, x_1, x_2, \ldots, x_K).$$

These first-round guesses can now be used to generate a second round (y^2) according to

$$y_1^2 = f_1\,(y_2^1, y_3^1, \ldots, y_G^1, x_1, x_2, \ldots, x_K)$$

$$y_2^2 = f_2(y_1^2, y_3^1, \ldots, y_G^1, x_1, x_2, \ldots, x_K)$$

$$\begin{array}{cc} \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{array}$$

$$y_G^2 = f_G(y_1^2, y_2^2, y_3^2, \ldots, y_{G-1}^2, x_1, x_2, \ldots, x_K).$$

This iteration scheme is repeated until some specified tolerance level is reached so that

$$|(y_i^k - y_i^{k-1})/y_i^{k-1}| \leqslant \delta$$

where δ is a small positive number.

Whether a solution exists for any given econometric model is a problem aside from the use of the Gauss-Seidel technique. One necessary but not sufficient condition is that the number of equations equals the number of unknowns. Although equality between the number of equations and the number of unknowns is no guarantee that a solution exists, in practice this is the main consideration. More common is the phenomenon of multiple solutions. A relation such as the "Phillips" curve

$$\dot{w} = \beta_0 + \beta_1 UR^{-1}$$

where $\dot{w}$ is the percent change in money wages and UR is the unemployment rate is defined in the first and third quadrants. Hence, it is possible to have solutions to the model which yield falling wages at negative unemployment rates. If multiple solutions should occur, computer program statements can be included to force the model solution out of those regions that do not apply or that are a priori unreasonable.

If we proceed on the basis that a unique solution exists for some simultaneous set of nonlinear equations, numerical routines such as the Gauss-Seidel algorithm provide many advantages over classical nonlinear methods. However, obtaining convergence with this technique is critically dependent on (1) normalization, (2) the ordering of the equations, and (3) the use of dampening factors.

The Problem of Normalization

Suppose that an equation has been fitted for a particular endogenous variable and that other endogenous variables are contained in this relationship. It is possible to renormalize this equation on one of the other endogenous variables. However, convergence of the system can be shown to depend on which of the variables is on the left side of the equation. This is demonstrated for a simple two-variable, two-equation case as follows:

$$y_1 + .2y_2 = 4$$

$$-y_1 + y_2 = 2$$

where the analytical solution is $y_1 = 3, y_2 = 5$.

The following normalization gives a converging system:

$$y_1 - 4 = .2y_2$$

$$y_2 = 2 + y_1$$

Let $y_0 = (y_1^0, y_2^0) = (15, 15)$.[3] Then the iteration sequence becomes:

$$y_1^1 = 4 - .2(15) = 1, y^1 = (1, 3)$$

$$y_2^1 = 2 + 1 = 3$$

$$y_1^2 = 4 - .2(3) = 3.4, y^2 = (3.4, 5.4)$$

$$y_2^2 = 2 + 3.4 = 5.4$$

$$y_1^3 = 4 - .2(5.4) = 2.92, y^3 = (2.92, 4.92)$$

$$y_2^3 = 2 + 2.92 = 4.92$$

$$y_1^4 = 4 - .2(4.92) = 3.016, y^4 = (3.016, 5.016)$$

$$y_2^4 = 2 + 3.016 = 5.016$$

$$y_1^5 = 4 - .2(5.016) = 2.9968, y^5 = (2.9968, 4.9968)$$

$$y_2^5 = 2 + 2.9968 = 4.9968$$

$$\vdots \qquad \vdots$$

It can be seen from the above sequence that for a given $\delta > 0$ this normalization will converge to a set such that $|(y_i^k - y_i^{k-1})/y_i^{k-1}| \leqslant \delta$ where δ is some predetermined tolerance level.

The following normalization of the same model, however, would result in a diverging system:

$$y_2 = 20 - 5y_1$$

$$y_1 = -2 + y_2$$

Let $y_0 = (15, 15)$. Then the iteration sequence becomes:

$$y_2^1 = 20 - 5(15) = -55, y^1 = (-57, -55)$$

$$y_1^1 = -2 + (-55) = -57$$

$$y_2^2 = 20 - 5(-57) = 305, y^2 = (303, 305)$$

$$y_1^2 = -2 + (305) = 303$$

$$y_2^3 = 20 - 5(303) = -1495, y^3 = (-1497, -1495)$$

$$y_1^3 = -2 + (-1495) = -1497$$

$$\vdots \qquad \vdots$$

Obviously this normalization choice will not converge to the solution set.

Use of the Gauss-Seidel algorithm requires a unique dependent variable for each equation. Experience has shown that convergence is enhanced if the dependent variable is the one which was normalized on in the regression used to obtain the estimate. If, for example, an equation is fitted by least squares with y_1 as the dependent variable and then solved for some other variable as dependent (say y_2), then the model will sometimes not converge. However, if the equation is refitted with y_2 as the dependent variable, the convergence problem frequently disappears. Furthermore, specification of an econometric model where each equation has a unique dependent variable makes a great deal of sense from the causal point of view.[4]

Ordering of the Equations

By the notion of ordering is meant the order in which the equations are positioned so that iterative computation can take place. The procedure suggested is to arrange the equations so that the matrix of endogenous variables would be as triangular as possible.[5] Consider, for example, the following set of five equations where y_i, $i = 1, \ldots, 5$ is the set of endogenous variables and z_i, $i=1, \ldots, 5$ is some set of exogenous variables:

$$y_1 = f(y_2, y_5, z_1)$$

$$y_2 = f(y_3, z_2)$$

$$y_3 = f(y_2, y_1, z_3)$$

$$y_4 = f(y_1, z_4)$$

$$y_5 = f(y_2, z_5)$$

A first attempt at a solution set for the above system could be determined by ordering the equations as outlined above. This ordering is illustrated in matrix form in exhibit A. The x's in the matrix represent the endogenous variables in each equation.

Exhibit A

Equation	Variable y_1	y_2	y_3	y_4	y_5
y_4	x			x	
y_5		x			x
y_3	x	x	x		
y_2		x	x		
y_1	x	x			x

As can be readily observed, it is not possible to order the equations so that a triangular matrix is obtained. However, this ordering is as triangular as possible. A first attempt at a solution would be to use the ordering y_4, y_5, y_3, y_2, y_1. If this sequence does not coverge it will most likely be caused by the position of y_4 and y_5 since these two equations have variables outside the triangular block. Should this ordering diverge, the ordering y_4, y_1, y_3, y_2, y_5 could be tried. Where convergence is not obtained after several orderings have been tried, the suspect equations should be pulled out of the system. If the remaining equations converge, then more careful attention should be given to the equations preventing convergence. It could be a problem of normalization or a simple mechanical error in the equation. It is also possible that the Gauss-Seidel technique cannot find a solution. However, inability to find a solution using this technique has not been a problem in models solved by the authors. Where convergence has been a problem, the use of a dampening factor has frequently been beneficial.

Use of a Dampening Factor

A dampening factor may be applied to any one or all of the equations to aid in obtaining convergence. An integer k, for $0 < k < 1$, is multiplied by y_i^m, the mth equation in the interdependent system, where m represents the iteration number. $(1-k)$ times y_i^{m-1} is then added to the equation, so that

$$y_i^m = k \cdot y_i^m + (1-k) y_i^{m-1} \quad i = 1, \ldots, G$$

A primary reason for using a dampening factor is that it helps prevent a diverging arrangement of the equations from dominating the system. A dampening factor in effect allows other equations more rounds to converge and tends to pull the diverging arrangement back toward convergence. As an example, consider again the diverging system presented earlier and let $k = .25$.

$$y_2 = 20 - 5y_1$$

$$y_1 = -2 + y_2$$

We have already seen that this normalization will not lead to convergence even though the system of equations has a solution set $y_1 = 3, y_2 = 5$.

Let $y^0 = (y_1^0, y_2^0) = (15, 15)$ where the dampened system is given by

$$y_2^m = y_2^{m-1}(1-.25) + .25\,(20-5y_1)$$

$$y_1^m = y_1^{m-1}(1-.25) + .25(-2+y_2)$$

Plugging y_0 in the initial starting set in the above equation will lead to convergence after approximately 25 iterations.

The comments presented cover the basic considerations in using the Gauss-Seidel algorithm to solve systems of simultaneous equations. Additional discussions of numerical techniques can be found in (*3, 6, 9, 11*). Computer programs incorporating the Gauss-Seidel algorithm have been prepared by Norman and also by Green and Pritchard (*5, 10*). The computer program prepared by Green and Pritchard varies slightly from the one suggested by Norman and was used by the authors in preparing computer simulation programs for beef, pork, broilers, turkeys, eggs, oranges, and soybeans. Though these models are relatively small (7 to 35 equations), the authors felt that substantial savings in computer costs and time could be attributed to the use of the Gauss-Seidel algorithm. Each of the models required extensive testing in its initial development. This frequently involved use of alternative equations which could be easily interchanged in the Gauss-Seidel routine as opposed to the more cumbersome matrix inversion approach to obtain the reduced-form solution. In addition, some of these models contain price ratios and other forms of nonlinearities in the endogenous variables which would have required linear approximations by the Taylor series if the reduced-form technique had been used to obtain a solution.

The typical cost of obtaining a model solution with the Gauss-Seidel technique ranged from $3 to $5 for these models, or about half the cost of the matrix inversion approach. A subroutine for obtaining the impact multipliers is included. The programs for these agricultural models are available from the author on request. A typical program for an 18-equation simultaneous model of the U.S. beef economy is given in the appendix to this report.

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Footnotes

[1] Considerations in the use of the Taylor series to obtain linear approximations have been discussed previously in this journal by Womack and Matthews (*12*). (Italic numbers in parentheses indicate items in the References, p. 73.)

[2] This approach is reported by C. Holt (*7*).

[3] Any real number for y_2^0 will suffice since y_2^0 is not used in the iterative computation.

[4] For further elaboration of this concept, see Fisher (*4*).

[5] Experience with interdependent model systems indicates that the more nearly the equations are aligned in a causal chain, stimulus-response form, the higher is the probability of obtaining convergence. This leads to a form that is as recursive and hence as triangular as possible.

Appendix: Computer Program for Beef With Gauss-Seidel Subroutine


```
            COMMON Y(160),YO(160),Y4(160),X(160),Z(160),N,XE,A(160)
C           1950-65 CLS BEEF
   22       FORMAT(1X,E14.8,7E15.8)
   8        FORMAT(1X,3E15.8,I10)
    903     FORMAT (8I10)
   190      FORMAT(1X,E14.8,7E15.8)
   191      FORMAT(1X,5E15.8,I10)
   2        FORMAT(5F15.6)
   291      FORMAT (I10)
   98       FORMAT(1H1)
            N=19
            N1=16
      55    READ(5,2)(Y(I),I=1,N)
      57    READ(5,2)(X(I),I=1,N)
      58    READ(5,2)(Z(I),I=1,N1)
            Z(15)=31908000.
            DO 893 I=1,N
            A(I)=Y(I)
   893      CONTINUE
            M=0
   3        CONTINUE
            DO 56 I=1,N
      56    YO(I)=Y(I)
            M=M+1
            WRITE(2,22)(Y(I),I=1,N)
            WRITE(2,22)(X(I),I=1,N)
            WRITE(2,22)(Z(I),I=1,N1)
   2021     N63=0
   502      CONTINUE
            DO 4 I=1,N
   4        Y4(I)=Y(I)
            NICK=600
            IF(N63-NICK) 201,202,202
   201      CONTINUE
            N63=N63+1
   192      FORMAT(I10)
            CALL CEN
            DO 503 I=1,N
            DIF1=(Y(I)-Y4(I))/Y4(I)
            IF(DIF1) 505,503,505
   505      CONTINUE
            DIF2=0.0
            DIF2=ABS (DIF1)
            IF(DIF2-.0002) 503,503,502
   503      CONTINUE
            DO 504 I=1,N
   504      Y4(I)=Y(I)/YO(I)
   290      FORMAT(1X,E15.8,I10)
```

```
202   DO 10 I=1,N
      Y4(I)=Y(I)/YO(I)
      WRITE(2,8) Y(I),YO(I),Y4(I),I
10    CONTINUE
      WRITE(2,291) N63
      WRITE(2,98)
      DO 126 I=1,N
      YO(I)=Y(I)
126   CONTINUE
      DO 125 I=1,N
      X(I)=YO(I)
125   CONTINUE
      READ(5,2)(Y(I),I=1,N)
      READ(5,2)(Z(I),I=1,N1)
      Z(15)=X(15)
      NZ=M-10
      IF(NZ) 3,3,5
5     CONTINUE
      STOP
      END
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```
0001                 SUBROUTINE CEN
0002                 COMMON Y(160),YO(160),Y4(160),X(160),Z(160),N,XE,A(160)
0003                 WC=.5
0004                 W=1.0-WC
              C
              C      TOTAL SUPPLY OF BEEF                              Y(1)      TSB
              C
0005                 Y(1)= Y(2) + Z(1) + Z(2)
0006                 Y(1)=WC*Y(1)+W*Y4(1)
              C
              C      TOTAL PRODUCTION OF BEEF                          Y(2)      TPB
              C
0007                 Y(2)= Y(3) + Y(4)
0008                 Y(2)=WC*Y(2)+W*Y4(2)
              C
              C      TOTAL PRODUCTION OF FED BEEF                      Y(3)      TPFB
              C
0009                 Y(3)=-678150140.0 + 631.29*Y(5) + 692321280.*Y(9)/Z(3)
0010                 Y(3)=WC*Y(3)+W*Y4(3)
              C
              C      TOTAL PRODUCTION OF NONFED BEEF                   Y(4)      TPNFB
              C
0011                 Y(4)=-1160867100. + 466.28*(Y(6)+Y(7)+Y(8)) + 1563341600.*Y(9)/Z(3
                    1)
0012                 Y(4)=WC*Y(4)+W*Y4(4)
              C
              C      FED BEEF HEIFER AND STEER SLAUGHTER               Y(5)      FBHSS
              C
0013                 Y(5)=Z(14)*Y(11)
0014                 Y(5)=WC*Y(5)+W*Y4(5)
              C
              C      NONFED BEEF HEIFER AND STEER SLAUGHTER            Y(6)      NFBHSS
              C
0015                 Y(6)= Y(11)-Y(5)
0016                 Y(6)=WC*Y(6)+W*Y4(6)
              C
              C      BEEF HEIFER AND STEER SLAUGHTER                   Y(11)     BHSS
              C
0017                 Y(11)=-2759537. + .68705*X(15) + .17975*Z(15) - 26354.9*X(14) + 26
                    1832.1*Y(14)
0018                 Y(11)=WC*Y(11)+W*Y4(11)
              C
              C      NONFED DAIRY COW SLAUGHTER                        Y(8)      NFDCS
              C
0019                 Y(8)=1427995.0 + .10650 *Z(4) + .26938 *X(8) +45527.1 *Y(10)-
                    1196830.8*X(10)
0020                 Y(8)=WC*Y(8)+W*Y4(8)
              C
```

```
                C      NONFED BEEF COW SLAUGHTER                    Y(7)      NFBCS
                C
0021                   Y(7)= 833297.4 + .18539*X(15) - 152481.8*Y(14)
0022                   Y(7)=WC*Y(7)+W*Y4(7)
                C
                C      TOTAL NUMBER OF CATTLE SLAUGHTERED           Y(12)     TNCS
                C
0023                   Y(12)= Y(8) + Y(7)  + Y(11)
0024                   Y(12)=WC*Y(12)+W*Y4(12)
                C
                C      BEEF HEIFERS FOR BREEDING                    Y(13)     BHFB
                C
0025                   Y(13)=-4976911.0 + 30760.9*X(14) + 104886.3*Z(7) + 9046.7*Z(8) +
                      1 .21127*X(15)
0026                   Y(13)=WC*Y(13)+W*Y4(13)
                C
                C      RETAIL PRICE OF BEEF                         Y(9)      RPB
                C
0027                   Y(9)=(.0021158-(.15586E-04)*Y(19)/Z(13)-(.83906E-05)*Y(18)/Z(13)-(
                      1.14793E-05)*Z(10)/Z(13)+.21576*Z(11)/(Z(9)/Z(13))-.6797*Z(5)/(Z(9)
                      1/Z(13)))*(Z(9)/Z(13))
0028                   Y(9)=WC*Y(9)+W*Y4(9)
                C
                C      PRICE OF NONFED CATTLE                       Y(10)     PNFC
                C
0029                   Y(10)= -3.2517 + .8422*Y(14) - .0785*Z(6)
0030                   Y(10)=WC*Y(10)+W*Y4(10)
                C
                C      BEEF COW INVENTORY                           Y(15)     BCI
                C
0031                   Y(15)=X(15)+Y(13)-Y(7)-.02*X(15)
0032                   Y(15)=WC*Y(15)+W*Y4(15)
                C
                C
                C      PRICE OF FED CATTLE                          Y(16)     PFEC
                C
0033                   Y(16)=Y(9)*(27.72 - (.3531E-06)*Y(11)+7.84*Y(9)/Z(16))
0034                   Y(16)=WC*Y(16)+W*Y4(16)
0035                   Y(17)=WC*Y(17)+W*Y4(17)
                C
                C
                C      TOTAL SUPPLY OF FED BEEF                     Y(18)     TSFB
0036                   Y(18)=Y(3)
0037                   Y(18)=WC*Y(18)+W*Y4(18)
                C
                C      TOTAL SUPPLY OF NONFED BEEF                  Y(19)     TSNFB
0038                   Y(19)=Y(1)-Y(18)
                C      PRICE OF FEEDER CATTLE                       Y(14)     PFC
```

```
                 C      NONFED BEEF COW SLAUGHTER                              Y(7)       NFBCS
                 C
 0021                   Y(7)= 833297.4 + .18539*X(15) - 152481.8*Y(14)
 0022                   Y(7)=WC*Y(7)+W*Y4(7)
                 C
                 C      TOTAL NUMBER OF CATTLE SLAUGHTERED                     Y(12)      TNCS
                 C
 0023                   Y(12)= Y(8) + Y(7)  + Y(11)
 0024                   Y(12)=WC*Y(12)+W*Y4(12)
                 C
                 C      BEEF HEIFERS FOR BREEDING                              Y(13)      BHFB
                 C
 0025                   Y(13)=-4976911.0 + 30760.9*X(14) + 104886.3*Z(7) + 9046.7*Z(8) +
                       1 .21127*X(15)
 0026                   Y(13)=WC*Y(13)+W*Y4(13)
                 C
                 C      RETAIL PRICE OF BEEF                                   Y(9)       RPB
                 C
 0027                   Y(9)=(.0021158-(.15586E-04)*Y(19)/Z(13)-(.83906E-05)*Y(18)/Z(13)-(
                       1.14793E-05)*Z(10)/Z(13)+.21576*Z(11)/(Z(9)/Z(13))-.6797*Z(5)/(Z(9)
                       1/Z(13)))*(Z(9)/Z(13))
 0028                   Y(9)=WC*Y(9)+W*Y4(9)
                 C
                 C      PRICE OF NONFED CATTLE                                 Y(10)      PNFC
                 C
 0029                   Y(10)= -3.2517 + .8422*Y(14) - .0785*Z(6)
 0030                   Y(10)=WC*Y(10)+W*Y4(10)
                 C
                 C      BEEF COW INVENTORY                                     Y(15)      BCI
                 C
 0031                   Y(15)=X(15)+Y(13)-Y(7)-.02*X(15)
 0032                   Y(15)=WC*Y(15)+W*Y4(15)
                 C
                 C
                 C      PRICE OF FED CATTLE                                    Y(16)      PFEC
                 C
 0033                   Y(16)=Y(9)*(27.72 - (.3531E-06)*Y(11)+7.84*Y(9)/Z(16))
 0034                   Y(16)=WC*Y(16)+W*Y4(16)
 0035                   Y(17)=WC*Y(17)+W*Y4(17)
                 C
                 C
                 C      TOTAL SUPPLY OF FED BEEF                               Y(18)      TSFB
 0036                   Y(18)=Y(3)
 0037                   Y(18)=WC*Y(18)+W*Y4(18)
                 C
                 C      TOTAL SUPPLY OF NONFED BEEF                            Y(19)      TSNFB
 0038                   Y(19)=Y(1)-Y(18)
                 C      PRICE OF FEEDER CATTLE                                 Y(14)      PFC
```

```
              C
0039                Y(14)=Y(9)*(16.879-(.2863E-06)*Y(11)+7.6285*Y(9)/Z(3)+11.05*Y(9)/Z
                   1(16))
0040                Y(14)=WC*Y(14)+W*Y4(14)
              C     BEEF IMPORTS                                      Z(1)      BI
              C     BEEF STOCKS                                       Z(2)      BS
              C     PRICE OF CORN                                     Z(3)      PC
              C     DAIRY COW INVENTORY                               Z(4)      DCI-1
              C     PRICE OF ALL OTHER ND+S                           Z(5)      PAO
              C     TIME 1965=16.0                                    Z(6)      T
              C     PRICE OF FEEDER CATTLE-2                          Z(7)      PFC-2
              C     PRICE OF FEEDER CATTLE-3                          Z(8)      PFC-3
              C     PERSONAL CONSUMPTION EXPENDITURES ND+S            Z(9)      PSCENDS
              C     TOTAL SUPPLY OF PORK                              Z(10)     TSP
              C     PRICE OF POULTRY                                  Z(11)     PCHICK
              C     PRICE OF CORN-1                                   Z(12)     PC-1
              C     POPULATION                                        Z(13)     POP
              C     RATIO OF FBHSS/BHSS                       Z(14)      R
              C     WAGE RATE IN MEAT PROCESSING                      Z(15)     WR
0041                RETURN
0042                END
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