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# **Investing in Agriculture as an Asset Class**

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### **Acknowledgements**

We wish to acknowledge Dr William Nganje for his comments and suggestions on earlier versions of this paper. The authors also extend appreciation to Dr. David Roberts for his constructive comments and suggestions.

This publication is available electronically at this web site: <http://agecon.lib.umn.edu/>. Please address your inquiries to: Department of Agribusiness and Applied Economics, North Dakota State University, P.O. Box 6050, Fargo, ND, 58108-6050, Ph. 701-231-7441, Fax 701-231-7400, E-mail [ndsu.agribusiness@ndsu.edu](mailto:ndsu.agribusiness@ndsu.edu).

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## Abstract

There has been massive investment in agricultural assets including farmland, handling and trading, technology, fertilizer, and others. Studies about investing in farmlands have been extensive, but have limited focus on investing IN non-farmland agricultural assets. This paper analyzes the role of farmland and other agricultural investments in class-specific portfolios. We use the Copula-VaR and Copula-VaR with restrictions methods to find, compare, and contrast the optimal portfolio compositions among US farmlands, classified agricultural equities, and grain futures. The results illustrate that farmland is attractive as an investment. However, as risk tolerance is increased, a shift to other agricultural assets would potentially bring higher returns.

EconLit Descriptors: G110, C150, D810.

## Investing in Agriculture as an Asset Class

### 1. INTRODUCTION

During the past decade there has been a substantial change in agriculture and agribusiness which ultimately is impacting investments in this sector. While numerous structural changes are occurring, most important are that allegedly demand growth is exceeding the rate of productivity growth, the spatial geography of production is changing both domestically and internationally, growth in non-traditional suppliers, agbiotechnology, greater risks and volatility, etc.<sup>1</sup> As a result of these changes there has been massive investment in agriculture broadly defined but inclusive of farmland, handling and trading, technology and others seeking to improve productivity. The investing industry is now suggesting a specific asset-class for agriculture investments (Clark, et al., 2012).

Much of the institutional and academic literature focus on investments in farmland. Indeed, there is substantial interest in farmland investments due to these changes. This has resulted in sharp increases in farmland values throughout the United States, as well as in many other countries. Sometimes these investments are compared to portfolios containing other non-specific financial assets. Focus on investing in non-farmland assets in agriculture has been limited. Of importance is that as demand growth escalates, there is a shift in demand for many technologies involved in agriculture including seeds, traits, machinery, information, logistics, etc., in addition to resources such as land. All of these benefit from changes in agriculture fundamentals resulting in increased demand for agricultural output. For these reasons there is interest in the role of investing in agricultural technologies and farmland. While there has been much attention to investing in farmland, agricultural technology has provided many investment opportunities as well.

The purpose of this paper is to analyze the role of farmland and other agricultural investments in class-specific portfolios. We derive optimal portfolios comprised of farmland and other agricultural investments to determine the extent that these are included in efficiently derived portfolios of agricultural assets. Alternative portfolios are specified under different assumptions. Specifically, we derived portfolios using Copula-VaR and Copula-VaR with restrictions on asset composition. These specifications are used to derive optimal portfolios of agricultural investments comprised of US farmland, equities in predominantly agricultural stocks, as well as investments in futures (oil, corn, and the S&P 500 index). The models were used to evaluate how farmland and other agricultural investments affect portfolio performance. Comparisons are made as risk tolerance is increased, as well as across portfolio specifications. The results illustrate that farmland is attractive as an investment. As risk tolerance is increased, there is a

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<sup>1</sup> A summary of these changes is contained in Wilson (2012).

shift to other agricultural assets. This is in part a result of the fact that returns in other agricultural assets are greater than farmland, but, in most cases they have greater risk.

The paper contributes to the evolving literature in a number of respects. First, the scope of the model allows for non-farmland agricultural specific assets to be included in the portfolio. As demand for agricultural products increases there is an escalation in demand throughout the sector to improve productivity. Generally, including other agricultural assets into an optimal portfolio has the impact of increasing returns for a given level of risk. Second, use of a Copula-VaR is novel to this literature. It allows for flexible joint distributions of returns rather than the more typical multivariate normal joint distribution assumption, a more general dependence structure and a better measurement of downside risk. As a result, these allocations provide superior portfolios than otherwise, which provide investors greater certainty regarding portfolio returns subject to downside risk.

The paper is organized as follows. First a summary of some of the literature on this topic is described below. The section that follows includes details on the model specification and data sources. Then, we present the results for each model and make comparisons. A summary and implications are described in the final section.

## 2. BACKGROUND AND PREVIOUS STUDIES

The changing fundamentals of the world agricultural market have now come to be referred to as the 9 billion people problem (*Economist*, 2011). This has evolved from increasing growth rates in consumption, declining area planted worldwide and ultimately that productivity growth rates are insufficient to meet demands. Demand has escalated due to population growth worldwide, as well as urbanization, and changes in diets with market maturity. This is in addition to the growth in use of grains and oilseeds in non-traditional uses including biofuels. These are compounded by reduced area planted in many countries and regions of the world, and that fertilizer use has increased dramatically since the early 1960s.

For all these reasons there is an alleged paradigm shift in commodity prices and profits resulting in a real appreciation of agricultural related assets which ultimately has spawned renewed interests in investing in agriculture. The *Economist* (2011a) suggested that there is massive investment in agriculture worldwide, broadly defined as farming, handling/trading, technology and logistics. The massive investments in agriculture are motivated in part by the fact that food security is rising to the top of the political agenda.

In part for these reasons, there has been growing attention to investing in agriculture. Indeed, there has been increasing attention to this topic at industry sponsored events looking to identify and describe the emerging investment alternatives

in agriculture.<sup>2</sup> A summary of all of these is that there is new interest in investing in agriculture, much of which is focused on the virtues of investing in farmland. Many of these events are focused on institutional investors looking for alternative asset classes and investment vehicles. While there is attention given typically on investing in other agriculture assets, most discussion is on farmland as an investment and limited attention is given to portfolios of agricultural assets.

## 2.1 Related Studies on Investing in Agriculture

There has been a long history of academic studies that have addressed investing in agriculture. Many of these emanate from the earlier Barry (1980) study that used the capital asset pricing model to analyze agricultural returns. Barry's analysis was extended (Irwin, Forster and Sherrick, 1988) to account for inflation and the results indicated farmland returns offered slight premiums relative to alternatives. Returns to a portfolio of Kansas farmland investments were analyzed in the early 1990s (Crisostomo and Featherstone, 1990). Their results indicated that swine and irrigated crop farms earned competitive results with common stocks and T-Bills. The role of farmland in a portfolio of stocks, bonds and business real estate indicated that farmland would continue to enter these optimal portfolios (Lins, Sherrick and Venigalla, 1992).

Bjornson and Innes (1992) analyzed returns to farmland owners relative to comparable risk in nonagricultural assets and Bjornson (1995) included time-varying conditions in their analysis of returns. Farmland was found to significantly improve the risk-efficiency of an optimal E-V frontier (Hennings, Sherrick and Barry, 2005) and fundamentally improved portfolio performance. Finally, a recent study used farm-level data from the University of Illinois endowment. Results indicated farmland plays a favorable role within an investment portfolio (Noland, Norvell, Paulson and Schnitkey, 2011).

Generally, these have been focused on farmland as an investment. Where they did include alternative assets, they were in the context of a broader non-agricultural portfolio. Painter (2010) used returns from a Farmland Real Estate Investment Trust (F-REIT) to show that both low and high risk portfolios would have benefitted from adding the F-REIT to the portfolio. The author also showed that one of the benefits of adding an F-REIT to the portfolio is the low correlation between farmland returns and other financial asset returns. Mandal and Lagerkvist (2012) found that investing in agricultural assets could provide diversification benefits during times of financial turmoil because of the low correlations between agricultural asset returns and financial asset returns.

## 2.2 Industry Studies on Agriculture Investments

Because of the dramatic increase in investment opportunities stemming from changes in agriculture, there have been several industry studies on these topics. The

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<sup>2</sup> As examples, see: Global AgInvesting (2013) and Terrapin (2013) amongst others. In addition, recent studies (FAO, 2012; and Garner and Brittain, 2012) have promoted and analyzed investments in agriculture.

general theme of these studies has been that the returns are attractive, primarily in farmland and that agriculture is emerging as an asset class.

Likely, many of these emanate from the theme developed by Hancock (2009) regarding their investments in agriculture. They show that agriculture (interpreted as ownership in farmland) is attractive as it has favorable returns, low risk, and returns are negatively correlated (or, uncorrelated) with equities and inflation. More recently Colvin and Schroeder (2012) indicated that farmland has favorable characteristics to make it an attractive investment option for private and institutional investors. They indicate that since the early 1970s, farmland has provided a 10-12% return, inclusive of current income, crop sales and lease payments, in addition to land appreciation. Further, they show that farm land has a much lower standard deviation vs. other assets (small cap equities, S&P 500, international equities, T-Bills).

Several studies have explored portfolios of varying agricultural assets. Kleinwort Benson (2010) developed a case for thematic investing in agriculture, specifically, with a focus on agricultural equities vs. soft commodities. Their results indicate that agricultural equities were attractive, liquid and their optimal agricultural portfolio was comprised of agriculture producers, suppliers, agri-services and agri-processors.

An alternative investment strategy was specified and promoted by Macquarie (2012). Their focus was outside the United States, with an emphasis on Brazil and looked for a pool of assets to diversify investment risk. Most of the emphasis is on farmland, in this case in Australia, United States and Brazil and, broad asset classes (vs. individual stocks). Specifically, they looked at broad aggregates of farm land, vs. other categories of equities and bonds.

German and Martin (2011) and Martin (2011) more recently studied the role of international farmland in portfolios and focused on returns for institutional investors. Farmland was viewed as an inflation hedge, as a diversifying source of returns, and price appreciation. Their empirical analysis examined the effect of including farmland in investment portfolios and other assets were defined as aggregates of stocks/bonds. The results indicated significant allocation of assets within portfolio including farmland in the United States and Brazil. Farmland was of value due in part to farm policies, technology (Carpenter, 2010), crop insurance as well as commodity prices, and macroeconomic measures.

In summary, there has been an evolution on the role and treatment of farmland. While there has been concern about farmland bubbles during the 1980's and again in 2013 (Pollock, 2012), there is a general notion that farmland in itself is attractive as an investment. Earlier studies began to emulate farmland as an investment relative to alternative aggregate assets and indexes. All point to the fact that farmland should comprise a share of an investment portfolio. This study extends this literature by analyzing the role of farmland relative to other agricultural assets, including technology, logistics, etc. It also extends the literature by analyzing the dependence structure that exists between farmland and other agricultural assets by using copulas.

### 3. Model Specification

We seek to analyze and determine the composition of a broadly defined agricultural portfolio inclusive of farmland, equities and other assets and to evaluate the role that land would contribute to such a portfolio. Portfolio theory provides a means to measure and manage risk. The traditional measure of risk used in portfolio problems is the variance (Markowitz, 1952). Diversification provides a method to manage risk based on reducing variance of the portfolio. Variance is a valid risk measure and linear correlation is the appropriate measure of dependence when returns to assets in the portfolio are normally distributed (Szegö, 2005). However, the normality assumption for asset returns in agriculture and outside of agriculture has proven to be limited (Just, and Weninger, 1999; Sun, et al., 2009). Alternative risk and dependency measures have been developed to account for non-normal data (Nelsen, 2006; Stoica, 2006).

Risk is sometimes measured as symmetric, treating upside and downside risk similarly. This symmetric view of uncertainty is not consistent with real-world observations (Alexander, and Baptista, 2004). Upside risk is a riskless opportunity for unexpectedly high returns. Investors are often not concerned with the upside risk but with the downside risk which is measured as the returns that fall below the individual's target return.

Use of downside risk measures in portfolio settings has been embraced in corporate finance and banking (Acerbi, 2007; Alexander and Baptista, 2002; Artzner, et al., 1999; Buch and Dorfleitner, 2008). For example, the Basel Committee on Banking Supervision utilizes a downside risk measurement in their evaluation of capital standards for banks (BCBS, 2009). They use the downside risk measure estimated by value-at-risk (VaR). Downside risk measures such as VaR have only been used in a number of agricultural applications (e.g., Manfredo, and Leuthold, 2001; Wilson, Nganje, and Hawes, 2007; among others).

We developed two portfolio optimization models to determine the allocation of investments amongst a group of agricultural assets. The assets are comprised of publicly traded agricultural equities, farm land, and other agricultural related assets. The models are specified and solved under different risk tolerance assumptions. The two models are Copula-VaR and Copula-VaR with restrictions. In the latter, maximum restrictions are imposed on shares of individual assets within the portfolio. The results illustrate the composition of agricultural related asset in the portfolio, the role of farmland in the portfolio, and how the composition of the portfolio changes with different levels of risk tolerance.

#### 3.1 Copula-VaR Optimization

The traditional approach to analyzing portfolios relies on multivariate normal distributions (Markowitz, 1952). However, normality assumptions for agricultural prices and yields have been shown to be invalid (Goodwin, and Ker, 2002; Just, and Weninger, 1999). Copulas are an alternative for modeling joint distributions that has been gaining popularity in financial literature including portfolio analysis (Alexander,

Baptista, and Yan, 2007; Alexander, Coleman, and Li, 2006; Bai and Sun, 2007; Bouyé, et al., 2001; Clemen and Reilly, 1999; Dias, 2004; Hennessy and Lapan, 2002). The main advantage of copulas is the asset's distributions can be specified as non-normal, in addition to the flexibility in specifying the dependency among the distributions of returns. While copulas have been used in finance for some time, the applications of copulas in the agricultural literature are recent (Vedenov, 2008; Zhu, Ghosh, and Goodwin, 2008). These studies use multivariate copula methodology to model the joint distribution of random variables of interest.

The Copula specification can be expanded by including VaR (Value-at-Risk) restrictions. VaR has become a popular method to manage risk [e.g. as discussed in Jorion (2007), Linsemeir and Person (1996) and Winston (1998) for applications using stochastic simulation, and used in agriculture by Manfredo and Leuthod (1999) and Wilson, Nganje and Hawes (2007), amongst others]. VaR can be defined as a single, summary statistic that measures the worst expected losses during a given time period, with a specified level of confidence, under normal market conditions (Jorion, 2007). Mathematically, VaR can be specified as:

$$VaR(x, \alpha) = \min\{\zeta \in \mathbb{R} | \Psi(x, \zeta) \geq \alpha\}, \quad (1)$$

where  $x$  is the random variable,  $\alpha$  is the confidence interval,  $\zeta$  is the lowest possible value, and  $\Psi$  stands for the cumulative probability of  $x$ . Calculation of VaR requires knowledge of the cumulative distribution function of portfolio returns, which in turn depends on the joint distribution of returns of all assets in the portfolio.

Our goal is to find the optimal allocation of investments to maximize the expected return subject to a tolerable maximum percentage of loss to the portfolio (i.e. portfolio VaR). The Copula-VaR method can be defined as using a copula to capture the joint distribution and dependence structure, while using Value at Risk (VaR) as the risk constraint. Mathematically, it is defined as:

$$\begin{aligned} & \text{Maximize } \mu(H(x_1, \dots, x_l)) \\ & \text{Subject to } VaR(H(x_1, \dots, x_l)) \leq V, \end{aligned} \quad (2)$$

where  $H(x_1, \dots, x_l)$  is the joint distribution of all assets specified from appropriate multivariate copula,  $\mu(H(x_1, \dots, x_l))$  is the expected return of the joint distribution, and  $V$  is the maximum tolerable percentage of loss for the portfolio. For the empirical model, the objective function and constraints are:

$$\begin{aligned} & \text{Maximize } \mu_p(\omega_{1, \dots, N}) \\ & \text{Subject to } \varepsilon_p(\omega_{1, \dots, N}) \leq V; \end{aligned} \quad (3)$$

$$\text{Max}(\omega_i) \leq W_i;$$

$$\text{and } \sum_{i=1}^N \omega_i = 1, \omega_i \geq 0, \forall i$$

where  $\mu_p$  and  $\varepsilon_p$  are the portfolio mean and 5% VaR respectively of different weights combinations,  $V$  is the maximum loss the investor willing to tolerate at 95% confidence level, and  $W_i$  is the maximum investment weight of each asset. For Copula-VaR model,  $W_i$  equals 1, while  $W_i$  is set to be 0.1 for the Copula-VaR with Restrictions model.

### 3.2 Copula Specification

The term Copula originates from the Latin term referring to link, join, or connect. Application of copulas to model multivariate distributions is described in numerous books (e.g., Vose 2008; Cherubini, Luciano, and Vecchiato, 2004; Nelsen, 2006).<sup>3</sup> A copula function is formally defined as an  $n$ -dimensional multivariate cumulative distribution function defined on the  $n$ -dimensional unit cube  $[0,1]^n$  with the properties (i)  $C(u_1, \dots, u_n) = 0$  if any  $u_i = 0, i = 1, \dots, n$ , and (ii)  $C(1, \dots, 1, u_i, 1, \dots, 1) = u_i$  for any  $u_i, i = 1, \dots, n$ . The copulas are related to joint distributions through Sklar's theorem, which (in a two-variable case) postulates that if  $H$  is a joint distribution function with margins  $F$  and  $G$ , then there exists a copula function  $C$  such that for all  $x, y \in \bar{R}$ ,  $H(x, y) = C(F(x), G(y))$  (e.g. (Nelsen, 2006)).

The Sklar theorem allows construction of a joint distribution of several random variables based on their marginal distributions and a specified copula function. There are an infinite number of copula functions and thus an infinite number of joint distributions that may be generated for given marginals. Various copula families have been used in risk research. As examples, Gaussian, Archimedean, etc. are discussed in (Hennessy, and Lapan, 2002)). Three copulas from the Archimedean family (Clayton, Frank, and Gumbel), Gaussian, and T copula are used in this research.

The Gaussian Copula is an extension of the multivariate normal distribution. It can be used to model multivariate data that may exhibit non-normal dependencies and fat tails. The Gaussian Copula is formally defined as:

$$C(u_1, \dots, u_n; \Sigma) = \Phi^K(\Phi^{-1}(u_1), \dots, \Phi^{-1}(u_n); \Sigma), \quad (4)$$

where  $\Phi(\cdot)$  is the cumulative distribution function of the standard normal distribution and  $\Sigma$  is the covariance matrix. In the two-dimensional case, the Gaussian copula density function can be written as:

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<sup>3</sup> For a complete review of copula theory refer to Joe (1997) and Nelsen (2006).

$$c(u, v) = \frac{1}{\sqrt{1-\rho^2}} \exp \left( \frac{(\Phi^{-1}(u))^2 + (\Phi^{-1}(v))^2}{2} + \frac{2\rho\Phi^{-1}(u)\Phi^{-1}(v) - (\Phi^{-1}(u))^2 - (\Phi^{-1}(v))^2}{2(1-\rho^2)} \right), \quad (5)$$

where  $\rho$  is the linear correlation between the two variables and  $\Phi(\cdot)$  is the cumulative density function of the standard normal distribution. One of the useful features of the Gaussian copula is that it is parameterized by a single parameter (correlation coefficient) which can be estimated from historical data.

The t copula is derived from the multivariate standardized t-Student distribution and is defined as:

$$C(u_1, \dots, u_n; \Sigma, \nu) = T_{\Sigma, \nu}(t_{\nu}^{-1}(\hat{u}_1), \dots, t_{\nu}^{-1}(\hat{u}_n))', \quad (6)$$

where  $T_{\Sigma, \nu}$  is defined as the standardized multivariate Student's t distribution function,  $\Sigma$  is the correlation matrix, and  $\nu$  are the degrees of freedom.  $t_{\nu}^{-1}(\hat{u})$  is used to denote the inverse of the Student's t cdf function. In the two dimensional case, the T copula density can be written as:

$$c(u, v) = |\Sigma|^{-1/2} \frac{\Gamma(\frac{\nu+n}{2})}{\Gamma(\frac{\nu}{2})} \left[ \frac{\Gamma(\frac{\nu}{2})}{\Gamma(\frac{\nu+1}{2})} \right]^n \frac{\left(1 + \zeta' \Sigma^{-1} \zeta\right)^{-\frac{\nu+n}{2}}}{\prod_{i=1}^n \left(1 + \frac{\zeta_i^2}{2}\right)^{-\frac{\nu+1}{2}}} \quad (7)$$

where  $\zeta$  is the vector of the T-student univariate inverse distribution functions. Both of these copulas are well formulated to take beyond the bivariate case.

The Archimedean copulas are defined by:

$$C(u, v) = \varphi^{-1}(\varphi(u) + \varphi(v)), \quad (8)$$

where  $\varphi$  is the generator of the copula. Here, we introduce its bivariate form. The reason we are choosing a two-dimension copula other than higher dimension is because the bivariate copula is easy to interpret, demonstrate, and estimate. But extension to higher dimensions through mixtures of powers and pairwise likelihood inference can be achieved. One of the most appealing features of Archimedean copulas is the relationship between the generator of the copula  $\varphi$ , and Kendall's tau.

This relationship can be defined by:

$$\tau = 1 + 4 \int_0^1 \frac{\varphi_\alpha(t)}{\varphi_\alpha(t)} dt, \quad (9)$$

where  $\tau$  is Kendall's tau. This provides a method of comparing rank correlation measures using different dependence structures. Three specific Archimedean copulas are used in this research, Clayton, Frank, and Gumbel. The Clayton copula is an asymmetric copula and exhibits greater dependence in the lower tail. The Frank copula on the other hand is a symmetric copula and weights the tails of the data equally. The

Gumbel copula is an asymmetric copula and exhibits greater dependence in the upper tail.

Implementation of copulas involves three steps including: 1) select and construct a copula, 2) estimate the parameters associated with the copula, and 3) sample from the parameterized copula. The Gaussian, t-copula, and Archimedean Copula are fitted and compared in this research. Details on their construction and selection are discussed in the next sections. Copula parameters are estimated through a maximum likelihood estimation method of the form of:

$$\hat{\delta}_2 = \operatorname{argmax}_{\hat{\delta}_2} \sum_{t=1}^T \ln c(\hat{G}_x(x_t), \hat{H}_y(y_t), \delta_2), \quad (10)$$

where  $\hat{\delta}_2$  is the estimated copula parameter,  $\operatorname{argmax}$  is the mathematical functions that provides the argument associated with the maximum,  $\ln$  is the natural logarithm, and  $\hat{G}_x(x_t), \hat{H}_y(y_t)$  are the estimated marginal distributions for  $x$  and  $y$ . To avoid distributional assumptions, a non-parametric distribution is used for the marginal distributions. Schwarz Information Criteria (SIC) and Akaike Information Criteria (AIC) were utilized for selecting the most appropriate multivariate copula. AIC and SIC are superior goodness of fit statistics to other fit ranking criteria (e.g. chi-squared), where AIC is less strict among the two. The final step is to simulate values from the estimated copula. Using this framework, a large sample of simulations can be generated to be used as input into the optimization routine.

Some bivariate copula relationships of interest are demonstrated in Figure 1 for illustration purposes (as our empirical analysis use multivariate copula). For CAT vs. the S&P, the best fit bivariate copula is Frank. This shows equal dependence on both tails. However, the Clayton copula was chosen to be the best fit for AGU vs. FMC. This stresses some dependence on the lower tails. Each pair may have different copula dependence, but we are more interested in the multivariate copula fitted for the whole data set. The VaR methodology complements copulas because the focus is on the lower tails of the data. The flexibility of the copula modeling allows the shape of each marginal distribution to be maintained and in theory more accurately capturing the risk (VaR) that exists in the lower tails.

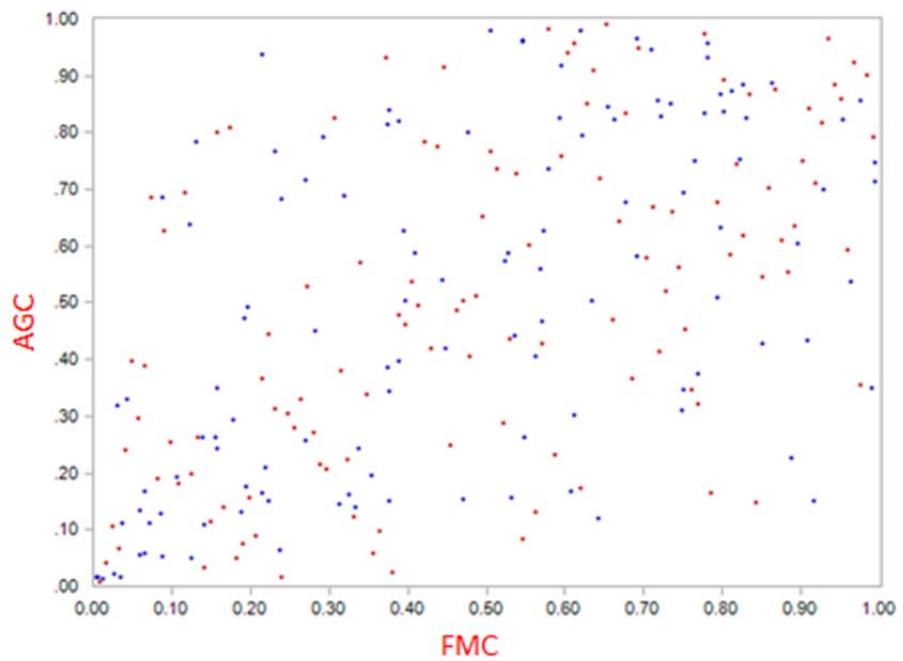
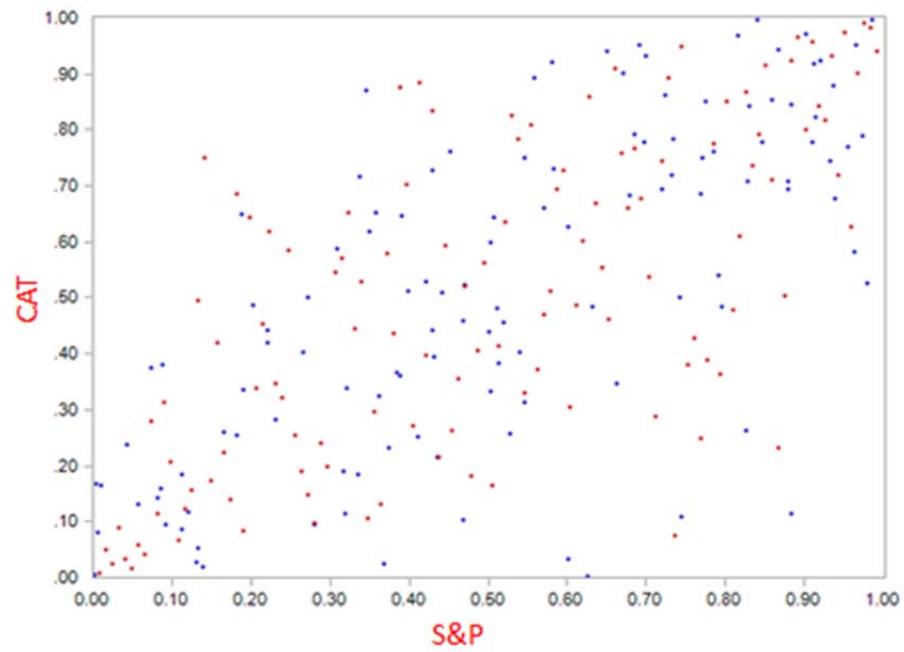


Figure 1. Bivariate Copula distributions: Upper Panel, CAT vs. S&P: Frank Copula, Lower Panel, AGU vs. FMC: Clayton.

### 3.3 Data Sources

The impetus behind specification of the publicly traded agricultural equities was from Patterson (2011) who reported returns to stocks in this group. This was the source of our initial definition of included stocks. These twenty-nine publicly traded companies were selected from different stock exchange markets, i.e. NYSE, NASDAQ, and Toronto. We further categorized these stocks into fertilizer, technology, chemical and seeds, grain trading, food safety, and transportation industry. The duration of analysis for the equity assets was monthly. Monthly futures prices were obtained from CME and NYMEX.

In total, we included thirty-seven agriculture related assets in the portfolio: five state level farmland values, CME corn, soybean, S&P Index and crude oil futures, and 29 agriculture-related publicly traded equity stocks. These assets' ticker name, company name, and description are in Table 1. The data period was from 2002 to 2011, though the life-span of some stocks was shorter. State average cropland values were for North Dakota, Michigan, Minnesota, Illinois, and Nebraska. This was from 2002 to 2011 quarterly data, which we converted into monthly data in concordance with the other assets.

**Table 1: Selected assets' name and description**

Ticker	Assets	Description	Best fit univariate distribution
CNH	CNH Global N.V.	Ag and Construction Equipment	Logistic(0.0087,0.0473)
DE	Deere and Co.	Ag and Forestry Equipment	Logistic(0.0134,0.0654)
BUI	Buhler Industries	Ag Equipment	Logistic(0.0230,0.0624)
CAT	Caterpillar, Inc.	Ag Equipment	Logistic(0.0151,0.0498)
AGCO	AGCO Corp.	Ag Equipment	Logistic(0.0040,0.0284)
RAVN	Raven Industries, Inc.	Ag Equipment--electronics	Logistic(0.0167,0.0528)
ADM	Archer Daniels Midland Co.	Ag Products	Logistic(0.0059,0.0328)
AGU	Agrium Inc.	Ag Products	Logistic(0.0077,0.0415)
AFN	Ag Growth International Inc.	Ag Products	Logistic(0.0122,0.0907)
MON	Monsanto Co.	Ag Products and Seeds	Normal(0.0103,0.0580)
FMC	FMC Corp.	Chemical	Logistic(0.0132,0.0459)
SYT	Syngenta Ag	Chemical and Seed	Logistic(0.0165,0.0504)
POT	Potash Corp Of Sask Inc.	Fertilizer	Logistic(0.0211,0.0487)
TNH	Terra Nitrogen Co. L.P.	Fertilizer	Normal(0.0063,0.0406)
CF	CF Industries Holdings Inc	Fertilizer	Normal(-0.0078,0.1712)
MOS	The Mosaic Co.	Fertilizer	Logistic(0.0164,0.0548)
IPI	Intrepid Potash, Inc.	Fertilizer	Logistic(0.0201,0.0509)
CAG	ConAgra Foods Inc.	Food	Logistic(0.0284,0.0591)
CBY	Canada Bread	Food	Normal(0.0224,0.1060)
GIS	General Mills, Inc.	Food	Logistic(0.0191,0.0383)

BG	Bunge Ltd.	Food	Logistic(0.0319,0.0833)
NEOG	Neogen Corporation	Food Safety	Logistic(0.0212,0.0670)
SOY	Sunopta, Inc.	Organic food	Laplace(0.0063,0.1706)
TRMB	Trimble Navigation Ltd.	Technology	Laplace(0.0257,0.1334)
HEM	Hemisphere GPS Inc.	Technology	Laplace(0.0341,0.1254)
VT	Viterra Inc.	Trading	Laplace(0.0104,0.0693)
AGT	Alliance Grain Traders Inc.	Trading	Laplace(0.0106,0.0915)
CNR	Canadian National Railway Co.	Transportation	Logistic(0.0148,0.0666)
CP	Canadian Pacific Railway Ltd.	Transportation	Laplace(0.0061,0.0656)
ND	North Dakota	Farmland	JohnsonB(-0.5267,0.1599,-0.0010,0.0158)
MI	Michigan	Farmland	Beta4(0.7184,0.4561,-0.0026,0.0102)
MN	Minnesota	Farmland	JohnsonB(-0.0100,0.0796,-0.0031,0.0184)
IL	Illinois	Farmland	JohnsonB(-0.5513,0.1814,-0.0028,0.0119)
NE	Nebraska	Farmland	JohnsonB(-0.4050,0.0890,0.0006,0.0137)
Corn	CBOT Corn	Futures	Logistic(0.0155,0.0529)
Oil	Crude Oil	Futures	Logistic(0.0191,0.0558)
S&P	S&P Index	ETF	Laplace(0.0093,0.04793)

We computed monthly logarithmic price returns for the assets. As it turns out, the data included a stable period (2002-2006), crisis period (2007-2009), and post crisis period (2010-2011), which are sufficient for us to investigate the potential tail

dependence. All the returns are calculated as the percentage logarithmic price ratio. Table 2 and Figures 2 and 3 reports the summary statistics for the assets, including mean return, standard deviation, skewness, and kurtosis.

There have been substantial returns for the agricultural equities and, these have exceeded those for the S&P, futures, as well as for farmland values in some cases. Raven Industry, Terra Nitrogen, and CF Industries are among the top three performing companies with the largest average return, but they have relatively high risk. For comparison, five cropland assets have the lowest average return, but their risks compared to stocks are almost risk-free. Measures for skewness and kurtosis suggest that most assets are not normally distributed.

**Table 2: Monthly logarithmic returns summary statistics**

Stock	Mean	Std Dev	Skewness	Kurtosis
ADM	0.0075	0.0869	-0.0608	1.6498
AGCO	0.0100	0.1186	-0.2426	1.0537
AGU	0.0178	0.1148	-0.7906	1.7934
BG	0.0102	0.1009	-1.2560	6.0587
BUI	0.0036	0.0548	-0.2494	2.9897
CAT	0.0134	0.1029	-0.8263	3.8653
CAG	0.0037	0.0600	-0.4810	0.9679
CBY	0.0073	0.0773	-0.0098	0.9878
CNH	0.0040	0.1773	-0.8788	2.8213
CNR	0.0103	0.0580	-0.1732	0.0676
CP	0.0121	0.0838	-0.4329	1.8882
DE	0.0128	0.0923	-0.7306	1.7361
FMC	0.0138	0.0925	-1.1928	2.6771
GIS	0.0063	0.0406	-0.4558	0.0157
HEM	-0.0078	0.1712	-0.0308	-0.0887
MON	0.0143	0.0989	-0.5300	1.7042
NEOG	0.0152	0.0917	-0.4502	0.4559
POT	0.0221	0.1137	-0.8702	2.7196
RAVN	0.0224	0.1060	-0.2674	-0.0441
SYT	0.0152	0.0719	-0.9078	1.8912

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TNH	0.0348	0.1490	0.3477	0.8992
TRMB	0.0178	0.1222	-0.3218	0.9185
SOY	-0.0072	0.1944	-1.4471	6.5410
MOS	0.0151	0.1532	-0.9279	2.0463
CF	0.0306	0.1476	-0.9724	2.2531
VT	0.0043	0.0916	-0.1843	0.9131
AFN	0.0167	0.1097	-0.4005	0.2276
IPI	-0.0142	0.1693	-0.1717	-0.0133
AGT	0.0084	0.1159	0.6670	1.6086
ND	0.0070	0.0053	0.1667	-0.9447
MI	0.0047	0.0038	-0.8612	-0.2247
MN	0.0074	0.0062	0.1305	-0.8852
IL	0.0071	0.0040	-1.2819	1.2076
NE	0.0078	0.0043	-0.1419	-1.4199
Corn	0.0095	0.0956	-0.5214	0.3090
Oil	0.0132	0.1008	-0.8798	2.2946
S&P	0.0010	0.0467	-0.8415	1.7353

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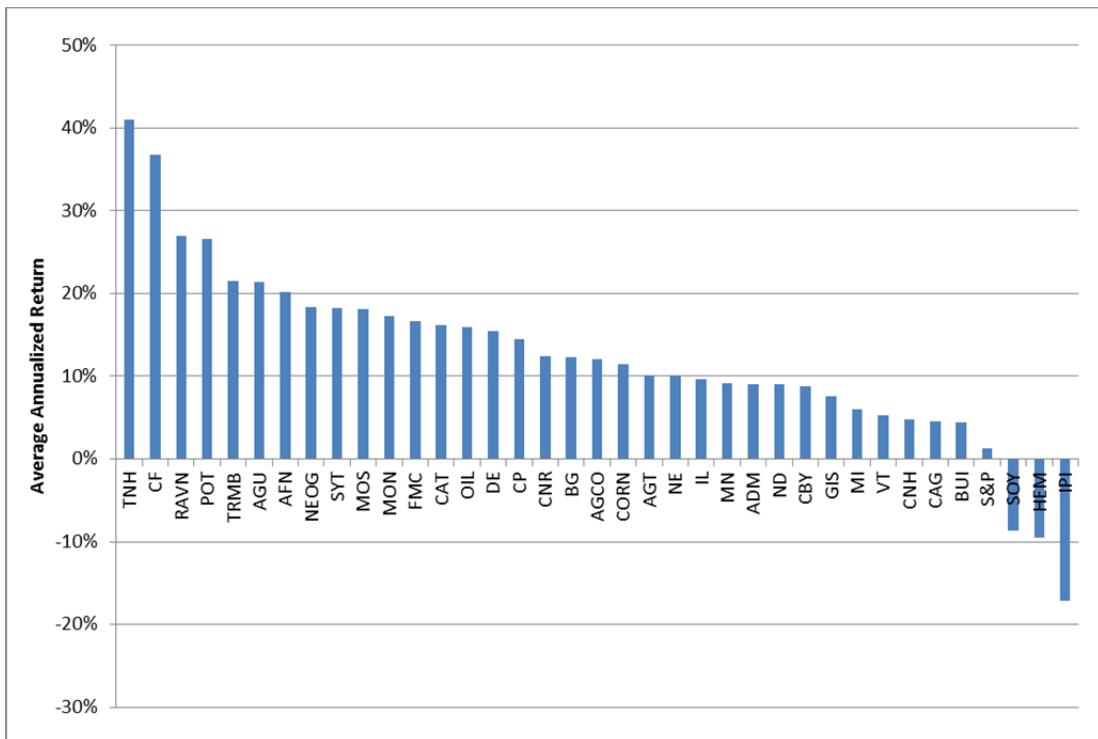


Figure 2: Average annualized returns by category (2001-2011)

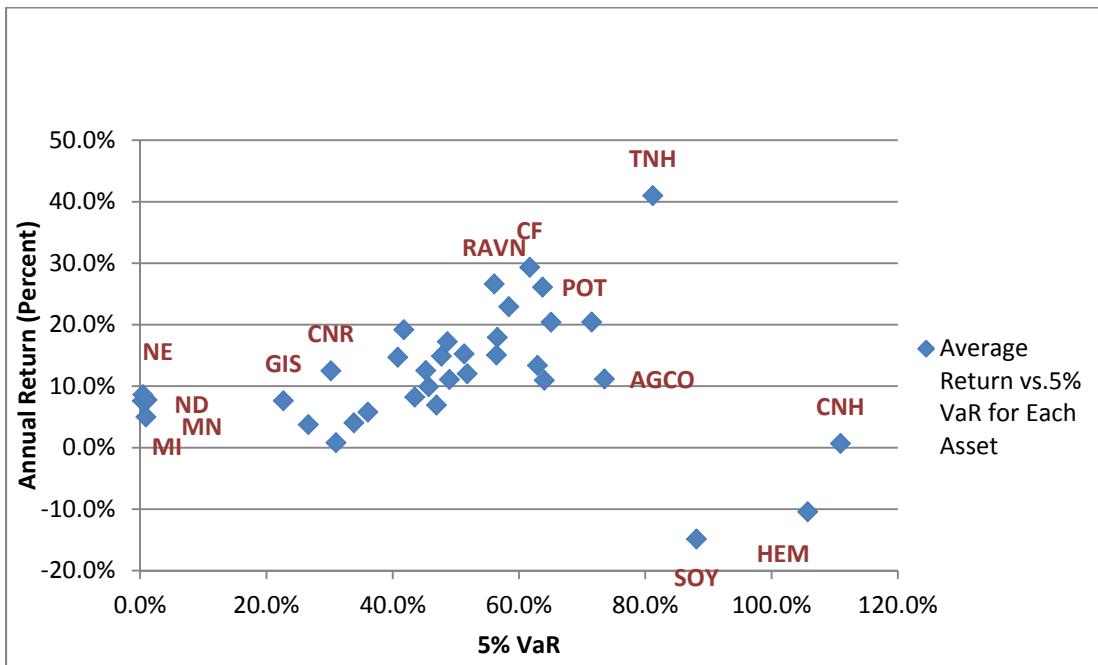


Figure 3: Annualized risk and returns by agricultural assets

#### 4. EMPIRICAL ANALYSIS

An important analytical derivation for the Copula-VaR for the empirical model is finding the appropriate univariate marginal distribution  $F_1, F_2, \dots, F_n$ . For each asset's logarithmic return, we fit the most appropriate marginal distribution. All the marginal distributions are listed in the last column of Table 1. The evidence suggests that most marginal distributions are not normal. Most assets can be represented as logistic, normal, laplace, Johnson, or beta distributions. The most common is the logistic distribution.

Maximum Likelihood estimation was used to determine the correct copula family, functional form and estimated the copula parameters. Next we select the best fit multivariate copula to the entire data set. All the major copulas were ranked, and Gaussian copula was chosen to be the best fit based on SIC information criteria.

For the Copula-VaR, we fit univariate marginals into Gaussian copula to yield joint distribution, and simulated 10,000 new observations. SAS was used to estimate the Gaussian Copula parameters and sample 10,000 simulated returns from the multivariate joint distribution. From the simulations we computed the portfolio mean  $\mu_p$  and 5% VaR  $\varepsilon_p$  for each possible portfolio combination. The 5% VaR is defined as having 95% confidence that the portfolio will not incur a loss below this specific value over a month under normal conditions. Lastly, we choose the optimal portfolio combination that yields the maximum portfolio mean subject to a certain level of 5% VaR tolerance.

#### 5. RESULTS & INTERPRETATION

Results from each model are compared first as risk tolerance is increased, and then across specifications.

##### 5.1 Copula- VaR:

Important to the Copula-VaR results are the characteristics of the dependence and joint distributions. Specifically, 1) Copulas more accurately capture the dependence and joint distribution among assets (without requiring assumptions for linearity dependence and normal distribution of any assets); 2) VaR provides investors' a simple measure to capture the maximum loss of any joint distribution. Thus, the Copula-VaR was chosen versus the traditional Mean-Variance method. We also computed the optimal portfolio selection with Mean-Variance (MV) approach for illustration and comparison of how they differ. The difference may result from strict assumptions and risk specification, i.e. multivariate normal joint distribution and linear correlation assumptions, and variance as a risk measurement to penalize both tails of distribution. From the methods and results comparison, Copula-VaR is considered a superior method to typical MV method.

The Copula-VaR results are shown in Table 3. For low levels of risk tolerance, these results have a portfolio comprised of land, primarily in Nebraska, but also smaller

shares in ND, Minnesota and Illinois. None of the futures assets were included in the optimal portfolio. There are minor holdings in Raven Industries (technology) and TNH (fertilizer).

Increasing the allowable risk results in increasing returns and changes in the composition of assets. Increasing the 5% VaR restrictions to 25, 30, 35, and 40%, results in increases in the return from 17% to 19.5%, 21%, 22.5%, and 23.33% per year, respectively. These are illustrated in the efficient frontier in Figure 4 and compared to returns and risks of the individual assets. As greater risk is allowed, the composition of the portfolio shifts. Specifically, there are shifts away from farmland with less of Nebraska and an increase in technology investments. Notably these include increases in technology (Ravn), food safety/testing (Neogen), seed technology and ag chemicals (Syngenta) and fertilizer (Potash, TNH). The higher risk portfolios consists of less land (notable reductions in Nebraska), and increases in Syngenta, CF Industries (fertilizer), and, interestingly, CN and CP Rail stocks.

For comparison we ran the model using a mean-variance optimization and the results differ. Land comprises a greater share of assets as risk increases in the Copula-VaR. Second, the mean-variance result has a greater share of assets in Raven (technology), Syngenta and TNY. Third, the mean-variance result has long oil and corn futures, while the Copula-VaR would not. No attempt was made to reconcile these as the models have fundamentally different underlying specifications and they are not expected to be the same.

**Table 3: Copula-VaR with no weight restrictions**

Mean Return	17.04%	19.47%	20.99%	22.46%	23.33%
5% VaR	20%	25%	30%	35%	40%
ADM	0.0000	0.0000	0.0000	0.0000	0.0000
AGCO	0.0000	0.0000	0.0000	0.0000	0.0000
AGU	0.0000	0.0000	0.0000	0.0000	0.0002
BG	0.0000	0.0000	0.0000	0.0000	0.0001
BUI	0.0000	0.0000	0.0000	0.0000	0.0000
CAT	0.0000	0.0000	0.0000	0.0000	0.0000
CAG	0.0000	0.0000	0.0000	0.0000	0.0000
CBY	0.0000	0.0000	0.0000	0.0000	0.0000
CNH	0.0000	0.0000	0.0000	0.0000	0.0000
CNR	0.0006	0.0011	0.0566	0.0005	0.0521
CP	0.0006	0.0013	0.0000	0.0000	0.0405

DE	0.0000	0.0000	0.0000	0.0000	0.0000
FMC	0.0000	0.0000	0.0000	0.0000	0.0000
GIS	0.0000	0.0000	0.0000	0.0000	0.0000
HEM	0.0000	0.0000	0.0000	0.0000	0.0000
MON	0.0000	0.0000	0.0000	0.0000	0.0052
NEOG	0.0000	0.0000	0.0000	0.0003	0.0002
POT	0.0004	0.0000	0.0000	0.0001	0.0005
RAVN	0.1162	0.1156	0.0753	0.1871	0.1378
SYT	0.0040	0.0090	0.0512	0.0008	0.1259
TNH	0.1652	0.2365	0.2649	0.2075	0.1506
TRMB	0.0015	0.0010	0.0000	0.0002	0.0036
SOY	0.0000	0.0000	0.0000	0.0000	0.0000
MOS	0.0000	0.0000	0.0000	0.0000	0.0012
CF	0.0160	0.0081	0.0369	0.1463	0.1930
VT	0.0000	0.0000	0.0000	0.0000	0.0000
AFN	0.0000	0.0000	0.0000	0.0000	0.0000
IPI	0.0000	0.0000	0.0000	0.0000	0.0000
AGT	0.0000	0.0000	0.0000	0.0000	0.0000
ND	0.0007	0.0001	0.0002	0.0004	0.0001
MI	0.0000	0.0000	0.0000	0.0000	0.0008
MN	0.0005	0.0001	0.0038	0.0002	0.0106
IL	0.0062	0.0001	0.0034	0.0144	0.0276
NE	0.6879	0.6270	0.5075	0.4420	0.2488
CORN	0.0000	0.0000	0.0000	0.0000	0.0000
OIL	0.0000	0.0000	0.0000	0.0000	0.0009
S&P	0.0000	0.0000	0.0000	0.0000	0.0000

## 5.2 Copula-VaR with Restrictions:

Adding restrictions to the weights for individual holdings in the Copula-VaR model generates a lower return at the same VaR value, while it allows investors to separate restrictions in holdings in different companies. The result of Copula-VaR with

restrictions is shown in Table 4 and a comparison with Copula-VaR is shown in Figure 4. With 10% maximum restriction on any one asset, the model allocates 50% of its assets in farm land, including holdings in each of the major states. Other assets are primarily in CNR, BUI, minor shares in Raven and positions in oil were nil. Increasing the risk tolerance reduces the overall position in farm land to 31%, and greater shares of fertilizer (Potash), Monsanto, and each of Ravn, Syngenta, TNH and TRMB.

Finally, comparing the Copula-VaR to Copula-VaR restricted indicates that the latter would have a lower return by about 3-5%, for a given level of risk tolerance. While there is lesser share of assets in farmland, the Copula-VaR has greater shares allocated across a number of states, vs. concentrating holdings in Nebraska. It would also hold a slightly different composition of fertilizer assets shifting amount Potash Corporation (+), CF Industries (+) and reductions in TNH (-) by about comparable amounts. The difference possibly results from individual's restricted investment for some better return assets with the same level of risk.

**Table 4: Copula-VaR with 10% weight restrictions**

Mean Return	12.50%	15.91%	18.20%	19.77%	20.87%
5% VaR	20%	25%	30%	35%	40%
ADM	0.0000	0.0000	0.0000	0.0000	0.0000
AGCO	0.0000	0.0000	0.0000	0.0000	0.0000
AGU	0.0000	0.0000	0.0000	0.0000	0.0000
BG	0.0000	0.0000	0.0000	0.0000	0.0000
BUI	0.0966	0.0000	0.0000	0.0000	0.0000
CAT	0.0000	0.0000	0.0000	0.0000	0.0000
CAG	0.0000	0.0000	0.0000	0.0000	0.0000
CBY	0.0000	0.0000	0.0000	0.0000	0.0000
CNH	0.0000	0.0000	0.0000	0.0000	0.0000
CNR	0.1000	0.1000	0.0030	0.0029	0.0000
CP	0.0000	0.0000	0.0000	0.0000	0.0000
DE	0.0000	0.0000	0.0000	0.0000	0.0000
FMC	0.0000	0.0000	0.0000	0.0000	0.0000
GIS	0.1000	0.0448	0.0000	0.0000	0.0000
HEM	0.0000	0.0000	0.0000	0.0000	0.0000

MON	0.0000	0.0000	0.0000	0.0000	0.0860
NEOG	0.0000	0.0000	0.0000	0.0000	0.0000
POT	0.0000	0.0000	0.0970	0.1000	0.1000
RAVN	0.0034	0.1000	0.1000	0.1000	0.1000
SYT	0.1000	0.1000	0.1000	0.1000	0.1000
TNH	0.1000	0.1000	0.1000	0.1000	0.1000
TRMB	0.0000	0.0000	0.0000	0.0838	0.1000
SOY	0.0000	0.0000	0.0000	0.0000	0.0000
MOS	0.0000	0.0000	0.0000	0.0000	0.0000
CF	0.0000	0.0552	0.1000	0.1000	0.1000
VT	0.0000	0.0000	0.0000	0.0000	0.0000
AFN	0.0000	0.0000	0.0000	0.0000	0.0000
IPI	0.0000	0.0000	0.0000	0.0000	0.0000
AGT	0.0000	0.0000	0.0000	0.0000	0.0000
ND	0.1000	0.1000	0.1000	0.1000	0.0140
MI	0.1000	0.1000	0.1000	0.0133	0.0000
MN	0.1000	0.1000	0.1000	0.1000	0.1000
IL	0.1000	0.1000	0.1000	0.1000	0.1000
NE	0.1000	0.1000	0.1000	0.1000	0.1000
CORN	0.0000	0.0000	0.0000	0.0000	0.0000
OIL	0.0000	0.0000	0.0000	0.0000	0.0000
S&P	0.0000	0.0000	0.0000	0.0000	0.0000

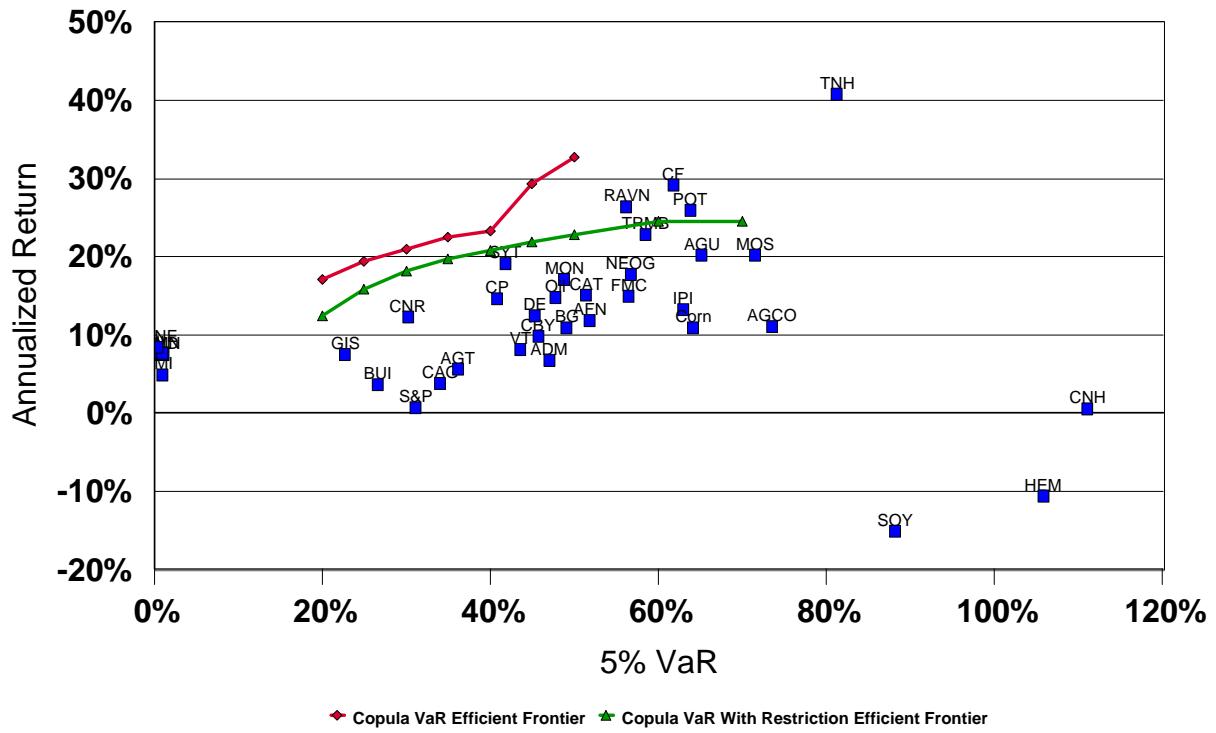


Figure 4: VaR-Copula with/without max weight constraint results comparison

## 6. SUMMARY AND IMPLICATIONS

There has been an escalating interest for investing in agriculture in recent years. Generally, this has focused on investing in farmland, though investments in other agricultural assets have been of similar interest. The growing interest in agricultural investments has been driven in part by the disparity between the growth in demand versus agricultural productivity. As a result, prices have escalated relative to costs, though there has been concurrent cost inflation, and profits throughout much of the agricultural industry have escalated. Alternative investments in agriculture range from investing in commodity hedge funds (Plevan 2012) which have shorter-term volatility but are more liquid, to farmland which has less volatility and liquidity. Farmland is one of the more limiting agricultural inputs. The greatest prospect for future returns in farmland are likely greatest in regions/countries going through new technology adoption whereby input values become capitalized in land from past technology and new technology increases value of inputs (land). Investment opportunities are similar in other agricultural inputs that reduce production and marketing costs, and improve technical efficiency of growers. These include fertilizer, farm machinery and equipment, agbiotechnology, seeds/germplasm, storage as well as investments in facilitating functions include food safety, GPS and farm management.

The purpose of this paper is to analyze the role of farmland and other agricultural investments in class-specific portfolios. We derived optimal portfolios comprised of farmland and other agricultural investments to determine the extent that these are included in efficiently derived portfolios of agricultural assets. Alternative portfolios are specified under different assumptions including Copula-VaR and Copula-VaR with restrictions. The advantage of using the Copula-VaR is that we can incorporate non-normal returns and dependencies. The combination of these provides greater flexibility of the distributional functions to capture the underlying relationship amongst assets.

The results indicate that land dominates the portfolios, particularly for lower risk tolerances. As risk tolerance increases, returns increase (substantially), and the composition of assets changes. For the Copula-VaR, there is less farmland and it is more diversified with holdings in Raven (technology), and TNH (fertilizer). The portfolio asset composition changes when the risk tolerance increases, with less land (notable reductions in Nebraska), and increases in Syngenta, CF Industries (fertilizer), and, interesting, CN and CP Rail stocks.

It is of interest that the S&P futures did not enter the portfolio as an asset. This suggests that agricultural assets have outperformed the broader market index. Further to this, our analysis used data through 2011. For comparison, we evaluated the return on our Copula-VaR portfolio during 2012, relative to the overall market. These results showed that the return during 2012 on our Copula-VaR was 22% (slightly different for different risk aversions), compared to the S&P which increased 12.6%. These results again support the robustness of agriculture, broadly defined to be inclusive of land, technology, seeds and traits, logistics, etc., is performing well as an asset class.

These results also differ compared to other recent studies. Other studies used MV and/or other comparative statistics and concluded that farmland should be an element of a portfolio of assets. The reason for this is mostly that it generates a better return that is lower in risk. However, these studies do not compare returns to investment in other agricultural assets. These results show that investment in other agricultural assets are important elements of an agricultural portfolio, and, that as risk tolerance increases, there are shifts generally toward technology, fertilizer, seeds and traits, and logistics.

The paper contributes to the growing literature in agricultural investing. First, it uses fairly state-of-the art methods for specifying portfolios. Second, it shows that other agricultural assets should be a part of a portfolio inclusive of farmland. As risk tolerance increases, more non-land assets enter the portfolio along with increased returns.

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