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Hospital Sizes for Rural Areas When Patient Arrivals Are Poisson Distributed

By Clark Edwards and Neville Doherty¹

This study applies the logic of queuing theory to the availability of hospital services in a rural-oriented, multicounty planning district in northwestern lower Michigan. The seven hospitals in the area had 611 beds and provided services for 489 patient-days in 1 year. The study found that the seven hospitals apparently cooperate with one another, at least partially, in providing services, because if they operated independently they would not have sufficient capacity for peak loads. The observed number of beds, which is about 5.5 standard deviations above the expected number of arrivals, is almost certainly adequate for peak loads if the hospitals cooperate fully.

Key words: Rural development; rural and health services; health facilities; statistical methods.

In rural areas, difficulties related to distance, sparse population, interjurisdictional rivalries, and low average incomes create problems in delivering certain community services. As a consequence, services that might feasibly be delivered efficiently to residents of a multicounty area by a single organization are frequently divided among several. This results in higher costs and poorer services than might be attained. Delivery of hospital services is an example. Below, we illustrate a method of appraising the availability of hospital services to residents of an eight-county, rural-oriented area in the northwestern portion of lower Michigan.

The study area is called the Grand Traverse Region. The counties are Antrim, Benzie, Crawford, Grand Traverse, Kalkaska, Leelanau, Missaukee, and Wexford. The area was selected because it forms a readily identifiable geographic and economic region functioning as a relatively closed trading and commuting area.

The region, with a population of 110,000, depends on fruit farming and tourism for most of its economic activity. It has some manufacturing in its main towns, Traverse City and Cadillac. Traverse City is the region's health services center. One of the region's six general hospitals and an osteopathic hospital, as well as a State

psychiatric hospital, are located there. The psychiatric hospital was not included in the study.

In general, the region evinces an air of apparent prosperity, but behind this front lie many typical rural problems: Low incomes, high unemployment, high proportions of old people, and emigration of young people.

Grand Traverse County contains two hospitals. Benzie, Crawford, Kalkaska, Leelanau, and Wexford each have one. And Antrim and Missaukee have none. Occupancy rates in the five larger hospitals were close to 80 percent in 1967 (table 1, col. 3). An average of 80 percent of the beds in use during a year is considered by many hospital planners as about optimal. A higher average occupancy rate might indicate inadequate emergency capacity. The two smaller hospitals in Kalkaska and Leelanau had occupancy rates of 58.3 percent and 66.1 percent respectively. Kalkaska's hospital, which had 20 beds and 4,259 patient-days, needed only 15 beds to obtain the 80 percent occupancy rate; Leelanau's, with 29 beds and 6,995 patient-days, would have had an optimum rate with 24 beds.

This criterion suggests that there were possibly 10 excess beds in the region in 1967, five in each of the two small hospitals in Kalkaska and Leelanau. These 10 excess beds are a small number from which to draw conclusions about underutilization. Moreover, the simple criterion used above implies that the preferred rate of occupancy is independent of hospital size. An alternative criterion used below suggests that small hospitals require more excess capacity than large ones.

¹ The application of Poisson analysis developed in this article was used by Doherty in the section on optimal hospital size in: Efficiency in the distribution and utilization of hospital services: A case study in rural Michigan. U.S. Dept. Agr., (unpublished).

Table 1.—Hospital size and occupancy rates, Grand Traverse Region, 1967, with comparisons

County with hospital	Actual				If each hospital operated independently		
	Beds	Patient-days	Occupancy rate	Mean patient-days	Probability of overflow with actual beds	Needed beds ¹	Implied occupancy rate
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	No.	No.	Pct.	No.		No.	Pct.
Benzie	43	12,342	78.6	34	0.0618	57	59.3
Crawford	68	20,561	82.4	56	.0526	86	65.2
Grand Traverse:							
General	250	73,730	80.8	202	.0004	259	78.0
Osteopathic	73	22,643	85.0	62	.0308	93	66.3
Kalkaska	20	4,259	58.3	12	.0104	26	46.4
Leelanau	29	6,995	66.1	19	.0110	36	52.1
Wexford	128	37,946	81.2	104	.0094	145	71.8
Total	611	178,476	80.0	489	—	702	69.7

¹ Calculated as: $m + 4\sqrt{m}$, see text, to reduce probability of overflow to less than .0001.

Sources: Hospitals, Jour. Amer. Hospital Assoc., Vol. 42, Aug. 1968; Michigan State Plan for Hospital and Medical Facilities Construction, Mich. Dept. Public Health, Lansing, 1968-69.

If each of the seven hospitals in the multicounty area functions as an independent unit, it must have sufficient capacity in terms of staff and equipment to handle its own peak load. For example, the general hospital in Benzie County had 43 beds and 12,342 patient-days in 1967 (table 1, col. 1 and 2). The expected number of patients on a random day was 34 (table 1, col. 4). If the arrival of patients is distributed by a Poisson distribution, then the probability of having n patients on a given day is

$$P_n = \frac{m^n e^{-m}}{n!}$$

where m is the expected number of patients. A Poisson assumption is appropriate when arrivals are distributed independently over time and we are concerned with the total number of arrivals during a time interval. That is, when we assume the probability of an arrival on one day is the same as the probability for any other day and is independent of the number of arrivals on a previous day.

The variance of the Poisson distribution is equal to the mean. Hence, for the Benzie County hospital, the mean number of patients is

$$m = 34$$

and the standard deviation is

$$s = 5.8$$

When m is large, the Poisson distribution is approximated by a normal distribution.² Hence, for the Benzie County hospital operating as an independent service center, we can expect between 28 and 40 patients two-thirds of the time. The 43-bed hospital has a capacity which is 1.54 standard deviations above the mean. Given the assumptions, we can expect the number of arrivals to exceed 43 patients with a probability of .0618 (table 1, col. 5). This indicates overcrowding possibly once every 2 weeks; it does not allow much safety margin. To insure that the number of arrivals does not exceed the number of beds more than once every 3 years or so, we would want to provide 51 beds, a capacity about three standard deviations above the mean. Four standard deviations extend the period to 30 years. Using four deviations, the Benzie County hospital would require 57 beds instead of 43 (table 1, col. 6). It would then have an occupancy rate of only 59 percent instead of 79 (table 1, col. 7).

Extending the analysis to all seven hospitals, we see that the needed number of beds in the eight-county area is 650 if each hospital operates independently, and if its capacity is such that the number of arrivals exceeds the number of beds only about once in 3 years for each hospital; or, 702 beds are needed as insurance against

² Feller, W. Probability theory and its applications. John Wiley and Sons, Inc., New York, 1950, p. 143-144.

overcrowding once in 30 years (table 1, col. 6). Either of these are more than the 611 beds available in 1967. A bed capacity four standard deviations above the mean for each hospital would result in an average occupancy rate of only 70 percent for the area as a whole (table 1, col. 7). This suggests that the seven hospitals do not operate independently; that efficient utilization of available facilities is gained by sharing services among hospitals in the commuting area. The general hospital in Traverse City is most likely to be able to function independently according to the probabilities in column 5 of table 1. It has the least likelihood of overflow. The most dependent hospitals in this sense are the osteopathic hospital in Traverse City and the general hospitals in Benzie and Crawford counties.

A widely held view in hospital planning is that hospital costs would be lower if there were greater cooperation among hospitals in the same area with a given population. Because maximum patient demands are unlikely to occur in all hospitals at once, the census for a group of hospitals functioning as a unit, or for a single large hospital, would vary less than for smaller independent hospitals. That is, fewer beds would be needed in the unit concept to provide the same level of protection as could be provided with a given number of beds in several independent hospitals.³ As independent units, each hospital must have the capacity, in terms of staff and equipment, to handle its own maximum load. In cooperation, however, all hospitals are open to all patients and practitioners, and consciously avoid unnecessary duplication of services and equipment. Large hospitals tend to offer more services than small hospitals. For many of these services the small hospital is an unsatisfactory substitute. But there are other services which can be adequately provided at both large and small hospitals. With full cooperation, each hospital requires less staff and equipment, compared with independent hospitals, because patients could be transferred to other hospitals in emergencies or for specialized treatment.

In the Grand Traverse region, the implications of cooperation for bed saving are as follows: The average daily census in the region's hospitals in 1967 was 489 patients. With the assumption that the demand for hospital facilities has a Poisson distribution, a single large hospital, or a group of fully cooperating hospitals, with 577 beds ($489 + 4\sqrt{489}$) would meet expected needs with the probability of less than .0001 that demand would exceed 577 on any given day. The expected occupancy rate would be 84.7 percent. The actual

number of beds in the region was 611, indicating 34 excess beds in 1967 by this criterion. This suggests that full cooperation among the seven hospitals could safely allow a modest reduction in plant and equipment.

We infer it is almost certain that the number of hospital beds in the Grand Traverse multicounty area is adequate to meet patient needs providing the hospitals cooperate fully, because the observed number of available beds is about 5.5 standard deviations above the expected number of arrivals. The analysis suggests that under the assumption of full cooperation, these seven hospitals may be paying a little more for capacity than they need to. They could have 34 fewer beds and an occupancy rate of around 85 percent compared with the observed rate of 80 percent and still have a very small probability of being overcrowded. A defense of the current capacity could be in terms of (1) the high cost of crowding in the sense that 5.5 standard deviations provide a more adequate safety margin than 4 deviations, or (2) the cost of full cooperation, which might be more than the cooperation is worth. We also infer that the hospitals must necessarily cooperate at least partially to take care of the local demand for hospital beds, otherwise there would not be enough bed capacity in the area to take care of emergency needs during peak loads.

The method used to appraise the availability of hospital services in the Grand Traverse multicounty area

Table 2.—Hypothetical optimal hospital sizes for alternative expected patient-days¹

Patient-days	Required beds ²	Occupancy rate
	No.	Pct.
1	7	14.3
5	16	31.3
10	24	41.7
15	32	46.9
20	37	53.5
50	77	64.6
100	139	72.0
200	255	78.5
300	367	81.7
400	478	83.8
500	587	85.2
1,000	1,123	89.1

¹ Optimal means the probability that the number of arrivals exceeds the number of beds is .0001. For the normal distribution, this is calculated as: $m + 3.88\sqrt{m}$; see text.

² From cumulative Poisson distribution tables for 1 to 15 patients per day. From cumulative normal distribution tables for 20 to 1,000 patient-days.

³ Long, M. F. Efficient use of hospitals. In: *Economics of Health and Medical Care*, Univ. Mich., 1964, p. 214.

can be used to appraise the availability of other kinds of community services to residents of rural areas. There are many community services for which a rural-oriented, multicounty planning district needs only one facility such as a public building, a community college, a shopping center, or an industrial park. Dividing a commuting and trade area into isolated markets with separate facilities may be less efficient than letting all residents of the area function as a single market using a single facility. The method would apply to a single facility for an area, providing the demand for use of the services may be assumed to be independently distributed over time. (A service for which demand is not independent is snow plowing.) The general principle involved is an aspect of economies of scale and of the kind of efficiencies associated with agglomeration. One source of service for all the customers in an area has economies not available if the market is divided into two or more sectors each with its own service channel. This is a well-known conclusion from the queuing theory problem; it deserves to become a widely applied principle of area development.

Table 2 illustrates the extent to which economies of

scale are attained. If a hospital expects an average of only one patient per day when arrivals are governed by the Poisson distribution, seven beds are needed to insure that the arrivals will exceed capacity only once in 1,000 chances. The occupancy rate in this extreme case is only 14 percent.

The data in table 2 are taken from a table of cumulative probabilities for the Poisson distribution for values from 1 to 15. The data for values from 20 to 1,000 were taken from probability tables for the normal distribution. To maintain comparability of calculated probability of .0001 throughout table 2, the required number of hospital beds is calculated to be 3.88 standard deviations above the expected number of patients. Notice that as the size of the delivery system expands to handle a mean of one to 1,000 patients per day, the occupancy rate rises from 14 to 89 percent. The norm of 80 percent occupancy frequently used in hospital planning is seen from this analysis to be optimally associated with a hospital of 301 beds handling an average of 241 patients per day. Lower occupancy rates should be expected for smaller independent hospitals, higher rates for larger ones.