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Empirical Determination of Optimum Quality Mix

By Masao Matsumoto and Ben C. French

Changes in the distribution of an agricultural commodity among quality classes may affect the total revenue obtained from a given quantity of product. Growers and marketing firms need to consider the best mix of qualities as well as the best level of output. This paper describes an approach to the empirical problem of selecting, from several alternative feasible mixes of qualities, the one which would yield the maximum net revenue. The paper focuses on a single attribute and a single commodity but the concepts and methodology used can be extended readily to other quality attributes and other commodities. Key words: Product quality, multiproduct demand analysis, brussels sprouts, price discrimination.

Most agricultural commodities are produced and marketed in a variety of quality classes defined by attributes such as size, color, shape, texture, sweetness, blemishes, percentage of defects, and the like. Since consumer preferences for quality are variable, changes in the distribution of a commodity among such classes may affect the total revenue obtained from a given quantity of product. The mix of qualities is determined to a considerable extent by biological and growth processes which are beyond the direct control of management. However, it can be influenced, at a cost, by cultural practices and by the handling, grading, and packaging practices employed by processors and shippers. In formulating production plans, growers and marketing firms thus need to consider not only how much to produce, but the best mix of qualities in view of both revenue and cost effects. Firms or industry groups are also faced with the problem of determining how quality classes shall be defined, including the number of separate classes to be distinguished. The present analysis takes the structure of quality classes as given.

This paper describes an approach to the empirical problem of selecting from among the alternative feasible mixes of qualities, the particular mix that may be expected to yield the largest net revenue. We illustrate the approach for a specific commodity—frozen brussels sprouts—and a single quality attribute, the size of the individual sprouts. The concepts and methodology may be extended readily to other quality attributes and commodities.

The focus of the analysis is on the potential gain in net returns to the industry (both growers and processors) rather than to an individual firm or a particular industry subgroup. The ultimate division of any net

gains from altered quality distribution would depend on the nature of grower supply response and the structure of competition in the raw product market.

Achievement of an improved or optimum quality position implies the development of a raw product pricing system that reflects final product demands back to growers in a manner consistent with the optimizing adjustments. This requires that there be a clearly defined relationship between final product quality and raw product quality. Given this relationship—which is direct in the case of size attributes—an appropriate raw product pricing system would reflect the differences in marginal production costs involved in shifting the quality mix among classes. The final level of raw product prices would depend on the characteristics of grower supply and competition among freezers.

Theoretical Framework

An appropriate theoretical framework for analyzing the quality mix problem may be derived from the theory of the multiproduct discriminating monopolist. If we think of each quality variation as a different product, or alternatively as a different market outlet, the problem may be viewed as the determination of the optimum distribution of total sales among markets or products with interrelated demands. While many microtheory texts deal with some aspects of the problem, perhaps the best mathematical treatment for our purposes is to be found in Waugh (5).¹ We shall follow his general line of

¹Italic numbers in parentheses indicate items in the References, p. 9.

development, except that costs associated with varying quality mixes are given explicit treatment in our model.²

Assume that the production of an agricultural commodity is distributed among K quality classes. The price received for the i th quality depends not only on the quantity in that class, but the quantity in each other class, plus other variables such as income and competing products which may shift the entire level of demand. The total quality demand system is described by K quotations, one of each quality class. Holding shift variables such as income constant and assuming linear relationships (as in our empirical analysis) a typical demand equation would have the form

$$(1) \quad P_i = b_{i0} + \sum_{j=1}^K b_{ij} Q_j \quad i, j = 1, 2, \dots, K$$

where P is price, Q is quantity, and b_{i0} and b_{ij} are coefficients of the demand equations.

The gross revenue equation, obtained by multiplying the price in each quality class by the quantity in that class and summing, may be written as

$$(2) \quad GR = \sum_{i=1}^K Q_i P_i = \sum_{i=1}^K Q_i \left(b_{i0} + \sum_{j=1}^K b_{ij} Q_j \right).$$

To obtain the net revenue equation we subtract the total cost from the gross revenue. For this purpose, the unit cost may be decomposed into two parts, a constant part \bar{c} , common to all quality classes, and a variable part, c_i , which varies among quality classes. In general, the c_i 's may vary as functions of quantities in all classes and these functions may have nonlinear forms.³ For simplicity, we shall regard the c_i 's as constants (they are so estimated in the empirical analysis). The basic conceptual model is not essentially altered by this simplification.

²The problem of determining optimum product quality for a firm producing a single quality of product has been well developed by Dorfman and Steiner (2). They introduce a continuously variable index of quality into both the demand and cost functions of the firm and determine the conditions for joint optimization of quality and price for a given quantity. They note that a range of qualities would be offered if the market consists of groups of consumers with differing quality demands. However, they do not develop this case explicitly, nor do they deal with the case where the firm (or industry) produces a spectrum of qualities as a consequence of biological factors involved in the production process.

³General theoretical considerations involved in incorporating product quality into the production function are well developed in Dano (1). However, he does not extend his analysis to the joint product case found in agriculture.

With these considerations, the net revenue equation is expressed as follows:

$$(3) \quad NR = \sum_{i=1}^K Q_i \left(b_{i0} + \sum_{j=1}^K b_{ij} Q_j \right) - \sum_{i=1}^K c_i Q_i - \bar{c}Q$$

where Q is the total quantity to be sold, obtained by summing over all classes.

We wish to find the values of Q_i which will maximize (3) for any given value of Q . To solve, we form a Lagrangian function, set the first partial derivatives equal to zero, and solve the resulting linear equations for values of Q_i as a function of Q .

$$(4) \quad L = \sum_{i=1}^K Q_i \left(b_{i0} + \sum_{j=1}^K b_{ij} Q_j \right) - \sum_{i=1}^K c_i Q_i + \bar{c}Q + \lambda \left(\sum_{i=1}^K Q_i - Q \right).$$

$$(5) \quad \frac{\partial L}{\partial Q_i} = b_{i0} + \sum_{j=1}^K (b_{ij} + b_{ji}) Q_j - c_i + \lambda = 0.$$

$$(6) \quad \frac{\partial L}{\partial \lambda} = \sum_{i=1}^K Q_i - Q = 0.$$

Solving for λ gives the value of marginal net returns which, for a maximum, must be the same for all quality classes.

To assure a maximum we must have, in addition to conditions (5) and (6), $d^2L < 0$ subject to

$$\sum_{i=1}^K dQ_i = 0.$$

The value of d^2L will be negative definite if the principal bordered minors of the Hessian determinant formed from L alternate in sign, starting with the minor formed by the first three rows and columns > 0 . (See Lloyd (3) or Yamane (6).)

Note that in solving equations (5) and (6) we are not required to know the value of \bar{c} (the costs common to all sizes). We need only the marginal costs (the c_i 's) associated with altering the quality distribution.

With no other restraints, the solution procedure outlined above could give negative values for some of the Q_i 's. If this should happen, either of two alternatives may be considered. In some cases it may be clear that

the values of the Q_i 's that were negative will always assume the minimum amount permitted. Quantities for these classes therefore may be set at zero (or some proportion of Q) and the problem solved again with the added restrictions as equalities. More generally, restraints may be added of the form $Q_i \geq 0$ for all i (or some $Q_i \geq a_i Q$) and a solution obtained as a quadratic programming problem. This, or course, could be done at the outset. The first procedure was followed in the frozen brussels sprouts model since it was evident that a proper solution would be obtained by this method.

Demand Interrelationships for Quality Attributes

Estimation of the parameters of the system of demand equations requires continuous series of data pertaining to prices and quantities in each quality class. For frozen brussels sprouts such data have, for the most part, never been published in any form. Consequently, it was necessary to acquire data directly from the records of processing firms in the industry.

Monthly sales and price data by size class were compiled from records of freezers covering the years 1961 and 1962.⁴ These firms packed about 55 percent of all the frozen brussels sprouts marketed by California firms during these 2 years and the data are believed to be representative of the California industry's experience. California freezers accounted for about 94 percent of U.S. frozen brussels sprouts production in these years. Data covering additional periods would have been desirable but lack of complete historical records in the firms surveyed, plus the sheer physical difficulties involved in the compilations, required that the survey be limited to a 2-year period.

Frozen brussels sprouts vary considerably in size, with the smaller sizes commanding a price premium. The brussels sprouts trade typically quotes prices by a range of count per pound which we have grouped into three classes: large (less than 25 per pound), medium (25 to 40 per pound), and small (over 40 per pound). There is a further breakdown by type of package—retail (1 pound or less) and institutional (over 1 pound). Thus, our basic

⁴This phase of the analysis was initiated in the early 1960's at the request of the members of the brussels sprouts industry. The survey of processors was completed in 1963 and preliminary analysis finished shortly thereafter. However, because of diverting assignments, the final phases were not completed until much later. A very limited amount of data has been obtained for more recent years and serves as a basis for checking the previous results. The major conclusions reached with the data appear to hold, at least approximately, for more current conditions.

data series consists of 24 monthly observations on prices and sales for three sprout size classes and two container classes.

The basic model to be estimated expresses the average monthly price in each size and container category as a linear function of the monthly quantity sold in each category. Nonlinear forms are of course equally possible. With many variables and cross-relationships, it is difficult to determine the nature of existing curvilinearity and to estimate any except the simple log form. Preliminary graphic explorations suggested that linear estimates would provide reasonable approximations within the range of the data.

In view of the relatively short period involved, the annual values of competing products and income were omitted.⁵ Quantities were expressed in total rather than per capita terms. Monthly carlot unloads of fresh brussels sprouts in 41 U.S. cities were included as a variable to allow for both seasonal shifts and the competitive effects of the fresh commodity. Initial regression estimates suggested the presence of a trend in some equations, possibly due to the omission of income and other competing products. Thus, a time trend also was added. Since there are six size-container classes, we have six equations each of the form:

$$(7) P_{it} = b_{i0} + b_{i1}Q_{RLt} + b_{i2}Q_{RMt} + b_{i3}Q_{RSt} \\ + b_{i4}Q_{ILt} + b_{i5}Q_{IMt} + b_{i6}Q_{ISt} + b_{i7}Q_{Ft} \\ + b_{i8}T + V_{it}$$

where

- Q is quantity in 1,000 pounds;
- R refers to retail size containers;
- I refers to institutional size containers;
- L, M, S are sizes of sprouts—large, medium, and small;
- P is average price (cents per pound);
- Q_F is monthly carlot unloads of fresh brussels sprouts in 41 U.S. cities;
- T is a time trend, varying from 1 to 24;
- $i = 1, 2, \dots, 6$, indicates a particular size-container category (e.g., $P_{1t} = P_{RLt}$);
- $t = 1, 2, \dots, 24$, indicates the number of the monthly observation; and
- v is an unexplained disturbance.

⁵Data pertaining to monthly sales of most competing frozen vegetables are not available. Explorations of monthly shipment data for fresh vegetables, except fresh brussels sprouts, failed to produce promising indicators of regular shifts in the monthly demand for frozen sprouts.

The choice of statistical procedure to use in estimating the parameters of this equation system involved both theoretical and empirical considerations. Since we are dealing with monthly observations our initial approach was to view the demand equations as part of a simultaneous system. That is, monthly quantities sold were regarded as determined simultaneously with monthly prices. This view required that we specify short-run supply equations and then estimate the parameters of the entire demand-supply system by simultaneous equations procedures.

Supply equations were formulated in which the monthly quantities of sprouts marketed, by size class, were expressed as linear functions of prices and beginning monthly inventory levels in each size class. The latter were adjusted for normal seasonal variation. Beginning inventories were viewed as predetermined variables, thus producing a system in which the coefficients of the demand equations were identified.

The parameters of this system of equations were estimated by two-stage least squares. The results were quite unsatisfactory, both by usual statistical criteria and in terms of the expected signs and magnitudes of the equation coefficients. This may have been due in substantial part to the very poor quality of our data pertaining to monthly quantities of each size category held in cold storage.⁶ In any case, it was necessary to abandon this procedure and turn to other methods of estimation.

Our second approach was to apply ordinary least squares directly to the estimation of the parameters of each structural equation. This method is justified if processors establish quantities to be sold each month without regard to current price. We would not argue that processor plans and adjustments are so completely inflexible, but processors may well be more concerned with orderly movement of total stocks and honoring buyer commitments than with adjustments to monthly price variations. Under these circumstances the degree of bias may be "reasonably" small.

The results of our initial application of ordinary least squares, although superior to the two-stage least squares model, still had some coefficients with positive signs or of low statistical significance. The main difficulty appeared to be that the sample was too small, the range of observations too limited, and the intercorrelation among quantity variables too high to isolate the separate

effects on price of each of the six alternative size-container classes in each equation. To reduce the intercorrelation problem and the number of coefficients to be estimated we combined several of the variables in each equation.

The most satisfactory results were obtained by a formulation which related price in each size-container class to the quantity in that class and the total quantity in all other classes. The regression equations are summarized in table 1. All of the coefficients are consistent with theoretical expectations—i.e., they are all negative and each price is affected more by changes in its own quantity than by quantities in other classes. Sales of fresh brussels sprouts (Q_F) had a highly significant effect on prices of medium sprouts and a somewhat significant effect on prices of medium sprouts in retail containers. The coefficient of Q_F was of quite low statistical significance in the equations for large sprouts and for small institutional-container sprouts. Therefore, Q_F was omitted in the final regressions for these classes. Possibly the extreme size classes are less sensitive to the fresh market. The trend factor clearly was not significant for institutional containers and so was omitted from the latter set of equations. For large sprouts in institutional containers, the coefficient of $Q_T - Q_{IL}$ was small and of very low statistical significance so it was dropped from this equation—i.e., its coefficient is regarded as zero.

Cost of Altering the Quality Mix

The primary means by which the size distribution of sprouts may be changed is through varying the time interval between harvests or the timing of once-over machine harvest. The major cost factors involved are the loss in yield with decreased size and the higher per pound harvesting cost with smaller sizes. The distribution could also be altered by simply discarding some of (say) the larger sprouts. To maintain production at a given level, the discarded sprouts would have to be replaced by quantities produced on additional acreage. This appeared to be excessively expensive compared with other procedures considered and so was rejected as an alternative.

With an inelastic demand, discarding quantities in lower quality classes would, of course, increase total revenue. In this case, the f.o.b. freezer demand is elastic, so revenue would be reduced. Moreover, our purpose is to determine the optimum quality mix for any given quantity, leaving open the determination of the total amount to be produced.

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Table 1.—Demand relationships for frozen brussels sprouts by size-container class: Final selected regressions

Dependent variable	Constant term	Explanatory variables ^a								<i>R</i> ^c	<i>d</i> ^d	
		<i>Q</i> _{RL}	<i>Q</i> _{RM}	<i>Q</i> _{RS}	<i>Q</i> _{IL}	<i>Q</i> _{IM}	<i>Q</i> _{IS}	<i>Q</i> _T - <i>Q</i> _{<i>i</i>} ^b	<i>Q</i> _F			<i>T</i>
		regression coefficients and <i>t</i> -ratios ^e										
<i>P</i> _{RL}	25.197	-.00136 (1.721)						-.00045 (2.421)		-.03450 (3.689)	.773	1.68
<i>P</i> _{RM}	27.887		-.00051 (1.710)					-.00027 (1.403)	-.00252 (2.294)	-.01249 (1.795)	.801	1.20
<i>P</i> _{RS}	29.914			-.00235 (2.756)				-.00024 (1.116)	-.00182 (1.487)	-.01628 (1.959)	.773	1.75
<i>P</i> _{IL}	22.827				-.00334 (4.152)						.663	2.76
<i>P</i> _{IM}	26.234					-.00132 (3.049)		-.00027 (1.942)	-.00406 (4.865)		.945	2.39
<i>P</i> _{IS}	27.064						-.00636 (3.906)	-.00013 (1.458)			.732	1.85

^aSee text for explanation of symbols.

^b $Q_T = Q_{RL} + Q_{RM} + Q_{RS} + Q_{IL} + Q_{IM} + Q_{IS}$. *Q*_{*i*} refers to the variable treated as dependent.

^cCoefficient of multiple correlation.

^dDurbin-Watson statistic. The hypothesis of no positive serial correlation of error terms is not rejected at the 5 percent significance level for the first, fourth, fifth, and sixth equations. The value of *d* falls in the indeterminant range for the second and third equations. There is a suggestion of negative serial correlation for the fourth equation.

^eFigures in parentheses are *t*-ratios.

essentially constant, regardless of sprout size. Thus, cost per pound is greater for the smaller sizes.

Our studies of these operations indicated that with conventional multiple-harvest cultural practices, shifting 1 pound of sprouts from the large to the medium size would increase the combined production and processing cost by roughly 0.9 cent per pound.⁷ Shifting a pound from the medium to the small class would increase costs by another 0.9 cent per pound. Since we have no direct observations to indicate how costs of shifting size distributions might be affected by one-over machine harvest and the introduction of new varieties, the cost figures should be regarded only as rough approximations. However, they are satisfactory for purposes of illustration and seem close enough to permit a tentative evaluation of potential gains or losses from changes in distribution.

Since our focus is primarily on size distribution, we have assumed that the average observed price difference between container types for the same sprout size reflects the difference in cost of packaging in retail and

institutional containers. This was 1.73 cents per pound higher for retail packages (for all sprout sizes) during the period for which data were obtained.

The *c*_{*i*}'s of our conceptual model—equations (3), (4), and (5)—may be derived by adding the estimated differences in costs to *c*, the unit cost common to all sizes. Thus, the total cost of production and processing may be expressed as

$$(8) \quad TC = \bar{c}Q + 1.73Q_{RL} + 2.63Q_{RM} + 3.53Q_{RS} + 0.0Q_{IL} + 0.9Q_{IM} + 1.8Q_{IS}.$$

Recall that, with total quantity (*Q*) constant, \bar{c} drops out of the marginal revenue equations.

Note that with costs expressed as a linear function of the *Q*_{*i*} (i.e., $TC = aQ + \sum_{i=1}^K a_i Q_i$) we can always rewrite

the equation as $TC = (a + a_j)Q + \sum_{i=1}^K (c_i - c_j)Q_i$. The

coefficient for *Q*_{*j*} will thus be zero. In (8), $Q_{IL} = Q_j$ and $\bar{c} = a + a_j$.

⁷For details of the cost estimates see Matsumoto (4).

Revenue Effects

A monthly gross revenue equation may be obtained by multiplying the equation in table 1 by the quantity corresponding to the dependent variable and summing equations.⁸ A net revenue equation is derived by subtracting equation (8) from the gross revenue equation. Following the procedure outlined in (3) to (6), the revenue-maximizing quality mix may be determined for each month as a function of the total quantity of frozen sprouts to be sold during the month, the monthly sales of fresh sprouts, and the value of the trend factor. The basic decision facing the industry, however, is not how to allocate predetermined monthly sales among quality classes (although eventually this must be done), but how much to produce in each quality class. Since production decisions are made annually rather than monthly, we need an annual revenue function. This is obtained by summing the monthly revenue equations over 12 months.

Given the annual total quantity of sprouts to be sold and the expected seasonal pattern of sales of fresh-market sprouts, the annual revenue function may be solved for the optimum allocation of quantities to each quality class for each month. The monthly quantities then may be summed to obtain the annual production mix. Our basically simple conceptual problem involving three quality classes (and two container classes) thus expands, in application, to the determination of the optimum allocation of a given annual quantity among 72

⁸Since prices are expressed in cents per pound and quantities in 1,000-pound units, the value obtained by this procedure must be multiplied by 1,000 to obtain the true value of revenue (in cents).

economic classes (12 months, three qualities, and two container types).

While this type of problem clearly can be handled without great difficulty on a computer, even as a quadratic programming problem, it is perhaps asking quite a bit of our limited empirical model, especially since it does not include consideration of inventory cost, customer relations, and expectation formation which might influence the distribution pattern. It also unnecessarily complicates the development of the points we wish to illustrate. To focus more directly on the quality mix problem—that is, the best distribution of qualities for a given quantity of annual industry production—we shall introduce some simplifications which enable us to abstract from the seasonal pattern of distribution, while still providing a basis for some rough generalizations.

Specifically, we shall hold the level of fresh marketings at the mean value for the period of the study and set T at zero.⁹ This gives us a set of typical demand equations, summarized in table 2, which illustrate the essential characteristics of the interrelated demand struc-

⁹Recall that the trend variable was inserted primarily to allow for effect of possible demand shifters, such as income or competing products, which were not included as specific variables in the demand model. In this sense, the trend has no significance beyond the study period. The choice of zero for the trend value was a matter of convenience in formulating illustrative demand and revenue equations. If T had been set 12 or 24, rather than zero, the demand for retail containers would have been reduced relative to the institutional containers and the demand for the retail large class would have been reduced relative to the medium and small. These small shifts would slightly alter the solutions to the problem of optimum distribution in favor of medium sizes, but the major conclusions would remain essentially the same.

Table 2.—Simplified demand model for frozen brussels sprouts by size-container class
($Q_f = 58.6$ and $T = 0$)

Dependent variable	Constant term	Explanatory variables					
		Q_{RL}	Q_{RM}	Q_{RS}	Q_{IL}	Q_{IM}	Q_{IS}
		equation coefficients					
P_{RL}	25.197	-.00136	-.00045	-.00045	-.00045	-.00045	-.00045
P_{RM}	27.379	-.00027	-.00051	-.00027	-.00027	-.00027	-.00027
P_{RS}	29.807	-.00024	-.00024	-.00235	-.00024	-.00024	-.00024
P_{IL}	22.827				-.00334		
P_{IM}	25.996	-.00027	-.00027	-.00027	-.00027	-.00132	-.00027
P_{IS}	27.064	-.00013	-.00013	-.00013	-.00013	-.00013	-.00636

ture and which may be easily converted to annual demands and revenues by a factor of 12. The total quality mix determined with these simplified equations would not be identical with a solution based on the more complex summation of monthly allocations, but we would expect it to be reasonably close.¹⁰

Although we are ultimately interested in the net revenue equations, we shall first briefly explore the gross revenue function since it may be used to develop some useful information pertaining to effects of changes in size allocation without immediate reference to cost. This had considerable merit in view of the rough nature of our cost-size-of-sprout relationships.

To extract the desired information we first take the total differential of the gross revenue equation, obtaining

$$(9) \quad dGR = \sum_{i=1}^6 \frac{\partial GR}{\partial Q_i} dQ_i .$$

The marginal revenues, $\partial GR/\partial Q_i$, are then calculated for each quality class with quantities held at the average levels for the sample of observations. The gross revenue effects of small shifts in allocation between pairs of classes (i and j) are obtained by setting $dQ_i = 1$, $dQ_j = -1$, and all other $dQ_r = 0$.

Table 3 summarizes these values. It indicates, for example, that shifting 1,000 pounds from the retail-large to the retail-small class would increase gross revenue by \$45.25. These marginal gains or losses may be compared with expected marginal costs to suggest something about desirable directions and potential gains from change.

If our estimates of transfer costs between classes are subtracted from the values in table 3, we obtain the net revenue estimates shown in table 4. With these adjustments, the marginal gain in shifting 1,000 pounds from the retail-large to the retail-small class, for example, is reduced to \$27.25, still a significant gain. As expected in view of our assumptions concerning cost differences, the potential gains or losses from shifts between container types for like sizes is quite small in all cases. In general, table 4 suggests that net revenue could be increased by a

¹⁰The solution by months would involve reduced total frozen-sprout sales in months of large fresh-market sales. In months of smaller frozen-sprout sales, the optimum solution would require relatively larger percentages of sprouts in small sizes and relatively smaller percentages in medium sizes, with large sizes at the minimum permitted. The weighted average quality mix for all months would be close to, but not necessarily the same as, the mix obtained with an average level of fresh market sales.

Table 3.—Changes in gross revenue associated with shifting 1,000 pounds of frozen brussels sprouts from one size-container class to another^a

1,000-pound decrease in	1,000-pound increase in				
	Q_{RM}	Q_{RS}	Q_{IL}	Q_{IM}	Q_{IS}
	dollars				
Q_{RL}	25.58	45.25	-24.42	10.19	24.20
Q_{RM}		19.67	-50.00	-15.39	-1.38
Q_{RS}			-69.67	-35.06	-21.05
Q_{IL}				34.61	48.62
Q_{IM}					14.01

^aStarting with average sample quantities in each class.

program aimed at shifting quantities from large into medium and small categories. However, it does not indicate how much should be shifted or what the final distribution should be. For this purpose, we need to make additional calculations.

The distribution among quality classes which maximizes the value of the net revenue function was obtained as indicated in equations (4), (5), and (6). The b coefficients of the revenue equation are contained in table 2 and the c coefficients are given in equation (8). The initial solution produced negative values for the quantities in the large size classes. Consequently, it was

Table 4.—Changes in net revenue associated with shifting 1,000 pounds of frozen brussels sprouts from one size-container class to another^a

1,000-pound decrease in	1,000-pound increase in				
	Q_{RM}	Q_{RS}	Q_{IL}	Q_{IM}	Q_{IS}
	dollars				
Q_{RL}	16.58	27.25	-7.12	18.49	23.50
Q_{RM}		10.67	-23.70	2.21	6.92
Q_{RS}			-34.37	-8.76	-3.75
Q_{IL}				25.61	30.62
Q_{IM}					5.01

^aStarting with average sample quantities in each class.

necessary to impose restrictions which would guarantee that quantities in all classes would be greater than or equal to zero. A solution could have been obtained by quadratic programming procedures. However, since it was clear that the quantity in the "large" class would be the minimum permitted, restrictions were imposed as equalities and solutions were obtained as before, except with the added restrictions. All other unrestricted quantities remained positive with this solution.

Table 5 shows the optimal distribution for the total quantity of average monthly sales observed during the same period. Three alternative solutions are presented, each involving different restrictions on the quantity of large sprouts. Solution 1 imposes only the restriction that the quantity in each class must be greater than or equal to zero. Because it may be unreasonable to assume that the quantity in large sizes can be reduced to zero (without abandoning parts of the production), we have computed two additional solutions which require that at least (a) 5 percent and (b) 10 percent of the production will fall in the large class. To focus entirely on the size allocation problem, solutions 2 and 3 also restrict the

distribution between retail and institutional containers to the proportions observed during the sample period.

The three solutions give similar results. They leave the proportions allocated to medium sizes with little change, reduce the quantity in large sizes, and increase the allocation to smaller sprouts. Solution 1 increases the California industry revenue, compared with the value obtained by applying sample proportions to the estimated revenue equations, by \$158,000 per year, solution 2 by \$145,000, and solution 3 by \$104,000. Comparisons with actual average revenue during the sample period gave similar results. These potential gains, obtained by expanding sample values to reflect total industry experience, are in the order of 1-1/2 to 1 percent of gross revenue.

The optimal allocation proportions in table 5 pertain only to the quantities sold during the sample period, 1961-62. As total sales change, so do the optimal allocations among size classes. By leaving the value of Q (the total quantity) unspecified, a more general solution can be obtained in which the quantities in each quality

Table 5.—Actual and optimal distribution of sprout sizes for the 1961-62 sample quantity of sales.

Size-container class	Actual distribution	Optimal distributions		
		Solution 1 ^a	Solution 2 ^b	Solution 3 ^c
		proportion of total		
Large:				
Retail	.096	.000	.023	.046
Institutional	.114	.000	.027	.054
Total	.210	.000	.050	.100
Medium:				
Retail	.488	.473	.483	.449
Institutional	.154	.204	.153	.142
Total	.642	.677	.636	.591
Small:				
Retail	.126	.270	.267	.263
Institutional	.022	.053	.047	.046
Total	.148	.323	.314	.309

$$\frac{Q_L}{Q_T} \geq 0.$$

$$\frac{Q_L}{Q_T} \geq .05 \text{ and } Q_{RL} = .457Q_L, Q_{RM} = .759Q_M, Q_{RS} = .852Q_S.$$

$$\text{Restrictions same as for solution 2 except } \frac{Q_L}{Q_T} \geq .10.$$

class are expressed as linear functions of Q .¹¹ They may be converted to a percentage basis by dividing through Q . These solutions suggest that as industry total quantities increase, the proportions allocated to medium sizes should be increased and the allocation to the smaller sizes slightly decreased, with large sizes at the minimum permitted.

Concluding Comments

The results of our study are encouraging. They demonstrate that it is possible to develop an empirically quantifiable model which may aid agricultural groups in formulating programs aimed at achieving the best—or at least a better—mix of qualities. The approach used would be appropriate for many quality attributes other than size.

The major limitation of this and possibly similar studies that might be attempted is in the data, especially the price and sales data. Although the demand model developed is significant in a statistical sense and is consistent with theoretical expectations, substantially more observations would be desirable to build confidence in the precision and generality of the estimates.

For many commodities priced at shipping points or central markets there may be fairly long series which could be tabulated from market news data. For most processed commodities, price and sales data unfortunately are not published by quality class, if at all.

¹¹Examination of the original unrestricted solution indicated that as crop size increased, the optimum quantity in the "large" class would eventually become positive. However, this would occur only for crop sizes far larger than any yet observed. Quantities in other categories would always remain positive.

Commodity groups interested in this type of analysis could initiate data collection programs through their trade associations. For storable commodities, this would need to include information on the quality mix of inventory holdings, as well as sales. Costs associated with quality changes would, of course, have to be obtained by special studies in each case.

The potential industry gains suggested by this study appear quite modest—in the neighborhood of 1 percent of gross revenue. Our guess would be that potential gains for other commodities and quality attributes would be found to be similarly modest. For many industry groups, however, the absolute magnitude could be substantial and such gains may be well worth striving for, especially in periods of tight margins and increasing costs.

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