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Speculation in Commodity Futures: An Application of Statistical Decision Theory

By Samuel H. Logan and J. Bruce Bullock

A recursive, monthly price forecasting model for live cattle serves as a basis for applying decision theory to speculation in cattle futures. The distribution of predicted futures prices is obtained from the standard error of the forecast of the cash price forecasting model in conjunction with the historical distribution of the difference between futures and cash prices during the month of futures contract delivery. Baumol's expected gain-confidence limit model is utilized in determining which of the available futures contracts offers the highest minimum payoff potential holding the probability of at least such a payoff constant.

Key words: Cattle futures, speculation, price forecasting, decision theory, safety-first.

Speculation in commodity futures contracts, such as those for live beef cattle, has many of the attributes of gambling. The speculator, by selling or buying futures, is essentially betting that he knows better than the current market what future price conditions will be for a particular commodity. If both the speculator and the market were in agreement as to what the future price would be, there would be little incentive to speculate.

Like the roll of the dice or the turn of a card, the outcome of speculation in futures is uncertain. However, unlike the situation at the gaming table, the probabilities of the outcome on a speculative venture generally cannot be calculated precisely. And, unlike the gamble with the cards, two persons' computations of the odds of making a given profit or loss may not be identical. Indeed, subjective probabilities may be even more important in the decision criteria with respect to commodity speculation than those calculated mathematically.

With many commodities for which futures contracts are traded, statistical decision techniques can be used to generate additional information for selecting among alternative buy-or-sell actions. This paper demonstrates the application of statistical decision theory to speculation in live beef cattle futures. The general underlying theoretical model for this analysis was outlined in a previous issue of this journal (4).¹

¹Underscored numbers in parentheses indicate items in the References, p. 103.

The Problem

The futures speculator faces several alternative actions—he may sell short, anticipating a price decline, after which he buys back the contract; or he may “go along” by purchasing a contract in anticipation of selling it later for a higher price. These two actions are compounded by the number of possible contracts for beef cattle being traded at any one time.² The anticipated profit or loss of a possible action, then, depends on the set of current prices and expected future prices for the various futures contracts, as well as on the costs of brokers' commissions.

Of course, the trader also has the option of utilizing his funds in some other unrelated venture.

The decision to buy or sell a futures contract generally is based on the speculator's knowledge of cash market conditions, since ultimately, in the delivery month, the futures price can be expected to bear some close association with the cash market price for commodities whose quality and market location are the same as those specified in the futures contract. If such a relationship fails to materialize, there will be inducement for traders either to make or take delivery of the live animals rather than cancel the contract with an offsetting action. Thus, since the current cash market

²Cattle futures contracts are deliverable every other month, and open interest frequently involves contracts calling for delivery a year and a half ahead of time.

conditions are more evident, trade in cattle contracts calling for delivery nearest to the current time period would be expected to have less risk, for example, than trade in contracts calling for delivery 6 months from now. In this sense, the expected profits from actions involving different futures contracts might be identical, but one action might involve higher variance (risk) than the other.

The relationship between the cash market and the futures prices is less than perfect. While the futures and cash markets generally are closely related in the delivery month, the two prices in most cases have not been equal. Furthermore, this difference between the futures price and the cash price at time of delivery has shown much variation historically, a factor which causes additional uncertainty for the trader who tries to apply his knowledge of the cash market to expected futures prices.

Analytical Framework

We shall assume that the speculator desires to take whatever action (buying, selling, or no action) will give him the largest expected profit, given some consideration of risk. To provide meaningful comparisons of alternative actions, we shall further assume that the speculator has a set amount of funds to invest. His actions will be limited, then, by the size of his funds and the commission and margin requirements for the futures trade.³ Also, for simplicity, we shall limit the relevant time horizon to 6 months, giving the speculator an option of three futures contracts.

Putting the problem in the usual statistical decision framework, we can define P_{t+K}^f as the price at which a particular futures contract is eventually terminated (at time $t+K$). This price is also the state of nature (θ_h) about which the uncertainty exists, where "h" indicates a particular price or range of prices. The actions a_{ij} refer to buying or selling where $i=1$ for sell and 2 for buy, and $j=1, 2,$ or 3 relating to the alternative contracts. The possible outcomes from actions a_{ij} and states of nature θ_h are given by λ_{ijh} , and are derived by the following:

$$\lambda_{ijh} = (-1)^i (P_{h,t+k}^f - P_t^f) - c$$

where c is the cost of the futures trade and all other

³Current cattle futures contracts traded on the Chicago Mercantile Exchange call for delivery of 40,000 pounds of Choice beef cattle. Margin requirements are \$400 per contract, and the brokerage or commission fee is \$36 per complete contract transaction.

terms are as defined above. The problem is shown in tabular form in table 1.

Given a marginal probability distribution of θ , $P(\theta)$, the problem as presented in table 1 could be solved as a "no data" problem by finding the expected payoff for each action. The objective would be simply to maximize the expected payoff of the various actions, where the expected payoff is given by

$$\begin{aligned} \text{E.P.}_{ij} &= \sum_b \lambda_{ijh} P(\theta_h) \\ &= \sum_b [(-1)^i (P_{h,t+k}^f - P_t^f) - c] P(\theta_h) \end{aligned}$$

Such a formulation overlooks changes in supply and demand conditions. It seems more logical to base our actions on some experimental result, Z , or predictive tool—such as a price forecast—which would yield a conditional probability distribution of θ_h with less variance than that of the marginal distribution, $P(\theta)$. The optimum action, then, would be the one that maximizes

$$\text{E.P.}_{ij} = \sum_b [(-1)^i (P_{h,t+k}^f - P_t^f) - c] P(\theta_h|Z).$$

Because of the relationship of the futures price to the cash market at time of delivery, a recursive system of equations developed to predict monthly live cattle cash prices was used as an "experiment" to generate a conditional distribution for futures prices. The distribution of estimation errors, given predicted cash prices (derived from the regression results), was combined with a marginal probability distribution relating to the "basis," or amount by which the futures market price differed from the cash market price at time of contract termination. The resulting joint a posteriori distribution was used to estimate the expected value of each alternative action, a_{ij} .⁴

In this particular problem, the distribution of the predicted cash price varies as the length of the projection varies (1, 2, or 3 months, etc.), and as the month for which the projection is being made varies (first, second, or third month of a quarter). The variances of the distributions generally increase as the length of projection increases, a factor which is not included in the usual Bayesian framework. In order to consider possible

⁴The mean of the sum of two variables, x and y , is given by $M_{x+y} = M_x + M_y$, and the variance by $\sigma_{x+y}^2 = \sigma_x^2 + \sigma_y^2 + 2\sigma_x\sigma_y r_{xy}$, where r_{xy} is the correlation between x and y . These parameters then can be used to derive the resulting joint a posteriori distribution.

Table 1.—Representation of a decision problem for speculating on three futures contracts

States of nature (prices in the future)	Outcomes of various actions on contracts for—					
	February		April		July	
	A_{11} (sell)	A_{21} (buy)	A_{12} (sell)	A_{22} (buy)	A_{13} (sell)	A_{23} (buy)
θ_1	λ_{111}	λ_{211}	λ_{121}	λ_{221}	λ_{131}	λ_{231}
θ_2	λ_{112}	λ_{212}	λ_{122}	λ_{222}	λ_{132}	λ_{232}
θ_3
.
θ_H	λ_{11H}	λ_{21H}	λ_{12H}	λ_{22H}	λ_{13H}	λ_{23H}

aversion to risk on the part of the speculator, the usual decision-making framework was extended to reflect the certainty equivalence modifications suggested by Baumol (1).

Basically, Baumol's concept involves relating the degree of risk, or variation, to the expected return by subtracting a uniform multiple of the population standard deviation from the expected return of alternative actions. The resulting figures are returns for which the probabilities of being exceeded are identical regardless of differences in variances of the distributions involved, given normal distributions of returns for the various investments. Thus, the decision strategy is to select that action which gives the highest value of the function:⁵

$$W = E - k\sigma$$

where

E = expected or mean return

k = some constant

σ = standard deviation of the distribution of returns

If k equals 1, then about 84 percent of the time the return will reach level W or higher.

In applying Baumol's development to the speculation problem, we can define the decision function for buying as

$$W = (E - k\sigma') - P_t^f - c$$

where

⁵This problem could also be formulated under the safety-first concept advanced by Roy (13), in which the probability of disaster is minimized, or under the safety-first principles offered by Telser (14), in which expected profit is maximized given some constraint on the probability of ruin.

$E = P_{t+k}^c + b$ (predicted cash price plus mean basis)
 σ' = standard deviation of the sum of forecasting error and basis
 $c, k,$ and P_t^f are as defined earlier.

The formulation for selling would be

$$W = P_t^f - (E + k\sigma') - c$$

This development can be used as a first step in evaluating speculation alternatives. Those ventures which do not show a positive expected profit after consideration of the variance factor are deleted. Those remaining can be analyzed further in terms of the relative probabilities and magnitude of gains.

Results

Price-Forecasting Model

The set of cash prices is estimated recursively from a pyramid of predicted values of certain independent variables. The general procedure is to forecast the Chicago prices of 900- to 1,100-pound Choice slaughter steers as a function of (1) previous monthly prices (1 and 12 months previous), (2) predicted marketings of fed cattle in major feeding regions of the United States, and (3) shift or dummy variables. Fed-cattle marketings, in turn, are predicted on the basis of additional equations.

The structure of the price-forecasting model is outlined by the following set of equations:⁶

⁶A detailed description of the price-forecasting model is given in (3); however, that study was based on El Centro, Calif., prices. The present study refers to Chicago prices and has been revised to include 1968 data.

Price-forecasting equation (P_{ji}):

$$P_{ji} = f_j(M_{ji}^k, P_{(ji)-1}, P_{(ji)-2}, Q_1, Q_2, Q_3)$$

Choice slaughter cattle prices (P_{ji}) are predicted as a function of projected marketings of fed cattle (M_{ji}^k) in various regions, lagged prices of Choice steers, and quarterly dummy variables.⁷ The subscript $j = 1, 2, \text{ or } 3$ and refers to the month of the quarter for which the projection is being made; the subscript $i = 1, 2, 3 \dots 6$ and refers to the length of the projection in months. Thus, P_{22} would indicate a 2-month prediction of the second month of a quarter.

Fed-cattle marketings (M_j^k):

$$M_j^k = g_j(W_h^k, Q_1, Q_2, Q_3, T)$$

Fed-cattle marketings (M_j^k) (j defined as above) in region k are projected as a function of either predicted or actual cattle on feed (W_h^k) by weight group h in the region, plus quarterly variables and linear time trend.⁸

Cattle on feed (W_h^k):

$$W_h^k = f_h(S_k, C_k, W_{1k}, W_{2k}, W_{3k})$$

The number of cattle on feed in weight group h for region k is a function of January 1 inventories of steers (S_k) and calves (C_k) and the number of cattle on feed by weight group (excluding $h = 4$) in region k in the current quarter, or the total for the most recent two quarters.⁹

The model revolves around estimates of cattle on feed from which predicted marketings of fed cattle are derived. Since cattle-on-feed data are available only for the first of each quarter, the model is segmented by quarters. Either the current number of cattle on feed, or a projection of cattle on feed at the beginning of a

⁷Quarterly dummy variables take on values of 1 for the quarter for which the projection is being made and zero for all others. Projections for the fourth quarter use -1 values for all three dummy variables. In this manner the coefficients of the dummy variables are forced to sum to zero.

⁸Weight groups are 1 = 500-699 pounds, 2 = 700-899, 3 = 900-1,099, and 4 = 1,100 or more pounds. The cattle feeding regions used are as follows: California, Arizona, Texas, Colorado, and North Central Region.

⁹Separate equations are used for projecting cattle on feed, depending upon from which quarter the projection is being made; i.e., equations used if the projection is made January 1 are different from those used if the prediction is made on, say, July 1.

quarter, is needed to estimate marketings of fed cattle for the months in that particular quarter.

In some instances, projected cattle and calves on feed may require estimates of January 1 steer and calf inventories. The functional relationships used to predict these numbers are:

Steer inventory ($S_{(t+1)k}$):

$$S_{(t+1)k} = g_k(C_{tk}, BC_{(t-1)k}, M)$$

Steer inventory on January 1 for the coming year in region k is a function of the January 1 inventory for calves for the current year in that region (C_{tk}), the January 1 inventory of beef cows for the previous year in that region ($BC_{(t-1)k}$), and the average Kansas City-Chicago feeding margin for the current year up to the time of projection.¹⁰

Calf inventory ($C_{(t+1)k}$):

$$C_{(t+1)k} = h_k(PP'_{(t-1)}, BC_{tk}, BH_{tk}, PP')$$

Calf inventory on January 1 for the coming year in region k is a function of the average price of feeder steers at Kansas City the preceding year ($PP'_{(t-1)}$), inventories of beef cows (BC_{tk}) and beef heifers (BH_{tk}) on January 1 of the current year in region k , and the average price of feeder steers at Kansas City for the current year up to time of projection (PP').

Ordinary least-squares regression techniques were used to estimate the parameters of the model with data for 1960 through 1968.

The estimated coefficients for the price-forecasting model are presented in table 2. Three equations are given for each of the six monthly situations. The equation to be used depends on the month in the quarter for which the prediction is being made and the length of the projection. Thus, predicting the price for August on March 1 would require a 6-month projection for the second month of a quarter, or equation (17).

The use of projections as regressors clearly violates a basic assumption of least squares—that the regressors are nonstochastic—and leads to biased and inconsistent estimates of the parameters (7, pp. 282-84; 11, pp. 331-34). However, despite the bias of the estimated coefficients, the least-squares prediction does yield a

¹⁰The feeding margin is specified as being the difference between the current price of Choice slaughter steers at Chicago and the price of feeder steers (all weight and grades) at Kansas City 6 months previous.

Table 2.—Monthly forecasting equations for Chicago prices of 900 to 1,100-lb. Choice slaughter steers^a

Equation	Length of price projection (months)	Month of Quarter	Constant	Lagged Prices		Projected marketings of fed cattle					Quarterly dummy variables			Standard error of estimate	R ²
				P _{t-1}	P _{t-12}	Calif.	Ariz.	Colo.	Texas	North Central	Q ₁	Q ₂	Q ₃		
1	1	1	25.5993	.8085	.0612	-.0296	.0373	.0560	-.0145	-.0212	.9752	-3.9476	.9255	.6575	.9003
				(.0636)	(.0813)	(.0141)	(.0680)	(.0328)	(.0117)	(.0126)	(.6886)	(1.6751)	(.7902)		
2	1	2	32.6199	.8138	.0949	-.0224	.0170	.1492	-.0121	-.0361	1.3568	-.5771	-.5711	.9307	.8178
				(.1096)	(.1373)	(.0203)	(.0556)	(.1052)	(.0290)	(.0238)	(1.2614)	(1.6448)	(.8133)		
3	1	3	14.1734	.9250	-.1086	-.0875	-.0119	.0735	.0110	-.0146	.1056	1.0965	.1730	.8422	.8772
				(.0944)	(.0893)	(.0157)	(.0543)	(.0353)	(.0139)	(.0066)	(.4925)	(1.0193)	(.4667)		
4	2	1	17.3889	.8234	.1358	-.0513	.0555	.1040	-.0370	-.0152	2.0954	-4.3642	-.0039	.9391	.8029
				(.1010)	(.1149)	(.0200)	(.0977)	(.0464)	(.0169)	(.0186)	(1.0101)	(2.4035)	(1.1725)		
5	2	2	51.1228	.7174	.1943	-.0231	.0196	.2415	-.0204	-.0633	2.4014	-.0217	-1.3913	1.1610	.7165
				(.1393)	(.1682)	(.0253)	(.0695)	(.1275)	(.0361)	(.0285)	(1.5342)	(2.0433)	(.9768)		
6	2	3	12.6095	.8998	-.1017	-.0124	-.0324	.0792	-.0043	-.0108	.3929	.3508	.1425	1.3091	.7032
				(.1798)	(.1458)	(.0244)	(.0845)	(.0571)	(.0221)	(.0110)	(.7829)	(1.6284)	(.7257)		
7	3	1	18.1717	.8995	.1025	-.0529	.0076	.1192	-.0416	-.0078	2.2022	-3.3785	-.4593	1.1243	.7175
				(.1438)	(.1386)	(.0239)	(.1161)	(.0553)	(.0203)	(.0230)	(1.2202)	(2.9289)	(1.4433)		
8	3	2	59.4453	.6407	.2751	-.0207	.0174	.2960	-.0289	-.0756	2.9922	.2553	-1.8443	1.3351	.6340
				(.1669)	(.1914)	(.0291)	(.0300)	(.1447)	(.0414)	(.0325)	(1.7489)	(2.3513)	(1.1899)		
9	3	3	10.3537	.8885	-.0567	-.0066	-.0218	.0777	-.0060	-.0097	.3007	.3461	.2420	1.4877	.6167
				(.2396)	(.1677)	(.0281)	(.0959)	(.0681)	(.0255)	(.0132)	(.8944)	(1.8788)	(.8280)		
10	4	1	60.5648	.6130	.0667	.0433	.0606	.0274	.0505	-.0653	-1.0472	-.0751	3.1680	1.2668	6414
				(.1929)	(.1664)	(.0379)	(.1087)	(.0904)	(.0325)	(.0320)	(1.5962)	(1.6555)	(1.6113)		
11	4	2	4.3746	.8542	-.0547	-.0077	.0335	-.0502	.0165	.0032	-.9880	-1.2046	1.0349	1.4628	.5605
				(.1805)	(.1538)	(.0237)	(.1098)	(.0911)	(.0366)	(.0190)	(1.8750)	(3.8320)	(2.3070)		
12	4	3	3.6059	1.0892	-.2180	.0197	-.0891	-.0025	.0045	.0007	.8582	.3656	-.1774	1.5327	.5875
				(.2372)	(.1446)	(.0294)	(.1505)	(.0550)	(.0262)	(.0125)	(1.0876)	(3.2264)	(1.2522)		
13	5	1	-13.1291	1.1543	.0839	.0195	-.1465	-.0562	.0228	.0130	-1.4973	3.5276	-1.5205	1.1524	.6994
				(.2303)	(.1486)	(.0337)	(.1072)	(.0584)	(.0184)	(.0190)	(1.1273)	(1.7810)	(2.3656)		
14	5	2	-11.1833	.8938	-.0817	-.0104	-.0341	-.0684	-.0123	.0258	-2.0870	-5.5810	3.6527	1.4234	.5862
				(.1921)	(.1579)	(.0230)	(.1083)	(.0889)	(.0367)	(.0192)	(1.8500)	(3.8732)	(2.3196)		
15	5	3	9.0047	1.0126	-.2604	.0294	.0125	.0416	.0148	-.0127	.5202	3.2248	-.7612	1.6753	.5276
				(.2796)	(.1652)	(.0335)	(.1641)	(.0582)	(.0285)	(.0130)	(1.2042)	(3.5599)	(1.4431)		
16	6	1	19.7344	.7336	-.0690	-.0083	.0481	.0257	.0078	-.0142	.6056	-2.0398	1.5775	1.4887	.5202
				(.2829)	(.1861)	(.0467)	(.1413)	(.0700)	(.0271)	(.0230)	(1.5260)	(1.7941)	(2.9146)		
17	6	2	-9.6748	.9617	.0794	-.0123	.0442	-.0193	-.0124	.0112	-.6951	-2.8970	1.3123	1.4248	.6014
				(.2067)	(.1723)	(.0320)	(.1659)	(.0986)	(.0352)	(.0205)	(1.8297)	(3.6153)	(1.2798)		
18	6	3	10.1827	1.0970	-.1849	.0269	-.0679	.0448	.0222	-.0143	.9736	4.2338	-1.1534	1.6415	.5634
				(.2859)	(.1633)	(.0330)	(.1658)	(.0617)	(.0278)	(.0135)	(1.2141)	(3.6229)	(1.4378)		

^aNumbers in parentheses below the coefficients are standard errors.

value which converges in probability to the conditional expectation of the dependent variable given the observed values of the predetermined variables.¹¹ As Johnston comments, "Thus, where this type of prediction is required, least squares is appropriate, even though it would not be used to obtain estimates of the structural parameters" (9, p. 164).

The usual statistical tests of significance for the coefficients (e.g., the *t*-test), however, are not valid when stochastic regressors are used. Therefore, substantive conclusions cannot be made about the magnitude or signs of the estimated coefficients.

In all likelihood, some of the variables could be omitted from particular equations (e.g., Arizona marketings or last year's price). These variables were retained in all equations, however, since it can be argued a priori that such variables logically could be expected to have some effect on price.

As can be seen from table 2, the R^2 values decrease as the span of prediction increases. Conversely, the standard error of the estimate increases with the length of prediction.¹² The standard error figures in table 2 compare with standard deviations of the basic price data of \$1.78, \$1.87, and \$2.06 for the first, second, and third months of the quarter respectively for 1960-68. The distribution of the residuals was tested for normality by a chi-square test and the null hypothesis was not rejected.

The Decision Model

The results of the regression analysis served as the basis for generating the conditional or a posteriori distribution for the states of nature (termination prices for the futures contract). To incorporate into the model the difference between futures price and cash price, the closing futures price on the last day of trade for a futures contract (usually the 20th of the month) was related to the Chicago daily cash price for 900- to 1,100-pound Choice slaughter steers, as reported by the U.S. Department of Agriculture. Contracts not closed out by the last day of trade are subject to delivery; hence, it was felt that this comparison would be the most meaningful, since the two markets at this point are closely related.¹³

¹¹ See Johnston (9), pp. 162-64 for proof.

¹² Similar tendencies in the R^2 and standard error values were found by Hayenga and Hacklander (8) and Rohdy, Hoffman, and Madsen (12) in their monthly price forecasting models.

¹³ Since July 1968, delivery has been permitted after the sixth calendar day of the month, thus giving overlapping trade

Data for the 29 closing dates from April 20, 1965, through June 20, 1969, yielded a mean excess of futures prices over cash prices of \$0.22 per hundredweight, with a standard deviation of \$0.55.¹⁴ The hypothesis that the distribution of differences was normal was not rejected by a chi-square test. The distribution is widely dispersed as indicated by the size of the standard deviation relative to the mean. The available data were insufficient to permit meaningful tests of seasonal changes in the basis or the development of other conditional distributions. Such additional analyses, however, might yield distributions with smaller standard deviations relative to their means than the one utilized here.

As indicated earlier, the expected futures price is based on the predicted cash price plus the expected basis. The distribution, then, of the expected futures price is the sum of the two separate distributions, and the variance is the sum of the two variances (assuming no correlation between the two series). The variance of predicted price $V(\hat{P}_{t+k}^c)$ is given by¹⁵

$$\sigma_{\hat{P}}^2 = \sigma^2 X_* (X_*' X_*)^{-1} X_*$$

where

σ^2 is the disturbance variance (derived from the standard error of the estimate),

X_* is the vector of new observations of the predetermined variables from which P_{t+k}^c is being predicted.

$(X_*' X_*)^{-1}$ is the inverse of the product of the matrix of predetermined variables used in the regression analysis and its transpose.

Thus, the variance of P_{t+k}^c increases as the values of the predetermined variables move further from their means.

In our case, however, a price prediction for one point in time may be viewed as a single random drawing from the conditional distribution of P_{t+k}^c given X_* in which case the variance of the forecast error is given by

$$\sigma_f^2 = \sigma_{\hat{P}}^2 + \sigma^2$$

and delivery of contracts. This factor tends to bring futures and cash prices into a more predictable relationship during the entire month rather than just on the last day. The last day, however, was selected as representative, since after the close of trade that day there is no option other than to take or make delivery.

¹⁴ Futures prices are given in Chicago Mercantile Exchange Yearbook (5), published annually.

¹⁵ For derivation of the variance of the predicted values, see Goldberger (7), pp. 168-70.

The latter function reflects that the error in forecasting a single drawing is "the sum of two uncorrelated errors: the error in estimating the expectation of the distribution from which the drawing comes and the deviation of the drawing from its expectation" (Goldberger (7, p. 170)).

The variance of the forecast, σ_f^2 , was then added to the variance of the basis to derive the standard deviations for the various alternative predictions. To solve the decision problem, then, cash prices are predicted for the months for which futures contracts are being traded, and \$0.22 is added to this figure to get the expected futures price at time of termination. If the predicted futures price is greater than the current futures price for a particular contract, then a buy action (a_2) is indicated. Conversely, a lower predicted price points toward a sell action (a_1).

In mid-January 1969, as an illustration, cash prices were predicted by the forecasting model for February, April, and June 1969, and then adjusted to include the futures-cash basis. Since these forecasts were higher than the relevant futures prices prevailing near the middle of January, the latter prices were subtracted from the predicted values to obtain expected payoffs from the alternative actions. The predictions and expected and actual payoffs are given in table 3.¹⁶ In all three instances, the forecasting model underestimated the actual prices that developed, and the actual payoff exceeded the expected value. Nonetheless, the action indicated by the decision model was correct. The underestimation of actual prices which moved beyond the range of the price data included in the regression analysis emphasizes the need to incorporate recent data continually in revising the coefficients of the forecasting equations.

The risk component, however, must be included in the speculation decision. The variances of the predicted cash prices, σ_p^2 , given the set of predetermined variables for February, April and, June, were 1.9691, 2.2492, and 4.8782 respectively. When added to the disturbance variances (the standard error of the estimate squared), these values resulted in estimated variances of a single forecast, σ_f^2 , of 3.3167, 3.8539, and 7.5727 for the 3 months. The standard deviations of predicted futures prices then were 1.9024 for February, 2.0387 for April, and 2.8063 for June.¹⁷

The probabilities of making a profit, given the predicted values and current futures prices, are given in

¹⁶Costs of futures trading have not been subtracted in this example; hence the profit figures are gross rather than net levels.

¹⁷These standard deviations are calculated as $\sqrt{\sigma_p^2 + (.55)^2}$ where $(.55)^2$ is the variance of the basis.

Table 3.—Predicted prices and expected payoffs for 3 futures contracts, January 1969

(Dollars per hundred weight)

Month	Predicted cash price	Predicted futures price	Current futures price ^a	Expected payoff ^b	Actual payoff ^c
February . . .	28.02	28.24	28.05	0.19	1.87
April	27.78	28.00	27.00	1.00	3.53
June	27.71	27.93	26.60	1.33	8.50

^aPrices at close of trade January 17, 1969.

^bBefore deduction of commission charge.

^cBased on prices at close of final day of trade.

table 4. Thus, April has the highest probability of profit (.688); June is about the same (.681). February is somewhat less favorable, although still more than 50-50 (.539).

Table 4.—Standard error of the forecast and probability of making a profit

Month	Standard error of predicted futures price ^a	Probability of a profit ^b	Required price level ^c
	σ_{f*}		
	Dol./cwt.		Dol./cwt.
February	1.9024	.539	26.96
April	2.0387	.688	26.62
June	2.8063	.681	26.04

^aJoint error for cash prediction and futures-cash differential.

^bConverted to a standard normal by $(\hat{P}_{(t+k)}^f - P_t^f) / S_{\hat{P}^f} = Z$ where Z = standard normal variate.

^cBased on requirement of probability of .75 of making a profit $\hat{P}_{(t+k)}^f - .6748\sigma_{f*}$.

If the speculator desires, say, a .75 probability of making a profit, he can evaluate the three buy alternatives by subtracting .6748 times the standard deviation of the futures price estimate from the expected value and comparing the resulting figure with the current futures price. The results of this procedure are shown in the last column of table 4. On this basis, no action would be taken since current prices of all three contracts are above the level needed to give a .75 probability of profit.

Either procedure can be used to evaluate the speculation alternatives.

Summary and Conclusions

The above development illustrates an application of statistical decision theory to speculation in live beef cattle futures contracts. Cash prices are forecast by equations derived from statistical analysis, and then are adjusted to predict futures prices up to 6 months ahead. The variance of the forecast, derived from the least-squares regression analysis involving cash prices, is combined with the variance of the historical distribution of the futures-cash price differential to obtain a measure of the distribution around the predicted futures price. The resulting joint standard error is used to evaluate the probability of profit, given the predicted futures price and the current price for that futures contract.

If the speculator requires a particular probability of making a profit, he can apply the Baumol formulation by subtracting some multiple of the standard error from the price forecast and comparing the remainder with the current futures price. Or, he can simply determine the probability of making a profit for each alternative and evaluate them accordingly.¹⁸

Any price-forecasting model, such as the one developed here, which is used for decision-making should be continually updated. This need has been particularly evident in projecting cash prices for live beef cattle where an upward trend in demand has pushed prices beyond the range included in the initial analysis. Although the price-forecasting model indicated the correct actions on the part of speculators for the first 6 months of 1969, it substantially underestimated prices for April and June.

¹⁸The speculator's subjective probabilities about price movements undoubtedly are important components of his decision process. He could inject his subjective probabilities into this model by computing $P(Z|0)$ as outlined in (4) and then apply his subjective estimate of $P(\epsilon)$.

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