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The Demand for Fertilizer, 1949-64: An Analysis of Coefficients From Periodic Cross Sections

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Econometric studies of the demand for farm inputs have accumulated large amounts of quantitative information in recent years. The usefulness of this information for various policy purposes, however, is questionable, since most of these studies have assumed that input demand coefficients have remained constant over time. The relaxation of this assumption should prove useful by allowing decision makers to take explicit account of the way in which demand coefficients are changing over time.

This analysis deals with the demand for a particular input, fertilizer, with major emphasis on analyzing the variations in demand coefficients by means of periodic cross-section data at the State level. A number of previous studies (2, 4)[1] based on time-series data have provided estimates of the demand for fertilizer, assuming the coefficients to be constant over the period we propose to investigate. A study by Griliches (3) was concerned with similar estimation based on cross-section data for a single year, 1954. The Griliches study provides the foundation for our article.

More specifically, our objectives are:

(1) To ascertain whether fertilizer demand coefficients based on cross-section data have remained constant over time.

(2) To determine whether any useful information is revealed by introducing dynamic and stochastic considerations. For example, does the long-run demand for fertilizer, based on static assumptions, differ from the long-run demand based on dynamic assumptions? Are the results substantially the same if coefficients of explanatory variables are assumed subject to random variation rather than constant?

(3) To determine whether introduction of variables signifying shifts for State or regional classifications and several cross-sections over time alter a previous conclusion by Griliches (3, p. 383) that a dynamic adjustment model may "not represent a very useful approach to cross-sectional data." Griliches suggests that a more useful approach than simply introducing a lagged value of the dependent variable 5 years earlier (an approach he employed) might consist of employing several cross sections and allowing dummy variables for each State.

Changes in Fertilizer Use

Fertilizer use in the United States has increased dramatically in recent years. For example, the use of the three principal plant nutrients--nitrogen, phosphoric acid, and potash--which either individually or in combination make up fertilizer "tripled between 1950 and 1965" (10, p. 1). Although the increase in magnitude for these three nutrients has been about the same, the use of nitrogen and potash has increased relative to the use of phosphoric acid (2, p. 595). At the same time, the average plant nutrient content of fertilizer rose from .20 in the earlier 1940's to .36 in 1965.

In contrast to other farm inputs, the price of fertilizer has not increased in recent years. In fact, as Griliches points out, the "real" price of fertilizer, measured as the price of a plant nutrient unit relative to the price received for all crops, "fell by 50 percent between 1940 and 1950" (2, p. 595). Moreover, there was a modest downward trend of this price from 1950 through 1964.

In addition to these changes at the aggregate U.S. level, there also exists a substantial range among States in the nutrient content of fertilizer, the proportion in which nutrients are consumed, and the levels of fertilizer use from State to State (3). Similarly, the pattern of fertilizer use has shifted considerably over major regions of the United States from 1945 to 1967. Pounds of fertilizer consumed per acre rose rapidly in the West North Central region and moderately in the East North Central, West South Central, Mountain, and Pacific regions, and fell rapidly in the New England, Middle Atlantic, South Atlantic, and East South Central regions.

[1] Underscored numbers in parentheses indicate items in the References, p. 56.

Data Models And Procedures

The specified form of the basic economic model is similar to Griliches' model (3) and the statistical model was confirmed by inspecting the original data with the aid of scatter diagrams. The basic form of the model is:

$$(1) \qquad Y = X_1^{\beta_1} X_2^{\beta_2} X_3^{\beta_3}$$

where Y = pounds of principal plant nutrient consumed per acre; X_1 = "real" price of fertilizer, i.e., price paid per plant nutrient unit relative to average value of crops produced per acre; X_2 = price of fertilizer relative to the average cash wage paid per day to hired farmworkers; and X_3 = price of fertilizer relative to average cash rent paid per acre. Thus, fertilizer use is treated as a function which is linear in logarithms of the price paid for fertilizer, the prices received for farm products, and prices paid for two other major inputs--labor and land.

Such a function is implied by a Cobb-Douglas production function between farm output and fertilizer, land, and labor, as well as profit maximization by individual farmers. Therefore, equation (1) may be regarded as the reduced form of the fertilizer demand equation resulting from a structural form which simultaneously determines output and levels of the various inputs.[2]

The data employed to estimate the various alternative formulations of equation (1) are cross-sectional from the 48 contiguous States for the years 1949, 1954, 1959, and 1964. Therefore, for observation i in year j, equation (1) becomes:

$$(2) \qquad Y_{ij} = X_{1ij}^{\beta_{1j}} X_{2ij}^{\beta_{2j}} X_{3ij}^{\beta_{3j}}$$

where i = 1, . . ., 48, denotes the contiguous States, and j = 1949, 1954, 1959, and 1964, denotes years for which cross-sectional data are utilized. Cross-sectional rather than time-series data were utilized because it was not feasible to assess the contribution of input prices (other than fertilizer prices) on fertilizer consumption for the latter data. In contrast, cross-sectional data allowed us to isolate the impact of other important input prices in addition to the real price of fertilizer.

Three measures of the variables included in equation (2) are employed in our analysis. These are pounds of principal plant nutrient used per acre or cropland harvested (Y); pounds of principal plant nutrient used per acre of cropland harvested and improved pasture (Y'); and pounds of principal plant nutrient, adjusted for the change in nutrient content by a price-weighted relationship used per acre of cropland harvested and improved pasture (Y'').[3]

Four basic formulations of the econometric models implied by equation (2) are estimated, each based on alternative assumptions. The first two employ the traditional assumptions of constant coefficients at a given point in time for successive observations across States. The first model, which we call the nonrandom static model, may be represented as:

$$(3) \qquad y_j = \beta_{1j}x_{1j} + \beta_{2j}x_{2j} + \beta_{3j}x3_j + \mu_j$$

where j = 1949, 1954, 1959, 1964 and the lowercase letters denote the logarithms of the variables in equation (2). To ascertain whether the coefficients of this model are constant over time, we formulated the following null hypothesis:

$$H_o^s : \beta_{49} = \beta_{54} = \beta_{59} = \beta_{64} = \beta$$

where $\beta_j' = (\beta_{1j}\ \beta_{2j}\ \beta_{3j})$ and $\beta' = (\beta_1 \beta_2 \beta_3)$.

The statistical model based on H_o^s may be stated as:

$$(3.a) \qquad Y = X\beta + U^{so}$$

where

$$Y' = (Y_{49}\ Y_{54}\ Y_{59}\ Y_{64}),$$

$$X' = (X_{49}\ X_{54}\ X_{59}\ X_{64}),$$

$$X_j = (X_{1j}\ X_{2j}\ X_{3j}), \text{ and}$$

$$U^{so'} = \left(U_{49}^{so}\ U_{54}^{so}\ U_{59}^{so}\ U_{64}^{so}\right)$$

denotes the vector of stochastic terms under the null hypothesis. Each element is assumed to

[2] Although the problem of simultaneous determination of the variables in the proposed model exists, it does not appear to be of major importance in the case of fertilizer demand. Griliches (2, p. 601) suggests that since fertilizer prices are largely "administered prices" and since prices of output and inputs may be regarded as predetermined in the fertilizer decision, the simultaneous interaction may be ignored. For a treatment of the interaction between output and various inputs, see Malinvaud (7, pp. 517-520).

[3] A paper providing a detailed discussion of the transformations, raw data sources, and comparisons of the data series, as well as a more complete development of the various econometric models estimated, will be supplied by the authors upon request.

be normally distributed, more precisely $U^{so}_{ij} \sim N(0,\sigma^2_{so})$.

A more general specification would suggest that $B_{j'} \neq B_j (j' \neq j)$. Under this alternative hypothesis, the coefficients are allowed to vary from one cross-section to another. More precisely,

$$H^S_a: \beta_{49} \neq \beta_{54} \neq \beta_{59} \neq \beta_{64}$$

and the corresponding model is:

$$Y = X^* \beta^* + U^{sa}$$

where X* denotes a diagonal matrix of which each nonzero element is

$$X_j, \beta^* = (\beta_{49}\ \beta_{54}\ \beta_{59}\ \beta_{64}), \text{ and}$$

$$U^{sa} = (U^{sa}_{49}\ U^{sa}_{54}\ U^{sa}_{59}\ U^{sa}_{64})$$

denotes the vector of stochastic terms under the alternative hypothesis. Each element is assumed to be normally distributed.

Coefficients are estimated under the null hypothesis as a pooled regression and the alternative hypothesis as separate regressions for each census year by ordinary least squares. This was done for the three measures: Y, Y', and Y''.

The test employed to determine whether the coefficients (elasticities) are constant over the years considered is based on an F-statistic. It is calculated as:

$$F = \frac{\hat{U}^{so'}\hat{U}^{so} - \hat{U}^{sa'}\hat{U}^{sa}/(df_{so} - df_{sa})}{\hat{U}^{sa'}\hat{U}^{sa}/df_{sa}}$$

where df_{so} and df_{sa} denote the degrees of freedom under the null and alternative hypothesis, respectively.

Another formulation of the static model introduced an intercept coefficient and allowed its value to vary over States. This was accomplished by introducing dummy variables into the models resulting from the null and alternative hypotheses. In estimating such models, we found that although many of the dummy variable coefficients for individual States are significant, they tend to group around certain specific values. Thus, it seems reasonable to employ regional dummy variables which have the added advantage of summarizing information from a group of States with fewer coefficients.

For this formulation, nine regions were utilized to group various States.[4] Such a regional definition was found to be generally consistent with the grouping pattern of the individual State dummy variable formulation. The resulting intercept coefficient over time is estimated as:

$$Y = X^*\beta^* + D\delta + U^{sr} \qquad (3.c)$$

where D denotes a 4x9 matrix of dummy variables, each element

$$D_{jr} = \begin{Bmatrix} 1, \text{ if the observation (State)} \\ \quad \text{is in the rth region} \\ 0, \text{ otherwise} \end{Bmatrix}, \text{ for all } j;$$

$$\delta' = (\delta_1, \ldots, \delta_9)$$

denotes the respective regional dummy coefficients; and

$$U^{sr'} = (U^{sr}_{49}\ U^{sr}_{54}\ U^{sr}_{59}\ U^{sr}_{64})$$

denotes the vector of stochastic terms under the static specification employing regional dummies. Each element is assumed to be normally distributed.

The second model is a simple dynamic version of equation (3) employed by Griliches (3) and is referred to here as the nonrandom dynamic model, its form is

$$y_j = \lambda_j \beta_{1j} x_{1j} + \lambda_j \beta_{2j} x_{2j} + \lambda_j \beta_{3j} x_{3j} + (1-\lambda_j) y_{j-5} + \lambda_j \mu_j .$$

The difference between these two models is simply that for the latter, λ_j (defined as the adjustment coefficient) is allowed to assume values other than 1. Moreover, the formulation in (3) interprets the interstate differences in plant nutrient use as "representing an adjustment to long-run differences in relative prices" (3, p. 377), while formulation (4) recognizes the possibility that interstate differentials may also reflect short-run considerations.

The derivation for equation (4) is based on the "partial adjustment" model. The basic premise is that an adjustment to a disequilibrium situation is not instantaneous. There presumably exists some sort of adjustment cost, such as a cost of being out of equilibrium

[4] As defined in Agricultural Statistics (11), these regions are: Region 1–New England; Region 2–Middle Atlantic; Region 3–East North Central; Region 4–West North Central; Region 5–South Atlantic; Region 6–East South Central; Region 7–West South Central; Region 8–Mountain; and Region 9–Pacific.

or a cost of change which justifies the observed inertia in the responsiveness of farmers to relative input prices, i.e., $y_t^* = f(x_t)$. That adjustment during a period of time is a function of the difference between the desired and the actual level of consumption, i.e.,

$$y_t - y_{t-1} = \lambda(y_t^* - y_{t-1}).$$

Substituting the first equation into the second and solving for y_t, we have

$$y_t = \lambda f(x_t) + (1-\lambda)y_{t-1}$$

which is nothing more than an alternative representation of (4). However, note that in (4) a 5-year lag is assumed. This is the result of having comparable data only at 5-year intervals and the lack of any substantial changes in cross-sectional differentials for 1-year periods.

Procedures similar to those employed in the case of the nonrandom static model are utilized to ascertain whether all coefficients, including the adjustment coefficient, are constant over time. However, here 1949 is omitted, since including the lagged dependent variable representing 1944 (a war year) might be expected to involve considerable structural changes. The relevant null hypothesis, therefore, is:

$$H_o^d: \begin{bmatrix}\lambda\beta \\ 1-\lambda\end{bmatrix}_{54} = \begin{bmatrix}\lambda\beta \\ 1-\lambda\end{bmatrix}_{59} = \begin{bmatrix}\lambda\beta \\ 1-\lambda\end{bmatrix}_{64} = \begin{bmatrix}\lambda\beta \\ 1-\lambda\end{bmatrix}$$

and the corresponding model is designated as (4.a). The general specification for the dynamic case follows from the alternative hypothesis in which all coefficients vary among years, i.e.,

$$H_a^d: \begin{bmatrix}\lambda\beta \\ 1-\lambda\end{bmatrix}_{54} \neq \begin{bmatrix}\lambda\beta \\ 1-\lambda\end{bmatrix}_{59} \neq \begin{bmatrix}\lambda\beta \\ 1-\lambda\end{bmatrix}_{64},$$

and the model accordingly is designated (4.b).

In addition to the above specifications, a dummy variable formulation of the nonrandom dynamic model was introduced which is operationally equivalent to (3.c) except that $\lambda_j \neq 1$, for all j. This specification employs the same regional dummy variables as (3.c) and is referred to as (4.c).

The second two models are based on the same distinction between static and dynamic but assume that the coefficients are subject to random variation. They are referred to as the random static and random dynamic models. The rationale for such an assumption has been fairly well outlined by Hildreth and Houck (5, p. 584) for the general case. For the case here, response of fertilizer use to relative prices depends on a number of factors in a given State such as soil types, climatic conditions, crops grown; natural stocks of nutrients in the soils; amount of capital possessed by, or available to, farmers; and type of land tenure. Unfortunately, these factors cannot always be observed and, when they are, they cannot be conveniently included in regression models. However, since these factors do vary across States and over time and are unobserved, it seems reasonable, as an alternative formulation to (3) and (4), to regard the coefficients as the mean of a random coefficient (elasticity). Thus, formulations (3) and (4) become:

(5) $$y_{ij} = (b_{1j} + v_{1ij})x_{1ij} + (b_{2j} + v_{2ij})x_{2ij} + (b_{3j} + v_{3ij})x_{3ij}$$

(6) $$y_{ij} = \lambda_j(b_{1j} + v_{1ij})x_{1ij} + \lambda_j(b_{2j} + v_{2ij})x_{2ij} + \lambda_j(b_{3j} + v_{3ij})x_{3ij} + (1-\lambda_j)y_{ij-5}$$

where i, j are as defined previously; b_{kj} denotes the mean response of fertilizer consumption to a unit change in the kth independent variable, k = 1, 2, 3; and v_{kij} denotes independently and identically distributed random errors with zero means. For these formulations, the actual response is $(b_{kj} + v_{kij})$ for the i, jth observation.

The procedures employed to estimate the unknown parameters of these two models require some further elaboration. Considering the random static formulation, to determine the mean response (elasticity) of fertilizer consumption to a unit change in one of the independent variables we reformulate (5) as:

(5') $$y_{ij} = b_{1j}x_{1ij} + b_{2i}x_{2ij} + b_{3i}x_{3ij} + \omega_{ij}$$

where

$$\omega_{ij} = \sum_{k=1}^{3} x_{kij}\, v_{kij};$$

i, j as defined previously. The b_{kj} are the mean elasticities and v_{kij} are the unobserved random disturbances with zero means.

More precisely, the v_{ki} are assumed to be independently and identically distributed with variance

$$E(v_{ki})^2 = \alpha_{kk}.$$

Moreover, for any i', i and $k' \neq k$, it is assumed that $v_{k'i'}$ is independent of v_{ki}. Thus, for any given j, the variance-covariance matrix of ω_j, $E(\omega_j\omega_j') = \theta$, is composed of $\theta_{i'i} = 0$, $i' \neq i$ and

$$\theta_{ii} = \sum_{k=1}^{3} x_{ik}^{2} \alpha_{kk} = \dot{x}_{(t)} \alpha,$$

where $\dot{x}$ is a 48 x 3 matrix whose elements are the squares of the corresponding elements of x, $\dot{x}_{(t)}$ is the tth row of $\dot{x}$, and α is a column vector with elements $\alpha_k = \alpha_{kk}$, $k = 1, 2, 3$. This development follows from Hildreth and Houck (5, pp. 585-589).

There are a number of consistent estimators of the unknown parameters of formulation (5'). Rao (8) has considered related models, and Rubin (9) has derived maximum likelihood estimators for model (5'). Because of the complexities of programming Rubin's estimators, we have utilized a less sophisticated consistent estimator suggested by Hildreth and Houck (5). The estimator is:

$$\text{(7)} \qquad b = (x' \hat{\theta}^{-1} x)^{-1} x' \hat{\theta}^{-1} y$$

where $\hat{\theta}$ denotes a diagonal matrix which is an estimate of the variance covariance matrix of ω. The elements of the diagonal are given by:

$$\text{(8)} \qquad \hat{\theta} e = \dot{x}\hat{\alpha}$$

where e denotes a T column vector, the transpose of which is 1,1, ..., 1), and $\dot{x}$ denotes a matrix whose elements are the squares of the corresponding elements of x.

The estimator, $\hat{\alpha}$, was adopted after consideration of three estimators, all of which are consistent. This estimator is derived by applying restricted least squares to a regression problem where the unknown parameters are the values of $\hat{\alpha}_k$. This is a simple quadratic programming problem (6, pp. 170-172) which selects the vector $\hat{\alpha}$ that minimizes the sum of squared errors subject to the condition that the elements of $\hat{\alpha}$ be nonnegative.

The basic approach to the derivation of equation (7) follows from generalized least squares (7, pp. 149-153), where α and thus θ are unknown. That is, the procedure is first to derive a consistent estimate of α by quadratic programming and utilize these estimates in equation (8) to derive the diagonal elements of θ which, in turn, are utilized in equation (7) to estimate the vector $\hat{b}$.

The same estimator employed for formulation (5') is utilized for model (6) which is similarly rearranged as

$$\text{(6')} \qquad y_{ij} = \lambda_j b_{1j} x_{1ij} + \lambda_j b_{2j} x_{2ij} + \lambda_j b_{3j} x_{3ij} + (1-\lambda_j) y_{ij-5} + \lambda_j \omega_{ij}.$$

As before, the essential difference between the two models is that for (5') we estimate the mean elasticities under the assumption that $\lambda_j = 1$, for all j, while for formulation (6'), we impose the estimates derived from the nonrandom dynamic model. Moreover, for each of the two random models, we estimate the coefficients under both the null and alternative hypotheses. In the case of the random static model, the equation corresponding to the null hypothesis is designated (5.a') while the equation corresponding to the alternative hypothesis is designated (5.b'); similarly, for the random dynamic model, the designations are (6.a) and (6.b').

In the following section, we present the estimates for each of these four basic models and a test of the hypothesis that the coefficients, estimated for formulations (3) and (4), are constant over the period for which we have cross-sectional data.

Empirical Results

NONRANDOM STATIC MODEL

In table 1, the results for the three different measures of the variables are reported under the null hypothesis, while only the results for the weighted price measure (Y'') are reported under the alternative hypothesis. In the case of Y'', $F = 7.15$ and is significant at the .05 level. Therefore, the null hypothesis, H_0^S, is rejected. For all three measurements, we come to the identical conclusion that the elasticities vary from one cross section to another. Least-squares estimates of the parameters give the expected signs regardless of the measurements employed. Although there is close statistical correspondence between all three measures, we select Y'' for all further investigation since it accounts for changes in fertilizer nutrient content and, thus, provides a more intuitively appealing measure of plant nutrients used per acre. Note that the estimates conform to the intuitive notion that labor is a complement to and land is a substitute for fertilizer since the cross elasticity for labor ($-\hat{\beta}_{1j}$) is negative and the cross elasticity for land ($-\hat{\beta}_{3j}$) is positive.

Further examination indicates that a major portion of the interstate variation represented by each series is explained by a few simple economic variables (high R^2). The standard errors of the estimates are low and all estimates are highly significant. It is clear that the absolute value of all coefficients trends downward over the period of investigation implying that fertilizer use per acre has become less responsive to relative changes in other input prices and crop value per acre. For example, in 1949, a 10-percent increase in crop value was associated with approximately an 8.9-percent increase in fertilizer use; a 10-percent increase in the price of labor was associated with approximately a 14.3-percent decrease in fertilizer; a 10-percent increase in the price of land services was associated with approximately a 10.4 percent increase in fertilizer use. In contrast, in 1964, the corresponding responses of fertilizer use to 10-percent increases in the variables referred to above are, respectively, a 6.2-percent increase, a 4.2-percent decrease, and a 5.1-percent increase.

In an attempt to shed further light on the parameter changes we tested to determine if all coefficients for each 2 successive years were stable. In each case, the null hypothesis of no parameter change could not be rejected. Thus, an isolation of parameter shifts between any successive census years was not possible.

Note, however, that fertilizer use has become more responsive to its own price over the same period. For example, in 1949, a 10-percent increase in the price of fertilizer led to approximately a 5-percent $[(-.891 + 1.427 - 1.038)*10]$ decrease in fertilizer use, while a similar increase in 1964 resulted in approximately a 7-percent $[(-.620 + .421 - .505)*10]$ decrease in fertilizer use.

The inference that fertilizer use is responsive to its own price is based on reformulating (3) as

$$\ln Y_j = (\hat{\beta}_{1j} + \hat{\beta}_{2j} + \hat{\beta}_{3j}) \ln P_j^f - \hat{\beta}_{1j} \ln V_j - \hat{\beta}_{2j} \ln P_j^w - \hat{\beta}_{3j} \ln P_j^r$$

where

$$X_{1j} = \frac{P_j^f}{V_j},\ X_{2j} = \frac{P_j^f}{P_j^w},\ \text{and}\ X_{3j} = \frac{P_j^f}{P_j^r},$$

and X_{kj} are as defined on p. 46, which implies that the sum of the estimated coefficients is the elasticity of fertilizer demand with respect to fertilizer price (P_j^f).

The regional dummy variable specification (3.c) in table 1 is reported here only for the formulation which allows intercept coefficients to vary across regions and elasticities to vary over time. This is the result of little additional information (i.e., insignificant) being provided by alternative designations where intercept coefficients vary over time as well as across regions for both stable and varying elasticities.

The results of this formulation may be regarded as superior to those of (3.a) or (3.b). Table 1 reveals that all β_{kj}($k = 1, 2, 3$; $j = 1949, \ldots, 1964$) coefficients are significant at the 5-percent level and lower in absolute value than the corresponding estimates for model (3.b). The reduction in these estimates is the result of including previously omitted variables which, for five of the nine regions, are significant at the 5-percent level. Moreover, as indicated by the F value of 18.5, the set of regional dummy variables is highly significant. This F value provides a test of the null hypothesis $\delta = (\delta_1 \ldots \delta_9) = (0 \ldots 0)$ against the alternative hypothesis $\delta = (\delta_1 \ldots \delta_9) \neq (0 \ldots 0)$ and implies that the former should be rejected in favor of the latter. Note also that the measure of fit provided by the value R^2 is considerably higher than the similar measure for the previous model. Thus, it appears the inclusion of regional dummy variables adequately summarizes the effects of States (grouped) on the amount of fertilizer used per acre.

Considering the values of the regional dummy coefficients, it is clear that the New England States (region 1) and Middle Atlantic States (region 2) have had the largest impact on U.S. fertilizer demand. A probable explanation lies in the intensive farming in these regions as well as their general acceptance of fertilizer practices. The South Atlantic States (region 5) are quite close to those mentioned above in their effect on demand; this may be due in part to the considerable amounts of fertilizer required for tobacco production. In contrast, the Mountain States (region 8) show the smallest impact on demand, which is most likely explained by the region's extensive farming in such enterprises as grains and range feeding of cattle. The values of the dummy variable coefficients for the other regions fall between those discussed above and similarly may be explained on the basis of the crops and types of farming found in such areas.

NONRANDOM DYNAMIC MODEL

Tests of the null hypothesis regarding stable slope and adjustment coefficients for the nonrandom dynamic model are shown in table 2 and indicate that it should be rejected at the .05 level of significance. In all regressions,

Table 1.--Nonrandom static models

Eq. no.	Dependent variable	N	Year	Coefficients of[a/] X_{1i}	X_{2i}	X_{3i}	R^2	F
3.a	Y (.071)	192	49-64	-.747* (.059)	.830* (.077)	- .728* (.070)	.735	6.94*
3.a	Y' (.063)	192	49-64	-.673* (.057)	.820* (.074)	- .807* (.067)	.748	6.00*
3.a	Y'' (.067)	192	49-64	-.693* (.056)	.817* (.072)	- .770* (.068)	.742	7.15*
3.b	Y'' (.089)	48	49	-.891* (.148)	1.427* (.199)	-1.038* (.180)	.708	
3.b	Y'' (.053)	48	54	-.733* (.090)	.913* (.117)	- .786* (.112)	.811	
3.b	Y'' (.045)	48	59	-.663* (.077)	.654* (.095)	- .666* (.093)	.820	
3.b	Y'' (.035)	48	64	-.620* (.059)	.421* (.073)	- .505* (.069)	.856	
3.c	Y'' (.043)	192	49	-.816* (.079)	1.284* (.177)	- .720* (.097)	.904	
			54	-.704* (.081)	.751* (.172)	- .408* (.100)		
			59	-.605* (.082)	.467* (.160)	- .321* (.100)		
			64	-.522* (.080)	.183 (.155)	- .181** (.096)		

Regional dummy variables 1	2	3	4	5	6	7	8	9	F
1.50* (.58)	1.42* (.58)	1.15* (.58)	.41 (.60)	1.40* (.53)	1.09* (.52)	.41 (.57)	.30 (.60)	.65 (.63)	18.5*

a/ Coefficients significantly different from zero at the 5-percent level are denoted by *; those at the 10-percent level by **. Standard errors of the estimates are reported in parentheses.

Table 2.--Nonrandom dynamic models

Eq. no.	Dependent variable	N	Year	Coefficients of[a/] X_{1t}	X_{2t}	X_{3t}	Y_{t-5}	λ	R^2	F
4.a	Y'' (.021)	144	54-64	-.220* (.028)	.043 (.036)	-.154* (.032)	.609* (.025)	.391	.962	3.59*
4.b	Y'' (.023)	48	54	-.289* (.050)	.096 (.077)	-.152* (.066)	.563* (.040)	.437	.966	
4.b	Y'' (.018)	48	59	-.062 (.048)	-.119** (.061)	-.015 (.055)	.826* (.052)	.174	.973	
4.b	Y'' (.019)	48	64	-.118* (.058)	-.056 (.060)	-.128* (.052)	.695* (.066)	.305	.958	
				Long-run coefficients						
4.a	Y''	144	54-64	-.567	.111	-.397				
4.b	Y''	48	54	-.661	.220	-.348				
4.b	Y''	48	59	-.356	-.684	-.086				
4.b	Y''	48	64	-.387	-.184	-.420				
4.c	Y'' (.020)	144	54	-.344* (.049)	.176 (.118)	-.122* (.059)	.535* (.041)	.465	.972	
			59	-.131* (.061)	-.045 (.117)	.022 (.059)	.784* (.068)	.216		
			64	-.179* (.066)	-.002 (.111)	-.083 (.059)	.652* (.079)	.348		

Regional dummy variables 1	2	3	4	5	6	7	8	9	F
.235 (.410)	.203 (.405)	.302 (.406)	.270 (.418)	.222 (.375)	.206 (.361)	.070 (.402)	.090 (.427)	.179 (.438)	1.24

Long-run coefficients (4.c)

X_{1t}	X_{2t}	X_{3t}
-.740	.378	-.262
-.606	-.208	.102
-.514	-.005	-.239

a/ Coefficients significantly different from zero at the 5-percent level are denoted by *. Standard errors of the estimates are reported in parentheses.

the introduction of the lagged dependent variable reduces both the absolute size and the statistical significance of all other coefficients. The decrease in absolute size is expected, since the nonrandom dynamic model treats these coefficients as short-run coefficients, while the previous static model assumes that the corresponding coefficients reflect primarily long-run considerations.

The reduction in statistical significance and the wrong signs on the estimates, in some cases, are more difficult to explain. The most reasonable explanation appears to be multicollinearity. Y_{j-5} is highly correlated with X_{1j} and X_{3j} and moderately correlated with X_{2j}. Thus, it is likely that the variance of the estimates relative to the value of the estimates is high, resulting in insignificant t-tests for the coefficients but, nonetheless, increasing values of R^2. Moreover, Y_{j-5} is clearly the dominant variable for all regressions of the nonrandom dynamic model and most likely "robs" from X_{1j}, X_{2j}, and X_{3j} in the explanation of interstate variation in fertilizer use per acre. This possibility is supported by the rather high estimated correlation between the estimates of $\lambda_j\beta_{kj}$, and $1-\lambda_j$, $k = 1, 2, 3$. These values are in the neighborhood of .4 to .7, with the highest values occurring between $\lambda_j\beta_{2j}$ and $1-\lambda_j$. Thus, it seems reasonable to suggest that our estimates of $\lambda_j\beta_{kj}$, $k = 1, 2, 3$, are biased downward and our estimates of $1-\lambda_j$ are biased upward. This implies that the estimates of the adjustment coefficients, λ_j, are biased downward.

The above discussion also partially explains the long-run coefficients (β_{kj}) shown in table 2, provided the degree of underestimation of $\lambda_j\beta_{kj}$, $k = 1, 2, 3$, and λ_j is not the same. If the relative underestimation were the same, then estimates of the long-run coefficients are unbiased. This follows, since the estimate of the various long-run coefficients is simply $\frac{\lambda_j\beta_{kj}}{1 - (1 - {}_j)}$.

However, if the relative underestimation of $\lambda_j\beta_{kj}$, $k = 1, 2, 3$, is greater than the relative underestimation of λ_j, then estimates of the long-run coefficients are biased downward. This appears to be the case, since estimates of the long-run coefficients for the dynamic model are smaller (in absolute terms) than estimates resulting from the static model under both the null and the alternative hypotheses. These differences, in some cases, are small (e.g., 1954 estimates of $\beta_{1,54}$) and may not be statistically significant, but for other estimates (particularly all estimates of β_{3j}), it would appear that either the magnitude of underestimation of the long-run coefficients is large or that the original reason for introducing Y_{j-5} was not valid.[5] That is, cross-sectional differences reflect adequately long-run differentials in fertilizer consumption, and the dynamic model may "not represent a useful approach to cross-section data" (3, p. 383). Note, however, as we would expect, the implied long-run coefficient of the price of fertilizer (sum of the coefficients on X_{1j}, X_{2j}, X_{3j}) is considerably larger than the corresponding coefficients for the static formulation (1954: -789 vs. -.502; 1959: -1.126 vs. -.606; 1964: -.991 vs. -.704). A decision on the adequacy of the "adjustment model" must await the empirical results of other models examined in this paper.[6]

As in the case of (3.c), the results for the dummy variable formulation of the dynamic model are reported in the bottom portion of table 2 only for the specification which allows intercept coefficients to vary across regions and elasticities to vary over time. The justification is the equivalent to that noted in the discussion of the empirical results for (3.c).

As shown in table 2, all coefficients of the regional dummy variables are roughly one-half their standard errors and thus insignificant. However, the presence of the lagged dependent variable confounds the separation of the individual regional effects from the effect induced by the lagged variable. The problem, as before, is that the lagged fertilizer use variable appears to reflect too much and thus reduces the coefficients of the dummy variables to too low a level. It also renders the set of dummy variables insignificant as indicated by the calculated F value of 1.24.

In any case, it is worth noting that the long-run coefficients for X_{1j} implied by this model (4.c) are generally equivalent to those given in table 1 for the static regional dummy variable model (3.c).

RANDOM MODELS

The results for the random static model are reported in table 3 under the null hypothesis (5.a') that the mean coefficients are constant and under the alternative hypothesis (5.b') that

[5] This discussion is based on the usual assumption that long-run elasticities are larger than the corresponding short-run elasticities.

[6] Tests to determine if the adjustment coefficients have been constant, while the elasticities have changed and vice versa resulted in rejection in favor of the alternative hypothesis (4.b). Thus, little information can be gained from more restrictive specifications than (4.b).

the mean coefficients vary from one cross-section to another. However, for this model and its dynamic version, a test of the null hypothesis is not provided, since the distribution properties of the estimator employed have not been determined (5, pp. 590-591). As in the case of (3.b), the (absolute) percentage mean response of fertilizer consumption to percentage changes in fertilizer price, crop value per acre, farm wages, and price of land services trended downward from 1949 to 1964. This again leads to the conclusion that fertilizer consumption, over the period of investigation, has become less responsive to relative changes in input prices and output price. More specifically, the estimators calculated from the quadratic programming formulation are quite similar to the corresponding nonrandom estimates (table 1) and show in 1949 a 10-percent increase in the farm wage rate being associated with a mean 14.6-percent decrease in plant nutrients used per acre; whereas, in 1964, a similar increase in the farm wage rate resulted in only a mean 4.4-percent decrease in fertilizer consumption per acre. Similar movements of smaller magnitude are associated with price of land, crop value, and fertilizer prices.

The values of the estimators for the random dynamic model are reported in the bottom portion of table 3. For the null hypothesis (6.a'), the nonrandom dynamic estimate, i.e., $\lambda_{54-64} = .391$ is imposed while for the alternative hypothesis, estimates from formulation (4.b) are imposed, i.e., $\lambda_{54} = .437$, $\lambda_{59} = .174$, and $\lambda_{64} = .305$ (see table 2). In comparing the quadratic program estimates to those of the nonrandom dynamic model (table 2), we find the estimated values (absolute) of the former to be generally higher. However, these random estimates suffer from the same difficulties as the nonrandom dynamic estimates. The correspondence between the long-run coefficients implied by the random static model and the random dynamic model is slightly better than correspondence between the nonrandom static (table 1) and the nonrandom dynamic (table 2) long-run coefficients (i.e., the differences between the two sets of estimates are generally smaller for random models). The difference in correspondence, however, can hardly be considered overwhelming.

Conclusion

The above results indicate that, for both the static and dynamic formulations of the demand for fertilizer, coefficients based on cross-sectional data have not remained constant over time. In fact, the coefficients consistently trend downward (absolutely) throughout the period of investigation. The static model estimated a 30-percent decline in the fertilizer-output price elasticity and a 51-percent decline in the fertilizer-land input price elasticity in 1949. Similarly, the dynamic model estimated declines of 59 percent, 41 percent, and 16 percent for the same elasticities from 1954 to 1964. Therefore, we conclude that fertilizer consumption per acre has become less responsive to changes in relative prices. One possible explanation is that fertilizer use has become an accepted practice, implying that farmers who employ fertilizer are less receptive to relative price changes. A further possible explanation with regard to the fertilizer-output price ratio is that farmers are operating on a production function which exhibits diminishing productivity, implying that a stage of production has been reached where output is less responsive to fertilizer input. Thus, there is less rationale for substantially changing fertilizer use in response to changes in relative fertilizer-output prices.

However, fertilizer use appears to have become more responsive to fertilizer price as measured by the general upward movement (absolutely) in the sum of the elasticities over time for all formulations considered. For the more reliable formulations (i.e., the static formulation, an example of which is model (3.b)), this sum has trended downward from -.5 in 1949 to -.7 in 1964. Thus, the sum has trended toward -1.0, the point at which the fertilizer supply industry maximizes total revenue. This may be the result of firms supplying fertilizer to farmers attempting to maximize total sales. Such an implication, however, deserves further study before it can be substantiated.

In comparing the results for the static and dynamic models, we find that the coefficients implied by the former model are generally larger than the long-run coefficient implied by the latter model (except for the elasticity on fertilizer price). On the basis that the static model reflects some short-run considerations, however, we would expect just the opposite. Moreover, these results are not altered substantially if the coefficients are assumed subject to random variation rather than being constant at a given point in time. In fact, the random coefficient formulations do not appear to be worth the effort in the case of fertilizer demand, i.e., a meaningful amount of additional information is not obtained.

Table 3.--Random models

Equation number	Dependent variable	N	Year	$\hat{b}_1$	$\hat{b}_2$	$\hat{b}_3$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_3$	
5.a'	Y''	192	49-64	-.714	.896	-.825	.005	.091	.246	
5.b'	Y''	48	49	-.973	1.459	-.948	.000	.086	1.151	
5.b'	Y''	48	54	-.739	.931	-.803	.000	.006	.013	
5.b'	Y''	48	59	-.661	.649	-.668	.000	.009	.006	
5.b'	Y''	48	64	-.638	.438	-.498	.000	.004	.005	
				$\lambda\hat{b}_1$	$\lambda\hat{b}_2$	$\lambda\hat{b}_3$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_3$	λ
6.a'	Y''	144	54-64	-.231	.063	-.162	.001	.012	.040	.391
6.b'	Y''	48	54	-.294	.120	-.174	.000	.006	.025	.437
6.b'	Y''	48	59	-.059	-.121	-.018	.000	.002	.001	.174
6.b'	Y''	48	64	-.128	-.040	-.130	.001	.006	.007	.305
	Long run:									
	Y''		54-64	-.591	.161	-.414				
	Y''		54	-.673	.275	-.398				
	Y''		59	-.339	-.695	-.103				
	Y''		64	-.420	-.131	-.426				

The dummy variable formulation of both the static and dynamic models leads to improved results for the former, but dubious results for the latter. The dynamic model results are affected by estimation problems confronted by including regional dummy variables as well as lagged values of the dependent variable in the same regression. Thus, Griliches' previous conclusion that a dynamic adjustment model may "not represent a very useful approach to cross-section data" (3, p. 838) is not altered by the introduction of dummy variables and several cross-sections over time.

Of those models considered, the most useful for explanation or prediction purposes is the nonrandom static regional dummy variable model (3.c). This reasonably simple model is not confronted with the estimation problems prevalent in the dynamic and random versions. It reflects adequately the long-run differentials in fertilizer demand while providing satisfactory estimates of the unknown parameters, and explains a major portion of the total variation in fertilizer use per acre.

References

(1) Chow, Gregory C. Tests of equality between sets of coefficients in two linear regressions. Econometrica 28:591-605, July 1960.

(2) Griliches, Zvi. The demand for fertilizer: An economic interpretation of a technical change. Jour. Farm Econ. 40: 591-606, August 1958.

(3) Griliches, Zvi. The demand for fertilizer in 1954: An interstate study. Jour. Amer. Statis. Assoc. 54: 377-384, June 1959.

(4) Heady, Earl O., and Martin H. Yeh. National and regional demand functions for fertilizer. Jour. Farm Econ. 41: 332-348, May 1959.

(5) Hildreth, Clifford, and James P. Houck. Some estimators for a linear model with

random coefficients. Jour. Amer. Statis. Assoc. 63: 584-595, June 1968.

(6) Judge, G. G., and T. Takayama. Inequality restrictions in regression analysis. Jour. Amer. Statis. Assoc. 61: 166-181, March 1966.

(7) Malinvaud, E. Statistical methods of econometrics. Rand McNally and Co., Chicago, 1966.

(8) Rao, C. R. The theory of least squares when the parameters are stochastic and its application to the analysis of growth curves. Biometrika 52: 447-458, 1965.

(9) Rubin, H. Note on random coefficients. In: Statistical inference in dynamic economic models, ed. by T. C. Koopmans, John Wiley and Sons, Inc., New York, Cowles Comm. Monog. 10, 1950, p. 419-421.

(10) Sundquist, W. B. Fertilizer and the cost-price squeeze. Natl. Plant Food Inst., Plant Food Rev. 13: 1, 1967.

(11) U.S. Department of Agriculture. Agricultural statistics, 1951, 1956, 1961, and 1966.