Estimating the Productivity of Irrigation Water

By Howard C. Hogg, Lloyd B. Rankine, and Jack R. Davidson

An attempt to incorporate known agronomic relationships in a production function estimated from field records is illustrated in this paper. The example considered is sugarcane irrigation on two Hawaii sugar plantations.

Deriving a meaningful functional relationship from records devised for management control depends upon the researcher's awareness of relevant agronomic concepts and the content of the records kept. Agronomic concepts are reflected by the results of physical experiments concentrating on one or more aspects of plant growth. Not all of these results agree nor do the scientists always agree on interpretation. Basic theories of how the biological and chemical agents interact in agricultural production are being constantly reworked as the sciences mature and the technology of measurement advances.

Field records provide insufficient detail to adequately measure all the separate effects considered relevant. Also, record systems tend to lag considerably behind changes in the state of the arts. Nevertheless, with increasing demands on available water supplies, the economist is frequently called upon to estimate the economic productivity of irrigation water. Time and budget considerations typically preclude the use of comprehensive experiments as the basis for these estimates.

Soil-Moisture-Plant Relationship for Sugarcane

Some of the agronomic principles associated with water and plant growth are as follows: A particular soil will hold a given amount of water against gravity when allowed to drain freely. This quantity is referred to as the moisture storage capacity of the soil, or simply as "field capacity." As this moisture evaporates or is transpired by the plant, the remaining moisture is held with increasing force or tension. When stored soil moisture is at a maximum (field capacity), moisture stress is perhaps 0.33 atmosphere (ATM) and when all of the moisture available to the plant is exhausted, it is about 15 ATM. A moisture stress, C, is defined as the point where plant growth is retarded. In addition to these factors, the amount of water required in transpiration changes over the crop cycle.

A simple variable incorporating several of these concepts has been proposed for estimating water-yield relationships for sugarcane (2).[1] A slightly modified version of this variable, which can be computed from plantation records and meteorological data, is defined by the equation:

$$W = \frac{E_a}{E_p} = \frac{(N x CS) + R_e}{E_p} \qquad (1)$$

where:

E_a = actual evapotranspiration

E_p = potential evapotranspiration estimated for each climatic zone from pan evaporation observations

N = number of irrigation rounds applied to the field

C = the percentage of available soil moisture that can be used prior to reaching the critical moisture stress for a particular soil

[1] Underscored numbers in parentheses indicate items in the References, p. 17.

S = available soil moisture storage associated with the soil type found in each field

R_e = effective rainfall which we defined as 75 percent of the actual rainfall for the climatic zone in which the field is located.

The variable W then is a ratio of water adequacy which states that maximum yield can be maintained by irrigating when the percentage of available soil moisture represented by C has been exhausted. A shorter irrigation interval would not increase yield and a longer interval would decrease yield. The variable W represents a single estimate of water adequacy for the entire crop. The yield-maximizing quantity of water is represented by E_p which is the sum of plant moisture requirements throughout the crop cycle, water not limiting. Current agronomic thought suggests a need to allow for differential effects on growth if water deficits occur during different stages of the crop cycle (2). This would require estimating a W for each stage of growth, then treating these as separate independent variables.

Figure 1 shows the hypothesized relationship between yield and W. In this example C is 60 percent and the moisture stress at this point is 1 ATM. There is theoretically a point (to the right of point Q in figure 1) where additional water will result in decreased cane yield. In this paper, we are interested in determining the position and shape of the rising portion of the function (point O to P).

An Empirical Example

The fields of a single sugar plantation on the island of Oahu were divided into four groups. These groups consist of those fields irrigated with brackish and those with nonbrackish water and containing one of two physically similar soil groupings which are referred to as soil A and soil B fields.[2] The relationship between water quantity and moisture stress for these fields is given in table 1.

Robinson (5) has published research results that support a critical moisture stress of 2 ATM

[2] Group A fields contain dark magnesium clays and related soils. Group B fields consist of low humic latosols and related soils.

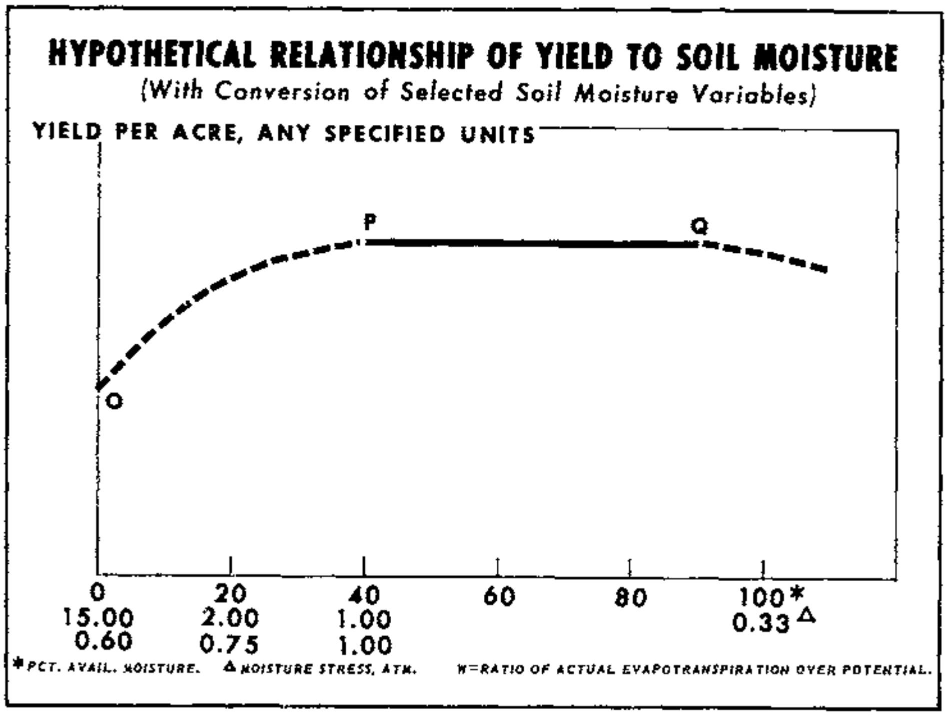

Figure 1

374-028 O - 70 - 2

Table 1.--Relationship between water quantity and moisture stress for A and B soils

A		B	
Soil moisture	Moisture stress	Soil moisture	Moisture stress
100	0.33	100	0.33
75	11	40	1
41	2	20	2
34	4	10	4
0	15	0	15

Sources: Data for soil A from (8), for soil B from (7).

for sugarcane. His findings are summarized in figure 2 where sugarcane stalk elongation is related to average moisture stress.

In addition to the C-factor, it was necessary to estimate potential evapotranspiration. This was done by first stating plant moisture requirements for different stages of growth, in terms of pan evaporation, then summing over the crop cycle. A detailed discussion of this procedure is provided elsewhere (1). The remaining components of W are available from plantation records.

Production functions for fields irrigated with nonbrackish and brackish water given in tables 2 and 3, respectively. Brackish water contains 3.24 or more grams of sodium chloride per gallon. The functions are stated for a given land productivity and all other variables (area of cane of harvest, planted or ratoon crop, N, P, and K) are held constant at their mean values to facilitate comparison. A reciprocal form is used because in most cases it provided the best statistical fit.

Equation (2) is given for comparative purposes. It is the result obtained from fitting a function to plantationwide data where the water variable is the reciprocal of total water (TW) applied rather than the reciprocal of W as in equations (3) through (8). The dependent variable in all cases in tons of cane per acre (TCA). The t-ratios for W in equations (3), (4), (5), and (8) all significant at the 1 percent level and are substantially larger than the t for TW in equation (2).

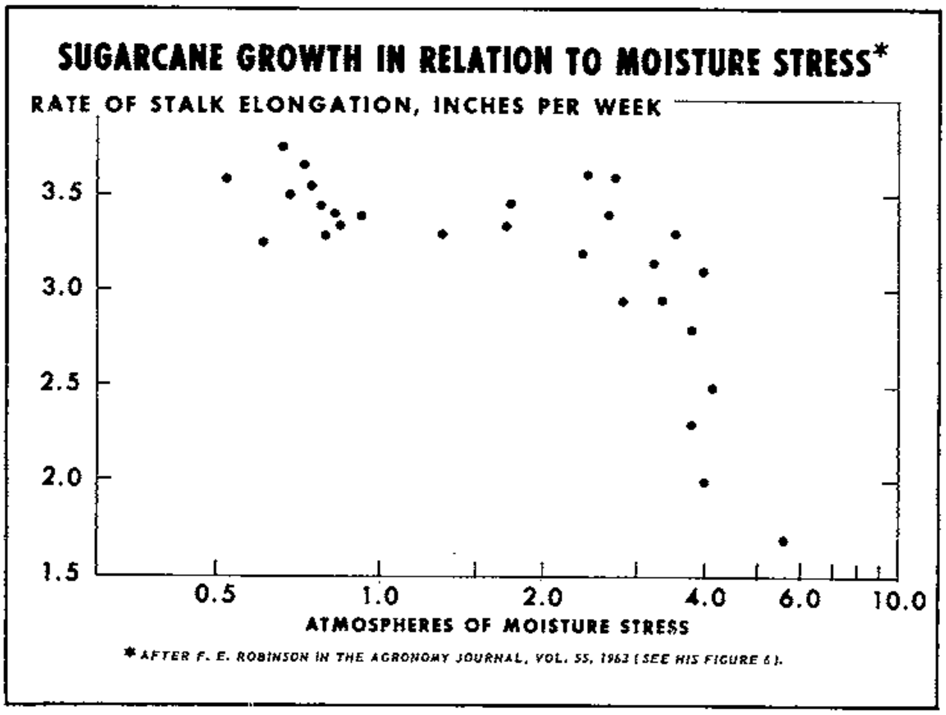

Figure 2

Table 2.--Production functions for fields irrigated with nonbrackish water [1]

Equation	Soil	C-factor	Growth-retarding moisture stress	Constant	Regression coefficient for 1/W
(3)	A	0.60	2 ATM	139.56	-18.31 (4.62)***
(4)	B	.60	1 ATM	167.10	-41.93 (5.44)***

[1] All other variables are held constant at their mean values.
*** t-ratio significant at the 1 percent level.

Table 3.--Production functions for fields irrigated with brackish water [1]

Equation	Soil	C-factor [2]	Constant	Regression coefficient for 1/W
(5)	A	0.25	134.07	-11.39 (4.89)***
(6)	A	.25	135.25	-10.31 (4.29)***
(7)	B	.30	127.53	-8.85 (2.32)**
(8)	B	.30	146.82	-12.54 (4.11)***

[1] All other variables are held constant at their mean values.
[2] The C-factors for these functions are largely arbitrary as the concentration of sodium chloride in the irrigation water is unknown.
** t-ratio significant at 5 percent level.
*** t-ratio significant at 1 percent level.

(2) $$TCA = 123.54 - 1{,}164.18\,(1/TW)$$
$$(2.27)^{**}$$

where:

TCA = tons of cane per acre
TW = total water in inches
** = t-ratio significant at 5 percent level.
(All other variables are held constant at their mean values.)

Although the C-factors for all moisture stresses occurring between 0.33 and 15 ATM were not tested, the functions given in table 2 appear to provide the best estimates for fields irrigated with nonbrackish water. These functions are superior to formulations employing different C-factors, as well as those fitted to plantationwide data (without distinguishing between soil types). The reason for the different growth-retarding moisture stresses between equations (3) and (4) is not known. However, figure 2 indicates that the growth-retarding moisture stress is actually somewhere between 1 and 2 ATM. Considering the nature of our data the above result is not surprising.

The only difference between equations (5) and (7) of table 3 and equations (3) and (4) of table 2 are the C-factors. In the former case, to maintain maximum yield, it was necessary to irrigate when only 25 and 30 percent of the available soil moisture had been exhausted, respectively. For equations (3) and (4), irrigation was not required for maximum yield until 60 percent of the available moisture was used. In effect this means that the critical moisture stress is reached earlier when irrigating with brackish water. This findng agrees with agronomic theory.

Many physical scientists believe that yields can be increased by overirrigating to leach accumulated salts when the irrigation water is brackish. To test for this possibility E_p, which in this example is 1.52 x annual pan evaporation, was increased 12.5 percent. This modification requires a 12.5 percent overirrigation to maintain yields at the maximum level. Equations (6) and (8) give the results of this change. For A soils the result is insignificant. However, for B soils a substantial improvement is indicated.

The record system maintained by the Oahu plantation in the above examples did not permit the estimation of multiple-stage production functions. A second plantation, on the island of Hawaii, was used for this purpose. Unfortunately, only a limited number of observations are available; consequently, a detailed analysis was impossible. Equation (9) represents an example of a two-stage production function fitted to these data (3).

$$(9) \qquad TCA = -.84 + \underset{(1.51)}{67.08}\, W_1 - \underset{(1.32)}{43.75}\, W_1^2 + \underset{(1.33)}{120.78}\, W - \underset{(1.30)}{59.42}\, W_2^2$$

where $W_i = E_a/E_p$ in ith period.

For this function, C equals 1.00 and 0.60 in stages 1 and 2, respectively. If the components of W_i are correctly specified, yield should be maximized at $W_i = 1.00$. The yield maximizing W_i's for equation (9) are 0.77 for stage 1 and 1.01 for stage 2. This may result from assuming C = 1.00 for stage 1 or from an error in the estimation of E_p for stage 1. While the procedure illustrated by equation (9) is not directly comparable to the functions presented earlier, it is sufficiently interesting to warrant consideration in work of this type.

Conclusions

These results suggest that when production functions are estimated from field records, a serious effort to incorporate known agronomic principles can lead to improved estimates. Analyses of this type provide management data in the form of a production function which can be optimized in a conventional manner. Reference (3) discusses the optimization of a function employing a similar version of the composite variable discussed in the present paper. In this discussion, the production function is used to estimate optimum irrigation level as a function of water cost, a static factor demand relationship for irrigation water, short-run cost curves, and product supply curves. In addition to providing management data directly, these analyses can serve as a basis for the refocusing of field records. That the plantation records could not support all the refinements suggested by recent developments in the agronomy of sugarcane culture is to be expected. The records were not designed for such analysis. However, relatively simple adjustments could greatly increase their analytical value. In the present case, knowing the date of each irrigation round for each crop would allow testing for the differential effects on yield of water deficits occurring during different stages of growth. Similarly, it would be helpful to know how consistently a particular level of irrigation is maintained for each field. A measure of moisture stress just prior to each irrigation round would be most helpful in this respect. Even a subjective judgment of high, low, or average by the irrigation superintendent would be better than no information at all.

This work also emphasizes the need for more comprehensive experiments to develop adequate water-yield relationships. Although the piecemeal approach can provide much useful data, serious attempts should be made to close the gaps.

References

(1) Campbell, Robert B. Sugar cane. In: Irrigation of agricultural lands, ed. by R. M. Hagan, H. R. Haise, and T. W. Edminster. Amer. Soc. Agron., Handb. 11, Ch. 33, 1967.

(2) Chang, Jen-hu, R. B. Campbell, and F. E. Robinson. On the relationship between water and sugar cane yield in Hawaii. Agron. Jour., Vol. 55, 1963.

(3) Hogg, Howard C., Jack R. Davidson, and Jen-hu Chang. Economics of a water yield function for sugar cane. Irrig. and Drainage Div. Jour., March 1969.

(4) Hogg, Howard C., Jack R. Davidson, and Lloyd B. Rankine. A composite variable for estimating the productivity of irrigation water. Proc. Comm. Econ. Water Resource Devel., Western Agr. Econ. Res. Council, 1967.

(5) Rankine, Lloyd B. Method for determining the productivity of irrigation water for the production of sugar in Hawaii. Unpublised Ph.D. thesis, Univ. Hawaii, 1969.

(6) Robinson, Frank E. Soil moisture tension, sugar cane stalk elongation, and irrigation interval control. Agron. Jour., Vol. 55, 1963.

(7) Thorne, M. D. Moisture characteristics of some Hawaiian soils. Proc. Soil Sci. Soc. Amer., 1949.

(8) Uehara, Goro. Unpublished data from personal files, Dept. Agron. and Soils, Univ. Hawaii.