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A Model for Decision Making Under Uncertainty

By J. Bruce Bullock and S. H. Logan

Decision theory usually is partitioned according to whether the decision is made under conditions of (a) certainty, (b) risk, or (c) uncertainty. These areas are defined as follows:

(a) Certainty if each action taken by the decision maker is known to lead invariably to a specific outcome.

(b) Risk if each action leads to one of a set of possible unknown outcomes, but each outcome occurs with a known probability distribution.

(c) Uncertainty if each action leads to one of a set of possible outcomes, but the probability of a particular outcome is not known to the decision maker.

Luce and Raiffa (11, p. 13)¹ suggest that we add a fourth classification (d), a combination of risk and uncertainty in the light of experimental evidence--the area of statistical inference.²

Decision making in the realm of certainty poses no particular problems since each action has a single-valued or known outcome. The decision maker simply selects the action with the most favorable outcome. However, decision problems under risk and uncertainty have several possible outcomes associated with each action. A set of decision rules, consistent with the decision maker's objective (utility) function, is needed to select the course of action that maximizes utility.

This paper presents one method of developing decision rules when the outcome of alternative actions cannot be specified with certainty. The model presented is applicable to a wide range of decision problems (1, 2, 5, 6).

¹Underscored numbers in parentheses refer to items in the References, p. 114.

²Classifications a, b, and c are similar to those specified by Knight (10).

Decisions Under Uncertainty

The problem of decision making under uncertainty can be characterized as a decision maker faced with choosing the optimal course of action, A_i , from a set of m possible actions. The outcomes of these various actions are dependent on the occurrence of alternative states of nature θ_j , $j = 1, 2, \dots, n$. The states of nature are values of one or more exogenous factors that directly affect the outcome of a particular action but cannot be controlled with certainty by the decision maker. For example, if the set of actions represent different fertilizer applications for corn, the states of nature might be alternative levels of rainfall.

For each possible action A_1, A_2, \dots, A_m , there are n potential outcomes, one for each state of nature. Uncertainty implies that the individual has no information about the likelihood of occurrence of any particular state of nature θ_j . Thus, the decision maker is faced with a set of unknown outcomes. Each outcome, λ_{ij} , can be represented as a point in an action-state plane, $\lambda_{ij} = (A_j, \theta_j)$, as shown in table 1.

For example, the outcome (profits) of a decision to feed high-quality steers will depend on the price of slaughter cattle at the end of the feeding period. Thus θ_1 may represent high slaughter cattle prices, θ_2 average prices, and θ_3 low prices. The outcome of decision A_1 (feed high-quality steers) and A_2 (feed low-quality steers) will depend on which value of θ occurs (cost per pound of gain is assumed to be known with certainty in both cases). We can represent this decision problem as shown in table 2, where λ_{12} is the profit per head from feeding high-quality steers when average prices are received at the end of the feeding period.

Table 1--Matrix representation of outcome plane

Action	States of nature									
	θ_1	θ_2	.	.	.	θ_j	.	.	.	θ_n
A_1	λ_{11}	λ_{12}	.	.	.	λ_{1j}	.	.	.	λ_{1n}
A_2	λ_{21}	λ_{22}	.	.	.	λ_{2j}	.	.	.	λ_{2n}
.
.
.
A_i	λ_{i1}	λ_{i2}	.	.	.	λ_{ij}	.	.	.	λ_{in}
.
.
.
A_m	λ_{m1}	λ_{m2}	.	.	.	λ_{mj}	.	.	.	λ_{mn}

Table 2.--Representation of a decision problem

Action	States of nature		
	θ_1 (high prices)	θ_2 (average prices)	θ_3 (low prices)
A_1 (feed high-quality steers)	λ_{11}	λ_{12}	λ_{13}
A_2 (feed low-quality steers)	λ_{21}	λ_{22}	λ_{23}

To make rational and consistent decisions relative to the action-state-outcome combinations, a utility index or some sort of preference ordering must be assigned to the set of outcomes. If the decision maker's preferences

among the outcomes are consistent with von Neumann-Morgenstern utility axioms (14, p. 26; 11, p. 22-31), it is possible to define a utility function, $u_{ij} = u(\lambda_{ij})$, that will map the outcomes into a utility plane.

Von Neumann and Morgenstern show that there exists a utility function u on the set of prospects

A. the individual has a complete and transitive preference ordering over the set of all possible prospects, that is,

(1) for any two prospects u and v , one and only one of the following relations holds:

$$u = v, u > v, u < v^3$$

(2) $u > v, v > w$ implies $u > w$

B. $u < w < v$ implies the existence of an α such that $\alpha(u) + (1-\alpha)v < w$, and $u > w > v$ implies the existence of an α such that $\alpha(u) + (1-\alpha)v > w$, where $0 < \alpha < 1$, and

C. it is irrelevant whether a combination of two prospects is obtained in two successive steps--first the probabilities $\alpha, 1-\alpha$, then the probabilities $\beta, 1-\beta$; or in one operation with the probabilities $\gamma, 1-\gamma$ where $\gamma = \alpha\beta$. (That is, complex choices can be partitioned into simpler choices to facilitate evaluating preferences.)

$$(1) \alpha u + (1-\alpha)v = (1-\alpha)v + \alpha u$$

$$(2) \alpha[\beta u + (1-\beta)v] + (1-\alpha)v = \gamma u + (1-\gamma)v.$$

In other words, for each prospect P_i there exists a number $u_i = u(P_i)$ which is called utility of P_i . This function has the following properties (4, ch. 4):

(1) $u(v) > u(w)$ if and only if the individual prefers v to w .

(2) If P_k is a prospect of receiving v with probability α or w with probability $(1-\alpha)$ then $u(P_k) = \alpha u(v) + (1-\alpha) u(w)$.

However, the derivation of such a utility function is no small undertaking. Thus, as a matter of practical application, it is usually assumed that the utility function is linear with respect to money over the relevant range. Consequently, maximization of monetary gain is equivalent to maximizing utility.

Thus the decision problem can be stated as follows: Given a set of possible actions, A , the set of alternative states of nature, θ , and the

³Where: = implies indifference between prospects
> is read as "is preferred to"
< is read as "is not preferred to"

utility index u_{ij} , associated with the selection of action A_i and the occurrence of θ_j (outcome λ_{ij}),⁴ select the action that is in some sense optimal--where optimality is defined by the particular decision criterion used. Possible decision criteria include maximizing the minimum gain (maximin), minimizing the maximum regret (minimax), and the "principle of insufficient reason."

The Maximin Criterion. Each action is appraised on the basis of its security level (i.e., its lowest possible utility payoff). In the example below, action A_1 has a minimum possible utility (security level) of one whereas A_2 has a security of two. The maximin criterion is to select the action associated with the maximum of these minimum values (maximin). Thus, action A_2 is selected:

Utility Payoff Matrix

Action	State		Security level
	θ_1	θ_2	
A_1	1	5	1
A_2	3	2	2

The Minimax Criterion. Each action is appraised on the basis of its "regret index." Regret is the utility foregone as a result of selecting a nonoptimal action, given θ_j as the true state. The regret index for each action is its maximum "regret" value or lost utility.

In the above example, there is no regret if action A_1 is selected and θ_2 is the true state nor if action A_2 is selected and θ_1 is the true state. However, three utility units are foregone (regret = 3) if action A_2 is selected when θ_2 is the true state. The regret payoff matrix for the above example is:

Regret Payoff Matrix

Action	State		Regret index
	θ_1	θ_2	
A_1	2	0	2
A_2	0	3	3

⁴The matrix formulation of the decision problem is obtained by replacing λ_{ij} with u_{ij} in table 1.

The minimax criterion is to select the action that minimizes maximum regret. This criterion defines A_1 as the optimal action since it has the lowest regret index.

The "Principle of Insufficient Reason." This criterion asserts that if the decision maker has no information about the relative frequencies of the states of nature, then the occurrence of each state should be considered as equally likely. The criterion is to select the action that has the highest expected utility index.

$$\bar{u}_i = \frac{u_{i1} + u_{i2} + \dots + u_{in}}{n}$$

Each of these decision criteria has serious shortcomings (11, p. 278-286; 3; 13). Moreover, few decision problems fall into the category of complete uncertainty, i.e., where the decision maker has no knowledge of the likelihood or distribution of θ . Given the volume of public and private information currently available, most well-informed decision makers will have at least a subjective⁵ estimate of the distribution of θ , particularly for decisions of a recurring nature.

Bayesian Decision Theory

Generally some a priori information regarding the relative frequency of θ in the past will be available. Thus, emphasis in decision theory has shifted to the estimation of Bayesian strategies (7, 9, 12, 15), i.e., the selection of optimal actions based on some a priori information (either objective or subjective) about the probability distribution of the states of nature, $P(\theta)$.

The Bayesian approach to decision making can be stated as follows: Given a set of m possible actions, the set n of alternative states of nature, and the utility index associated with each outcome (table 1), along with a vector of a priori information about the relative frequency of θ ,

⁵ For an analysis using subjective probability estimates, see Carlson (2).

$$P(\theta) = \begin{bmatrix} P(\theta_1) \\ P(\theta_2) \\ \vdots \\ P(\theta_j) \\ \vdots \\ P(\theta_n) \end{bmatrix}$$

where $P(\theta_j)$ is the a priori probability that state θ_j will occur, select the action A_1 for which expected utility $\hat{u}_i = \sum_j u_{ij} P(\theta_j)$ is a maximum.

The a priori information can be any information that the decision maker has about the relative frequency of θ . This information is expressed in the form of a probability distribution $P(\theta)$ that provides some indication of the likelihood of a particular value of θ (states of nature) occurring. It may be nothing more than a subjective evaluation of the probabilities by the decision maker, or it may be derived from a histogram showing the relative frequencies of θ in the past.

In addition to the a priori knowledge of the probability distribution $P(\theta)$, it may be possible for the decision maker to gain additional information about the likelihood of a particular state θ by performing an experiment Z (with results Z_k , $k = 1, 2, \dots, n$) that serves as a predictor of θ . That is, it may be possible to construct a conditional probability distribution $P(\theta | Z)$ which incorporates the a priori information, $P(\theta)$, with information about the past performance of Z as a predictor of θ . The a posteriori probability distribution, $P(\theta | Z)$, is calculated using Bayes' formula (8),

$$(1) \quad P(\theta | Z) = \frac{P(Z | \theta) (P\theta)}{P(Z)}$$

where $P(Z)$ is the probability of observing a particular experimental result.

Table 3.--Matrix of a posteriori information

States	Experimental results									
	Z_1	Z_2	.	.	.	Z_k	.	.	.	Z_n
θ_1	$P(\theta_1 Z_1)$	$P(\theta_1 Z_2)$.	.	.	$P(\theta_1 Z_k)$.	.	.	$P(\theta_1 Z_n)$
θ_2	$P(\theta_2 Z_1)$	$P(\theta_2 Z_2)$.	.	.	$P(\theta_2 Z_k)$.	.	.	$P(\theta_2 Z_n)$
.
.
.
θ_j	$P(\theta_j Z_1)$	$P(\theta_j Z_2)$.	.	.	$P(\theta_j Z_k)$.	.	.	$P(\theta_j Z_n)$
.
.
.
θ_n	$P(\theta_n Z_1)$	$P(\theta_n Z_2)$.	.	.	$P(\theta_n Z_k)$.	.	.	$P(\theta_n Z_n)$

The experiment, Z, can be anything that is used as an estimator of θ . It may consist of simply observing the current state of nature θ_j and assuming that the value of θ at the time of payoff will also be θ_j . The experiment may consist of an elaborate model used to project future values of θ . For example, if the states of nature are future prices, the experiment would consist of some price forecasting mechanism.

The experimental information expands our knowledge about the likelihood of θ from the $P(\theta)$ vector to an $(n \times n)$ matrix of conditional probabilities (table 3). $P(\theta_j|Z_k)$ is the probability of θ_j occurring given Z_k as the experimental result (prediction of θ).⁶ If the experiment Z is a perfect predictor of θ , table 3 will consist of ones along the diagonal and zeros elsewhere.

With data provided by the experiment, the Bayesian strategy becomes: Given a projection

⁶ $P(\theta_j|Z_k)$ is estimated by the relative frequency over the historical period with which θ occurred as the true state of nature when Z_k was the experimental result. For applications of this procedure see (6, 5, 1).

of θ (for example, Z_k) select the action A_i for which the expected utility

$$(2) \quad \hat{u}_i^k = \sum_j u_{ij} P(\theta_j|Z_k)$$

is a maximum. Thus the Bayesian strategy consists of a set of optimal actions, at least one for each experimental result.⁷

Value of the Data

The derivation of Bayesian decisions by using only the a priori probability distribution $P(\theta)$ is referred to as the "no data" problem. Decision problems using a posteriori distribution are called "data" problems. The difference in expected incomes resulting from using the "data" strategy bundle relative to the "no data" strategy can be interpreted as the value of the data, i.e., the value of the information provided by the experiment.

⁷ It is possible that two or more actions could have the same expected utility for a given experimental result.

The expected value of the "no data" strategy is defined above as $u_i = \sum_j u_{ij} P(\theta_j)$. The expected value of following the "data" strategy is calculated by multiplying the expected value of the optimum action for each experimental result by the probability of observing the appropriate experimental result, $P(Z)$, and summing over all possible results.

$$(3) \quad \sum_k \left[\sum_j u_{ij} P(\theta_j | Z_k) \right] P(Z_k)$$

The expression in brackets was defined in equation 2 as \hat{u}_i^k (expected utility of action A_i given Z_k as a prediction of θ). Thus equation 3 reduces to $\sum_k \hat{u}_i^k P(Z_k)$. Therefore, the value of the data is defined as

$$(4) \quad V = \sum_k \left[\sum_j u_{ij} P(\theta_j | Z_k) \right] P(Z_k) - \sum_j u_{ij} P(\theta_j)$$

$$V = \sum_k \hat{u}_i^k P(Z_k) - \hat{u}_i$$

The value of the data can then be compared with the cost of performing the experiment to evaluate the net contribution of the experimental information to expected income.

The Bayesian decision model presented above provides a framework for developing decision criteria for problems characterized by uncertain outcomes. The model incorporates the available objective and/or subjective information into the decision process. Data requirements are modest; a priori information is generally available from past experience and published information. Additional information can be obtained from experiments such as econometric forecasting models.

Few decision problems do not contain at least some element of uncertainty. This is particularly true of production and marketing decisions in the agricultural sector. The outcome of alternative actions depends on such factors as rainfall, yield, feedlot performance, and future prices. The Bayesian decision model is a method of systematically incorporating available information about the frequency distribution of these factors directly into the decision process.

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