PRODUCTIVITY GROWTH IN WESTERN AUSTRALIA

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February 1992 Draft

ABSTRACT

Sheep industry survey data from the Australian Bureau of Agricultural and Resource Economics (ABARE) is used to investigate productivity growth in broadacre agriculture in Western Australia over a 35 year period from 1953/54 to 1987/88. Tornqvist indices of five output groups (crops, wool, sheepmeat, cattle and others) and five input groups (livestock, materials and services, labour, capital and land) are constructed. These indices are used to investigate productivity growth through the construction of an index of total factor productivity which is observed to grow at an annual rate of 2.7 percent over the 35 year period. In the second stage of the study the input and output indices are used in the estimation of output supply and input demand equations derived from a Generalized McFadden profit function. The results provide information on the biases in technical change along with price elasticity estimates.

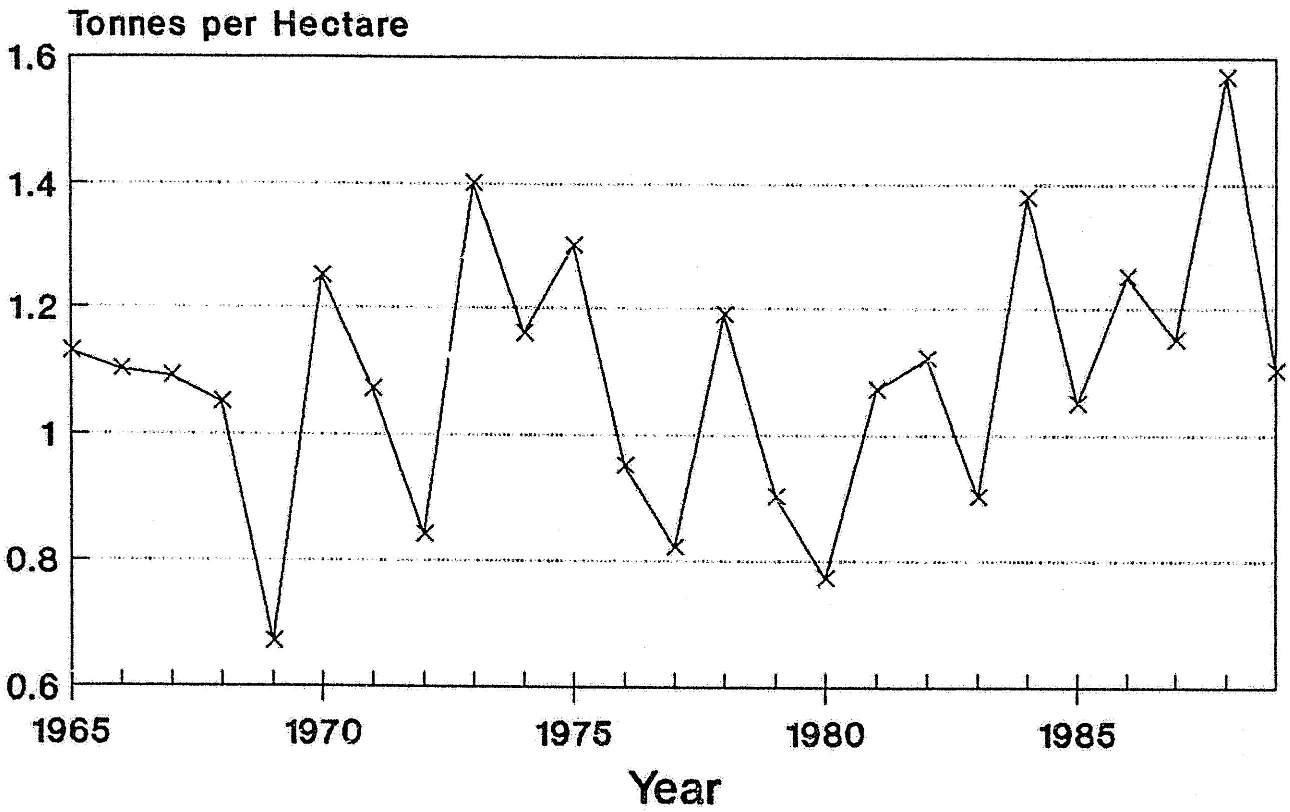
1. Introduction

The term "productivity" is often miss-used. In the manufacturing sector it is regularly used interchangeably with "labour productivity" whilst in agriculture the terms "productivity" and "yield" are generally treated as synonymous. The consideration of yield trends alone as a measure of productivity may provide a misleading indication of the degree of productivity improvement. For example, consider the plot of Western Australian wheat yields in Figure 1. This plot indicates an average yield increase of approximately 5 kg per year or about 0.5% per year. If this is an accurate measure of productivity growth, then the performance is poor when compared with rates estimated for other countries and industries. For example, ABS (1989) estimates the rate of change in multi-factor productivity for the Australian market sector to be 1.5 percent per year over the period from 1974/75 to 1987/88, while Lawrence and McKay (1980) estimate the rate of change in total productivity of the Australian sheep industry from 1952/53 to 1976/77 to be 2.9 percent per year.

The purpose of this paper is to attempt to measure the rate of change in productivity of Western Australian broadacre agriculture over a 35 year period from 1953/54 to 1987/88 using sheep industry survey data from the Australian Bureau of Agricultural and Resource Economics (ABARE). This study is divided into two stages. The first stage, in Section 2, involves the construction of Tornqvist indices. Indices for five output groups and five input groups are constructed and discussed before being aggregated to form indices of total output and total inputs. The ratio of these two aggregate indices produce an index of total factor productivity.

In Section 3 the indices constructed in the first stage are used to estimate an econometric model in an attempt to investigate possible biases in the rate of technological change

Figure 1
Average Wheat Yield (WA)



(Source: ABS)

between inputs and between outputs.[1] The possibility of non-neutral technological change in both outputs and inputs is considered by using a dual approach to production modeling. A Generalized McFadden (GM) profit function is specified and a set of output supply and input demand equations are derived using Hotelling's Lemma. The resulting system of equations are estimated using the indices derived in Section 2. The biases in technological change with respect to each pair of inputs and each pair of outputs are calculated from these estimates. In the final section, conclusions are made and directions for future work suggested.

2. Tornqvist indices

Introductory economics texts define productivity as the ability of a productive unit to combine and utilize inputs in the best possible way to produce their output(s). Yield per hectare is a deficient measure of productivity in that it only accounts for the input of land and overlooks labour, capital and other production inputs. Also, as many broadacre farms in Western Australia produce more than one output (e.g. wheat, barley, wool and sheepmeat), consideration of a single output in isolation without considering the jointness of production is bound to provide misleading measures of 'productivity'. In this section an index of agricultural productivity is formulated which is not restricted to a single input or output but considers the ratio of all outputs to all inputs.

To simplify the description of the method, consider an

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[1]Measures of productivity change derived from index numbers are generally assumed to capture technological change, scale economies, changes in the quality of inputs, and so on. It is conventional, however, to assume the coefficient of a time trend in an econometric analysis of production measures technological change, even if no attempt has been made to remove the effects of the other factors listed. This study shall maintain this convention.

example involving two inputs (land and labour) and two outputs (grain and livestock). To obtain a measure of (all outputs)/(all inputs) the aggregation of outputs and inputs into single measures is necessary. For our simple two input, two output example, a crude input index (II_t) could be defined by:

(1) $$II_t = [X_{1t} + X_{2t}] / [X_{10} + X_{20}] \times 100 \qquad ,t=1,...,T,$$

where X_{1t} = labour input in period t,

X_{2t} = land input in period t,

and time period 0 refers to the base period which is chosen arbitrarily. This index is simply the sum of the quantities of all inputs used in the t-th period over the sum of those used in the base period. The output index (OI_t) may be similarly defined:

(2) $$OI_t = [Y_{1t} + Y_{2t}] / [Y_{10} + Y_{20}] \times 100 \qquad ,t=1,...,T,$$

where Y_{1t} = grain output in period t,

Y_{2t} = livestock output in period t.

Given these indices a total factor productivity index ($TFPI_t$) is defined as the ratio of the two indices multiplied by 100. That is:

(3) $$TFPI_t = OI_t / II_t \times 100, \qquad ,t=1,...,T.$$

To illustrate this method and to also point out one of its shortcomings, consider the following simple example. In the base year (year 0) Farmer Jim produces 110 tonnes of grain on 100 hectares using 500 hours of labour. He has no livestock enterprise. In year 1 he produces the same amount of grain on half the area using the same amount of labour. Thus

$$OI_1 = [110 + 0] / [110 + 0] \times 100 = 100,$$

$$II_1 = [50 + 500] / [100 + 500] \times 100 = 91.67$$

and hence

$$TFPI_1 = 100 / 91.67 = 109.10.$$

These figures suggest the farmer has become approximately 9% more efficient. It could be argued that the improvement achieved is greater than what this figure indicates. Given that the price of a hectare of land is much greater than that of an hour of labour we may be able to improve the measure of productivity by allocating different weights to land and labour to reflect these price differentials. For example, assuming a price of \$50 per hectare for land (leasing fee for one year) and \$10 per hour for labour, a weighted input index (WII_t):

(4) $WII_t = [W_1X_{1t} + W_2X_{2t}] / [W_1X_{10} + W_2X_{20}] \times 100$

for the given data could be:

$$WII_1 = [50(50) + 10(500)] / [50(100) + 10(500)] \times 100 = 75$$

providing a TFP index of:

$$TFPI_1 = 100 / 75 \times 100 = 133.33.$$

This implies a 33 percent increase in productivity, an intuitively more reasonable result than the 9 percent result we obtained without weighting.

Given that relative prices tend to vary through time, what prices should be used to determine values of the weights? If the weights are determined by prices in the base period (and are assumed fixed through time), the above index becomes the widely used Laspeyres index:

(5) $LII_t = [P_{10}X_{1t} + P_{20}X_{2t}] / [P_{10}X_{10} + P_{20}X_{20})] \times 100$

where P_{10} = the price of land in the base period and

P_{20} = the price of labour in the base period.

Similar indices have often been used in productivity studies for much of this century. Its popularity, however, has diminished since it was noted by Christensen (1975) and others, that behind all productivity indices are assumption(s) regarding the form of the production function (i.e. the relationship between inputs and outputs).

A Laspeyres index implicitly assumes that the production function is linear. The unrealistic nature of this assumption with respect to agriculture becomes obvious when one considers that the linear production function can predict a positive output when land is zero. Furthermore, the marginal products of the linear production function are constant over all values. This implies, for example, that on a fixed quantity of land the 1st, 2nd and 1000th hours of labour will add the same amount to production. Thus, even though the Laspeyres index is a substantial improvement over partial or simple productivity indices, it still suffers from a number of fundamental deficiencies.

If one were estimating an econometric production function, one would turn to a more flexible (i.e. less restrictive in its assumptions) functional form. Similarly, there exists index numbers which correspond to these more flexible functional forms. The Tornqvist index (Tornqvist, 1936) can be shown to be a derivative of the homogeneous translog production function, which provides a second order approximation to an arbitrary production function at any given point (Christensen, Jorgenson and Lau, 1973). This index has been used widely in studies of agricultural productivity in the last decade (Lawrence and McKay 1980, Kingwell 1982, Ball 1985, Rayner, Whittaker and Ingersent 1986, Rahuma and Veeman 1988 and Wong 1989). A Tornqvist input quantity index for the case of two inputs is defined as follows:

(6) $$TII_t = \exp[A_{1t}\log(X_{1t}/X_{1t-1}) + A_{2t}\log(X_{2t}/X_{2t-1})] \times 100$$

where $$A_{1t} = (C_{1t} + C_{1t-1})/2,$$
$$A_{2t} = (C_{2t} + C_{2t-1})/2 \text{ and}$$
$$C_{is} = P_{is}X_{is} \;/\; [P_{1s}X_{1s} + P_{2s}X_{2s}].$$

The C_{is}'s are the budget shares of the i-th good in the s-th time period. Thus the A_{it}'s are the averages of the budget shares across the time periods t and t-1.

The principal advantage of the Tornqvist index is that it is not based upon simplistic linear production assumptions as are the Laspeyres, Paasche and others. It does however suffer from a few disadvantages in that it is more difficult to compute and is not as intuitive as the Laspeyres to interpret. The primary disadvantage is in the extra data it requires. When calculating the Laspeyres index only prices in the base year are required. The Tornqvist, however, requires prices from all years.

In this study we draw heavily upon the experiences of two earlier studies, Lawrence and McKay (1980) and Kingwell (1982), which have employed Tornqvist indices in analyses of productivity change in agriculture. Lawrence and McKay (1980) studied the productivity of the Australian sheep industry (all farms with more than 200 sheep) from 1952/53 to 1976/77 while Kingwell (1982) considered all Western Australian agriculture from 1950/51 to 1977/78. A study covering all agricultural enterprises would involve the aggregation of data from industries as diverse as broadacre sheep/wheat and market gardening. It was decided to follow the lead of Lawrence and McKay (1980) and restrict our attention to the sheep industry so as to minimize aggregation problems.

The data used in this study was taken from the Australian Agricultural and Grazing Industry Survey (AAGIS) conducted by the Australian Bureau of Agricultural and Resource Economics (ABARE) (see ABARE 1989). Our data request asked for sample means per farm from 1952/53 to 1987/88 for all farms:

- in Western Australia;
- in the wheat/sheep zone (medium rainfall);
- with more than 200 sheep; and
- with estimated value of agricultural operations greater than $20,000 in 1988/89 dollars.

The majority of farms in Western Australia would be described in the above way. In 1988 we estimate that 7,500 farms out of a total of 13,000 would satisfy the above criteria.

As a first step, inputs and outputs were each divided into five groups and indices constructed for each group. The five output groups constructed were crops, wool, sheepmeat, cattle and other. The five input groups were livestock, materials and services, labour, capital and land. The components of each of these groups are listed in Table 1. As noted in the previous section, a quantity and a price is required for each commodity in each year to construct Tornqvist indices. In ABARE's AAGIS a number of commodities (notably materials and services and

TABLE 1

COMPONENTS OF COMMODITY GROUPS

<u>OUTPUTS</u>:

- Crops
  - Wheat
  - Barley
  - Oats
- Wool
- Sheepmeat
  - Sheep sales plus abs(negative operating gains)
- Cattle
  - Cattle sales plus abs(negative operating gains)
- Other

<u>INPUTS</u>:

- Livestock
  - Sheep purchases plus positive operating gains
  - Cattle purchases plus positive operating gains
  - Sheep opening stock
  - Cattle opening stock
- Materials and Services
  - Fuel and electricity
  - Fertilizer
  - Seed
  - Fodder
  - Chemicals and medicines
  - Plant maintenance
  - Freight
  - Improvements maintenance
  - Rates and taxes
  - Insurance
  - Interest
  - Other
- Labour
  - Hired labour and contracts
  - Shearing and crutching
  - Stores and rations
  - Operator labour
  - Family labour
- Capital
  - Water
  - Fencing
  - Buildings
  - Plant
- Land

capital) did not have a quantity as well as a value recorded for obvious reasons. In these cases we have used ABARE's price index information (see ABARE 1990) to derive implicit quantities by dividing the value by the relevant price index. The specification of measures of the capital items was a difficult task. Many capital stock items needed to be converted into flows. The approach of Lawrence and McKay (1980) was used. Capital items were divided into depreciation, maintenance and opportunity cost. For further discussion of methods used refer to the appendix in Lawrence and McKay (1980).

Indices of the five output groups along with an index of total output are presented in Table 2. The per farm output in the Western Australian sheep industry has steadily increased over the 35 years of the study period. On average an increase of 5.3 percent per year has been observed with some small deviations from this trend due to seasonal influences. The main outputs of crops, wool and sheepmeat have moved in a similar way to total output. The two exceptions have been cattle and 'other outputs'. After a period of low wheat and wool prices in the early 70's, many farmers in the wheatbelt diversified into cattle. This surge in cattle production is evident in Table 2 in the mid to late 70's. Following this was a noticeable decline in cattle production as world beef prices slumped badly resulting in many farmers moving out of cattle. The 'other outputs' group is most likely dominated by grain legumes - lupins and peas. There has been a dramatic increase in these outputs during the 80's. The average annual growth in 'other outputs' was estimated to be 23.1 percent over the 35 years.

Indices of the five input groups along with an index of total inputs are presented in Table 3. Inputs, like outputs, have steadily increased during the study period. The average annual rate of growth in inputs was 2.6 percent. The volatility of the input indices is not as great as that observed by the output indices as season does not have the same influence. The

TABLE 2

INDICES OF OUTPUTS

Year	Grain	Wool	Sheepmeat	Cattle	Other	Total
1953/54	28	29	30	10	0	23
1954/55	22	25	25	7	0	19
1955/56	37	31	34	5	0	27
1956/57	24	32	39	11	0	23
1957/58	24	43	47	35	0	28
1958/59	42	44	35	32	1	35
1959/60	42	50	36	38	1	36
1960/61	40	44	38	32	0	34
1961/62	38	46	37	45	0	34
1962/63	45	43	40	44	1	36
1963/64	45	39	34	14	2	33
1964/65	41	47	48	43	2	38
1965/66	58	56	61	49	2	49
1966/67	60	65	73	55	4	55
1967/68	42	61	75	27	4	45
1968/69	55	75	71	42	4	54
1969/70	28	67	66	33	5	41
1970/71	39	71	75	42	5	49
1971/72	40	82	74	61	9	55
1972/73	42	76	68	70	8	51
1973/74	39	81	72	123	18	55
1974/75	56	88	62	123	10	64
1975/76	69	94	68	106	22	76
1976/77	145	101	98	190	27	123
1977/78	71	79	74	122	36	77
1978/79	83	88	93	159	40	88
1979/80	81	88	106	98	77	88
1980/81	100	100	100	100	100	100
1981/82	144	83	91	63	151	117
1982/83	131	92	110	60	156	115
1983/84	131	85	105	71	160	112
1984/85	166	96	105	37	160	128
1985/86	125	94	110	46	134	109
1986/87	138	107	92	45	198	119
1987/88	118	108	94	27	290	112
Average Annual Percentage Change	5.1	3.8	3.9	5.1	23.1	5.3

TABLE 3

INDICES OF INPUTS

Year	Livestock	M & S	Labour	Capital	Land	Total
1953/54	24	35	60	40	59	41
1954/55	18	37	59	44	59	41
1955/56	28	37	55	43	59	42
1956/57	29	37	58	45	63	44
1957/58	36	46	70	59	75	54
1958/59	32	45	70	52	75	53
1959/60	24	47	69	63	77	53
1960/61	28	41	66	54	73	49
1961/62	38	44	69	56	76	53
1962/63	39	47	75	60	77	56
1963/64	25	49	59	60	77	51
1964/65	55	53	60	70	75	60
1965/66	78	60	62	71	75	67
1966/67	90	72	68	73	76	76
1967/68	65	57	63	81	65	65
1968/69	59	61	69	81	66	67
1969/70	67	59	65	81	67	67
1970/71	68	55	59	81	68	64
1971/72	80	56	62	71	71	65
1972/73	80	73	83	76	81	78
1973/74	59	73	71	76	81	73
1974/75	52	52	69	53	72	59
1975/76	55	62	78	56	75	66
1976/77	138	79	89	88	43	78
1977/78	101	75	89	86	98	87
1978/79	106	91	94	93	99	95
1979/80	87	92	102	92	99	95
1980/81	100	100	100	100	100	100
1981/82	78	117	97	97	102	103
1982/83	95	108	102	95	97	100
1983/84	79	106	99	93	88	95
1984/85	70	106	93	86	94	94
1985/86	81	90	93	81	93	89
1986/87	65	93	93	77	100	90
1987/88	66	96	98	72	125	95
Average Annual Percentage Change	4.0	3.3	1.7	1.9	1.4	2.6

input groups individually appear to move in a similar way to total outputs with the exception of livestock. The behavior of livestock is not unexpected given our previous discussion of the cattle industry in the 70's. There are two noticeable blips in the trend in total inputs. The first was a sharp fall in the mid 70's, most likely in response to the high levels of inflation at that time. The second is a milder fall in the mid 80's. This slow down in input usage was most probably due to the poor world commodity prices at the time.

The indices of total outputs and total inputs from Tables 2 and 3 are reproduced in Table 4 along with the total factor productivity index - the ratio of the output index over the input index times 100. These three indices are also plotted in Figure 2. We observe that TFP has increased at an average annual rate of 2.7 percent. This results from average increases of 5.3 percent in outputs and 2.6 percent in inputs per annum.

The estimates of the average annual percentage change in each index, presented at the bottom of Tables 2, 3 and 4, are obtained by regressing the logarithm of the index upon a time trend. The value of 2.7% obtained for TFP may be compared to the values of 2.9% obtained by Lawrence and McKay (1980) for the Australian sheep industry (all zones) and 3.1% estimated by Kingwell (1982) for Western Australian agriculture. The values 2.7% and 2.9% are very similar. The slightly slower growth rate could be due to one or more of three factors:

- i) The rate of productivity growth in Western Australia could be slightly slower than that of Australia as a whole.
- ii) The drought years of the early 1980's are included in this study but were not included in the Lawrence and McKay paper.
- iii) Small differences in survey methods and in variable definitions could have an influence.

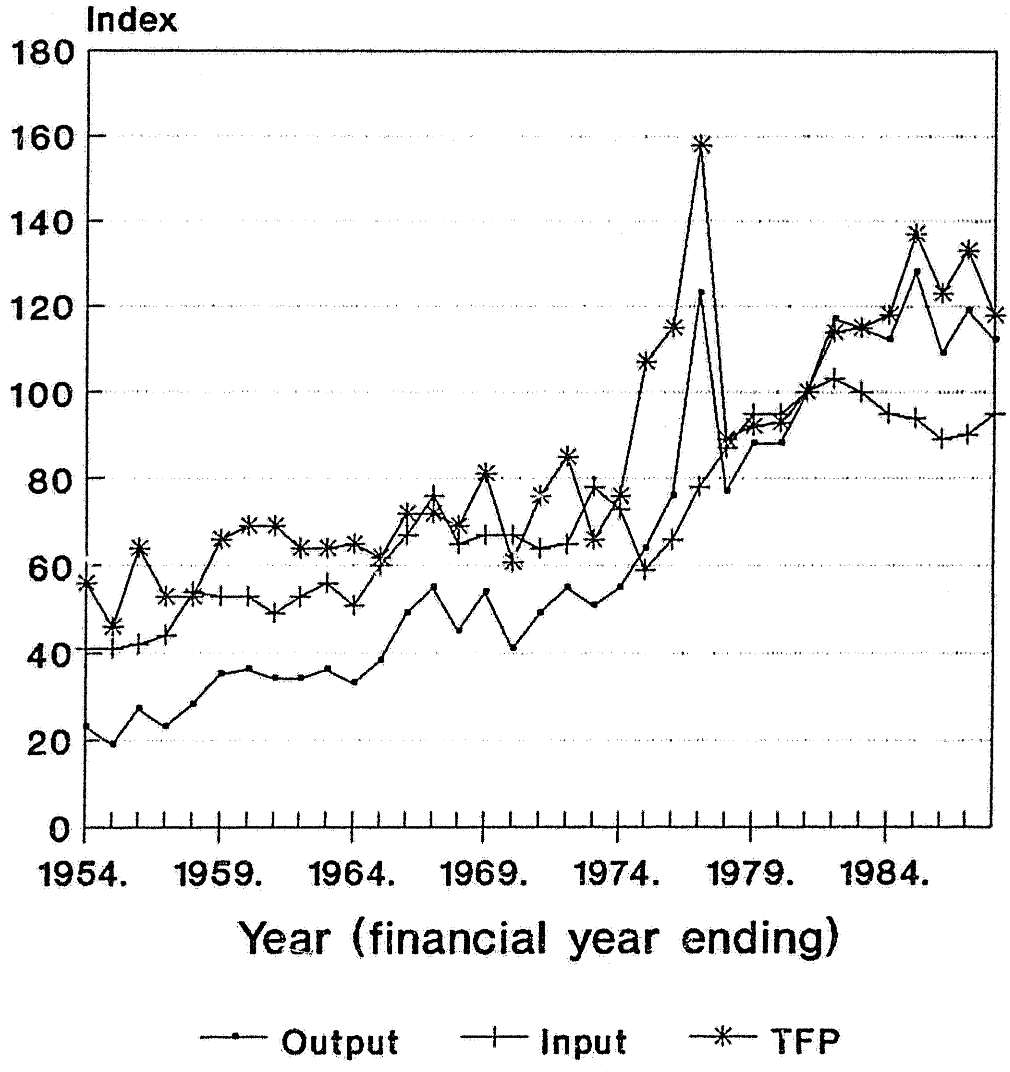
The difference between our figure of 2.7% and Kingwell's 3.1%

TABLE 4

TOTAL FACTOR PRODUCTIVITY

Year	Output	Input	TFP
1953/54	23	41	56
1954/55	19	41	46
1955/56	27	42	64
1956/57	23	44	53
1957/58	28	54	53
1958/59	35	53	66
1959/60	36	53	69
1960/61	34	49	69
1961/62	34	53	64
1962/63	36	56	64
1963/64	33	51	65
1964/65	38	60	62
1965/66	49	67	72
1966/67	55	76	72
1967/68	45	65	69
1968/69	54	67	81
1969/70	41	67	61
1970/71	49	64	76
1971/72	55	65	85
1972/73	51	78	66
1973/74	55	73	76
1974/75	64	59	107
1975/76	76	66	115
1976/77	123	78	158
1977/78	77	87	89
1978/79	88	95	92
1979/80	88	95	93
1980/81	100	100	100
1981/82	117	103	114
1982/83	115	100	115
1983/84	112	95	118
1984/85	128	94	137
1985/86	109	89	123
1986/87	119	90	133
1987/88	112	95	118
Average Annual Percentage Change	5.3	2.6	2.7

Figure 2
Total Factor Productivity



could be due to points (ii) and (iii) mentioned above and/or due to the inclusion of pastoral and high rainfall properties in the Kingwell analysis.

3. Econometric analysis

The neo-classical production function specifies the relationship between an output and the vector of inputs used to produce that output. To estimate the single output form of this function the index of total outputs derived in the previous section could be used as the output and the five indices of inputs would form the vector of inputs. The Cobb-Douglas production function has been the most often used functional form in econometric analyses of production. Its general form is:

$$(7) \qquad Y_t = \alpha \prod_{i=1}^{k} X_{it}^{\beta_i} e^{\gamma t} e^{u_t} \qquad ,t=1,\ldots,T,$$

where Y_t is output, X_{it} is the i-th input, t is a time trend reflecting technological change and u_t is a disturbance term. The popularity of this function is due primarily to its ease of estimation. A logarithmic transformation provides a function which is linear in parameters:

$$(8) \qquad \ln Y_t = \ln\alpha + \sum_{i=1}^{k} \beta_i \ln X_{it} + \gamma t + u_t \qquad ,t=1,\ldots,T.$$

The Cobb-Douglas has been often criticized for its restrictive properties, such as its unitary elasticity of substitution and Hicks-neutral technical change. Many more flexible functional forms have been proposed, such as the translog which can accommodate other substitution possibilities and non-neutral technological change in the inputs.

$$(9) \qquad \ln Y_t = \ln\alpha + \sum_{i=1}^{k} \beta_{i0} \ln X_{it} + \frac{1}{2}\sum_{i=1}^{k}\sum_{j=1}^{k} \beta_{ij} \ln X_{it} \ln X_{jt}$$

$$+ \sum_{i=1}^{k} \beta_{iT} \ln X_{it} \ln t + \gamma_1 \ln t + \gamma_2 (\ln t)^2 + u_t \qquad ,t=1,\ldots,T.$$

Direct ordinary least squares estimates of the above translog production function have been obtained in past studies, however problems with multicollinearity and degrees of freedom have often been encountered. Furthermore, the above function is still defined in terms of a single output. The vast majority of broadacre farms in Western Australia are multi-output firms. They generally produce wheat, wool and sheepmeat as minimum, and may also produce oats, barley, lupins, beef or any number of products. A method which can properly account for the multi-product nature of the industry and also permit the investigation of biases in technological change between outputs, as well as inputs, would be preferred.

The approach we use exploits the dual relationship between production and profit functions in describing the production technology. The advantages of the dual profit function approach to the modeling of multiple output production systems in Australian agriculture are well documented by McKay, Lawrence and Vlastuin (1983), Wall and Fisher (1987), Lawrence and Zietsch (1990), and Low and Hinchy (1990). We use the dual profit function approach to ascertain whether technological change is neutral and if not, which inputs and/or outputs have been favored by technical change over the 35 year period under consideration.

It has been necessary to reduce the total number of inputs and outputs in this profit function analysis from 10 to 8 due to degrees of freedom problems which shall be explained shortly. To this end we have aggregated wool and sheepmeat into a single index and cattle and 'other' into a single index also. This incidentally provides the same 8 groups considered by McKay, Lawrence and Vlastuin (1983) in their study of the Australian sheep industry. Using a similar approach to Diewert and Wales (1987) and Lawrence and Zietsch (1990) we define the following GM profit function for our 8 netputs:[2]

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[2]We follow the usual convention of using "netputs" or "net

$$\Pi_t = \frac{1}{2}\sum_{i=1}^{7}\left[\sum_{j=1}^{7} \beta_{ij}p_{it}p_{jt}/p_{8t}\right] + \sum_{i=1}^{8} \beta_i p_{it} + \sum_{i=1}^{8} \beta_{ic}p_{it}t + \beta_{cc}\left[\sum_{i=1}^{8} \gamma_i p_{it}\right] t^2, \quad t=1,\ldots,T, \tag{10}$$

where $\beta_{ij} = \beta_{ji}$ for all $i,j=1,\ldots,7$;

γ_i = mean of the i-th netput quantity (q_{it});[3]

and the β's are unknown parameters which need to be estimated. Note that the eight netput, the land input, has been arbitrarily chosen to provide the normalizing price for the first summation term in the above profit function.

Hotelling's Lemma is used to derive the set of 8 netput supply equations from this profit function. In fact we derive 3 output supply equations and 5 input demand equations. We obtain the first partial derivatives of the profit function with respect to each of the 8 prices. These are then set equal to the respective quantities to provide the 8 netput supply functions:

$$q_{it} = \beta_i + \sum_{j=1}^{7} \beta_{ij}p_{jt}/p_{8t} + \beta_{ic}t + \beta_{cc}t^2 \quad ,i=1,\ldots,7, \quad t=1,\ldots,T, \tag{11}$$

$$q_{8t} = \beta_8 + \sum_{j=1}^{7}\sum_{i=1}^{7} \beta_{ij}p_{it}p_{jt}/p_{8t}^2 + \beta_{8c}t + \beta_{cc}\gamma_8 t^2 \quad ,t=1,\ldots,T. \tag{12}$$

Disturbance terms, which are assumed to be distributed multivariate normal with zero means and variance-covariance matrix Φ, are added to each of these equations for estimation.

One argument for the estimation of such a system of netput

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outputs" where they are either outputs or inputs entered as negatives.

[3]This is done to conserve degrees of freedom, as suggested by Lawrence and Zietsch (1990) and Low and Hinchy (1990).

supply equations, over the direct estimation of a production function, is that the right hand side variables in these supply equations are prices, which may be reasonably assumed to be exogenous in the case of the Western Australian sheep industry. The direct estimation of a production function, such as that specified in Equations 8 or 9, could involve simultaneity problems with the (endogenous) input quantities appearing on the right hand side of the equation.[4]

The GM functional form is relatively new to the applied literature. The two most popular functional forms specified for profit and cost functions in dual analyses of agricultural production over the past 15 years have been the Translog (Ray, 1982; McKay, Lawrence and Vlastuin, 1983; Weaver, 1983; Antle, 1984; Kuroda, 1988; and Glass and McKillop, 1989) and the Normalized Quadratic (Shumway, 1983; Shumway and Alexander, 1988; Shumway, Saez and Gottret, 1988; and Huffman and Evenson, 1989). The principal advantage of the GM functional form, over these two functional forms, is its ability to deal with the violation of the necessary curvature conditions, which have plagued so many applied studies in the past. Diewert and Wales (1987) use the results of Lau (1978) to establish that a GM cost function will satisfy the necessary curvature conditions if the matrix of second order terms is negative semi-definite. For the case of a GM profit function the matrix of second order terms must be positive semi-definite (Lawrence and Zietsch, 1990).

Following Diewert and Wales (1987) and Lawrence and Zietsch (1990), we observe that if our 7 by 7 matrix of quadratic terms $B=\left[\beta_{ij}\right]$ is not positive semi-definite then we can impose positive semi-definiteness by using the method due to Wiley, Schmidt and Bramble (1973), which involves the replacement of B with the

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[4]Many recent studies which involve the direct estimation of a production function in an agricultural application, use the arguments of Zellner, Kmenta and Dreze (1966), regarding the maximization of expected rather than actual profit, to answer this criticism.

product of a lower triangular matrix A and its transpose:

(13) $B = AA'$

where $A = \left[\alpha_{ij}\right]$ is a 7 by 7 matrix with all values above the leading diagonal set to zero. This will ensure the GM profit function is globally convex in prices whilst retaining its flexibility (Diewert, 1985, cited in Lawrence and Zietsch, 1990). The primary disadvantage of this transformation is that the GM is no longer linear in parameters and hence computationally burdensome non-linear least squares estimation must be used. This is illustrated by the following expansion of Equation 13 for a simple 3 by 3 (4 netput) example.

$$(14) \qquad B = \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} \\ \beta_{21} & \beta_{22} & \beta_{23} \\ \beta_{31} & \beta_{32} & \beta_{33} \end{bmatrix} = \begin{bmatrix} \alpha_{11} & 0 & 0 \\ \alpha_{21} & \alpha_{22} & 0 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{bmatrix} \begin{bmatrix} \alpha_{11} & \alpha_{21} & \alpha_{31} \\ 0 & \alpha_{22} & \alpha_{32} \\ 0 & 0 & \alpha_{33} \end{bmatrix}$$

$$= \begin{bmatrix} \alpha_{11}^2 & \alpha_{11}\alpha_{31} & \alpha_{11}\alpha_{21} \\ \alpha_{21}\alpha_{11} & \alpha_{21}^2+\alpha_{22}^2 & \alpha_{21}\alpha_{31}+\alpha_{22}\alpha_{32} \\ \alpha_{31}\alpha_{11} & \alpha_{31}\alpha_{21}+\alpha_{32}\alpha_{22} & \alpha_{31}^2+\alpha_{32}^2+\alpha_{33}^2 \end{bmatrix}$$

Thus all of the β_{ij}'s in the netput supply equations, similar to Equations 11 and 12, are replaced by their corresponding expressions in the α_{ij}'s, providing equations which are non-linear in the parameters. The expressions for our 8 netput case are obviously more involved than those presented in Equation 14. This is apparent in the computer instructions listed in the appendix. The SHAZAM econometric computer package (see White et al., 1990) was used to estimate the system of equations.

The degrees of freedom problem which was mentioned earlier is evident in the expression for the supply equation of the numeraire netput (Equation 12). The number of parameters to be estimated in Equation 12 is:

$7(7+1)/2 + 3 = 24.$

If there were 9 netputs then the the numeraire equation would have:

$$8(8+1)/2 + 3 = 39$$

parameters to be estimated from 34 observations.[5] This is why the maximum number of netputs that can be considered in this application is 8.

Netput supply elasticities are calculated to provide a method of comparing these results with past dual analyses. The elasticity of the supply of the i-th netput with respect to a change in the price of the j-th netput is defined by:[6]

(15) $\varepsilon_{ij} = (\partial q_i/\partial p_j)(p_j/q_i) \quad ,i,j=1,...,8,$

where $\partial q_i/\partial p_j$ is the first partial derivative of the i-th netput supply equation with respect to the j-th netput price. Following Lawrence and Zietsch (1990) these derivatives are:

(16) $\partial q_i/\partial p_j = \beta_{ij}/p_7 \quad ,i,j=1,..,7;$

(17) $\partial q_i/\partial p_8 = \partial q_8/\partial p_i = \sum_{j=1}^{7} \beta_{ij} p_j/p_8^2 \quad ,i=1,...,7;$ and

(18) $\partial q_8/\partial p_8 = \sum_{i=1}^{7} \sum_{j=1}^{7} \beta_{ij} p_i p_j/p_8^3.$

The estimated elasticities will be of interest to policy makers, however the primary reason for estimation of this system is to investigate biases in technological change. The profit function, defined by Equation 10, permits technological change to be non-neutral in both inputs and outputs. Technological change will be Hicks-neutral with respect to any two inputs (q_i,q_j) if the technological change causes a homothetic shift of the isoquants in (q_i,q_j) space (Weaver, 1983 p47). Thus

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[5]Only 34 observations are used in estimation because output prices have been lagged by one period in our estimating equations. This is done to reflect the fact that output prices are generally unknown to a farmer when he/she must make the production decisions at the beginning of a season. A more sophisticated method of dealing with supply response dynamics would have been used if time and resources permitted.

[6]The time subscript, t, will be implicit from this point forward.

technological change is Hicks-neutral with respect to (q_i, q_j) if:

(19) $B_{ij} = \partial(q_i/q_j)/\partial t = 0$

or equivalently:

(20) $B_{ij} = (\partial q_i/\partial t)/q_i - (\partial q_j/\partial t)/q_j = 0$.[7]

If B_{ij} is positive then technological change is Hicks-saving in q_i relative to q_j. The relevant partial derivatives for the GM are:

(21) $\partial q_i/\partial t = \beta_{ic} + 2\beta_{cc}\gamma_i t \qquad ,i=1,\ldots,8,$

thus providing bias measures of:

(22) $B_{ij} = (\beta_{ic} + 2\beta_{cc}\gamma_i t)/q_i - (\beta_{jc} + 2\beta_{cc}\gamma_j t)/q_j \qquad ,i,j=1,\ldots,8.$

A table containing estimates of the B_{ij}'s are calculated to provide an indication of the directions of any technological change biases.

When the unrestricted system of equations defined by Equations 11 and 12 were estimated using SHAZAM's SYSTEM command, three of the eigenvalues of the matrix of second order coefficients were negative, indicating the estimated technology was not convex in prices. Hence the restricted system of equations defined by Equations 11 to 13 were estimated using SHAZAM's NL command. The restricted estimates are listed in Table 5. The majority of the estimated coefficients are significant at the five percent level with the signs of the technology coefficients indicating growth in the use of all inputs and in the production of all outputs.

The price coefficients are not meaningful in their present form. The netput supply elasticities, defined by Equations 15 to 18, are listed in Table 6, evaluated at the sample means. The own-price elasticities are all of the expected signs as this is imposed by the specification. The crops elasticity of 0.488 is almost identical to the value of 0.50 reported in McKay et al. (1983). The McKay et al. study differs from this study in that

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[7]These expressions follow directly from Weaver (1983) and Shumway and Alexander (1988).

TABLE 5

NON-LINEAR LEAST SQUARES ESTIMATES

Coefficient	Estimate	Std. Error	t-ratio
β_1	-20.014	10.284	-1.95
α_{11}	5.917	.537	11.02
α_{21}	-1.166	.297	-3.92
α_{31}	-.080	.349	-.23
α_{41}	-.107	.551	-.20
α_{51}	-1.661	.471	-3.52
α_{61}	-3.598	.498	-7.22
α_{71}	-1.211	.508	-2.38
β_{1c}	4.511	.517	8.72
β_{cc}	.000223	.000152	1.47
β_2	59.087	6.470	9.13
α_{22}	1.418	.162	8.72
α_{32}	-.495	.439	-1.13
α_{42}	-3.043	.350	-8.70
α_{52}	-.602	.587	-1.03
α_{62}	1.328	.642	2.07
α_{72}	-1.407	.561	-2.51
β_{2c}	1.250	.482	2.59
β_3	-21.699	6.307	-3.44
α_{33}	.935	.279	3.35
α_{43}	1.431	.351	4.08
α_{53}	-3.586	.485	-7.39
α_{63}	1.155	.589	1.96
α_{73}	-3.225	.422	-7.65
β_{3c}	3.529	.303	11.64
β_4	-68.514	14.401	-4.76
α_{44}	.153	.554	.28
α_{54}	.877	.705	1.24
α_{64}	3.127	.525	5.96
α_{74}	.443	.461	.96
β_{4c}	-.282	.552	-.51
β_5	-39.459	6.532	-6.04
α_{55}	-.512	1.148	-.45
α_{65}	.281	1.808	.16
α_{75}	-.437	1.201	-.36
β_{5c}	-2.076	.430	-4.83
β_6	-67.427	4.764	-14.15
α_{66}	-.320	.909	-.35
α_{76}	.034	.742	.05
β_{6c}	-.893	.423	-2.11
β_7	-89.531	8.412	-10.64
α_{77}	.085	1.240	.07
β_{7c}	-.114	.490	-.23
β_8	-57.513	4.908	-11.72
β_{8c}	-.698	.490	-1.42

Log-Likelihood Function = -935.23

TABLE 6

ESTIMATED PRICE ELASTICITIES (ε_{ij}'s)

	1.	2.	3.	4.	5.	6.	7.	8.
1. Crops	.488	-.096	-.008	-.009	-.136	-.295	-.107	.162
2. Sheep and Wool	-.083	.040	-.008	-.050	.013	.072	-.007	.024
3. Cattle and Other	-.010	-.012	.027	.058	-.059	.014	-.049	.030
4. Livestck	.010	.064	-.050	-.173	.045	.023	.002	.080
5. Services and Materials	.139	-.015	.048	.043	-.239	-.051	-.229	.305
Labour	.266	-.076	-.010	.019	-.045	-.322	0.000	.169
Capital	.095	.008	.034	.002	-.197	0.000	-.202	.262
Land	-.133	-.022	-.020	.060	.243	.153	.242	-.523

it uses Australia wide data instead of Western Australian; it utilizes the translog rather than the GM; and it uses a slightly shorter time-series of data. Otherwise, the applications are almost identical in construction. The similarities in the results of the two studies are limited. The primary concern with the results of our study is the small values obtained for the sheep and wool and cattle and other elasticities.[8] McKay et al. did report a small value of 0.12 for their cattle and other elasticity but their elasticity of sheep and wool was 0.72, substantially larger than the 0.04 obtained in this study. Our results also differ greatly from those of Lawrence and Zietsch (1990) and Low and Hinchy (1990). These differences with past studies, which themselves differed substantially, indicate that our results should be viewed cautiously.

The measures of bias in technological change (B_{ij}), defined by Equation 22, are listed in Table 7, evaluated at the sample means. Recall that a positive value of B_{ij} indicates that technological change is Hicks-saving in q_i relative to q_j. Looking firstly at inputs, the largest positive values are all in the materials and services row, indicating that the rate of technological change in the use to this input has been greater than any other input. That is, technological change has been 'materials and services' saving relative to the other four input groups. This result is consistent with Weaver's (1983) analysis of the U.S. wheat region. The remaining input bias effects all appear quite small relative to those associated with materials and services. We note, however, that technological change has been labour saving relative to all other inputs, with the exception of materials and services. This observation does not conflict with what one would expect.

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[8]Output prices for sheep and wool and cattle and other were lagged by more than one period in an attempt to improve our elasticity estimates. This follows Low and Hinchy (1990) who specified lags of two and three periods, for sheep and cattle, respectively. As the change had little impact upon the elasticity estimates, the original single period lags were retained.

TABLE 7

ESTIMATES OF BIAS IN TECHNOLOGICAL CHANGE (B_{ij}'s)

INPUTS:

	Livestock	Materials and Services	Labour	Capital	Land
Livestock	0.000	-.026	-.007	.003	-.004
Materials and Services	.026	0.000	.019	.029	.022
Labour	.007	-.019	0.000	.010	.003
Capital	-.003	-.029	-.010	0.000	-.007
Land	.004	-.022	-.003	.007	0.000

OUTPUTS:

	Crops	Sheep and Wool	Cattle and Other
Crops	0.000	.050	.135
Sheep and Wool	-.050	0.000	.085
Cattle and Other	-.135	-.085	0.000

The output bias effects are substantially larger than those calculated for inputs. The largest positive value is for the cattle and other group relative to crops, with the next largest for the same group relative to sheep and wool. This bias towards the cattle and other group is most likely dominated by the advances in lupin and pea technology which have influenced much of the Western Australian wheatbelt over the past 20 years. The positive bias in wool and sheepmeat relative to crops, indicate that technological advances in crop (wheat, oats and barley) production have been the slowest of the three output groups considered.

The tabulated measures of the biases in technological change provide information on the directions of biases in technological change. The significance of these effects could be discussed if estimates of the standard errors of the B_{ij}'s were obtained. Tests for the Hicks-neutrality of technological change, similar to those presented in Shumway and Alexander (1988), will be included in a later draft of this paper, along with estimates of the standard errors of the B_{ij}'s.

4. Conclusions

Tornqvist index numbers were used to construct an index of total factor productivity growth for Western Australian broadacre agriculture. Indices of five output groups and 5 input groups were constructed from 35 years of ABARE's AAGIS data. These indices were then used to construct indices of total outputs, total inputs and total factor productivity. Total factor productivity was observed to grow at an average rate of 2.7 percent per year, the result of annual growth of 5.3 percent in outputs and 2.6 percent in input use. These results compare favorably with other estimates of Australian productivity growth.

The estimated growth in productivity suggested that technology had advanced at a significant rate over the sample period. Was this technological change uniform over all outputs and inputs in the industry, or had some advanced at faster rates than others? This question prompted the estimation of a system of output supply and input demand equations derived from a Generalized McFadden profit function. The tornqvist input and output indices calculated in the first half of the paper provided the data for the estimation of the system.

Own-price elasticities, cross-price elasticities and measures of the bias in technological change were derived from the estimated system. The own-price elasticities of the sheep and wool and cattle and other output groups were substantially smaller than any obtained in past studies. This has prompted us to view the results with caution. The measures of bias in technological change suggested that materials and services and labour were Hicks-saving relative to the other input groups. The measures of biases for the outputs were much larger than those obtained for the inputs and indicated that the cattle and other group, most likely dominated by legumes, had experienced faster technological growth than wool and sheepmeat and crops.

There is much more that can be done in this analysis. The correction or rationalization of the unusual elasticity estimates is my first priority. The calculation of standard errors of the bias estimates and tests for Hicks-neutrality should also be completed shortly. I also have an interest in measuring the rate of return to research expenditure. One approach under consideration is that of Huffman and Evenson (1989) who include research, education and extension variables directly into their system of output supply and input demand equations. Furthermore, I have data from four other states in Australia other than Western Australia which I have not begun to analyze yet. This extra data will add a further dimension to the study.

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APPENDIX

SHAZAM INSTRUCTIONS

```
par 1000
file 6 nlgm8.out
file 4 p92t8.dta
read(4) q1-q8 p1-p8
genr t=time(0)
set nodoecho
do #=4,8
genr q#=-q#
endo
do #=1,8
?stat q# / mean=mq#
?stat p# / mean=mp#
genr t#=t**2*mq#
endo
genr z=lag(p1)
genr p1=z
genr z=lag(p2)
genr p2=z
genr z=lag(p3)
genr p3=z
do #=1,7
genr p#=p#/p8
do %=1,7
genr p#b%=-p#*p%/p8**2
endo
genr p#b#=p#b#/2
endo
sample 2 35

sys 8 / dn iter=200 piter=50 rest coef=b
ols q1 p1-p7 t t1
ols q2 p1-p7 t t2
ols q3 p1-p7 t t3
ols q4 p1-p7 t t4
ols q5 p1-p7 t t5
ols q6 p1-p7 t t6
ols q7 p1-p7 t t7
ols q8 p1b1 p1b2 p1b3 p1b4 p1b5 p1b6 p1b7 &
 p2b2 p2b3 p2b4 p2b5 p2b6 p2b7 &
 p3b3 p3b4 p3b5 p3b6 p3b7 &
 p4b4 p4b5 p4b6 p4b7 &
 p5b5 p5b6 p5b7 &
 p6b6 p6b7 &
 p7b7 t t8
rest t1:1-t2:2=0
rest t1:1-t3:3=0
rest t1:1-t4:4=0
rest t1:1-t5:5=0
rest t1:1-t6:6=0
```

```
rest t1:1-t7:7=0
rest t1:1-t8:8=0
rest p1:2-p2:1=0
rest p1:3-p3:1=0
rest p1:4-p4:1=0
rest p1:5-p5:1=0
rest p1:6-p6:1=0
rest p1:7-p7:1=0
rest p2:3-p3:2=0
rest p2:4-p4:2=0
rest p2:5-p5:2=0
rest p2:6-p6:2=0
rest p2:7-p7:2=0
rest p3:4-p4:3=0
rest p3:5-p5:3=0
rest p3:6-p6:3=0
rest p3:7-p7:3=0
rest p4:5-p5:4=0
rest p4:6-p6:4=0
rest p4:7-p7:4=0
rest p5:6-p6:5=0
rest p5:7-p7:5=0
rest p6:7-p7:6=0
rest p1b1:8-p1:1=0
rest p1b2:8-p2:1=0
rest p1b3:8-p3:1=0
rest p1b4:8-p4:1=0
rest p1b5:8-p5:1=0
rest p1b6:8-p6:1=0
rest p1b7:8-p7:1=0
rest p2b2:8-p2:2=0
rest p2b3:8-p3:2=0
rest p2b4:8-p4:2=0
rest p2b5:8-p5:2=0
rest p2b6:8-p6:2=0
rest p2b7:8-p7:2=0
rest p3b3:8-p3:3=0
rest p3b4:8-p4:3=0
rest p3b5:8-p5:3=0
rest p3b6:8-p6:3=0
rest p3b7:8-p7:3=0
rest p4b4:8-p4:4=0
rest p4b5:8-p5:4=0
rest p4b6:8-p6:4=0
rest p4b7:8-p7:4=0
rest p5b5:8-p5:5=0
rest p5b6:8-p6:5=0
rest p5b7:8-p7:5=0
rest p6b6:8-p6:6=0
rest p6b7:8-p7:6=0
rest p7b7:8-p7:7=0
end

dim bb 7 7
```

```
copy b bb/frow=1;7 trow=1;7 fcol=1;1 tcol=1;1
copy b bb/frow=10;16 trow=1;7 fcol=1;1 tcol=2;2
copy b bb/frow=19;25 trow=1;7 fcol=1;1 tcol=3;3
copy b bb/frow=28;34 trow=1;7 fcol=1;1 tcol=4;4
copy b bb/frow=37;43 trow=1;7 fcol=1;1 tcol=5;5
copy b bb/frow=46;52 trow=1;7 fcol=1;1 tcol=6;6
copy b bb/frow=55;61 trow=1;7 fcol=1;1 tcol=7;7
matrix e=eigval(bb)
print e
print bb

dim el 8 8

matrix el(8,8)=0

do #=1,7
matrix el(#,8)=0
matrix el(8,#)=0
do %=1,7
matrix el(#,%)=bb(#,%)/mp8*mp%/mq#
matrix el(#,8)=el(#,8)-bb(#,%)*mp%/mp8**2*mp8/mq#
matrix el(8,#)=el(8,#)-bb(#,%)*mp%/mp8**2*mp#/mq8
matrix el(8,8)=el(8,8)+bb(#,%)*mp#*mp%/mp8**3*mp8/mq8
endo
endo

format(1x,8f8.3)
print el/format

dim c 8 8 bt 8

genl bt:1=b:8
genl bt:2=b:17
genl bt:3=b:26
genl bt:4=b:35
genl bt:5=b:44
genl bt:6=b:53
genl bt:7=b:62
genl bt:8=b:92
genl bts=b:9

do #=1,8
do %=1,8
matrix c(#,%)=(bt(#)+2*bts*mq#*17)/mq#-(bt(%)+2*bts*mq%*17)/mq%
endo
endo

print c/format

delete b bb el

nl 8/ncoef=45 coef=b iter=200 piter=50
eq q1=b1+a11**2*p1+a11*a21*p2+a11*a31*p3+a11*a41*p4+a11*a51*p5+a11*a61*p6 &
 +a11*a71*p7+b1t*t+btt*t1
```

```
eq q2=b2+a11*a21*p1+(a21**2+a22**2)*p2+(a21*a31+a22*a32)*p3 &
 +(a21*a41+a22*a42)*p4+(a21*a51+a22*a52)*p5+(a21*a61+a22*a62)*p6 &
 +(a21*a71+a22*a72)*p7+b2t*t+btt*t2
eq q3=b3+a11*a31*p1+(a21*a31+a22*a32)*p2+(a31**2+a32**2+a33**2)*p3 &
 +(a31*a41+a32*a42+a33*a43)*p4+(a31*a51+a32*a52+a33*a53)*p5 &
 +(a31*a61+a32*a62+a33*a63)*p6+(a31*a71+a32*a72+a33*a73)*p7+b3t*t+btt*t3
eq q4=b4+a11*a41*p1+(a21*a41+a22*a42)*p2+(a31*a41+a32*a42+a33*a43)*p3 &
 +(a41**2+a42**2+a43**2+a44**2)*p4+(a41*a51+a42*a52+a43*a53+a44*a54)*p5 &
 +(a41*a61+a42*a62+a43*a63+a44*a64)*p6+(a41*a71+a42*a72+a43*a73+a44*a74)*p7 &
 +b4t*t+btt*t4
eq q5=b5+a11*a51*p1+(a21*a51+a22*a42)*p2+(a31*a51+a32*a52+a33*a53)*p3 &
 +(a41*a51+a42*a52+a43*a53+a44*a54)*p4+(a51**2+a52**2+a53**2+a54**2+a55**2) &
 *p5+(a61*a51+a62*a52+a63*a53+a64*a54+a65*a55)*p6+(a71*a51+a72*a52+a73*a53 &
 +a74*a54+a75*a55)*p7+b5t*t+btt*t5
eq q6=b6+a11*a61*p1+(a21*a61+a22*a62)*p2+(a31*a61+a32*a62+a33*a63)*p3 &
 +(a41*a61+a42*a62+a43*a63+a44*a64)*p4+(a51*a61+a52*a62+a53*a63+a54*a64 &
 +a55*a65)*p5+(a61**2+a62**2+a63**2+a64**2+a65**2+a66**2)*p6 &
 +(a71*a61+a72*a62+a73*a63+a74*a64+a75*a65+a76*a66)*p7+b6t*t+btt*t6
eq q7=b7+a11*a71*p1+(a21*a71+a22*a72)*p2+(a31*a71+a32*a72+a33*a73)*p3 &
 +(a41*a71+a42*a72+a43*a73+a44*a74)*p4+(a51*a71+a52*a72+a53*a73+a54*a74 &
+a55*a75)*p5+(a61*a71+a62*a72+a63*a73+a64*a74+a65*a75+a66*a76)*p6 &
 +(a71**2+a72**2+a73**2+a74**2+a75**2+a76**2+a77**2)*p7+b7t*t+btt*t7
eq q8=b8+a11**2*p1b1+a11*a21*p2b1+a11*a31*p3b1+a11*a41*p4b1+a11*a51*p5b1 &
 +a11*a61*p6b1+a11*a71*p7b1+(a21**2+a22**2)*p2b2+(a21*a31+a22*a32)*p3b2 &
 +(a21*a41+a22*a42)*p4b2+(a21*a51+a22*a52)*p5b2+(a21*a61+a22*a62)*p6b2 &
 +(a21*a71+a22*a72)*p7b2+(a31**2+a32**2+a33**2)*p3b3 &
 +(a31*a41+a32*a42+a33*a43)*p4b3+(a31*a51+a32*a52+a33*a53)*p5b3 &
 +(a31*a61+a32*a62+a33*a63)*p6b3+(a31*a71+a32*a72+a33*a73)*p7b3 &
 +(a41**2+a42**2+a43**2+a44**2)*p4b4+(a41*a51+a42*a52+a43*a53+a44*a54)*p5b4 &
 +(a41*a61+a42*a62+a43*a63+a44*a64)*p6b4+(a41*a71+a42*a72+a43*a73+a44*a74) &
 *p7b4+(a51**2+a52**2+a53**2+a54**2+a55**2)*p5b5+(a61*a51+a62*a52+a63*a53 &
 +a64*a54+a65*a55)*p6b5+(a71*a51+a72*a52+a73*a53+a74*a54+a75*a55)*p7b5 &
 +(a61**2+a62**2+a63**2+a64**2+a65**2+a66**2)*p6b6 &
 +(a71*a61+a72*a62+a73*a63+a74*a64+a75*a65+a76*a66)*p7b6 &
 +(a71**2+a72**2+a73**2+a74**2+a75**2+a76**2+a77**2)*p7b7+b8t*t+btt*t8
end

dim bb 7 7 aa 7 7
copy b aa/frow=2;8 trow=1;7 fcol=1;1 tcol=1;1
copy b aa/frow=12;17 trow=2;7 fcol=1;1 tcol=2;2
copy b aa/frow=20;24 trow=3;7 fcol=1;1 tcol=3;3
copy b aa/frow=27;30 trow=4;7 fcol=1;1 tcol=4;4
copy b aa/frow=33;35 trow=5;7 fcol=1;1 tcol=5;5
copy b aa/frow=38;39 trow=6;7 fcol=1;1 tcol=6;6
copy b aa/frow=42;42 trow=7;7 fcol=1;1 tcol=7;7
matrix bb=aa*aa'
matrix e=eigval(bb)
print e
print bb

dim el 8 8

matrix el(8,8)=0
```

```
do #=1,7
matrix el(#,8)=0
matrix el(8,#)=0
do %=1,7
matrix el(#,%)=bb(#,%)/mp8*mp%/mq#
matrix el(#,8)=el(#,8)-bb(#,%)*mp%/mp8**2*mp8/mq#
matrix el(8,#)=el(8,#)-bb(#,%)*mp%/mp8**2*mp#/mq8
matrix el(8,8)=el(8,8)+bb(#,%)*mp#*mp%/mp8**3*mp8/mq8
endo
endo

format(1x,8f8.3)
print el/format

genl bt:1=b:9
genl bt:2=b:18
genl bt:3=b:24
genl bt:4=b:31
genl bt:5=b:36
genl bt:6=b:40
genl bt:7=b:43
genl bt:8=b:45
genl bts=b:10

do #=1,8
do %=1,8
matrix c(#,%)=(bt(#)+2*bts*mq#*17)/mq#-(bt(%)+2*bts*mq%*17)/mq%
endo
endo

print c/format

stop
```