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Estimation of Stochastic Frontier Production Functions With Time-Varying Parameters and Technical Efficiencies Using Panel Data from Indian Villages

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A B S T R A C T

A stochastic frontier production function with time-varying technical efficiencies is estimated using panel data from ICRISAT's Village Level Studies in three Indian villages. A Cobb-Douglas functional form is initially defined in which linear combinations of irrigated and unirrigated land and hired and family labour are included as explanatory variables.

Given the specifications of a linearized version of the Cobb-Douglas production frontier with coefficients which are a linear function of time, the hypothesis of time-invariant technical inefficiency is rejected for one of the three villages involved. The hypothesis of time-invariant coefficients of the explanatory variables is rejected for two of the three villages. Further, the hypothesis that hired and family labour are equally productive is accepted in only one of the three villages.

The technical efficiencies of individual farms exhibited considerable variation, both in the cases of time-varying and time-invariant technical efficiencies.

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1. Introduction

Frontier production functions and technical efficiency of individual firms have been considered in a large number of papers in economic, statistical and econometric journals. Battese (1991) presents a review of the concepts and models which have been suggested and surveys applications which have appeared in agricultural economics journals.

Frontier production functions assume the existence of technical inefficiency of the different firms involved in production such that, for specific values of factor inputs, the levels of production are less than what would be the case if the firms were fully technically efficient. The majority of the earlier applications of frontier production functions involved cross-sectional data. However, more recently attempts have been made to apply frontier production functions in the analysis of time-series data on firms involved in production. Initially the firm effects associated with the existence of technical inefficiency were assumed to be time-invariant random variables or independent and identically distributed over time. Models for frontier production functions have been proposed in which the firm effects associated with technical efficiency are assumed to be time varying [see Kumbhakar (1990), Cornwell, Schmidt and Sickles (1990) and Battese and Coelli (1991)].

In this paper, we apply the model proposed in Battese and Coelli (1991) in the analysis of panel data collected by the International Crops Research Institute for the Semi-Arid Tropics (ICRISAT) from sample farmers in three villages in India.

2. The Econometric Model

The model proposed by Battese and Coelli (1991) assumes that the production of firms is defined by a stochastic frontier production function in which the firm effects are an exponential function of time, such that the firms are not required to be observed in all the time periods involved. The model is defined by

$$Y_{it} = f(x_{it};\beta)\exp(V_{it} - U_{it}) \quad (1)$$

and

$$U_{it} = \eta_{it}U_i = \left\{\exp[-\eta(t-T)]\right\}U_i \quad (2)$$

where $t \in \mathcal{G}(i)$ and $i = 1,2,\ldots,N$;

Y_{it} represents the production for the i-th firm at the t-th period of observation;

$f(x_{it};\beta)$ is a function of a vector, x_{it}, of factor inputs and other relevant variables, associated with the production of the i-th firm in the t-th period of observation, and a vector, β, of unknown parameters;

the V_{it}'s are assumed to be independent and identically distributed $N(0, \sigma_V^2)$ random errors;

the U_i's are assumed to be independent and identically distributed non-negative truncations of the $N(\mu, \sigma^2)$ distribution:

η is an unknown scalar parameter; and

$\mathcal{J}(i)$ represents the set of T_i time periods among the T periods involved for which observations for the i-th firm are obtained. (If the i-th firm was observed in all T time periods, then $\mathcal{J}(i) = \{1,2,\ldots,T\}$, otherwise $\mathcal{J}(i)$ is a subset of the set of integers from 1 to T, which indicate the time periods for which observations on the i-th firm were obtained.)

The firm effects, U_{it}, are non-negative random variables which are associated with the existence of technical inefficiency of the firms. That is, the observed production, Y_{it}, is less than the stochastic frontier production, $f(x_{it};\beta)\exp(V_{it})$, for the given set of inputs in the vector, x_{it}. The model for the firm effects, defined by equation (2), specifies that the firm effects, U_{it}, approach U_i as t increases towards the last time period, T, involved in the panel. If the parameter, η, is positive then the firm effects, U_{it}, decline towards U_i a t increases towards T. This situation would indicate a decline in the level of technical inefficiency and, hence, an increase in technical efficiency over time.

As stated in Battese and Coelli (1991), the exponential specification of the behaviour of the firm effects over time is a rigid parameterization. It implies that the technical efficiency of the firms involved, $TE_{it} = \exp(-U_{it})$, is a double exponential function of time for the given firm, i. Kumbhakar (1990) assumed that the firm effects, U_{it}, were a more general exponential function of time involving two parameters. No empirical applications of Kumbhakar's (1990) model have yet appeared because the model has not been successfully programmed. Cornwell, Schmidt and Sickles (1990) assumed that the firm effects were a quadratic function of time in which the coefficients were random draws from a trivariate normal distribution.

The model for the firm effects, U_{it}, defined by equation (2), assumes that the rankings of the firm effects remain the same over time. In order to permit different orderings of the firm effects, U_{it}, for the firms at different time periods, a more complicated model than that of equation (2) would be required.

Battese and Coelli (1991) propose that the technical efficiency of the i-th firm at the t-th time period be predicted by the conditional expectation of the technical efficiency, $\exp(-U_{it})$, given the vector of the values of $V_{it} - U_{it}$ for the i-th firm up to time period t. The expression involved is

$$E\left[\exp(-U_{it})|E_i\right] = \left\{\frac{1 - \Phi\left[\eta_{it}\sigma_i^* - (\mu_i^*/\sigma_i^*)\right]}{1 - \Phi(-\mu_i^*/\sigma_i^*)}\right\}\exp\left[-\eta_{it}\mu_i^* + \tfrac{1}{2}\eta_{it}^2\sigma_i^{*2}\right] \quad (3)$$

where E_i represents the $(T_i \times 1)$ vector of E_{it}'s associated with the time periods observed for the i-th firm, where $E_{it} \equiv V_{it} - U_{it}$;

$$\mu_i^* = \frac{\mu\sigma_V^2 - \eta_i' E_i \sigma^2}{\sigma_V^2 + \eta_i'\eta_i\sigma^2} \quad (4)$$

$$\sigma_i^{*2} = \frac{\sigma_V^2\sigma^2}{\sigma_V^2 + \eta_i'\eta_i\sigma^2} \quad (5)$$

where η_i represents the $(T_i \times 1)$ vector of η_{it}'s associated with the time periods observed for the i-th firm; and

$\Phi(\cdot)$ represents the distribution function for the standard normal random variable.

The estimation of the stochastic frontier production function (1)-(2) and the prediction of the technical efficiencies of the different firms over time is achieved by the use of the computer program, FRONTIER, written by Coelli (1991). The FRONTIER program assumes that the stochastic frontier production function (1) is of Cobb-Douglas type.

Battese and Coelli (1991) illustrated the use of the FRONTIER program with the analysis of a subset of the data on a panel of sample farms from the village of Aurepalle in India. In this paper, we consider the complete data sets obtained over the ten-year period in which ICRISAT collected data from the three villages of Aurepalle, Kanzara and Shirapur.

3. ICRISAT's Village Level Studies

The data used in this study were obtained from the International Crops Research Institute for the Semi-Arid Tropics (ICRISAT) near Hyderabad in the Indian state of Andhra Pradesh. The data are from the village level studies (VLS) in which ICRISAT personnel collected a range of data from households engaged in agricultural production in different villages in India. As a part of its mandate, ICRISAT initiated its village level studies in 1975 to obtain reliable data on traditional agricultural methods in the Semi-Arid Tropics (SAT) of India so that improved technological methods could be introduced [see Jodha, Asokan and Ryan (1977) and Binswanger and Jodha (1978)].

The three villages involved in the VLS studies of ICRISAT were selected from districts which represented the broad agroclimatic subregions in the SAT of India. The main factors considered in the selection of the districts included soil types, rainfall and cropping pattern. Accessibility to agricultural universities or research stations and development programs and proximity to the ICRISAT's headquarters at Patancheru near Hyderabad were also given important consideration. Within the selected districts, talukas (subdivisions of a district) were selected which represented the typical characteristics in terms of land-use pattern, cropping, irrigation, livestock, infra-structural development, population, etc. Villages which were located near large towns or have special government or other programs were not considered in the sample. The data used in this study were collected from three villages, Aurepalle, Shirapur and Kanzara, during the years 1975 to 1985. Data on factor inputs and total production were obtained for a random sample of households in each village.

All households in each village were divided into two main groups. The agricultural labour group consisted of households operating less than 0.2 hectares of operated land and the cultivator group consisted of households operating at least 0.2 hectares of land. The cultivator group was further classified into three equal groups and ranked as small, medium and large farmers depending on the size of their holdings.

A random sample of ten households was selected from each group of farmers including the agricultural labour group so that 40 sample farmers were selected from each village. However, this study does not include the agricultural labour group for the purpose of the analysis of the frontier production function. During the ten-year period involved, some households which were originally classified as labour farmers became small farmers in the later years and hence were included in the sample. Farmers who refused to provide information or ceased to be members of the sample were replaced by other farmers. Hence the numbers of sample households in each village, as well as the number of time-series observations for each household, were not necessarily equal.

The villages of Aurepalle, Shirapur and Kanzara were selected from the districts of Mahbubnagar, Sholapur and Akola, respectively, and are located approximately 70 km south, 336 km west and 550 km north of Hyderabad, respectively. There were 3141 people in Aurepalle, 2017 people in Shirapur and 1380 people in Kanzara in 1985.

Considerable soil heterogeneity is a characteristic of the SAT of India. Aurepalle has medium and shallow alfisols (red soils) with low water retention capacity. Soil heterogeneity is remarkably high in Aurepalle compared to Shirapur and Kanzara. Shirapur has medium and deep vertisols (black soils) with high moisture retention capacity. Kanzara has mainly medium-deep black soils and shallow vertisols with medium moisture retention capacity. Soils in Kanzara are more homogeneous than in Aurepalle and Shirapur.

Rainfall in the SAT of India is generally erratic in distribution and the mean annual rainfall ranges from about 400 mm to 1200 mm. In the years 1975 to 1985 the average annual rainfall was 611 mm for Aurepalle, 629 mm for Shirapur and 850 mm for Kanzara. Rainfall is very erratic and uncertain in Aurepalle and Shirapur.

Walker and Ryan (1990) report that during four years of the study period Aurepalle and Shirapur had very little rainfall. Rainfall is relatively higher and less variable in Kanzara. Agriculture is predominantly dryland with two main seasons, the rainy season (kharif) which spans the months of June to October followed by the post-rainy (rabi) season.

In Aurepalle, dryland crops include sorghum, pearl millet, pigeonpea, castor and high-yielding variety (HYV) paddy. Sorghum, pearl millet, pigeonpea are intercropped, usually with one row of pigeonpea to four rows of cereal crops. The high-yielding variety paddy is mostly grown under irrigated conditions. Of the total cropped land, about 21 per cent is irrigated in Aurepalle, compared to 9 per cent and 7 per cent in the villages of Shirapur and Kanzara, respectively.

The rabi season has more reliable rainfall in the village of Shirapur. During the rabi season farmers grow mainly sorghum and chickpea. Local wheat and safflower are also grown. Irrigation is used for onions, chillies and other vegetables. However, the use of high-yielding varieties is very limited in Shirapur.

The village of Kanzara has relatively favourable rainfall in the kharif season and the crops grown include cotton, pigeonpea, hybrid sorghum, local sorghum, groundnut, green gram and black gram. Wheat and chickpea are mainly planted in the rabi season. Intercropping is more prevalent in Kanzara than in the other two villages. The use of improved technology, such as high-yielding varieties of sorghum and cotton, fertilizers and pesticides, is also high in Kanzara compared to the villages of Aurepalle and Shirapur.

There exists a large variation in the cropping patterns among the three villages. This variation is associated with differences in soil heterogeneity, rainfall pattern and other factors between the villages. Shirapur has the highest proportion of area cropped under cereals of which local sorghum contributes about 62 per cent of the total cultivated land in the village. The area under cereals in Aurepalle and Kanzara is about 50 and 30 per cent respectively. Oil crops play an important role in Aurepalle, where castor contributes about 35 per cent of the total cropped land, followed by sorghum and paddy which contribute about 20 per cent each. Cotton is a sole crop in Kanzara. It occupies about 40 per cent of the cultivated land in the village.

A similar variation exists in the marketed output of crops in the three villages. The crops which have the largest proportion of marketed output are castor in Aurepalle, cotton in Kanzara and sunflower in Shirapur. The cereal crops, sorghum, pearlmillet, paddy and wheat, are mainly subsistence crops.

The labour market includes cultivators and agricultural labourers which comprise about two-thirds of the active workers in SAT India. The labour market is active in the three villages. However, the use of labour (family and hired) varies from village to village, as well as from year to year, depending on rainfall, soil type, type of crop, irrigation, etc. Farm households heavily depend on hired labour to cultivate their land. In

Aurepalle and Kanzara, hired labour provides the majority (60 to 80 per cent) of the total labour used in crop production. The high demand for hired labour is due to the activities of paddy transplanting in Aurepalle and cotton picking in Kanzara [see Walker and Ryan (1990)]. The labour force comprises men, women and children, but the latter only make a very small contribution. The contribution of men to the total family labour in crop production is substantially higher than women, while women dominate the hired labour market.

In all the villages, cultivation such as plowing, harrowing and interculturing is carried out using animal draft power, usually involving bullocks. However, many households which own small areas of land do not have bullocks. Seasonal hiring is common, especially by small farmers. It is most common in Shirapur where bullock-to-land ratios are significantly lower than in the other two villages [Walker and Ryan (1990)]. Single bullock owners often pool their bullocks and cultivate on an exchange basis.

Fertilizer is used almost entirely for irrigated agriculture in the study villages. However, the use of fertilizer in dryland agriculture is increasing in the rainfall-assured village of Kanzara and, to some extent, in Aurepalle. For example, the use of fertilizer in dryland farming has increased from 3 per cent in 1975-76 to 50 per cent by 1985-86. However, application rates per hectare remained very low.

Manure plays an important role in the study villages. Many farmers apply manure to their land every year. However, the supply of manure is constrained by limited availability of fodder which restricts livestock production as well as the use of its byproduct for fuel.

Pesticides are applied mainly in irrigated agriculture, although the expenditure on fertilizers is much higher (about nine times) than the expenditure on pesticides. Pesticides are widely applied in the villages of

Aurepalle and Kanzara.

The following section deals with the empirical analyses of the data obtained from the three villages, Aurepalle, Shirapur and Kanzara. It is expected that different parameter values and technical efficiencies are likely because of the substantial differences in the agro-climatic environments among the three villages.

4. The Frontier Production Function

A stochastic frontier production function of Cobb-Douglas type, which involves four basic explanatory variables, is initially considered in this study. The frontier production function is defined by:

$$Y_{it} = \alpha_0 X_{1it}^{\beta_1} X_{2it}^{\beta_2} X_{3it}^{\beta_3} X_{4it}^{\beta_4} \exp(V_{it} - U_{it}). \qquad (6)$$

where Y_{it} is the total value of output for the i^{th} farmer in the t^{th} year of observation (in Rupees, expressed in terms of 1975-76 value terms)[1];

$X_{1it} \equiv a_1 UL_{it} + (1 - a_1) IL_{it}$ is a land variable in which UL_{it} and IL_{it} are the hectares of unirrigated and irrigated land under production, respectively, for the i^{th} farmer in the t^{th} year of observation and a_1 is a parameter, such that $0 < a_1 < 1$;

$X_{2it} \equiv a_2 FL_{it} + (1 - a_2) HL_{it}$ is a labour variable in which FL_{it} and HL_{it} are the hours of family labour and hired labour for the i^{th} farmer in the t^{th} year of observation (in male equivalent units)[2];

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1 The values of output (and input costs) were deflated using price indices which were constructed from data on prices and quantities of commodities grown in the three villages.

2 Labour hours were converted to male equivalent units based on the conversion rule that female and child labour hours are equivalent to 0.75 and 0.50 male hours, respectively. These factors are used by ICRISAT in empirical analyses involving labour of different family members.

$X_{3it} = OB_{it} + HB_{it} \equiv Bullock_{it}$ is the bullock labour variable in which OB_{it} and HB_{it} represent the hours of owned and hired bullock labour (in pairs), respectively, for the i^{th} farmer in the t^{th} year of observation;

$X_{4it} \equiv \exp(Cost_{it})$ is the exponent (or anti-logarithm) of the total cost of inputs (involving inorganic fertilizer, organic matter applied to land, pesticides and machinery costs) for the i^{th} farmer in the t^{th} year of observation; and

V_{it} and U_{it} are random variables having the distributional properties, as defined for equations (1) and (2).

The model, defined by equation (6), is formulated from the work of Bardhan (1973), Deolalikar and Vijverberg (1983, 1987) and Battese, Coelli and Colby (1989). Bardhan (1973) considered a production function of Cobb-Douglas type in which the variables, total labour (family plus hired labour hours) and the proportion of hired labour to total labour, were separately included as explanatory variables. Bardhan (1973) used Indian farm-level data and concluded that hired and family labour were heterogeneous in some cases.

Deolalikar and Vijverberg (1983) defined a more general model of CES type in the analysis of district-level data for Indian farms. Several special cases of the CES model were considered. They concluded that the model in which hired and family labour were included as separate explanatory variables was the best one. Deolalikar and Vijverberg (1983) also considered unirrigated and irrigated land in their production function. They concluded that the best model had a weighted average of the unirrigated and irrigated areas operated as the land variable.

Battese, Coelli and Colby (1989) considered the model in which labour and land variables were the weighted averages of their respective hired and family labour and unirrigated and irrigated land. Battese, Coelli and Colby

(1989) included cost of inputs in the Cobb-Douglas production function, provided input costs were positive. However, if input costs were not positive (as is the case for a large proportion of farms in this study), then input costs were not included in the function. This dummy-variable approach may be criticised because as input cost approaches zero then production should also approach zero according to the Cobb-Douglas production function.

The modified production function (6), in which costs of inputs enters exponentially, implies that as costs approach zero, the term, exp(Costs), approaches one, not zero. Hence, the model (6) implies that farms with positive input costs have a different intercept value than those with zero input costs (of those which are measured), but the latter farms do not have zero level of production.

The model of equation (6) is alternatively expressed by

$$Y_{it} = \alpha_0 \times a_1^{\beta_1}(Land_{it})^{\beta_1}\left[1 + (b_1 - 1)\left(\frac{IL_{it}}{Land_{it}}\right)\right]^{\beta_1}$$

$$\times\, a_2^{\beta_2}(Labour_{it})^{\beta_2}\left[1 + (b_2 - 1)\left(\frac{HL_{it}}{Labour_{it}}\right)\right]^{\beta_2}$$

$$\times\, (Bullock_{it})^{\beta_3}[\exp(Cost_{it})]^{\beta_4}\exp(V_{it} - U_{it}); \qquad (7)$$

where $Land_{it} \equiv UL_{it} + IL_{it}$ is the total hectares of land operated by the i^{th} farmer in the t^{th} year of observation;

$Labour_{it} \equiv FL_{it} + HL_{it}$ is the total hours of human labour for the i^{th} farmer in the t^{th} year of observation;

$IL_{it}/Land_{it}$ is the ratio of irrigated land to the total land in hectares operated by the i^{th} farmer in the t^{th} year of observation;

$HL_{it}/Labour_{it}$ is the ratio of hired labour to total labour for the i^{th} farmer in the t^{th} year of observation; and

b_1 and b_2 are parameters defined by

$$b_1 \equiv (1 - a_1)/a_1 \quad \text{and} \quad b_2 \equiv (1 - a_2)/a_2.$$

It is noted that if unirrigated and irrigated land were equally productive (an unlikely occurrence) then the parameter, a_1, would be 0.5, which implies that the parameter, b_1, would be equal to 1.0. Similarly, if hired and family labour were equally productive, then the parameter, b_2, would be equal to 1.0.

We, in fact, estimate a linearized version of the model of equation (7) obtained by considering the first-term of the Taylor expansion for the land and labour variables, namely

$$\begin{aligned} \log Y_{it} = \beta_0 &+ \beta_1 \log(Land_{it}) + \beta_2 \log(Labour_{it}) + \beta_3 \log(Bullock_{it}) \\ &+ \beta_4 Cost_{it} + \beta_1(b_1-1)\left(\frac{IL_{it}}{Land_{it}}\right) + \beta_2(b_2-1)\left(\frac{HL_{it}}{Labour_{it}}\right) \\ &+ V_{it} - U_{it} \end{aligned} \tag{8}$$

The parameter, β_0, is a simple function of α_0, β_1, a_1, β_2 and a_2.

It should be noted that the model of equation (8) is not equivalent to that of equations (6) or (7). The function (8) would be a close approximation to that of equation (7) if the land- and labour-ratio variables had values which were close to zero.

If hired and family labour were equally productive , then the coefficient of the labour-ratio variable, $HL_{it}/Labour_{it}$, would be zero. Thus testing that the coefficient of the labour-ratio variable is zero provides a procedure for testing whether hired and family labour are equally productive in the villages involved.

5. Empirical Results

A summary of the data on the different variables in the frontier production function is given in Table 1. It is evident from these statistics that Aurepalle farmers tend to be smaller in terms of value of output and total land operated. Kanzara farmers had the highest mean value of output, human labour and bullock labour. Kanzara farmers have the least amount of irrigation because of the relatively assured rainfall, whereas Aurepalle farmers have the greatest amount of irrigation because of the prevalence of growing paddy.

Bullock labour is used considerably more in Kanzara and Aurepalle than in Shirapur. Cost of inputs had a high proportion of zero observations in all three villages and so the sample means were not very large in all three cases.

The stochastic frontier production function (8) consists of ten parameters, six being associated with the explanatory variables of the function and four being parameters which specify the distributions of the random variables, V_{it} and U_{it}. The maximum-likelihood estimates for the parameters of the frontier production functions with time-invariant parameters for the three villages are presented in Table 2.

Tests of hypotheses about the distribution of the random variables associated with the existence of technical inefficiency and residual error are of interest. The frontier production function is equivalent to the traditional response function if the parameters, γ, μ and η, are simultaneously equal to zero. Hence a test of the null hypothesis, H_0: $\gamma=\mu=\eta=0$, is desirable. Further, if the parameter, η, was zero, then the farm effects associated with the existence of technical inefficiency would be time invariant. Also, if the parameter, μ, was zero, then the farm effects associated with the last period of observation in the panel would have

Table 1. Summary Statistics for Variables in the Stochastic Frontier Production Function for Farmers in Aurepalle, Shirapur and Kanzara

Variable	Sample Mean	Sample Standard Deviation	Minimum Value	Maximum Value
Value of Output (Rs, in 1975-76 values)				
- Aurepalle	3,559.9	4,482.7	7.2	18,094
- Shirapur	3,689.1	3,437.2	22.0	26,423
- Kanzara	5,206.7	7,207.7	121.6	39,168
Land (hectares)				
- Aurepalle	4.23	3.80	0.16	20.97
- Shirapur	6.63	5.45	0.61	24.19
- Kanzara	5.99	7.38	0.40	36.34
Land Ratio, IL/Land				
- Aurepalle	0.14	0.21	0	1.0
- Shirapur	0.13	0.24	0	1.0
- Kanzara	0.06	0.13	0	1.0
Human Labour (hours)				
- Aurepalle	2,133.5	2,697.4	18	12,916
- Shirapur	1,658.9	1,558.6	40	11,146
- Kanzara	2,565.7	3,138.7	58	15,814
Labour Ratio, HL/Labour				
- Aurepalle	0.42	0.29	0	0.98
- Shirapur	0.45	0.20	0.06	0.98
- Kanzara	0.56	0.27	0.016	0.996
Bullock Labour				
- Aurepalle	518.9	592.8	8	4,316
- Shirapur	340.6	280.5	14	1,240
- Kanzara	567.3	763.5	12	3,913
Cost of Inputs				
- Aurepalle	626.4	963.3	0	6,205
- Shirapur	458.8	1,023.8	0	6.746
- Kanzara	626.0	975.8	0	5.344

Table 2: Maximum-likelihood Estimates for Parameters of Stochastic Frontier Production Functions with Time-Invariant Coefficients for Farmers in Aurepalle, Shirapur and Kanzara[1]

Variable	Parameter	M.L. Estimates for Production Frontiers in Aurepalle	Shirapur	Kanzara
Constant	β_0	1.47 (0.58)	2.81 (0.52)	1.62 (0.66)
Log(Land)	β_1	0.36 (0.11)	0.183 (0.061)	0.102 (0.079)
Log(Labour)	β_2	1.27 (0.12)	0.781 (0.086)	0.836 (0.096)
Log(Bullocks)	β_3	-0.557 (0.069)	-0.104 (0.054)	0.049 (0.064)
Cost	β_4	0.00011 (0.00099)	0.00123 (0.00073)	0.00387 (0.00075)
IL/Land	$\beta_5 \equiv \beta_1(b_1-1)$	0.38 (0.29)	-0.11 (0.13)	0.44 (0.20)
HL/Labour	$\beta_6 \equiv \beta_2(b_2-1)$	-0.28 (0.12)	0.12 (0.13)	-0.16 (0.11)
	$\sigma_s^2 \equiv \sigma_v^2 + \sigma^2$	0.248 (0.073)	0.324 (0.044)	0.132 (0.012)
	$\gamma \equiv \sigma^2/\sigma_s^2$	0.30 (0.21)	0.638 (0.029)	0.146 (0.052)
	μ	-0.89 (0.99)	-2.87 (0.92)	0.46 (0.38)
	η	0.19 (0.22)	0.269 (0.062)	0.011 (0.024)
	Loglikelihood	-172.06	-131.18	-111.24

[1] The estimated standard errors for the maximum-likelihood estimates are presented below the corresponding estimates. These values are generated by the computer program, FRONTIER.

half-normal distribution. Hence the null hypotheses that η and μ were zero, either individually or together, are of interest if the stochastic frontier function is significantly different from the traditional response function.

Tests of these various hypotheses associated with the parameters, γ, μ and η, are presented in Table 3. The generalized likelihood-ratio test statistic is calculated after obtained the loglikelihood value associated with the restricted maximum-likelihood estimates for the special cases when the appropriate parameters are zero.

The results presented in Table 3 imply that, given the specifications of the stochastic frontier production function (8) with time-invariant parameters, then the model is significantly different from the traditional response function for all three villages. Further, for farmers in Aurepalle and Shirapur no sub-model in which the parameters, μ and η, are zero, either individually or jointly, is an adequate representation of the data. That is, technical inefficiency not only exists, but the farm effects are not time invariant, nor is the half-normal distribution an adequate representation. However, for farmers in Kanzara, the null hypothesis that the farm effects are time invariant and have half-normal distributions would be accepted under the model assumptions.

The above model for the stochastic frontier production function associated with panel data on sample farmers from the three villages is likely to be inappropriate. That is, the assumption that the coefficients of the explanatory variables in the frontier function (8) are time invariant, but that the farm effects associated with technical inefficiency have a particular time-varying structure may be regarded as objectionable. Hence we now consider a modification of the model in which the coefficients of the explanatory variables in the frontier production function are time varying and are, in fact, a linear function of the year of observation. That is, we

Table 3: Tests of Hypotheses for Parameters of the Distribution of the Farm Effects, U_{it}, Associated With the Stochastic Frontier Production Function With Time-Invariant Coefficients for the Farmers in Aurepalle, Shirapur and Kanzara

Null Hypotheses	Loglikelihood	χ^2-statistic	Decision
H_0: $\gamma = \mu = \eta = 0$		$(\chi^2_{3,0.95} = 7.81)$	
Aurepalle	-180.65	17.18	Reject H_0
Shirapur	-210.80	159.24	Reject H_0
Kanzara	-119.01	15.54	Reject H_0
H_0: $\mu = \eta = 0$		$(\chi^2_{2,0.95} = 5.99)$	
Aurepalle	-179.43	14.74	Reject H_0
Shirapur	-196.62	130.88	Reject H_0
Kanzara	-113.48	4.48	Accept H_0
H_0: $\mu = 0$		$(\chi^2_{1,0.95} = 3.84)$	
Aurepalle	-175.19	6.26	Reject H_0
Shirapur	-138.35	14.34	Reject H_0
Kanzara	-113.12	3.76	Accept H_0
H_0: $\eta = 0$		$(\chi^2_{1,0.95} = 3.84)$	
Aurepalle	-178.92	13.72	Reject H_0
Shirapur	-194.89	127.42	Reject H_0
Kanzara	-111.56	0.64	Accept H_0

consider the stochastic frontier production function, defined by

$$\log Y_{it} = \beta_{0t} + \beta_{1t} \log(\text{Land}_{it}) + \beta_{2t} \log(\text{Labour}_{it})$$
$$+ \beta_{3t} \log(\text{Bullock}_{it}) + \beta_{4t}\text{Cost}_{it}$$
$$+ \beta_{5t}(\text{IL}_{it}/\text{Land}_{it}) + \beta_{6t}(\text{HL}_{it}/\text{Labour}_{it}) + V_{it} - U_{it} \quad (9)$$

where

$$\beta_{jt} = \beta_j + \delta_j(\text{Year}_{it}) \equiv \beta_j + \delta_j xt, \; j = 0,1,\ldots,6. \quad (10)$$

For this more general specification of the stochastic frontier production function, there would be interest in testing if the coefficients of the production frontier were time invariant, or the elasticities with respect to the factor inputs were time invariant, after investigating whether the farm effects were time invariant and/or the half-normal distribution was a reasonable assumption.

The maximum-likelihood estimates for the parameters of the stochastic frontier model (9)-(10) with time-varying parameters and time-varying farm effects (2) are presented in Table 4. Tests of hypotheses about the distribution of the farm effects associated with the stochastic frontier production functions with time-varying coefficients are obtained from the data in Table 5.

The statistics in Table 5 suggest the following conclusions about the technical inefficiencies associated with farmers in the three villages:

(i) Given the specifications of the stochastic frontier production function with time-varying coefficients (9)-(10), the frontier is not significantly different from the traditional response function for farmers in Aurepalle. Hence it could be concluded that technical inefficiency is not evident for Aurepalle farmers.

Table 4: Maximum-likelihood Estimates for Parameters of the Stochastic Frontier Production Functions with Time-Varying Coefficients for Farmers in Aurepalle, Shirapur and Kanzara

Variable	Parameter	M.L. Estimates for Production Frontiers in Aurepalle	Shirapur	Kanzara
Constant	β_0	2.16 (0.86)	3.02 (0.86)	3.11 (0.96)
Year	δ_0	-0.21 (0.13)	-0.21 (0.13)	-0.10 (0.89)
Log(Land)	β_1	0.47 (0.18)	0.29 (0.14)	0.40 (0.68)
Year x Log(Land)	δ_1	-0.041 (0.034)	-0.033 (0.026)	-0.03 (0.54)
Log(Labour)	β_2	1.14 (0.23)	0.62 (0.12)	0.73 (0.68)
Year x Log(Labour)	δ_2	-0.055 (0.043)	0.043 (0.022)	-0.001 (0.87)
Log(Bullock)	β_3	-0.57 (0.22)	-0.06 (0.14)	-0.10 (0.54)
Year x Log(Bullock)	δ_3	-0.003 (0.047)	-0.007 (0.022)	0.02 (0.19)
Cost	β_4	0.16 (0.12)	-0.28 (0.16)	0.79 (0.71)
Year x Cost	δ_4	-0.16 (0.12)	0.28 (0.16)	-0.76 (0.71)
IL/Land	β_5	0.84 (0.66)	0.51 (0.31)	0.94 (0.87)
Year x (IL/Land)	δ_5	-0.16 (0.12)	-0.127 (0.049)	-0.07 (3.5)
HL/Labour	β_6	-0.56 (0.19)	0.61 (0.28)	-0.35 (0.99)
Year x (HL/Labour)	δ_6	0.043 (0.032)	-0.077 (0.045)	0.05 (2.5)
	$\sigma_S^2 = \sigma_V^2 + \sigma^2$	0.270 (0.032)	0.146 (0.028)	0.11 (0.59)
	$\gamma = \sigma^2/\sigma_S^2$	0.37 (0.19)	0.21 (0.13)	0.26 (0.72)
	μ	-0.49 (0.75)	-0.22 (0.15)	0.59 (1.7)
	η	-0.10 (0.10)	0.226 (0.061)	0.002 (8.6)
	Loglikelihood	-158.07	-121.87	-74.53

Table 5: Tests of Hypotheses for Parameters of the Distribution of the Farm Effects, U_{it}, Associated With the Stochastic Frontier Production Function With Time-varying Coefficients for the Farmers in Aurepalle, Shirapur and Kanzara

Null Hypotheses	Loglikelihood	χ^2-statistic	Decision
$H_0: \gamma = \mu = \eta$		$(\chi^2_{3,0.95} = 7.81)$	
Aurepalle	-159.54	2.94	Accept H_0
Shirapur	-166.69	44.82	Reject H_0
Kanzara	-90.69	32.32	Reject H_0
$H_0: \mu = \eta = 0$		$(\chi^2_{2,0.95} = 5.99)$	
Aurepalle	-159.09	2.04	Accept H_0
Shirapur	-142.86	41.98	Reject H_0
Kanzara	-77.59	6.12	Reject H_0
$H_0: \mu = 0$		$(\chi^2_{1,0.95} = 3.84)$	
Aurepalle	-159.10	2.06	Accept H_0
Shirapur	-139.94	35.94	Reject H_0
Kanzara	-77.63	6.20	Reject H_0
$H_0: \eta = 0$		$(\chi^2_{1,0.95} = 3.84)$	
Aurepalle	-158.36	0.58	Accept H_0
Shirapur	-141.69	39.64	Reject H_0
Kanzara	-74.57	0.06	Accept H_0

(ii) For Shirapur farmers, the frontier model is significantly different from the traditional response function, but the null hypotheses, that the farm effects associated with technical inefficiency are time invariant and/or have half-normal distribution, would be rejected.

(iii) For Kanzara farmers, the null hypothesis that the farm effects are time invariant would be accepted by the data.

Given these conclusions about the time-varying nature of the farm effects (when present) in the stochastic frontier production functions, hypotheses as to whether the coefficients of the frontiers are time invariant are considered. The relevant test statistics are presented in Table 6. From these results, it follows that the null hypothesis that the coefficients of the explanatory variables, other than year of observation, are time invariant would be accepted at the one-percent level of significance for Aurepalle farmers only.

Given the specifications of the stochastic frontier production functions with time-varying parameters and technical efficiencies which are considered in this paper, the estimated parameters for the preferred frontier models are presented in Table 7. Hence only farmers in Shirapur have time-varying technical efficiencies, whereas farmers in Kanzara have time-invariant technical efficiencies. Predictions for the corresponding technical efficiencies for Shirapur and Kanzara farmers are presented in Tables 8 and 9, respectively. The technical efficiencies of Shirapur farmers in the first year of observation showed great variability (from 0.191 to 0.898) but increased over the ten-year period to as high as 0.985. However, the Kanzara farmers had constant technical efficiency over time which varied from 0.445 to 0.817. In both cases, the level of technical inefficiency of the farmers involved was considerable for most farmers.

Table 6: Tests of Hypotheses for Coefficients of the Explanatory Variables of the Appropriate Stochastic Frontier Production Functions for Farmers in Aurepalle, Shirapur and Kanzara

Null Hypotheses	Loglikelihood[1]	χ^2-statistic	Decision
H_0: $\delta_0 = \delta_1 = \ldots = \delta_6 = 0$		($\chi^2_{7,0.99} = 18.48$)	
Aurepalle	-172.06	25.04	Reject H_0
Shirapur	-131.18	18.62	Reject H_0
Kanzara	-113.48	77.82	Reject H_0
H_0: $\delta_1 = \delta_2 = \ldots = \delta_6 = 0$		($\chi^2_{6,0.99} = 16.81$)	
Aurepalle	-166.02	12.96	Accept H_0
Shirapur	-131.91	20.08	Reject H_0
Kanzara	-113.30	77.46	Reject H_0

1 The loglikelihood values are calculated assuming that the stochastic frontier production functions for the three villages have $\gamma=\mu=\eta=0$ for Aurepalle; γ, μ and η are free parameters for Shirapur; and $\eta=0$ for Kanzara.

Table 7: Maximum-likelihood Estimates for Parameters of the Preferred Stochastic Frontier Production Functions with Time-Varying Coefficients for Farmers in Aurepalle, Shirapur and Kanzara

Variable	Parameter	M.L. Estimates for Production Frontiers in Aurepalle	Shirapur	Kanzara
Constant	β_0	0.50 (0.41)	3.02 (0.86)	3.110 (0.010)
Year	δ_0	0.0545 (0.0099)	-0.21 (0.13)	0.10 (0.14)
Log(Land)	β_1	0.289 (0.086)	0.29 (0.14)	0.39 (0.14)
Year × Log(Land)	δ_1	0	-0.033 (0.026)	-0.032 (0.023)
Log(Labour)	β_2	1.434 (0.095)	0.62 (0.12)	0.73 (0.18)
Year × Log(Labour)	δ_2	0	· 0.043 (0.022)	-0.001 (0.028)
Log(Bullock)	β_3	-0.619 (0.074)	-0.06 (0.14)	-0.09 (0.14)
Year × Log(Bullock)	δ_3	0	-0.007 (0.022)	0.021 (0.023)
Cost	β_4	-0.00040 (0.00097)	-0.28 (0.16)	0.761 (0.089)
Year × Cost	δ_4	0	0.28 (0.16)	-0.076 (0.089)
IL/Land	β_5	0.03 (0.23)	0.51 (0.31)	0.94 (0.47)
Year × (IL/Land)	δ_5	0	-0.127 (0.049)	-0.074 (0.056)
HL/Labour	β_6	-0.36 (0.11)	0.61 (0.28)	-0.35 (0.18)
Year × (HL/Labour)	δ_6	0	-0.077 (0.045)	0.053 (0.028)
	$\sigma_S^2 = \sigma_V^2 + \sigma^2$	0.191	0.146 (0.028)	0.112 (0.012)
	$\gamma = \sigma^2/\sigma_S^2$	0	0.21 (0.13)	0.257 (0.071)
	μ	0	-0.22 (0.15)	0.59 (0.65)
	η	0	0.226 (0.061)	0
	Loglikelihood	-166.02	-121.87	-74.57

Table 8: Predicted Technical Efficiencies of Shirapur Farmers from 1975-76 to 1984-85[1]

Technical Efficiencies

Farmer	75-76	76-77	77-78	78-79	79-80	80-81	81-82	82-83	83-84	84-85
1	-	.585	.650	.708	.758	.801	.837	.868	.893	.913
2	-	.242	.321	.403	.484	-	-	-	-	-
3	-	.859	.885	.906	.924	.939	.951	.960	.968	.974
4	-	-	-	-	-	.807	.842	.871	.895	.915
5	.309	.390	.471	.548	.618	.681	.736	.783	.822	-
6	.790	.827	.859	.885	.907	.924	.939	.951	.961	.968
7	.690	.742	.786	.824	.857	-	-	-	-	-
8	.265	.346	.428	.507	.581	.648	.788	.759	.802	.839
9	.351	-	-	-	.255	.336	.419	.499	.574	.642
10	.533	.604	.667	.723	.771	.812	-	.876	.899	.919
11	.632	.691	.743	.788	.827	.859	.885	.907	.925	.940
12	.908	.925	.940	-	-	-	-	-	-	-
13	.572	.638	.698	.749	.794	.831	.863	.889	.910	.927
14	.738	.783	.821	.853	.881	.903	.922	.937	.949	-
15	.554	.622	.683	.737	.784	.823	.856	.883	.905	.924
16	.488	.563	.631	.691	.744	.790	.828	.860	-	-
17	.522	.593	.658	.715	.765	.807	-	-	-	-
18	.539	.609	.671	.727	.775	.815	.849	.878	.901	.920
19	.605	.668	.723	.771	.812	.847	.875	.899	.918	.934
20	.894	.914	.930	.943	.954	.963	.971	-	-	-
21	.383	-	-	.609	-	.728	.776	.816	.850	.878
22	.818	.851	.879	.901	.920	.935	.948	.958	.966	.973
23	.191	-	-	-	.505	-	-	-	-	-
24	.637	.695	.747	.791	.829	.861	.887	.908	.926	.941
25	.801	.836	.866	.891	.912	.929	.942	.954	.963	.970
26	.530	.601	.664	.721	.770	.811	.846	.875	.899	.918
27	.898	.917	.933	.946	-	.965	.972	.977	.982	.985
28	.760	.802	.837	.867	.892	.920	.929	.943	.954	.963
29	.686	.739	.784	.822	.855	.882	.905	.923	.938	.950
30	.858	.884	.905	.923	.938	-	-	-	-	-
31	.909	.926	.940	-	-	-	-	-	-	-
32	.538	-	-	-	-	-	-	-	-	-
33	-	.345	.426	.505	.579	.646	-	-	-	-
34	-	-	-	-	-	.781	.812	.852	.879	.902
35	-	.719	.767	.808	.842	-	-	.916	.932	.945
36	-	-	-	-	-	-	-	.897	.916	.932
37	-	.871	.895	.914	.931	.944	.955	.964	.971	.977
38	-	-	-	-	-	-	-	-	-	.929
Mean	.592	.647	.699	.745	.787	.823	.854	.881	.903	.921

[1] Values of technical efficiencies are not obtained in years when no observations are observed.

Table 9: Predicted Technical Efficiencies of Kanzara Farmers

Farmer	Technical Efficiency
1	0.499
2	0.471
3	0.582
4	0.549
5	0.549
6	0.515
7	0.599
8	0.483
9	0.632
10	0.677
11	0.637
12	0.650
13	0.604
14	0.453
15	0.648
16	0.476
17	0.501
18	0.672
19	0.455
20	0.554
21	0.491
22	0.531
23	0.470
24	0.535
25	0.737
26	0.445
27	0.523
28	0.817
29	0.597
30	0.502
31	0.607
32	0.601
33	0.647
34	0.539
35	0.582
Mean	0.564

A stated in the discussion of the frontier production function with time-invariant coefficients of the explanatory variables, defined by equation (8), if family and hired labour were equally productive, then the parameter, b_2, would have value 1.0 and so the coefficients of the labour-ratio variable, $HL_{it}/Labour_{it}$, and its interaction with year of observation would be zero in the production frontier with time-varying coefficients of the explanatory variables. Test statistics for the null hypotheses of equal productivity of family and hired labour are presented in Table 10, given the specifications of the preferred frontier production functions for the three villages, reported in Table 7. These statistics imply that the hypothesis of equal productivity of family and hired labour would be rejected for the villages of Aurepalle and Shirapur, but accepted for the village of Kanzara.

It is noted that the coefficient of the logarithm of Bullock labour in the preferred production frontiers have negative values for all three villages, but the coefficient is significantly different from zero in Aurepalle only. Negative elasticities of bullock labour have been found in other studies [e.g., Saini (1979), Battese, Coelli and Colby (1989) and Battese and Coelli (1991)]. Various explanations have been suggested for this phenomenon.

Table 10: Testing the Hypothesis of Equal Productivity of Hired and Family Labour, Given the Specifications of the Preferred Frontier Production Functions for Farmers in Aurepalle, Shirapur and Kanzara

Village	Loglikelihood[1]	χ^2-statistic	Decision
Aurepalle	-171.55	11.06	Reject H_0
Shirapur	-130.89	18.04	Reject H_0
Kanzara	-76.49	3.84	Accept H_0

[1] These loglikelihood values are obtained if the null hypothesis, H_0: $b_2 = 1$, is true. This implies that $\beta_6 = 0$ for Aurepalle and $\beta_6 = \delta_6 = 0$ for Shirapur and Kanzara.

5. Conclusions

Our application of frontier production functions in the analysis of panel data from three Indian villages has indicated a number of important findings:

(i) When the data are analysed using a frontier model with coefficients that are constant over time (including the intercept parameter), then technical inefficiencies are found to be highly significant in all three villages and to be time varying in two of the three villages;

(ii) If the frontier production function contains time-varying coefficients (intercept and elasticities) then different conclusions about technical inefficiency were obtained in the different villages. In Aurepalle, the frontier function was not significantly different from the traditional response function (which can be estimated efficiently by ordinary least-squares regression). In Kanzara technical inefficiency was not significantly different over time. However, in Shirapur it could not be concluded that technical efficiency was time invariant for the farmers involved.

(iii) The hypothesis that the coefficients of the explanatory variables (other than the intercept) were time invariant was rejected for two of the three villages.

The above results indicate that the inclusion of year-of-observation as an explanatory variable in the frontier model to account for neutral technological change, provided that it is appropriate, does not necessarily mean that technical inefficiency will be found to be removed from the data.

Our analysis of the farm-level data from the three villages has not proceeded to the point of being able to explain or justify the different results which have been obtained for the three villages. Further investigations are required to deal with such issues.

The application of the frontier production function models considered in this paper has not included the possible effect of farm- or farmer-specific variables, such as education of the farmer, access to credit, etc. Further analyses incorporating such variables in the frontier models is being undertaken.

The empirical application of stochastic frontier production functions for the analysis of panel data requires that the deterministic component of the functions be appropriately modelled, in addition to the stochastic elements associated with technical inefficiency and random error. This is obviously a challenging exercise.

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