A Comparative Review of Some Firm Growth Models

By George D. Irwin

NATIONS, STATES, corporations, farm operators, and even a core of agricultural economists have the growth fever. Prominent among the symptoms in the last mentioned group are a freshet of computer-oriented models.

This article explores the growth features included in three types of models used in studies of recent vintage--multiperiod linear programming, recursive linear programming, and a family of simulator models--and then makes a summary comparison.[1] In the process, I hope to accomplish several things: (1) to touch on what is implied about growth in some recent efforts in comparative statics and identify some unresolved problems; (2) to review several types of models from the "early-finishers" among the current crop of growth projects, to see how they handle these problems; and (3) to make a few suggestions about how to use the growth concept in formulating research problems.

Growth Fundamentals

One of the mainstays of industrial economic life has long been growth and merger. Farming has seen some of this, but on a much lesser scale. Yet, with growing farm-nonfarm interdependence, it is at least conceivable that the dominant historical theme of the last half of the 20th century will be growth in size of production units.

[1] Case studies, production function analyses, or other econometric approaches may also have merit in studying growth. However, I restrict my assignment here to a discussion of the computer-oriented models, leaving the broader discussion for a time when my intellectual digestion has proceeded beyond its present state.

A fundamental issue in studying growth is the interrelating of the short run production theory, which involves at least some fixed resources, and the longer run investment theory, which varies them. The wedding of these two must necessarily recognize that, to paraphrase Boulding, the firm has a balance sheet as well as a production mechanism. The nature of the balance sheet items, in combination with the efficiency of the production mechanism in generating cash flows, are the interface with an off-farm capital market. This market, together with the rate at which the production mechanism generates residual funds internally and the rate of consumption withdrawal, determines a maximum for the investment process. Externalities such as off-farm growth rates in markets, resource supplies, and technology further constrain the environment.

In its simplest terms, the principle of growth is to acquire control of the services of additional productive resources by paying a price less than they will earn. The process of growth is, at its core, obtaining funds to purchase these resources, either internally or from external sources. All the other variables we might name--family consumption levels, profitability of the business, price and yield variability, lender attitudes, tax management, etc.--are merely the bounds within which the process can operate.

Thus, the two crucial aspects in considering growth are (1) the concept of the decision process used, and (2) the handling of internal and external flows of funds.

To sort through the details of various types of growth models, it may be useful to think in terms of the kinds of reports made annually to corporate stockholders. These are basically the net worth, the profit-and-loss, and the cash flows statements.

In the context of planning within the usual framework of marginal economic theory, restrictions generally relate to the physical and financial balance sheets, and the objective function generally to the profit and loss. The third category, cash flows, is an amalgamation of the first two. It appears in the model as the activities and equations involving money. It has probably been the least adequate of any feature of past models, except perhaps for the failure to consider in greater detail borrowing capacity as an asset. Flows affect the balance sheet in the following period, and thus are a key to building in dynamics.

The job of growth researchers is to identify variables hypothesized to have significant influence on these accumulation processes, to test the hypotheses, and to use the results to mold conclusions. These conclusions may include, for example, (1) information for farmers, or (2) better estimates of the parameters of aggregate supply, or (3) recommendations for changes in farm credit structure.

This hypothesis testing can be done at many levels, varying from case studies in the field to the construction of elaborate computer models. Both an advantage of and a danger in the model approach is that the appropriate model obviously depends upon your conception of how farmer decisions are made. This itself is a researchable problem, and one of the facets that must develop as a part of firm growth research. Studies using the models may be either or a combination of two types: of the growth process, per se, in a mechanical sense and largely abstracted from human talents and goals; or of which firm grows and why, including all the human variables. It is also useful to distinguish, conceptually, between the kind of growth an individual operator makes in adjusting to a given technology, and that he makes in adjusting to changes in technology.

Use of Optimizing Models

In a broad sense, all models are optimizing and all are simulators. It is just a matter of how. But a narrower definition of each term will prove useful here. First, the familiar question--why optimizing? George Kuznets (20)[2] has made some interesting observations:

> Virtually all of the analytical concepts used in agricultural economics are derived or are derivable from one or another optimizing model . . . The great attractiveness of optimizing models, one might almost say their fatal charm, is their deductive fertility . . . the main point . . . observed relations between time series and between variables in cross section can be explained by micro models that are not of the optimizing variety.

Optimizing applies to single-goal mathematical models. The price paid for analytic convenience is the ruling out of some realism at the beginning. For this reason, the multiple-goal, nonanalytic approach we will distinguish as simulation has intrinsic appeal, if less deductive elegance (27). This article will consider both kinds of models.

It would not be fruitful to conduct a drumming-out ceremony for either type at our current stage of understanding. The important problem is to explore how each is able to describe or represent the situation we are interested in examining. Each of several approaches has unique features. Whether or not a particular combination is appropriate depends on the problem. To state it in another way, the kit of research tools does not contain one universal model for studying growth.

Comparative Static Supply-Adjustment Models

We can set the stage for examining the various models by asking: How have the usual comparative static frameworks of the regional supply-adjustment studies handled these problems? Ordinarily they started with a representative farm, which translates to the potentially growing firm. The first step was to "cash in" all livestock, and other assets, and place them in an investment capital fund. Land, on the other hand, was usually considered fixed.

[2] Underscored numbers in parentheses refer to items in the References, p. 99.

Usually machinery was associated with the land. A matrix of production processes was set up, including various combinations of annual inputs and durables, such as buildings and breeding herds. The solution obtained was for some average year, say 5 to 10 years in the future.

A truncated form of the balance sheet served the limited function of setting up the initial resource restrictions. Its role in explaining liabilities and net worth and in the generation of funds from external sources was virtually ignored.[3] About the only way in which external capital was handled was via an assumed rate against which the assets could be pledged for borrowing. This may or may not have been augmented by an annual liquidity equation, depending on the study. The assets cashed in before the model run were assumed to be converted, over the time span, into the combination at the end, often involving completely different enterprises. Thus, the solution tells nothing about the process of getting to the new position, surely an item of great interest. A second feature was that during the period, it was assumed that the firm did not generate any funds internally which could be used to expand the business.[4] Yet the solution frequently showed large "net returns." Clearly, there are some difficulties in not knowing anything about the intermediate steps, the process of growth.

Growth Models

MULTIPERIOD LINEAR PROGRAMMING

Swanson model. In 1955, Earl Swanson (33) published what was probably the first linear programming model which, as he described it, "attempts to deal with the problem of planning over time. That is, more than one period of production is considered . . . a long run farm plan with a transition plan is . . . specified." The farm model was for 5 years ahead, with some activities continuing over the entire period, others representing the transition year only, and still others embracing all years following the transition. The model had an activity to transfer part of income from one year to the next, above a $5,000 minimum consumption and fixed cost allowance. One-half of the income above the minimum was made available to the business in the following year. The criterion function was maximum present value of the plan. Although primarily an enterprise choice model, investment in a grain combine was considered as an alternative. Interestingly, the word growth did not then get emphasized. The focus was on making models more realistic over time. Yet it is clearly an ancestor of more recent growth models.

Loftsgard-Heady model. About 4 years later, Loftsgard and Heady (21, 22) demonstrated a more detailed version of the multiperiod (they called it dynamic) linear programming model as an aid in farm and home planning. Their example allowed annual expansion of hog production on a fixed-acreage farm, and after livestock capacity was utilized it demonstrated internal generation of surplus funds.

The model involves basically a block diagonal matrix. That is, if activities and restrictions are arranged by years, the nonzero coefficients group diagonally. The only row overlapping is that net income for one year transfers to operating capital for the next year. The increase in operating capital between years is the difference between the net return for all activities and certain fixed charges and a household consumption allowance. The model was initiated with a given supply of operating capital. The amount available the following year was increased by the net return generated by the plan for the year, less living and fixed costs. The variable maximized was the sum of present value of net revenues from all production in all years, using a 6 percent discount rate.

The simplified structure of the model is illustrated in table 1. Note that the only entries off the diagonal block arrangement are in the capital row for the following year. The value entered is the sum of net revenue and operating capital, except that after the first year, the computation

[3] One study I am aware of, on Oklahoma dairy farms, did use a balance sheet to derive a more sophisticated supply-of-funds schedule. See Clark Edwards and H. W. Grubb (5).

[4] The study of Edwards and Grubb (5) assumed initial borrowing to make immediate adjustment and also estimated payback periods. It did not, however, consider the possibility of subsequent investments or increases in indebtedness.

Table 1.--Simplified version of the Loftsgard-Heady multiperiod linear programming farm planning model

Item	P_o	Year 1			Year 2			Year 3		
		Hogs	Beef	Fixed cost	Hogs	Beef	Fixed cost	Hogs	Beef	Fixed cost
Net revenue ...	max.	$\frac{60}{1.06}$	$\frac{36}{1.06}$	--	$\frac{60}{(1.06)^2}$	$\frac{36}{(1.06)^2}$	--	$\frac{60}{(1.06)^3}$	$\frac{36}{(1.06)^2}$	--
Year 1:										
Capital......	7,000	150	120	1						
Labor........	700	20	10	0						
Fixed cost...	3,600	0	0	1						
Year 2:										
Capital......	0	-210	-156	0	150	120	1			
Labor........	720				20	10	0			
Fixed cost...	4,000				0	0	1			
Year 3:										
Capital......	0				-210	-156	0	150	120	1
Labor........	750							20	10	0
Fixed cost...	3,900							0	0	1

Source: (22).

is made before the net revenue function is discounted. In complete detail, of course, the model had a large number of production activities, and a total of 8 years. By making the fixed charge row an equality, available capital in the first period is reduced by the amount of fixed costs and consumption--$3,600.[5] The remainder is available for allocation between hogs and beef. Each unit produced of either generates capital for the succeeding period, which starts at a zero capital level.

This simplified short-run example makes it easier to see the several parameters available to be manipulated for growth purposes: family consumption and fixed obligations, labor supply, price cycles, yield cycles, enlargement of the activity set with more farmer experience in later years, improvement of technical efficiency over time, or increases in initial capital supply.

What are some features one might like to alter in such a model for present purposes? (1) It depicts the short run; no land, building, or machinery investments are considered. (2) The external capital market is not considered. The capital transfer process assumes all owned funds. No borrowing, rental, or lease is considered. (3) No allowance is made for risk and uncertainty. (4) The goal is assumed to be maximization of the discounted sum of net revenue. (5) Social security and income taxes are omitted. (6) Consumption cannot be made a function of current or past years' income. Many of the same limitations apply to the models to be discussed below. Each, however, adds certain important dimensions to the analysis.

Irwin-Baker model. A further development emerges in a polyperiod model with explicit and detailed consideration of the external

[5] For the first year only, the row and column could be omitted from the computational matrix, and the fixed cost could be deducted from the initial capital supply. For later years, the row and column could be eliminated by entering the fixed cost as a negative P_0 value in the capital row, reversing the direction of the inequality sign, and then reversing the signs on all coefficients in the row (P_0 must be nonnegative). Answers would be the same, but interpretation would differ slightly, since the negative slack value would represent unused capital.

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Table 2.--Simplified two-period structure of the Irwin-Baker programming model incorporating financial and production decisions

	Feeder cattle	Fertilize		Sell corn		Savings account	Transfer cash to funds for--				Transfer cash		Borrow for--				Repay loan		Carry-over loan
		Level 1	Level 2	Period I	Period II		Fert.	Cattle	Oper. I	Oper. II	Cash II	Profit	Fert.	Cattle	Oper. I	Oper. II	Oper. I	Oper. II	
	1 head	1 acre	1 acre	1 bushel	1 bushel	$1	$1	$1	$1	$1	$1	$1	$1	$1	$1	$1	$1	$1	$1
Objective	Gross	Gross	Gross									+1.0	-i	-i	-i	$-\frac{1}{2}i$	$\frac{1}{2}i$	-1.0	
Maximum loan																			
Period I													$(1+k_1)$[1]	$(1+k_2)$	$(1+k_3)$				
Period II													$(1+k_1)$	$(1+k_2)$	$(1+k_3)$	$(1+k_3)$			
Operating loan balance																			
Period I															-1.0		1.0		1.0
Period II																-1.0		1.0	
Cash Balance[2]																			
Period I						1.0	1.0	1.0	1.0		1.0								
Period II				-Price		-(1+i)				1.0	-1.0	1.0					1.0		
Total balance[3]																			
Fertilizer		Costs	Costs				-1.0						-1.0						
Cattle	Costs							-1.0						-1.0					
Operating I									-1.0						-1.0				
Operating II										-1.0						-1.0			
Labor input																			
Period I	Input	Input	Input																
Period II	Input	Input	Input																
Feedlot capacity	1.0																		
Corn acreage		1.0	1.0																
Corn inventory	Input	-Yield	-Yield	1.0	1.0														

[1] k_i is defined as a discount factor $\frac{\text{Maximum loan for cattle less maximum loan for the purpose}}{\text{Maximum loan for the purpose}}$

[2] Cash receipts less expenses in parts of business assumed fixed.

[3] Cash plus borrowing.

capital market (14, 15).[6] It is polyperiod only in the sense of having transfers among the four calendar quarters of a single year. No investment theory is involved. Further, the set of production activities is severely limited because of the nature of the problem being investigated.

The model is illustrated in table 2. This matrix, too, would exhibit the general block diagonal characteristic if we rearranged the various rows and columns into periods. As presented, it illustrates the components arranged functionally.

To understand the structure, we need to summarize the situation being studied: A farmer has made all decisions for a year except (1) how many cattle in excess of one carload he will feed, (2) at what level he will fertilize corn, and (3) how he will finance operating expense. He has been able to work out income flows to this point, and has found he has insufficient funds to meet operating expenses if he feeds additional cattle and fertilizes corn at the same time. He has gone to the bank with his financial and income statements, and has obtained a commitment that a certain number of dollars can be borrowed for each purpose, taken alone. His problem is to choose among the three purposes. Operating expense must be met, and thus has implicitly a very high marginal return. Fertilizer, in turn, returns more per dollar spent than feeder cattle.

To handle the problem of considering the purposes simultaneously, the concept of an asset called credit reserve was introduced. It is equal to the largest number of dollars that can be borrowed for any purpose, given the financial and income statements. In most cases this is the amount for feeder cattle. Maximum loans available for the other purposes are smaller. Interrelating is accomplished if we define a "discount" factor which measures the increased rate at which the other purposes use up the farmer's credit reserve. This shows the importance of the balance sheet concept and a method for bringing it into a production analysis.

Four quarters[7] were used, since operating expenses occur periodically, as budgeted and shown in the P_0 row, and it is possible to sell stored corn or to borrow to meet them. In addition, the fertilizer loan is required only for the growing season. The cash available equations specify the difference between income from sale of other crops and livestock and the family living expenses. The value for a period can represent a negative balance. It is possible to repay loans in any period in the model. Cattle sales contribute to cash in the final period, when sold, and the fertilizer contributes to profit in the amount of its marginal net yield contribution. Interest charges on initial loans assume they will be outstanding for the year. Thus "prepayment" during the year returns part of the interest charged with borrowing. Finally, an activity takes the net accumulated cash in period 4, including unused returns from cattle, and transfers them to the net returns function.

Rod Martin's model. Another contribution provides guidance on incorporating the longer run investment aspects of growth (23, 24). In fact, it used the optimal farm solution from a minimum resource study as a composite single activity called operating plan, defined on a per acre basis. All other activities were to invest in resources, or to handle the transfer of funds between years.

A two-period condensed version of the model is presented in table 3. The firm has some surplus resources, i.e., it is an established 426-acre farm, and all resources are at least adequate to allow the operating activity to come into the solution at a level of 426 acres. However, machinery capacity is available for up to 700 acres, and fixed costs are constant up to this size. Expansion may be by buying for cash, on mortgage, or by renting. The model assumes livestock must be purchased to maintain the present crop-livestock balance if acreage is increased. Such a requirement is necessary

[6] For subsequent applications of this type of model, see the following theses prepared under Dr. C. B. Baker at the University of Illinois: D. F. Neuman, Effects of Nonreal Estate Loan Policy of Primary Lenders in the Organization of Farms in Central Illinois, 1963; L. F. Rogers, Effects of Merchant Credit on Farm Organization, 1963; J. M. Vandeputte, Farm Mortgage Debt Management on Low Equity Dairy Farms, 1967; A. G. Smith, Alternative Strategies for Financing Growth of a Grain Farm, 1968.

[7] The simplified two-period version in table 2 is for illustration only.

Table 3.--Resource levels, activities, and restriction requirements of capital accumulation model

Item	Unit	Resource or restriction level	Operating activity	Land acquisition			Hire labor	Borrow capital	Buy equipment	Fixed cost	Buy livestock	Consume 25% of net returns	Save capital	Transfer reinv. capt. 1 to op. capt. 2
				Cash buy	Amortized loan	Rent land								
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Prod. period 1:														
Land...........	ac.	426	-1.0	-1.0	-1.0	-1.0								
Labor..........	hr.	1,900	3.14											
Operating capt..	dol.	6,106	18.11	242.00	16.05	12.00	-1.0	-1.00	3.95	1.00	5.26	.25		
Collateral......	dol.	102,240			285.53		2.0							
Equipment.......	ac.	700	1.0						-1.0					
Livestk. Invest.	ac.	426	1.0								1.0			
Fixed cost......	dol.	4,532								1.0				
Net returns.....	dol.	0	-24.97	2.00	13.31	12.00	1.00	.06	2.79	.34		1.0		
Reinv. capt.....	dol.	6,106	-24.97	242.00	16.05	12.00	1.00	.06	3.95	1.0	5.26	.25	1.0	1.0
Prod. period 2:														
Land...........	ac.	426		-1.0	-1.0									
Labor..........	hr.	1,900												
Operating capt..	dol.	0			16.05				3.95					-1.0
Collateral......	dol.	102,240		-240.00	279.58									
Equipment.......	ac.	700												
Livestk. Invest.	ac.	426							-1.0					
Fixed cost......	dol.	4,532												
Net returns.....	dol.	0			13.16				2.72					
Reinv. capt.....	dol.	6,106	-24.97	242.00	32.10	12.00	1.00	.06	3.95	1.0	5.26	.25	-.04	
Subsequent time periods		⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
Objective function (net returns)			24.97	-2.00	-282.96	-12.00	-1.00	-.06	-73.22	-.34	0.0	0.0	.04	0.0

Source: (23).

because of the enterprise balance assumed in the coefficients of the operating activity. The model calls for $3,000 consumption base plus a marginal propensity to consume of 25 percent. Borrowing is limited to 50 percent of mortgage debt.

The relationship between years has three important aspects: (1) Land or machinery capacity added in any period is available for future periods, and creates loan collateral in those periods. (2) A reinvestment capital equation in the first year accumulates net capital generated, adds operating capital initially available, and subtracts the amounts committed for resource purchase, consumption, and saving. This amount is required and available in all succeeding periods, since the amount is also subtracted from the profit function. Hence, it can be inserted in the reinvestment capital row for succeeding years. (3) The only additional obligation if the firm operation remains unchanged the second year is for amortization of land and equipment loans. Thus only surplus reinvestment capital from the first period is transferred to the second to meet additional operating requirements.

The net returns equation insures that annual obligations on durables can be met. The particular objective function illustrated is to maximize net returns. The overall study examined six different objectives, but found little basis for choosing among them, since the structure of the model tended to overwhelm any differences among the functionals. A 30-year planning period was assumed, and several variations of the model were employed.

What are some of the desirable features of the model? (1) Investment in durables is considered. (2) Relationships to the external capital market are fairly explicit. Borrowing is allowed, based on equity and the type of asset to be purchased. (3) Several objectives are examined. (4) The consumption function is an explicit part of the model. (5) Both one-shot investment funds and annual liquidity requirements are specifically accounted for in the model. On the other hand, no opportunities for disinvestment and no risk elements are assumed.

S. R. Johnson's model. A model similar in many ways to Martin's brings the concept of risk into the analysis (17). Crop yields are assumed to consist of a base value (or average yield) plus a random component. First, a Monte Carlo simulation procedure is used to draw a sample value from the known crop yield distribution for each year of a 15-year planning period. This sample value represents the sum of base and random components. Then, using the series of yields, the model is solved for the 15-year period. Doing these two steps 20 times gives a distribution of outcomes based on yield variance. The model maximizes net worth (undiscounted accumulated wealth), and is recursive (interrelated between years) only on credit reserve.

Johnson points out that a major problem is in "choice between using probability distributions of raw data or using the best theoretical fit that can be obtained to the data. In the first case, all that one is doing is simulating the past (18, 19).

Boehlje-White model. A further development expands on the Martin-Johnson approach by reintroducing the enterprise choice question each year (3). It does not have stochastic elements, but does attempt to incorporate both annual production and investment into a single model, as illustrated conceptually in table 4.

Boehlje tried both net worth and disposable income objective functions. The former is illustrated here. Each of 10 time periods is described by four submatrices: (1) A production and annual input matrix, corresponding to the conventional monoperiod LP model. (2) An investment matrix, corresponding to investment theory. It makes increased capital stock available, converting financial assets to fixed facilities. It is related to production through durable input supplies, to credit through liquidity, credit, and net worth constraints, and to income division through net worth. (3) A credit matrix considering both long and intermediate term borrowing, principal repayment, and interest repayment activities. All short term funds are assumed to be borrowed at 7 percent. (4) A division-of-income matrix which apportions consumption and investment. The consumption plus taxes function has a constant plus a marginal withdrawal rate of 0.5. The reinvestment funds can be transferred to either intermediate or long term investment funds for future periods. The blocks of constraints for a particular year

Table 4.--Simplified two-period structure of the Boehlje-White linear programming model

Item	Production and annual input purchase	Investment	Credit	Division of income	Production and annual input purchase	Investment	Credit	Division of income	Net worth
Objective[1]				C				C	
Period 1 constraint sets: [2,3]									
Liquid capital		A	A						
Credit		A	A						
Annual inputs	A								
Durable inputs	A	A							
Income division	A			A					
Net worth		A	A	A					
Period 2 constraint sets: [2,3]									
Liquid capital				A		A	A		
Credit		A	A			A	A		
Annual inputs					A				
Durable inputs		A			A	A			
Income division					A			A	
Net worth		A				A	A	A	A

[1] This illustrative example is set up for optimizing on disposable income. To optimize on net worth, the C values indicated in the example would be replaced by zeroes, and a positive value would be entered in the objective for the net worth activity (which is evaluated at the end of the final period).

[2] Each row and each column represents a group of constraints or activities of the type indicated by the title. Details on the formulation of individual submatrices A are described in the source thesis.

[3] The income division row is an equality. The others carry greater-or-equal signs.

include categories for annual inputs, durable inputs, credit, liquidity, annual disposable income, and annual net worth. The latter two are tied to the objective function. Consecutive years are related through the effects of investment on (1) the supply of durables in later periods, (2) remaining capacity to borrow, and (3) the reinvestment capital transferred between periods.

It should be apparent that this sort of model lends itself to testing a wide variety of hypotheses. One hypothesis tested was that a simultaneous solution to a series of 10 years, each with a 79 x 49 matrix, exceeded available computer feasibility limits. After many false starts, this was disproved on an IBM 7094 with LP/90 when a feasible solution was reached after 4 hours, and successive optima required an additional 15 minutes from restart. To obtain even this speed, it was necessary to first solve the model with only the objective function for the first year, then add a second, and so on, using the disposable income criterion. A surprising result was that the solution for a year did not change as new year objectives were added and solved for. Apparently the simultaneity was not as strong as expected, at least in the backward direction.[8]

Other than for computing time, the results seem to describe a reasonable pattern of behavior over 10 years. Capital is gradually substituted for labor as the internally and externally generated funds are plowed into the business. The disposable income criterion tends to favor internal generation of funds, while the net worth criterion allows external generation. Additional work is possible, much in the same manner as already described for the original Loftsgard-Heady approach. The prime targets might include managerial ability and stochastic solutions. However, this model clearly points out a major frustration (viewed differently, a great potential benefit) in doing dynamic analysis--there are so many alternatives that one hardly knows where to start. One is literally forced to go to the real world for promising hypotheses to test.

[8] A suggested improvement is to incorporate more objective function values directly, rather than putting them in an accumulating equation and then transferring the value to the objective via a single activity. Apparently the indirect imputation process requires many iterations and much cycling to make explicit the Z-C values only implied for each activity in the present structuring.

RECURSIVE PROGRAMMING

A model that takes a different approach to describing the decision process of the farm firm and the attendant growth process is recursive linear programming. The LP model is for a single year, but it is solved a number of times in a sequence, with slight alterations each step. Restrictions for any given year depend on the optimum solution for the previous year. Flexibility constraints, consisting of upper and lower bounds on certain variables, are used to represent temporary limits placed upon the growth process by external factors.

The early applications by Richard Day (4), Schaller and Dean (32), and those developing the "FPED national model" (31) were based on a model for an area, and the growth concepts used were thus of an aggregate regional nature. Flexibility constraints consisted of restrictions on the rate at which profitable new technology became available, and on the rate at which labor would exit. On some external factors, both upper and lower limits were included.

A more recent published work by Theodor Heidhues (11) applies the same mathematical structure to the individual farm growth process. He then uses the model to examine some effects of possible EEC policies on different types of farms in northern Germany.

The model is set up as monoperiod programming models have been, except for three important features: (1) Detailed accumulative equations are provided to handle financial terms; (2) Investment and disinvestments, involving a fixed asset concept, are included in the model; (3) The model is related to another model for the following period by the fact that the P_0 values are functionally related to the optimum solution of the previous period. Solutions follow a sequential pattern, and the objective maximized is present value of returns.

Heidhues specifically considers two dynamic factors in farm adjustment: the environmental effect of technological and price variations, and the effect of a rising nonfarm standard of living on farmer income expectations. The basis include categories for annual inputs, durable inputs, credit, liquidity, annual disposable income, and annual net worth. The latter two are tied to the objective function. Consecutive years are related through the effects of investment on (1) the supply of durables in later periods, (2) remaining capacity to borrow, and (3) the reinvestment capital transferred between periods.

It should be apparent that this sort of model lends itself to testing a wide variety of hypotheses. One hypothesis tested was that a simultaneous solution to a series of 10 years, each with a 79 x 49 matrix, exceeded available computer feasibility limits. After many false starts, this was disproved on an IBM 7094 with LP/90 when a feasible solution was reached after 4 hours, and successive optima required an additional 15 minutes from restart. To obtain even this speed, it was necessary to first solve the model with only the objective function for the first year, then add a second, and so on, using the disposable income criterion. A surprising result was that the solution for a year did not change as new year objectives were added and solved for. Apparently the simultaneity was not as strong as expected, at least in the backward direction.[8]

Other than for computing time, the results seem to describe a reasonable pattern of behavior over 10 years. Capital is gradually substituted for labor as the internally and externally generated funds are plowed into the business. The disposable income criterion tends to favor internal generation of funds, while the net worth criterion allows external generation. Additional work is possible, much in the same manner as already described for the original Loftsgard-Heady approach. The prime targets might include managerial ability and stochastic solutions. However, this model clearly points out a major frustration (viewed differently, a great potential benefit) in doing dynamic analysis--there are so many alternatives that one hardly knows where to start. One is literally forced to go to the real world for promising hypotheses to test.

[8] A suggested improvement is to incorporate more objective function values directly, rather than putting them in an accumulating equation and then transferring the value to the objective via a single activity. Apparently the indirect imputation process requires many iterations and much cycling to make explicit the Z-C values only implied for each activity in the present structuring.

RECURSIVE PROGRAMMING

A model that takes a different approach to describing the decision process of the farm firm and the attendant growth process is recursive linear programming. The LP model is for a single year, but it is solved a number of times in a sequence, with slight alterations each step. Restrictions for any given year depend on the optimum solution for the previous year. Flexibility constraints, consisting of upper and lower bounds on certain variables, are used to represent temporary limits placed upon the growth process by external factors.

The early applications by Richard Day (4), Schaller and Dean (32), and those developing the "FPED national model" (31) were based on a model for an area, and the growth concepts used were thus of an aggregate regional nature. Flexibility constraints consisted of restrictions on the rate at which profitable new technology became available, and on the rate at which labor would exit. On some external factors, both upper and lower limits were included.

A more recent published work by Theodor Heidhues (11) applies the same mathematical structure to the individual farm growth process. He then uses the model to examine some effects of possible EEC policies on different types of farms in northern Germany.

The model is set up as monoperiod programming models have been, except for three important features: (1) Detailed accumulative equations are provided to handle financial terms; (2) Investment and disinvestments, involving a fixed asset concept, are included in the model; (3) The model is related to another model for the following period by the fact that the P_0 values are functionally related to the optimum solution of the previous period. Solutions follow a sequential pattern, and the objective maximized is present value of returns.

Heidhues specifically considers two dynamic factors in farm adjustment: the environmental effect of technological and price variations, and the effect of a rising nonfarm standard of living on farmer income expectations. The basis of the model is that adjustments come with a time lag. This reflects both quasi-fixed factor supply limits and uncertainty. Thus a learning period can be built in.

Investment-disinvestment follows the G. Johnson acquisition-salvage concept, with the modification that decisions are made on current expectation of annual income and cost (16).

The restrictions on the equations are of the following form:

$$\begin{bmatrix}\text{b value}\\ \text{for time}\\ t\end{bmatrix} = \begin{bmatrix}\text{Amount available}\\ \text{at beginning of}\\ \text{previous period}\end{bmatrix} - \begin{bmatrix}\text{Depreciation}\\ \text{if any}\\ \end{bmatrix}$$

$$+ \begin{bmatrix}\text{Amount added to}\\ \text{optimum solu-}\\ \text{tion of previous}\\ \text{period}\end{bmatrix} + \begin{bmatrix}\text{Exogenous}\\ \text{adjustment}\\ \\ \end{bmatrix}$$

Conceptually, this is very similar to the transfer equations already seen in the multiperiod models. The difference is that the Heidhues model solves one period at a time, sequentially. Thus choice between the two approaches depends on your conception of the decision process. Future expected returns are implicitly assumed to be constant for investments. The objective function maximizes ability of the farm to accumulate investment capital, subject to the consumption function and other requirements. Between successive years in the model, in addition to the changes in resource restriction, annual hired wage rate increases at the rate of 10 percent per year and consumption level increases to reflect a growing economy. Yields are increased according to projections from trends.

As illustrated in table 5, the model is tied together across production, labor hire, investment, fixed obligation, and money market sectors by a pair of equations controlling internal flow of funds. One insures that downpayments and total payment can be met, the other that annual commitments can be paid. The first category is of a one-time nature, while the latter is regularly recurring.

The time period of a single model was 3 years, and was run for each farm for 2 periods only. Matrix size each period ranged from 25 to 40 constraints and 40 to 60 activities.

As Heidhues asserts, "The ability to handle stocks and flows of money capital is a measure of the usefulness of a farm growth model" (16, p. 675). We will thus examine them in some detail. The requirements are that (1) certain annual fixed obligations, plus payment of principal and interest, must be met, (2) the remainder must be distributed between purchase of annual inputs, durables, and saving, and (3) liquidity must be maintained in terms of annual obligations. Some behavioral limits on borrowing must be observed.

Internal flow requires first a liquidity or flow-of-funds equation to insure that annual cash in-flow is adequate to meet current commitments. Second, an investment capital equation guarantees that long-term funds committed to new investment, to saving, and to additional liquidity requirements do not exceed those made available from disinvestments, borrowing, bank accounts, and surplus in the previous year's liquidity account. Current production returns are not available for current investment. Transfer of funds between years is made by accumulating in the liquidity equation and transferring to the P_0 values for investment funds the following year.

External flow includes three equations. The total debt limit equation is set by the balance sheet of the farmer, as a percentage of pledgable assets adjusted for previous commitments. In addition, the rate-of-borrowing equation can be set to insure that total debts do not increase beyond certain absolute limits. Finally, a repayment equation insures that commitments on both principal and interest are met.

Among the growth parameters that can be manipulated are depreciation or obsolescence of durables, rate of growth in either private consumption or the general wage rate, variations in externally determined fixed expenditures, and rate of increase in borrowing permitted. Others could be incorporated--as illustrated by some of the models discussed earlier. In addition, it would be possible to include some pairs of lower bound-upper bound behavioral restrictions on the individual, as are found in the aggregate applications of recursive programming. These might express inability to

Table 5.--Simplified structure of the Heidhues individual firm recursive programming model

Item	Production, annual purchase and sale	Labor hire	Investment and disinvestment	Fixed obligations	Borrow and repay (money sector)
Land Crop Seed Livestock Labor	A_{11}	A_{12}	0	0	0
Technical equipment Machinery Buildings	A_{21}	0	A_{23}	0	0
Consumption Fixed charges	0	0	0	A_{34}	0
Capital: Internal flow: Liquidity Investment	A_{41}	A_{42}	A_{43}	A_{44}	A_{45}
External flow: Debt limit Borrowing rate Repayment	0	0	0	0	A_{55}

Details on internal flow of capital:

Liquidity: Hired labor expense + Fixed obligation ≦ Production return + Interest income + Transfer from investment capital

Investment: Investment commitment + Transfer to liquidity account = Disinvestment + Borrowing + Bank account + Surplus liquidity of previous period

Details on external capital equations:

Total debt: Borrowing ≤ Previous years debt limit + Loans paid off in previous period - Previous period new borrowing

Rate of borrowing: Borrowing rate = α(Previous limit) + Previous repayment - Increase in previous period

Repayment: Repayment = Fraction of last period borrowing due + Fraction of repayment due from commitment of earlier periods

Source: (11).

312-914 O-68—3

make very large changes in individual enterprises over the short run.

SIMULATION MODELS

Still another approach to handling firm behavior over time is found in simulation models. In a broad sense, all the models attempt to simulate behavior of a farm firm over time. But one definition confines the term to models which are not strictly optimizing--do not have an analytic optimizing procedure--however close the approximation procedure may come. Of course, even optimizing may be severely constrained by all sorts of side restrictions, but the distinction used here is whether or not the criterion is maximized via an analytic mathematical technique. Eisgruber[9] and Hutton (13) both make the distinction that simulation models are nonanalytic (that is, they do not guarantee an optimum), and if analytic-optimizing models can handle the situation they are to be preferred. Thus the simulation models have their place when the decision process to be described is extremely complex, and analytic approaches either have not been or cannot be developed. These include situations with (1) multiple goals, (2) indivisibilities, (3) sequential decisions within the planning period, using different criteria, (4) nonlinear functions, and (5) concepts of organizational, managerial, and behavioral theories (8, 12).

Simulator decision models are ordinarily written in some computer user's language. They tend to call for less abstraction than the analytic models, so that it is a much more difficult question to decide how far to go on details of the decision process being modeled. Because of the sequential nature of the models and a growing computer inventory, the computer time restriction is simply much less pressing than for an LP problem of the same size--though with extremely large problems the core memory capacity may be taxed (7).

Once the computer program describing the desired decision process has been produced, we may think of its use in evaluating a policy as a sequence of experiments (27, ch. 9). Though the procedure does not guarantee an optimum and may not even seek one, research procedures may include the requirement that several alternatives be generated and the best of these chosen. The experimental aspect gets introduced by varying some independent or policy variables and evaluating the effect on outcome from the model. Another type of use of these models is to describe a particular decision process to allow tracing through effects of different inputs. Here the primary interest is in an algorithm to simplify computational burdens.

With so much flexibility available, the range of alternatives is as broad as the set of decision models one can specify and quantify. Because of this flexibility, the simulator has found use in research efforts designed to try to study the growth process of the individual firm. These studies may require a more detailed decision model than do more aggregate representative firm or supply studies--a model bringing in concepts of managerial, behavioral, and decision theory. These models lend themselves to progressive development, as is apparent in the following paragraphs. Because of their nature, the discussion that follows is necessarily more conceptual and abstract than that of the optimizing models.

To illustrate, we turn briefly to the general structure of a series of related models developed at Purdue University. Eisgruber initially wrote a program to simulate a farm operation (1, 6). Alternatively it could be used in management games as a classroom device. It was strictly an operating model, built around analyzing the effects of yearly plans and of land purchase decisions. The decision procedure followed the general land use planning approach. Input variables required for each year included acreage of each crop, fertilization levels, livestock number and types, and decision on land purchase. An option for stochastic yield and price coefficients was provided.

Subsequently, George Patrick (28) built a model simulating the entire farm business. His annual farm operation submodel was developed from Eisgruber's model, dropping the stochastic variable generator. The Patrick

[9] L. M. Eisgruber, Seminar on Simulation, unpublished notes, Purdue University, Department of Agricultural Economics, April 1967.

model draws extensively from behavioral concepts advanced by Simon and others, and uses four goals of the family, expectations on prices, and a consumption function related to family size and income level.[10]

Figure 1 shows a simplified version of the decision process used by Patrick. Figure 2 is a detailed breakdown of the planning process involved in the lower left quadrant of figure 1. Parameters studied in the model are the starting farm resource situation, plus a specification of which of three alternative levels to use for each of the following controlled (studied) variables: (1) managerial ability, (2) interest rate, (3) long-term loan limit, and (4) intermediate-term loan limit. Combinations of the variables were simulated for the farm over a 20-year period. The model itself incorporates many of the features discussed in the first paragraph of this section.

Edward Harshbarger has reintroduced stochastic yield and price variations to the model, has added land purchase and machinery enlargement, and has developed certain other modifications.[11] The variables he is studying include four land procurement policies, different equity ratio limits on borrowing, and different long and intermediate term loan limits. Land strategies range from buying at every opportunity to rental only. Liberal and conservative borrowing policies of 80 and 40 percent are considered. A third project, sponsored jointly by Economic Research Service and Purdue University, will probably build in additional detail in the direction of financial leverage and more sophistication in the area of income taxation and tax policy.[12] [13]

[10] See Patrick (28) for additional diagrams and details of the decision process assumed in the model.

[11] These results are to be reported in a Ph.D. thesis in preparation: C. E. Harshbarger, The effect of alternative strategies used in decision making on firm growth and adjustment (Purdue University).

[12] Virden Harrison (ERS), under the direction of W. H. M. Morris and George D. Irwin, is the project investigator.

[13] For a simulator including farm corporation tax policy, see N. E. Harl (10).

Research Strategy

Logic dictates the sequence-of-projects approach to programming and simulation, rather than so many "one shot" efforts. Any type of model building is expensive work, but the marginal cost of expanding a relatively satisfactory model is small. Further, several studies are required to realize the potential of a new model. A criticism of most growth studies to date, including both programming and simulation, is that they utilize the models so little for analyses after going to so much trouble to build them. This, of course, is partly a reflection of the dominance of dissertation work in the total agricultural economics research program. In addition, a group of roughly comparable studies offer the possibilities of comparative analyses.

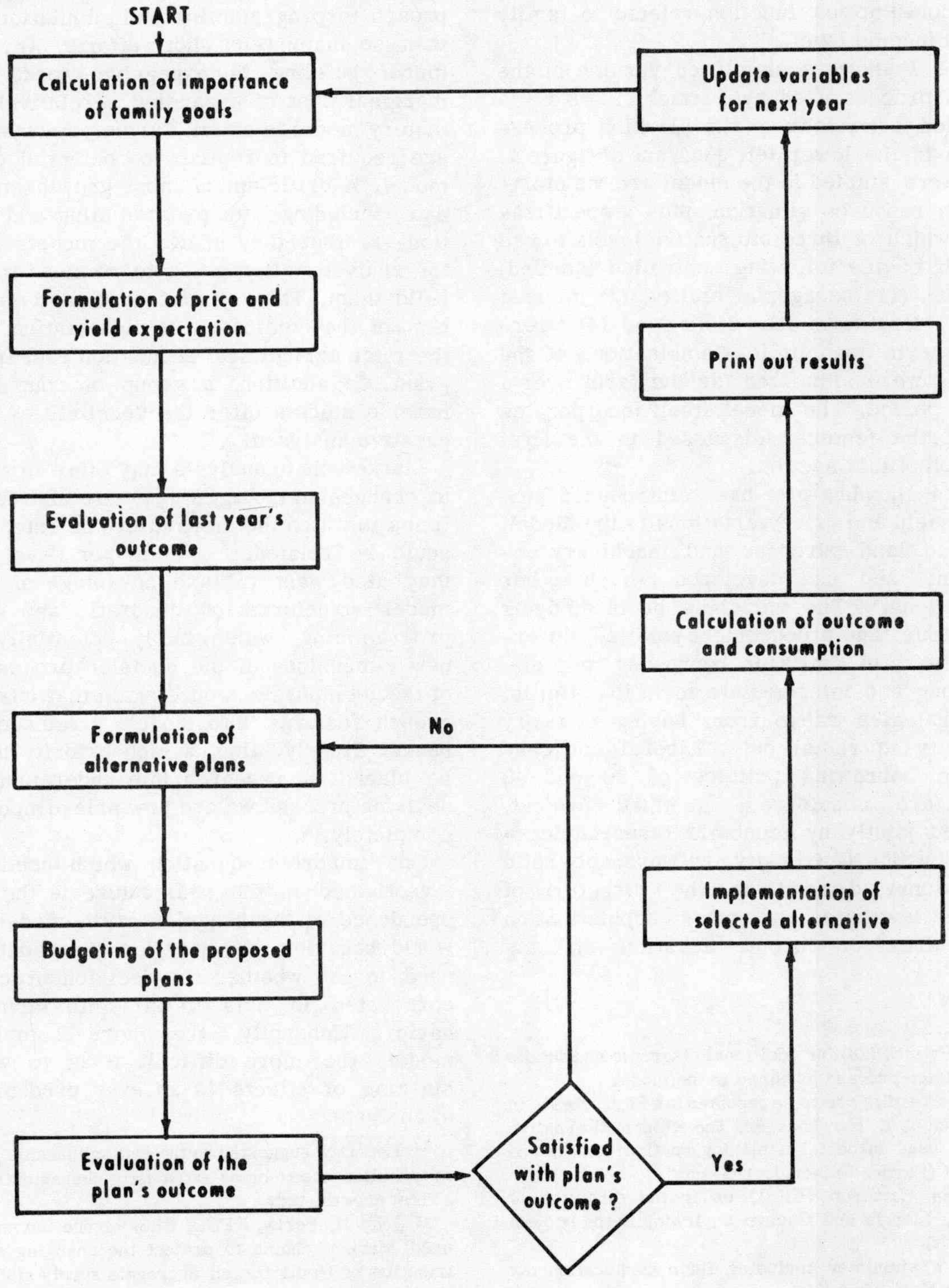
Markov chain analysis may offer an approach to changes in the aggregate size distribution of firms to which the more micro oriented studies could be related.[14] [15] Further developments may also want to take advantage of the new model structures of quadratic and dynamic programming, which imply potentially useful new dimensions of the decision process. Each of these emphasizes once again that introducing growth features into models strains our data banks. Clearly, also, a high priority needs to be placed on research into understanding the decision process we are now able to model more completely.

One important question which needs to be investigated in the near future is the correspondence of the behavior of the model to real world behavior, the question of validation. We need to ask whether the decision process incorporated is able to simulate farmer behavior. Unhappily, the more complex the model, the more difficult it is to validate. Masking of effects is an ever present pitfall to analysts.

[14] Lee Day suggested to the author that this approach might allow one to bring extra-firm characteristics into a firm growth study.

[15] John H. Berry, FPED, ERS-Purdue University has used Markov chains to project the changing size distribution of firms for an aggregate supply study, based on a representative firm approach.

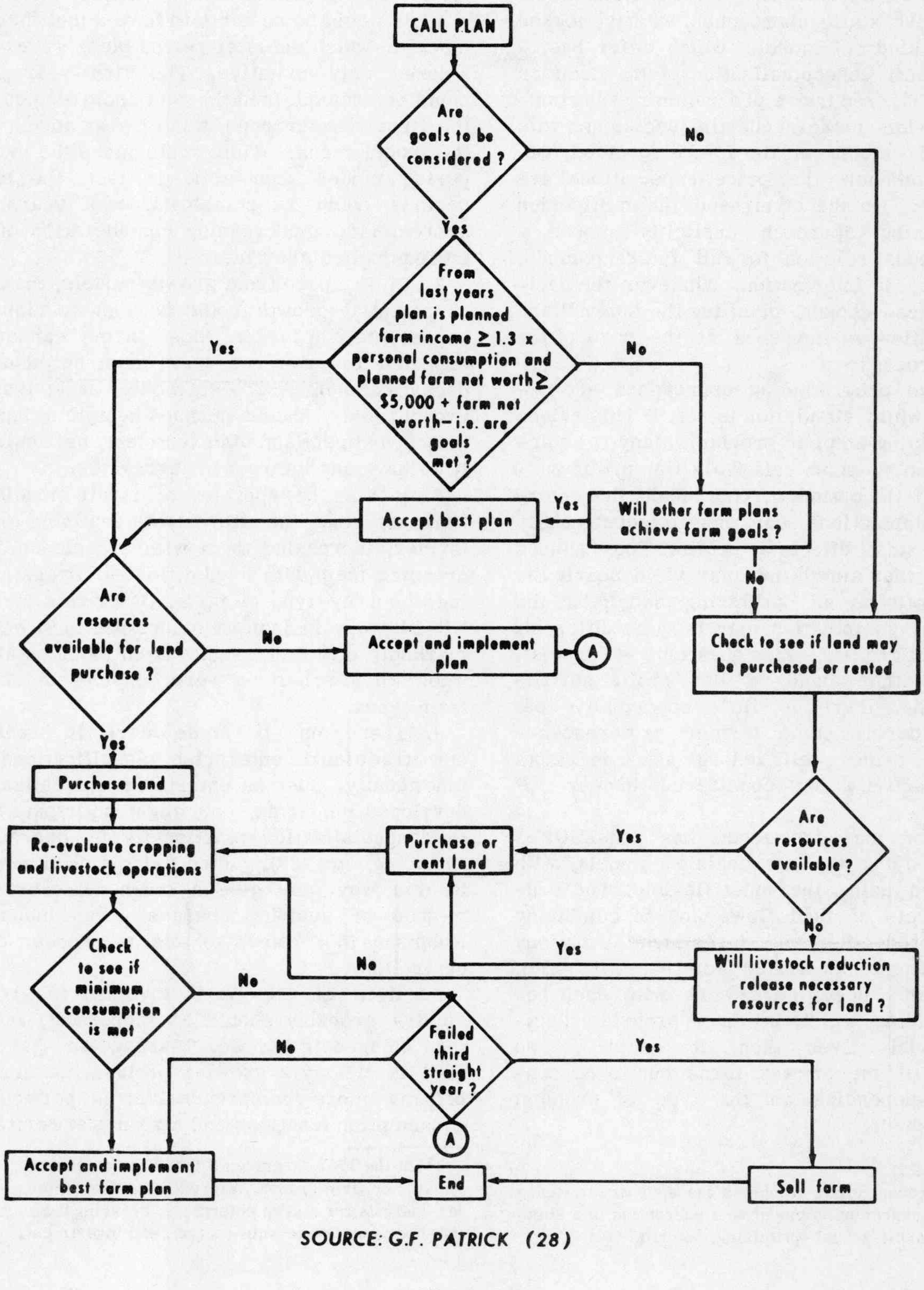
SCHEMATIC DIAGRAM OF PATRICK'S SIMULATOR



SOURCE: G.F. PATRICK (28)

Figure 1

SCHEMATIC DIAGRAM OF PLANNING PROCESS IN PATRICK'S SIMULATOR



SOURCE: G.F. PATRICK (28)

Figure 2

In Summary

After a brief review of growth principles and comparative static approaches, we have looked at three kinds of models, which differ basically in their conceptualization of the decision process. The recursive programming and simulation models make sequential decisions, with a certain amount of the future involved, depending on how the price expectations are formulated. On the other hand, the multiperiod programming approach explicitly makes a simultaneous solution for all future periods, given present information. Whatever the decision process chosen, detailing the internal and external flow of funds is at the core of the growth process.

The two programming approaches are optimizing while simulation is not.[16] This raises an interesting empiric problem. Many LP solutions seem to show relatively flat profit surfaces near the optimum. This means that enterprise combinations can change substantially with only small effects on profits. Thus chances are good that simulation may yield nearly the same profit as an optimizing model; but the enterprise combination may be quite different. In the aggregate, the question is crucial. On the other hand, a flat profit surface means that there is little opportunity cost in considering some farmer preferences--so they could well belong as constraints on the activity set considered in our LP models.

Each of the approaches has possibilities for embodying various decision models, with simulation being the most flexible. The crucial aspects of fund flows and of combining annual production and investment equations have been demonstrated for each. In sum, we probably need more work with each before deciding which is the appropriate decision model. Even then, it would be no surprise if the answer turns out to be conditional--depending on the type of problem being studied.

Finally, let us close with a few suggestions on placing these models in larger perspective for evaluating research proposals:

1. It would be possible to have a multiperiod model in which the later period plans were considered only tentative. The first-year plan could be adopted; then the data could be updated, the first year dropped, a later year added, and the model rerun. This would make the multiperiod model sequential. In fact, the later periods could be combinations of years, to approximate a decreasing consideration of an increasingly distant future.

2. Quite apart from growth models, the idea of potential growth could be used to classify representative farms. Most farms cannot be expected to grow and some must necessarily decay (2, 26). A growth model, or at least a growth study, should perhaps be able to handle both investment and disinvestment, particularly if we have any interest in aggregates.

3. It may be appropriate in all models to limit the range of alternatives available to the farmer, increasing them with experience, decreasing them with fixed attitudes, or classifying them by type of farm. Using this for the NC-54 study in Indiana (described in 9) we got markedly different results from those obtained when all alternatives were considered for all farm types.

4. There may be some merit in breaking our traditional enterprise classifications up functionally. Just as enterprise specialization developed on farms, one might anticipate further specialization functionally by operating, financing, etc. (29).[17] Identifying growth sectors in this way may give a much more precise method of studying changes in an industry. Robinson (30) refers to this as process concentration.

5. Much of the work involved in growth studies probably should be supporting, rather than on models per se. "Garbage in--garbage out" is clearly a growing problem as models become more comprehensive. In particular, consumption functions and tax rates are critical.

16 One recent study combines the approaches, using a linear programming model as a subroutine in a simulator, to handle annual optimizing. See Harl (10).

17 At the 1967 American Farm Economics Association session on firm growth, Warren Bailey presented a similar analysis for a crop enterprise, breaking the functions into operating, ownership control, and investment.

Note that the studies reviewed here had marginal propensities to consume varying from 0 to 1 and none considered social security tax at all. Unfortunately, a corresponding characteristic of more complex models is that these assumptions become easier to conceal.

6, There needs to be more followthrough analysis after heavy investment is made in construction of a model.

7. Growth models require more detailed tie-in with the larger, macro environment than static models, with allowance for feedback if all farms in a region do the same thing at the same time.

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