An Iterative Linear Programming Procedure for Estimating Patterns of Agricultural Land Use [1]

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THE BENEFITS DERIVED from an irrigation or land development project depend on the pattern of crop production on project land. In some instances, all the land of a particular quality in a project area can be devoted to the specialized crop which budget analysis shows to have the highest net return, without perceptibly affecting the price of the crop. This is because the added production represents an insignificant part of the total supply. In such instances, project planning can usually proceed on the assumption that land in a particular classification will be devoted to the crop showing the highest net return.

In other instances, however, the additional production of a crop on land served by a project may be large enough to reduce the price of the product below the level that would be realized if there were no production of the crop on this land. For example, the Cachuma Project of the U.S. Bureau of Reclamation at Goleta, Calif., greatly increased the potential supply of irrigated lemon and avocado land. If all the land in the project area well suited to these crops had been planted to them, the product prices would have declined considerably. Many similar situations could be cited. If the public agencies responsible for developing these projects ignore the effect of production on price, they will obtain poor estimates of the resultant crop pattern and will overestimate direct project benefits.

In many cases, an irrigation project is designed to serve an area where there are a number of land classes with differing yields and costs of production for each of a number of crops which might be grown on project land. If some or all of the potential crops are likely to decline in price because of the production on project land, the estimation of the pattern of crop production becomes complex. Classical rent theory suggests that a static competitive equilibrium is achieved when each land class earns the same rent or net return for each crop grown on it. The theory requires that all land earning rent be fully utilized. Marginal land need not be fully utilized since, by definition, it does not earn rent. An added requirement is that a crop must have the same price on all land classes where it is grown to supply any given market. Reaching an equilibrium under these conditions requires the simultaneous adjustment of prices and production for all crops while observing the imposed restraints.

The Molokai Project, at Hoolehua on the island of Molokai, Hawaii, affords an unusually clear example of the need to consider the effect on product prices of production on land served by an irrigation project. About 1,000 acres of high-quality land, previously uncultivated, are to be served with irrigation water from the project, and additional areas will be supplied with supplemental water. The land is well suited to vegetable crop production, and since it is possible to develop relatively large, highly mechanized farms (in comparison to existing vegetable farms in the State), the cost of producing many crops should be lower than in other areas of production. The consequent increase in supply will probably lower prices in the small Honolulu market, since the supply from project lands will be a sizable portion of the total. Most of the Molokai Project facilities have been built but not all of the land has been opened for development. In this paper we present an outline of a method used for determining the equilibrium pattern of land use

[1] Hawaii Agricultural Experiment Station, Technical Paper No. 889.

for this project. The criteria for this equilibrium pattern of land use are identical to those given earlier for classical rent theory.

The Estimating Procedure

The equilibrium land use pattern was estimated by successive approximation in a linear programming model that maximized net returns to the lands served by the project. An initial solution was obtained in which the per acre net return to each crop grown on each land class was predicated on the price which that crop would command if it were the only crop grown on project land. This initial price depends on the demand curve for the product, on the product supply curve pertinent to existing producers (those presently producing the crop on land not served by the project), and on the amount of the crop which can be grown on project land without depressing the price below the cost of production on marginal land (for that crop) within the project area. The initial prices are lower than or equal to the equilibrium prices, since competition among the crops for use of the land in the project should usually raise the price and reduce the quantity of each crop produced. Hence net returns, which comprise the c-values in the objective function of the linear program, are also lower than those expected to prevail in the area.

The initial prices were adjusted upward until supply equaled demand for all crops. For crops that could not compete for project land the price was determined by the intersection of the demand curve and the supply curve for existing producers. For crops that could successfully compete for project land, the supply forthcoming from existing producers was augmented by production from project land. At each step of the adjustment process, each crop earned the same rent as any other crop on any land class where both were grown. Each adjustment was a progressively diminishing fraction of the difference between the price of a crop at that stage of the adjustment process and the price of the crop without project production.

There were two types of restriction in the simplex table used in the linear programming model: acreage of each land class, and quantity of each crop produced on project land. The real activities were production of each crop on each land class. If the actual quantity of a crop produced was less than the quantity needed from the project to yield the market price shown for the crop, a portion of the quantity needed would be in disposal. This was the signal that price of that crop was too low and should be raised in the next adjustment. If acreage of a land class was in disposal, and if any crop earned rent on this land class, it was the signal that the price of that crop was too high and must be lowered in the next adjustment. When both tests were met within a threshold of tolerance, the equilibrium position had been reached.

Figure 1 illustrates steps in the solution of a two-crop, two-land-class problem. Diagram (a) shows the initial solution for crop 1. The supply curve SA has been shifted in two discrete increments, reflecting production on land classes 1 and 2 as price of the crop rises to the point where production on project land is possible. Note that all of land class 1 and a portion of land class 2 would be devoted to crop 1 if it were the only crop grown. The price of crop 1 at this point is PA_2, which is the cost of production on land class 2.

Diagram (b) shows the initial solution for crop 2. Without competition from crop 1, crop 2 will be grown only on land class 1, and the cost of production on this land class determines the price at this point in the solution.

In the initial solutions, land class 1 is used for both crops. In subsequent iterations of the computer program, the two crops compete on land class 1, successively increasing crop prices until the demand restriction reduces quantities demanded to levels achievable with the available amount of land. In the final solutions as shown in diagrams (c) and (d), the two crops share land class 1, but only a portion of land class 2 is used by crop 1. The rent earned by crop 1 on land class 1 is $PA_2 - PA_1$ per unit of product and the rent earned by crop 2 on land class 1 is $PB_2 - PB_1$ per unit. These price differences times the respective yields give the per acre rents, which must be equal for the two crops. Since not all of land class 2 is used, it is marginal and earns no rent. The curves SA' and SB' appear to be

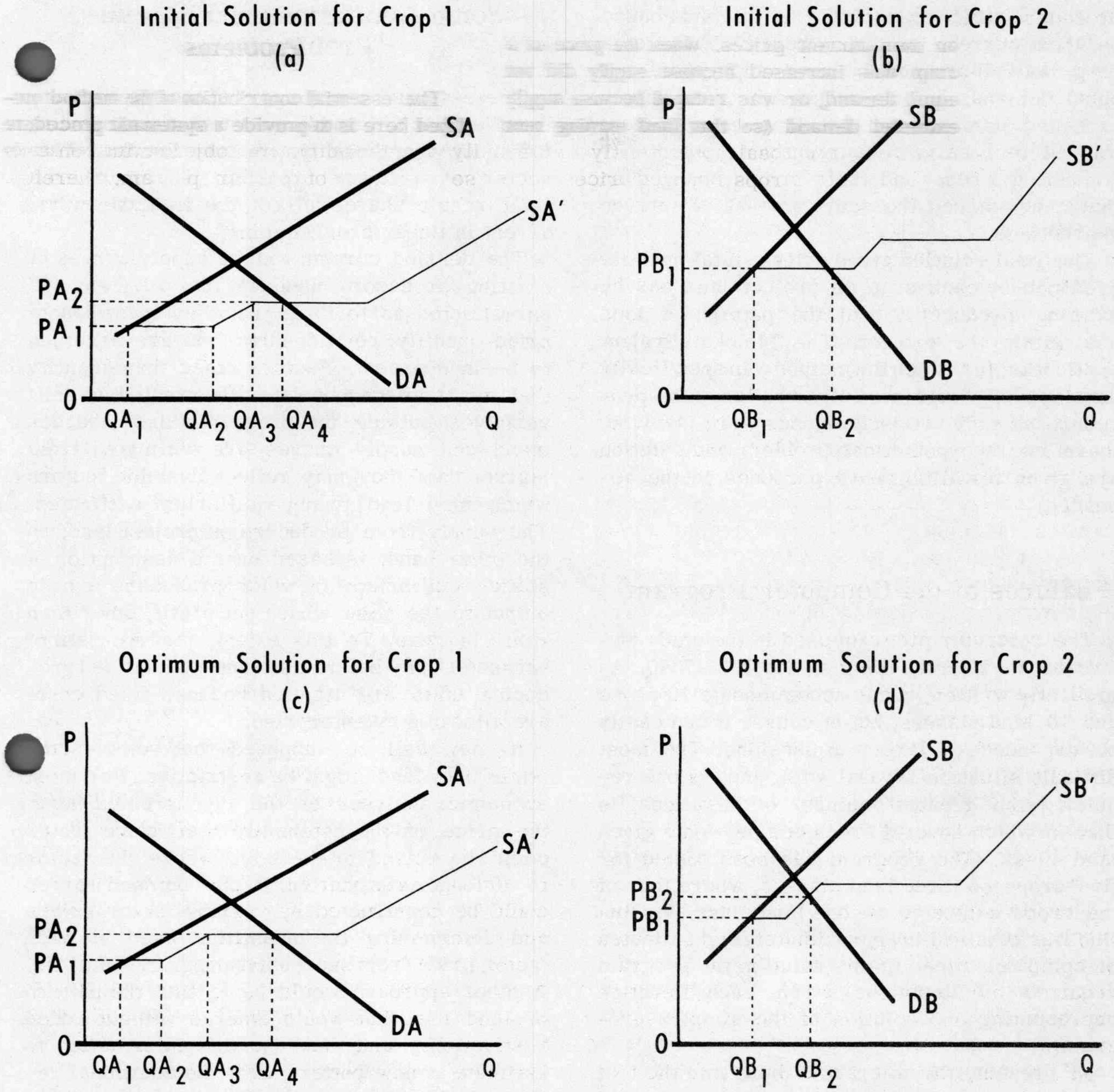

Figure 1.--Graphic representation of a two-crop, two-land-class problem.

stepped supply curves, but it should be observed that they relate only to the single final equilibrium point. That is to say, a shift in one of the demand curves would not result in a new equilibrium at the intersection of the new demand curve and the existing S' curve, but would normally lead to a shift in the S' curve as well.

It may be seen that the method employs a linear programming procedure, embedded in an adjustment procedure that systematically alters the c-values of the objective function and the levels vector as product prices and quantities are made to converge to their equilibrium values. The land use pattern at each step of the adjustment process was determined by the simplex procedure which maximized net revenue based on then current prices. When the price of a crop was increased because supply did not equal demand, or was reduced because supply exceeded demand (so that land earning rent was in disposal), all currently or potentially competing crops had their prices changed so that they earned the same amount of rent on shared land.

The final solution gives prices, total quantity produced of each crop on project land and by existing producers, and the pattern of land use within the project. The Molokai Project application for which the method was specifically developed (3)[2] did not fully challenge the procedure as only two land classes were involved, therefore, a hypothetical problem and solution are given for illustrative purposes in the appendix.

Features of the Computer Program

The computer program used in the study was written in Fortran IV for the IBM 7040. As presently written, it can accommodate 10 crops and 10 land classes, but of course it can easily be expanded to larger dimensions. The most difficult situation to deal with, and the one requiring the greatest number of iterations, is that in which several crops compete on a given land class. The program has been tested for five crops on three land classes, where four of the crops competed on one land class. A solution was obtained in approximately eight minutes of computer time. In this solution the program required 50 iterations with each iteration representing one solution of the simplex program.

At present, the tolerances built into the test of whether supply equals demand are 0.1 acre of land or 1.0 pound of product. While these are probably small enough for most practical purposes, they could be reduced to any desired level of precision with some increase in computer time needed. Since any level of precision can be achieved by practical means, the solution can be viewed as exact.

[2] Underscored numbers in parentheses refer to items in the References, p. 24.

Some Limitations and Special Problems

The essential contribution of the method outlined here is to provide a systematic procedure for modifying the objective function and levels vector of a linear program. Therefore, the final result shares all of the limitations inherent in linear programming.

The demand curves and the supply curves of existing producers used in the program are unrestricted as to form, but they must denote price-quantity relationships and are assumed to be independent. Factors other than quantity that might affect price must be treated as shift variables outside the program. Since the demand and supply curves are estimated from market data they may reflect dynamic factors which may lead to nonequilibrium estimates. The supply from producers on project land, on the other hand, is based on the assumption of static equilibrium in which producers expand output to the point where net profit, other than rent, is zero. To this extent, the two sets of estimates may be inconsistent. Specialized producing units are implied because fixed costs are allocated by enterprise.

It may well be supposed that some factor other than land might be restrictive. For most economic analyses of the type treated here, the price of the ostensibly restrictive factor could be raised to the point where the factor is no longer restrictive. Factor demand curves could be constructed by varying factor prices and determining the quantities used at each factor price from successive program solutions. Another approach would be to find the pattern of land use that would emerge without added restrictions, and then use this information to estimate a new pattern with the additional restrictions, using a standard simplex procedure. A comparison of the results would indicate the degree of distortion resulting from the restrictive factor.

Relationship to Other Available Methods

Interregional equilibrium models are closely related to the procedure outlined here. This similarity exists because the regions of an interregional model are usually treated as areas of uniform physical productivity, and thus are analogs of land classes. Most of the early interregional studies incorporating demand functions were restricted to a single commodity. Apparently, the earliest empirical study of this type was published by Fox (1) in 1953. His model employed a demand function for livestock feed but assumed livestock numbers, livestock prices, and feed production to be fixed in each region. The solution gave the price and consumption (per animal unit) of feed in each region and the pattern and volume of interregional feed shipments. Judge and Wallace (4) formulated an equilibrium model of the livestock economy, in 1959, which incorporated product demand curves for a single commodity and fixed regional product supplies. Subsequent work appears to have concentrated on generalizing this basic model to include factor as well as product shipments and prices (5, 6).[3] In 1964, Takeyama and Judge (8) presented a model that allows estimation of interregional production patterns and commodity flows and accommodates product supply and demand relationships for several commodities. Their formulation is not suited to the present problem because it assumes independent regional supply curves that are determined outside the model.

Superficially, the procedure outlined here appears to be closely related to reactive programming as developed by Tramel and Seale (9). There are, however, several significant differences between the two models. Our model incorporates land restrictions, by quality, as well as the quantity restrictions of reactive programming. Also, in our model it is possible to determine the quantity that will be supplied by existing producers and by producers on project lands directly from the model. Finally, the Tramel-Seale model yields an approximate solution while our model specifies the exact solution within the limits of accuracy achieved in the land classification system, budgeted production costs, the estimated market demand and supply functions for existing producers, and the tolerance levels specified.

An alternative to the present procedure for including demand functions for several commodities has been described by Yaron and Heady (10). Their procedure is based on a stepped approximation of the net revenue function with a solution achieved by considering each step of the function as a separate activity (a subactivity of the activity or commodity being considered). The difficulty with this procedure is that the solution is approximate and, as formulated, the objective is to maximize profit, which does not incorporate our equal rent criterion for a competitive equilibrium. Although the possibility was not explored, perhaps their procedure could be recast as a rent minimization problem as suggested by Smith (7).

Appendix--An Example

This example illustrates the data required and the results obtained from the computer program used in this study. In the example, there are five crops which might be grown in a project area consisting of three land productivity classes. The commodities are sold on a single market.

The required input data are given in tables 1 and 2.

The initial solution to this problem is given in tables 3 and 4. Table 3 provides the initial acreages and rents while table 4 contains initial prices and quantities.

The final solution of this problem resulted in the pattern of land use and per acre rents indicated in table 5. Equilibrium product prices and quantities are given in table 6. The complete program and a manual of instructions for its use are available from the authors.

[3] Fox and Taeuber (2) published a study in 1955 which allowed a single factor, feed, and livestock products to be shipped between regions. This study can be viewed as perhaps the first to consider multiple commodities.

Table 1.-- Market demand functions and supply functions for existing producers[1]

Crop	Demand functions		Supply functions	
	Price intercept	Slope	Cost intercept	Slope
Crop 1	250	-.003	-1,125	0.25
Crop 2	300	-.05	-33,330	17.0
Crop 3	400	-.08	-1,143	1.4
Crop 4	250	-.03	-1,500	2.0
Crop 5	300	-.03	-2,500	1.0

[1] Prices are estimated in dollars per 1,000 pounds and quantities are in 1,000-pound units. In this example the demand and supply curves are linear.

Table 2.--Production costs and yields by land class[1]

Crop	Production costs per acre			Per acre yield		
	Land class 1	Land class 2	Land class 3	Land class 1	Land class 2	Land class 3
	Dollars	*Dollars*	*Dollars*	*1,000 pounds*	*1,000 pounds*	*1,000 pounds*
Crop 1	7,000	6,500	6,000	50	45	40
Crop 2	5,000	4,500	4,000	55	50	45
Crop 3	7,000	6,500	6,000	40	30	20
Crop 4	10,000	9,500	9,000	100	90	80
Crop 5	10,500	10,000	9,500	95	90	85

[1] In this example, land classes 1, 2, and 3 show progressively lower yields for all crops, but this is not a requirement of the program.

Table 3.--Initial acreages and rents for five-crop, three-land-class problem

Crop	Acreage			Per acre rents		
	Land class 1	Land class 2	Land class 3	Land class 1	Land class 2	Land class 3
	Acres	*Acres*	*Acres*	*Dollars*	*Dollars*	*Dollars*
Crop 1	550.0	57.9	--	222.22	--	--
Crop 2	--	--	50.1	--	--	--
Crop 3	--	--	--	--	--	--
Crop 4	--	--	--	--	--	--
Crop 5	--	--	--	--	--	--
Total available	550	100	100	--	--	--

Table 4.--Initial product prices and quantities

Crop	Product prices	Quantity supplied by project producers	Total quantity supplied
	Dollars per 1,000 pounds	*Pounds*	*Pounds*
Crop 1................	144.44	30,107,408	35,185,186
Crop 2................	88.89	2,256,405	4,222,222
Crop 3................	175.00	1,871,071	2,812,500
Crop 4................	100.00	4,200,000	5,000,000
Crop 5................	110.53	3,705,263	6,315,789

Table 5.--Final acreages and rents for five-crop, three-land-class problem

Crop	Acreage			Per acre rents		
	Land class 1	Land class 2	Land class 3	Land class 1	Land class 2	Land class 3
	Acres	*Acres*	*Acres*	*Dollars*	*Dollars*	*Dollars*
Crop 1................	467.2	100.0	6.9	514.15	262.73	11.32
Crop 2................			50.0			11.32
Crop 3................	42.5			514.15		
Crop 4................	40.3			514.15		
Crop 5................			43.0			11.32
Total available......	550	100	100			

Table 6.--Equilibrium product prices and quantities

Crop	Product prices	Quantity supplied by project producers	Total quantity supplied
	Dollars per 1,000 pounds	*Pounds*	*Pounds*
Crop 1................	150.28	28,137,882	35,185,186
Crop 2................	89.14	2,251,360	4,222,222
Crop 3................	187.85	1,701,219	2,812,500
Crop 4................	105.14	4,026,047	5,000,000
Crop 5................	111.90	3,658,173	6,315,789

References

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