Irrigation Policy and Long-Term Growth Functions

By George A. Pavelis

A method for projecting the acreage of farmland irrigated in the 22 major water resource regions of the United States is described in this paper.[1] The method incorporates: (1) A statistical analysis of historical rates of irrigation development observed between 1939 and 1959 in the Census of Agriculture; (2) economic limits on irrigation that recognize regional variations in soils and water supplies; and (3) estimates of the absolute and relative importance (in terms of acreage) of new Federal and non-Federal irrigation development. The method is then used to quantify regional variations in the sensitivity of irrigation to three postulated irrigation policies, ranging from one involving minimal Federal and modest non-Federal development to one postulating no policy constraints on either type of development.

Background and Concepts

Figure 1 shows the distribution of the 33 million acres or so of irrigation reported for the United States in the 1959 Census of Agriculture (2).[2] About 141,000 acres were reported for Hawaii in 1959 and only 360 acres for Alaska. Figure 2 shows approximate national totals for the last five Censuses, as well as the acreage irrigated from streams, reservoirs, or other surface sources compared with the acreage supplied from wells. About 20 percent of the gross value of all crops in 1959 was attributed to irrigation--practiced on about 8.5 percent of the total acreage of harvested crops and on about 7.4 percent of all land then classed as usable for crop production. As shown in figure 3, these percentages vary widely among States and regions. Additional details on regional characteristics of irrigation are available elsewhere, notably in official Census reports (2) and in a Department of Agriculture report by Wooten, Gertel, and Pendleton (10). A straightforward extrapolation of 1939-59 regional trends indicates that a total of about 38.7 million acres of irrigated land in farms might be reported in the Census for 1964. Preliminary county and State returns are scheduled for release beginning in April 1965.

Concerning the future, a number of completed research studies have been addressed to the problem of projecting irrigation's general role in agricultural production, and also its special importance in regional patterns of water supply and demand (4, 5, 6, 8, 9). In pointing up the sensitivity of projections of regional water demands to technologic and economic assumptions concerning irrigation, the studies have all shown a major concern with the relatively indefinite future, but somewhat less concern with irrigation policy as it relates to achieving the levels of farm output calculated as being needed at specified dates. Given various hypotheses of the extent to which additional private or public development might be encouraged by policymakers, this paper develops corresponding continuous hypothetical growth functions, enabling one not only to estimate the time path of irrigation as conditioned by specified policies, but also to identify policy constraints consistent with acreages of irrigated land computed as being optimal for specified dates.

[1] Summarized from an unpublished report presented in the Natural Resources session of the joint 1964 meeting of the Operations Research Society of America and The Institute of Management Science, Minneapolis, Minn., October 7-9, 1964. The research described is underway in collaboration with the Economic Research Service in a North Central Region cooperative project (NC-57) on "Economic and Legal Factors in Providing, Using, and Managing Water Resources in Agriculture," and in a Western Region project (W-81) on "The Economics of Water Transfer: an Appraisal of Institutions." The author appreciates the comments and suggestions of Emery N. Castle, Harold H. Ellis, Karl Gertel, Robert C. Otte, and Gordon D. Rose. Also appreciated is the assistance of Jeremiah R. Williams in the statistical phases of the study reported.

[2] Underlined numbers in parentheses refer to Literature Cited, p. 60.

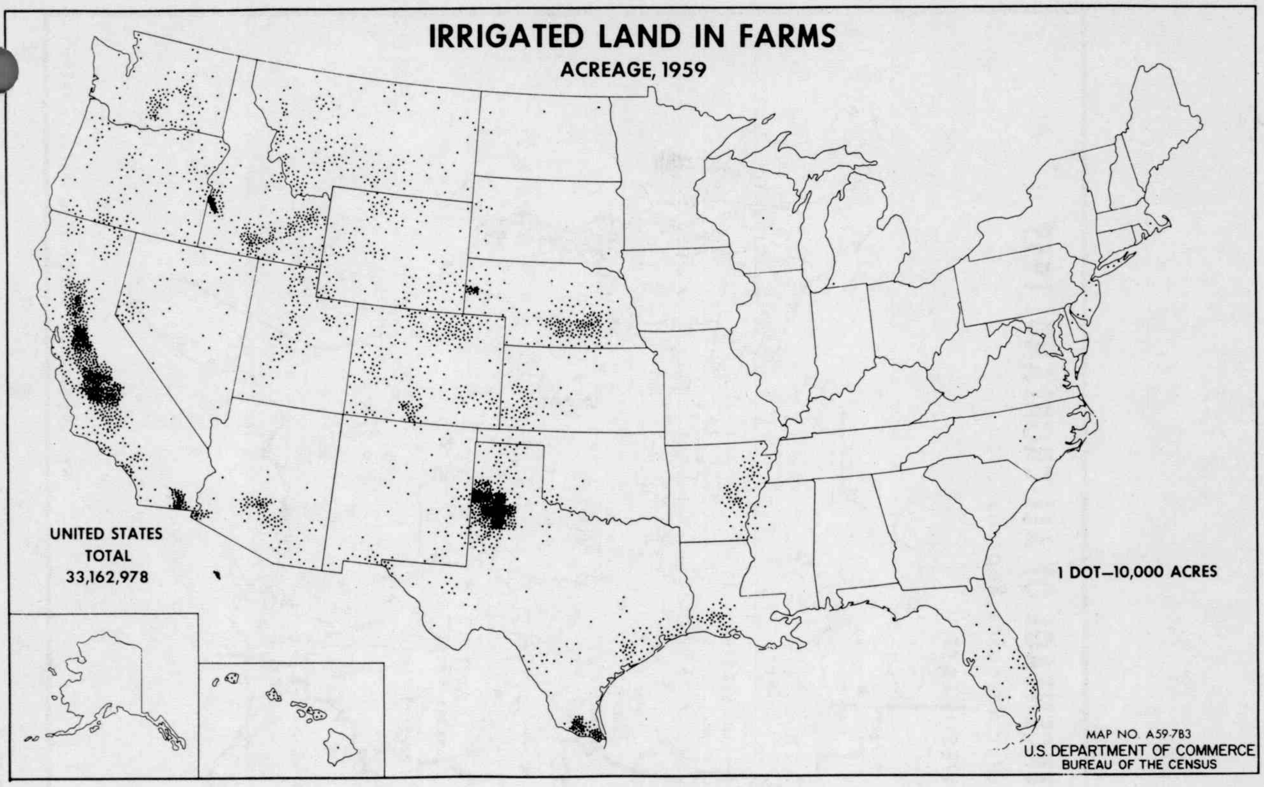

Figure 1

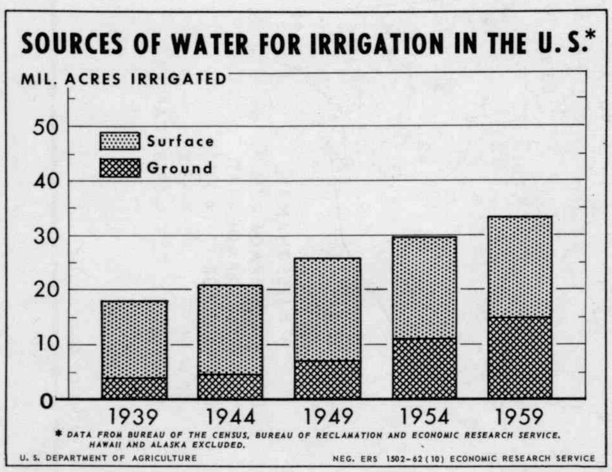

Figure 2

General Method

First, a maximum economic limit on the acreage of land irrigable in the indefinite future is postulated for each of the 22 regions in figure 3. Second, the present acreage is specified as a benchmark, and the difference between it and the maximum limit is considered to be the maximum remaining potential increase in each region. The present acreage is taken as the trend value for 1959, the most recent year of completely published observations. It is further assumed that the maximum potential increase will never accrue fully for any region; this means that acreages in the indefinite future will become asymptotic to the maximum initially specified. A final "premise" involves the selection of equation types thought best for combining information on the defined limits to irrigation, the acreage recorded to the present, historical rates of change, and anticipated rates of change.

Policy constraints are introduced simply by reducing maximum remaining potentials in

IRRIGATED LAND IN FARMS AS A PERCENTAGE OF ALL CROPLAND, 1959

By State-parts of Water Resource Regions

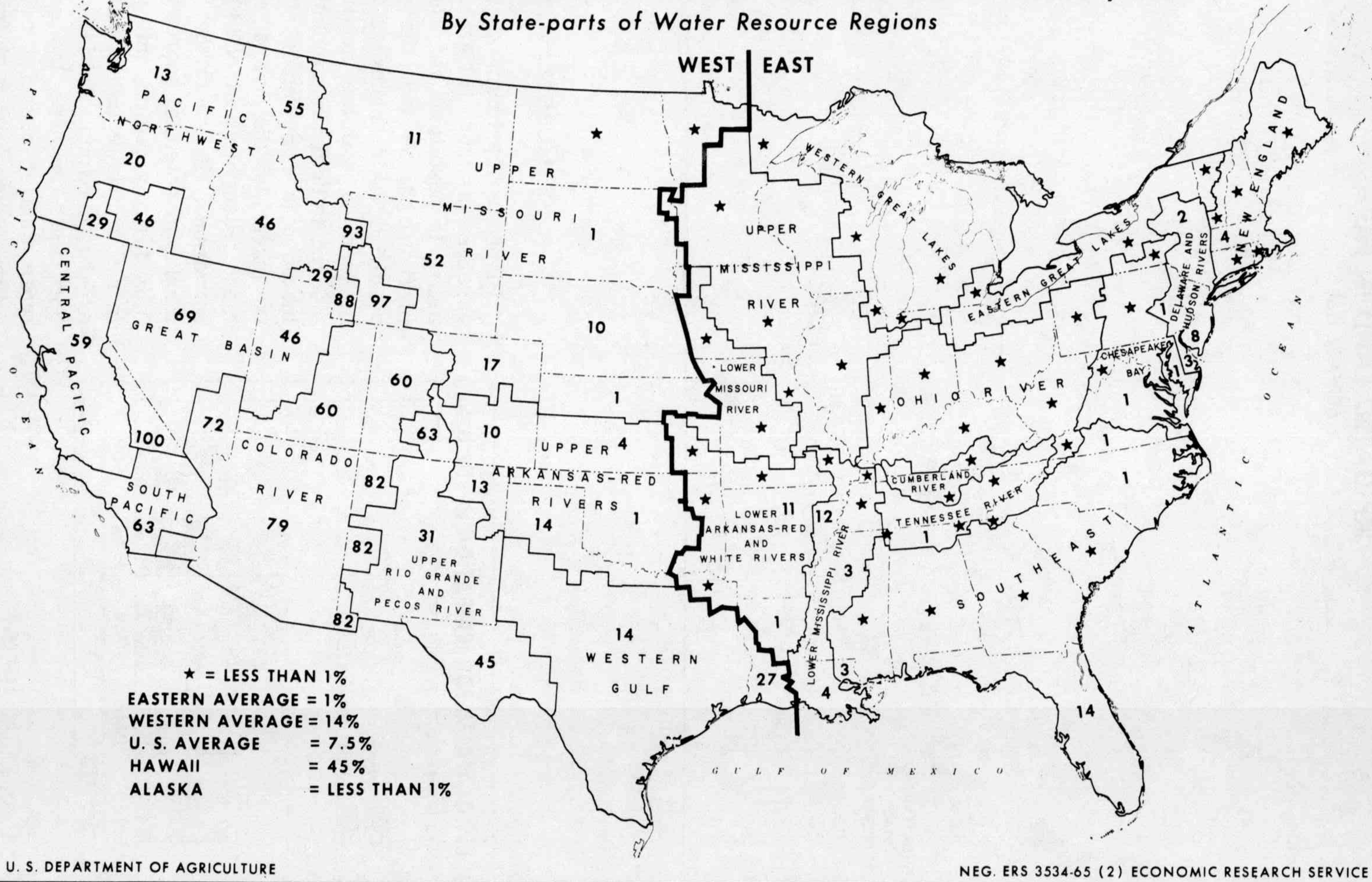

Figure 3

accordance with various hypotheses on the extent to which privately and publicly undertaken development in each region might be dissuaded. No judgments are made on the propriety of such constraints actually being invoked.

Historical and Projective Growth Functions

Growth functions have been synthesized for each of the 22 major water resource regions and the two region groupings of the United States as shown in figure 3. The eastern mainland is taken to include the Upper Mississippi, Lower Missouri, Lower Arkansas, Lower Mississippi, and all regions east thereof--13 regions in all. The western mainland includes the remaining 9 regions. Data for Hawaii and Alaska have been considered, but acceptable functions for these two States could not be derived. They are omitted from this summary report, except as noted later.

The total time span of the analysis is from the year 1939 (the beginning of the historical acreage series) to the year 2000. The period 1939-59 encloses the historical series of five observed acreages published (6) or developed (2) as necessary for each region for 1939, 1944, 1949, 1954, and 1959. For ease of computation, the year 1949 as the midpoint of the historical series is taken as the statistical origin of trends and also retained as the mathematical origin of the time variable t in regional long-term growth functions. That is, $t = (\text{year} - 1949)/5$, so that $t(1939) = -2$, $t(1944) = 1$, $t(1949) = 0$, $t(1954) = 1$, and $t(1959) = 2$. The latter as the most recent year of published record is the point of departure in projecting growth functions to $t(2000) = 10.2$.

DERIVATION PROCEDURE

Incorporating the concept of eventual acreage limits balanced against trends computed for 1939-59, the basic projective growth function for any region or group of regions is

$$(1) \quad A_t = L - \left[(L - A_2)\, e^{B(t-2)}\right], \; e = 2.71828, \; t \geqslant 2.$$

A_t is the acreage projected for any future time t. It is the difference between the parameter L (the eventual limit specified) and that part of L not reached by time t; namely, less the bracketed portion of (1). The latter depends in turn on the potential acreage remaining in 1959, $(L - A_2)$, when $t = 2$, and on the proportion of $(L - A_2)$ estimated to remain in years beyond 1959 when $t > 2$. Call this proportion $e^{B(t-2)}$ the damping factor, which is unity in 1959 when $t = 2$. It decreases asymptotically toward zero as t increases indefinitely. Thus, the limiting acreage not reached in distant years tends to zero, or is negligibly damped, and A_t approaches L.

Assigning the factor t-2 in (1), to give $e^{B(t-2)}$ a value of 1 for 1959 when t was 2, links any historical function ending in 1959 to its corresponding projective function beginning in 1959.

The parameter B in (1) represents the continuous constant percentage decline in remaining potential, imputed from the average percentage decline in remaining potential noted between 1954 and 1959 as the most recent interval of record, when t ranged from +1 for 1954 to +2 for 1959. This basis for projecting B beyond 1959 is given explicitly by

$$(2) \quad (L - A_2) = (L - A_1)\, e^B, \text{ in which}$$

$$(3) \quad B = \log_e \left[(L - A_2) / (L - A_1)\right]$$

With B and t thus known, the damping factor as used in (1) is taken from standard tables of $e^x = e^{B(t-2)}$.

Recognizing that the terms A, L, and B in (1) to (3) have unique values for each water resource region i, the aggregation of (1) to totals for the eastern mainland (E), the western mainland (W), and the United States (US) is simply

$$(4) \quad A_t(US) = A_t(E) + A_t(W) = \sum_{i=1}^{13} A_t(i) + \sum_{i=14}^{22} A_t(i).$$

Composite projective growth functions for region groupings can also be fitted directly. Results either way did not differ substantially in this study, but significance tests did consider directly fitted historical functions.

The historical function for 1939-59 for any region from which A_1 and A_2 are computed in calculating $L-A_1$, $L-A_2$, and B in (3) is expressed as

$$(5) \quad A_t = a b^t c^{t^2}, \; -2 \leqslant t \leqslant 2, \; t = (\text{year} - 1949)/5.$$

This complex exponential allows for variable or invariable percentage rates of change in A_t, depending on the statistical significance of the coefficients. Other functions, including $A_t = a + bt + ct^2$, were also considered in preliminary graphic plotting, but appeared to be less appropriate.

The constants a, b, and c in (5) are evaluated with least-squares techniques, determining the regression of acreage reported in the N = 5 censuses of 1939, 1944, 1949, 1954, and 1959 on time, for simplicity using Fisher's method of orthogonal polynomials (1).

The orthogonal equation is

$$(6) \quad \log A_t' = \log a' + \log b' \xi_1 + \log c' \xi_2,$$

where for all regions $\xi_1 = t$ (since N is odd), and $\xi_2 = t^2 - 2$. The standard logarithmic form of (5) is

$$(7) \quad \log A_t = \log a + \log bt + \log ct^2.$$

For this analysis it can be shown (1) that

$$(8) \quad \log a = \log a' - 2 \log c = 1/5 \sum_{t=-2}^{2} (\log \underline{A}_t) - 2 \log c$$

$$(9) \quad \log b = \log b' = 1/5 \sum_{t=-2}^{2} (t \log \underline{A}_t)$$

$$(10) \quad \log c = \log c' = 1/14 \sum_{t=-2}^{2} (t^2 - 2) \underline{A}_t$$

The $\underline{A}_t$ in (8), (9), and (10) denote acreages reported in the Census of Agriculture as contrasted with A_t, the computed acreage for time t.

HISTORICAL GROWTH RATES

Because the historical growth functions are necessary to estimate A_2, A_1, and B in the projective function (1), the historical estimators are discussed first. Table 1 summarizes the statistical analysis of irrigated acreage in each of the 22 mainland water resource regions and presents trend estimators converted to their natural form corresponding to (5).

Table 1.--Estimators of regional irrigation in the United States, 1939-59

Water resource regions	[1] $A_t = ab^t c^{t^2}$		Annual acreage change[3]	
			1939	1959
	[2] a	b	*Percent*	*Percent*
New England..........	31	1.7513	$\underline{50.40}$	$\underline{-6.35}$
Delaware and Hudson...	47	1.8108	12.61	12.61
Chesapeake Bay........	12	2.0291	15.20	15.20
Southeast.............	380	1.4477	$\underline{17.50}$	$\underline{2.19}$
Eastern Great Lakes...	10	1.5971	9.82	9.82
Western Great Lakes...	15	1.9743	$\underline{22.04}$	$\underline{10.26}$
Ohio Basin[4]...........	12	1.6043	9.91	9.91
Cumberland[4]...........	1	1.6043	9.91	9.91
Tennessee Basin[4]......	11	1.6043	9.91	9.91
Upper Mississippi.....	17	1.8426	13.80	13.80
Lower Mississippi.....	243	1.5152	8.67	8.67
Lower Missouri........	1	3.8730	31.10	31.10
Lower Arkansas........	290	1.4990	8.43	8.43
Eastern mainland....	1,070	1.4040	7.02	7.02
Upper Missouri........	4,605	1.1397	2.65	2.65
Upper Arkansas........	1,010	1.4105	$\underline{3.39}$	$\underline{9.42}$
Western Gulf..........	2,976	1.4859	$\underline{14.96}$	$\underline{3.97}$
Upper Rio Grande......	1,042	1.1169	2.24	2.24
Colorado Basin........	2,848	1.0374	0.74	0.74
Great Basin...........	1,740	1.0477	0.94	0.94
Pacific Northwest.....	3,894	1.1069	2.05	2.05
Central Pacific.......	5,239	1.1428	$\underline{4.87}$	$\underline{1.43}$
South Pacific.........	641	1.1360	2.69	2.69
Western mainland....	23,995	1.1254	2.39	2.39
United States.........	25,065	1.1436	2.72	2.72

[1] A_t = computed thousands of acres irrigated at time $t \leq 2$, t = (year - 1949)/5. The term 'c' was significant ($\geq$ 80 percent level in F statistics) for only 6 regions, as follows: New England (0.74373); Southeast (0.91640); Western Lakes (0.93883); Upper Arkansas (1.03610); Western Gulf (0.93513); and Central Pacific (0.97941).

[2] Thousands of acres irrigated in 1949, when t = 0.

[3] Significant variable rates of increase between 1939 and 1959 underscored; significant constant rates not underscored.

[4] Ohio, Cumberland, and Tennessee regions pooled in computing 'b' and 'c'.

The annual rates of increase estimated in table 1 lend quantification to regional shifts in irrigation (and associated water use) observed since World War II. Along with an average annual national increase of 2.72 percent, there was a marked relative shift to the eastern mainland associated with the annual increase there of 7.02 percent. However, the current acreage in the East is still only about 7 percent of the U.S. total. And in the East there was a pronounced shift to the Lower

Missouri, Chesapeake, Upper Mississippi, and Delaware-Hudson regions and, with the exception of New England, some shifts in all eastern regions at the expense of the Southeast. However, about 25 percent of all eastern irrigation in 1959 was still in the Southeast States, with another 25 percent in the Lower Mississippi Valley, and 30 percent in Lower Arkansas. Note the decreasing rate of increase in the Western Lakes region.

For the western mainland, with 93 percent of the total national acreage in 1959, a large part of the 2.39 percent annual increase between 1939 and 1959 was due to new irrigation development in the Upper Arkansas, Western Gulf, and Upper Missouri regions. Together these accounted for about 42 percent of the acreage irrigated in the West in 1959. The annual rate of increase itself increased in the Upper Arkansas--from 3.39 percent per year in 1939 to 9.42 percent in 1959. The yearly increase in the Western Gulf, however, fell from nearly 15 percent annually in 1939 to around 4 percent in 1959. This was due in part to depletion of ground water reserves in the High Plains of Western Texas. The Central Valley of California, currently accounting for 20 percent of all western irrigation and thus for between 18 and 19 percent of all the irrigated land in the United States, experienced a similar drop in its increase rate, from 4.87 percent per year in 1939 down to 1.43 percent in 1959. Urbanization, nearly full use of readily available water supplies, and other factors explain the relative decline in this irrigated region.

Economic Limits to Irrigation

In this study, the economic limit to irrigated acreage in each water resource region was construed to be the maximum acreage of soils feasibly irrigated (i.e., costs $\geq$ benefits at the extensive margin), given prevailing notions of natural moisture and yearly (or seasonal) moisture deficiencies, future irrigation returns in relation to costs, and foreseen limits on water supplies. With respect to the definition accepted, the studies of the Department of Agriculture for the Senate Select Committee on National Water Resources (6), the Department's own National Inventory of Soil and Water Conservation Needs (7), and a study by the Bureau of Reclamation (3) are relevant. Moreover, these works either directly or indirectly (and fairly independently) consider remaining potentials as divided into two major components: (a) The additional acreage irrigable from water supplies feasibly developed by individual farm operators, local irrigation districts, or State agencies; and (b) the additional acreage irrigable from water supplies feasibly developed in connection with large-scale multipurpose water projects, with Federal financing a matter of considerable importance.

Estimates of remaining irrigation potentials given in these studies have been collated and summarized by regions in table 2. In general, the remaining Federal potentials in the East are assumed to be limited by the acreages possibly irrigated with Federal assistance authorized by the multipurpose Watershed Protection and Flood Prevention Act (P.L. 83-566). Although this Act applies to all States and Puerto Rico, remaining Federal potentials in the West are approximated both from the

Table 2.--Remaining irrigation potentials in 1959 in the United States with Federal and non-Federal distribution

Water resource regions	Total	Federal	Non-Federal
	1,000 acres	*Percent*	*Percent*
New England.........	106	30	70
Delaware and Hudson.	288	36	64
Chesapeake Bay......	622	21	79
Southeast...........	4,515	45	55
Eastern Great Lakes.	284	42	58
Western Great Lakes.	456	7	93
Ohio Basin..........	922	32	68
Cumberland..........	14	76	24
Tennessee Basin.....	206	34	66
Upper Mississippi...	1,125	15	85
Lower Mississippi...	3,365	17	83
Lower Missouri......	1,212	9	91
Lower Arkansas......	2,309	26	74
Eastern mainland..	15,504	28	72
Upper Missouri......	4,819	84	16
Upper Arkansas......	2,436	36	64
Western Gulf........	3,174	96	4
Upper Rio Grande....	738	100	0
Colorado Basin......	925	96	4
Great Basin.........	757	78	22
Pacific Northwest...	4,515	85	15
Central Pacific.....	6,721	32	68
South Pacific.......	847	13	87
Western mainland..	24,932	65	35
United States.......	40,436	51	49

USDA Conservation Needs Inventory and from Bureau of Reclamation investigations.

For the eastern regions as a group, table 2 indicates that roughly 75 percent of the remaining acreage potentials can be developed independently by individual farm operators. Federal assistance through multipurpose small watershed programs under P.L. 566 is seen to have its greatest probable importance for supplemental irrigation in the Southeast, Lower Mississippi, and Lower Arkansas regions, considering both the percentages and acreages involved.

For the western regions, where Bureau of Reclamation projects now include about one-fourth of the total area irrigated, table 2 shows that roughly two-thirds of the remaining acreage potential is associated with Federal projects that may be completed, with new Federal reclamation of greatest relative importance to the Colorado, Western Gulf, Pacific Northwest, and Upper Missouri regions. Future State activity is of notable importance in California. The Upper Arkansas region stands out as perhaps most important from the standpoint of additional private development in the West.

For the United States as a whole, table 2 indicates that the expansion of irrigated acreage seems about equally dependent on Federal and non-Federal activity. Total remaining potentials by region in table 2 are added to acreages computed for 1959 in column 1 of table 3 to indicate the maximum economic limits to irrigation in the various regions (L_{100} in col. 4 of table 3). The derived limits as constrained are given in the columns headed by L_{25} and L_{50} in table 3.

Table 3.--Irrigation in 1959 and regional limits on irrigation in the United States based on selected proportions of remaining Federal and non-Federal potentials over 1959 being developed

Water resource regions	1959[1]	Regional limits[2]		
	A_2	L_{25}	L_{50}	L_{100}
	1,000 acres	*1,000 acres*	*1,000 acres*	*1,000 acres*
New England..........	29	74	82	135
Delaware and Hudson...	147	265	291	435
Chesapeake Bay........	49	327	360	671
Southeast.............	561	2,309	2,818	5,076
Eastern Great Lakes...	26	139	168	310
Western Great Lakes...	45	265	273	501
Ohio Basin............	30	416	491	952
Cumberland............	3	44	50	97
Tennessee Basin.......	28	114	131	234
Upper Mississippi.....	57	576	620	1,182
Lower Mississippi.....	559	2,102	2,242	3,924
Lower Missouri........	13	592	619	1,225
Lower Arkansas........	651	1,657	1,806	2,960
Eastern mainland....	2,198	8,880	9,951	17,702
Upper Missouri........	5,981	7,379	8,390	10,800
Upper Arkansas........	2,315	3,314	3,533	4,751
Western Gulf..........	5,024	5,852	6,611	8,198
Upper Rio Grande......	1,300	1,484	1,671	2,038
Colorado Basin........	3,065	3,305	3,528	3,990
Great Basin...........	1,910	2,141	2,289	2,667
Pacific Northwest.....	4,770	6,063	7,028	9,285
Central Pacific.......	6,296	9,110	9,657	13,017
South Pacific.........	827	1,223	1,251	1,674
Western mainland....	31,488	39,871	43,958	56,420
United States.........	33,686	48,751	53,909	74,122

[1] Computed as shown in and transferred from table 1.
[2] L_{25} assumes eventual development of 25 percent of remaining Federal potentials but 50 percent development of remaining non-Federal (private and State) potentials; L_{50} assumes both Federal and non-Federal development at 50 percent of their remaining potentials; and L_{100} assumes 100 percent for both.

Alternative Policies and Long-Term Growth Functions

Alternative policies and their possible consequences are reviewed here in terms of the basic growth function (1). They are considered in decreasing order of stringency with respect to future irrigation growth, and thus in increasing order of acreages irrigated at specified dates. Figure 4 illustrates aggregate U.S. historical and long-term functions, with the latter conditioned by the three policies considered. Some detail for the East, the West, and the United States as a whole is given in table 4, for 1959, 1964, 1980, and 2000.

L_{25}--minimal Federal and modest non-Federal development:

This limit, assumed to be operational in 1959-2000, identifies successful efforts to hold the eventual area irrigated in all regions to a level such that only 25 percent of the remaining Federal potentials shown in table 2 would be developed, and such that 50 percent of the remaining non-Federal potentials would be irrigated.

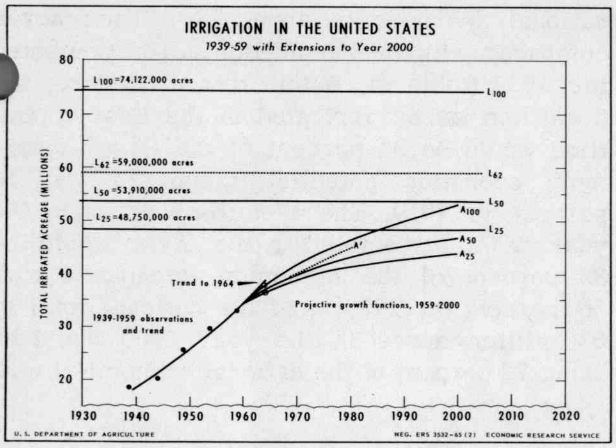

Figure 4

For each region the long-run growth function (1) now has the form:

$$(11)\ A_t = L_{25} - (L_{25} - A_2)\, e^{B(t-2)},\ t \geq 2,\ (\text{year} \geq 1959).$$

From tables 1 and 3 the empirical function (A_t in millions of acres) for the mainland 48 States is approximately

$$(12)\quad A_t = 48.751 - 15.065\, e^{-0.175\,(t-2)}.$$

By evaluating (12) for any year beyond 1959, the long-run consequences of a hypothesis of minimal Federal and modest non-Federal development in the United States can be estimated. The time path of irrigated acreage between 1959 and 2000 conditioned by this hypothesis is plotted as the curve A_{25} in figure 4. Subscripts in this chart and also in table 4 now refer to controlling limits over time, rather than to time t as such.

As expected, this hypothesis (combined with a recognition of past growth rates) indicates an accelerated relative shift of irrigation to the Eastern States, with the East accounting for about 12 percent of the total U.S. acreage projected to the year 2000. At that time about 58 percent of the limit L_{25} would be reached in the eastern mainland, 97 percent of the corresponding western limit would be reached, and, nationally, 90 percent of the L_{25} limit would be reached. But in relation to maximum economic potentials the percentages for the year 2000 would be considerably lower; that is, 29 percent for the East, 69 percent for the West, and 59 percent nationally.

Table 4.--Selected estimates of irrigated acreage in the United States, 1959 to 2000, based on reported Census data, 1939-59 trends, and alternative long-term limits

Year and basis	Eastern mainland	Western mainland	United States
	Million acres	*Million acres*	*Million acres*
1959:			
Reported..........	1.87	31.15	[1] 33.02
1939-59 trend.......	2.20	31.50	[2] 33.70
1964:			
1939-59 trend......	3.09	35.44	[3] 38.53
A_{25} (on L_{25})........	2.74	33.86	[4] 36.60
A_{50} (on L_{50})........	2.75	34.22	36.97
A_{100} (on L_{100})......	2.78	34.65	37.43
1980:			
A_{25} (on L_{25})........	4.06	37.34	41.40
A_{50} (on L_{50})........	4.15	39.10	43.25
A_{100} (on L_{100})......	4.41	41.80	46.21
2000:			
A_{25} (on L_{25})........	5.15	38.75	43.90
A_{50} (on L_{50})........	5.30	41.65	46.95
A_{100} (on L_{100})......	6.05	47.00	53.05

[1] Add 141,000 acres for Hawaii and 360 acres for Alaska.
[2] Data insufficient for Alaska and trend not significant for Hawaii. U.S. total is about 33.88 with reported data for these States added.
[3] U.S. total is about 38.71 if 1959 reported acreages for Hawaii and Alaska are added.
[4] Hawaii and Alaska excluded from consideration in this and subsequent U.S. totals.

L_{50}--modest Federal and non-Federal development:

This limit identifies efforts to hold the eventual area irrigated in all regions to a level such that 50 percent of both the remaining Federal and non-Federal potentials given in table 2 would be developed. As constrained by this limit the long-run growth function (1) for any region has the form

$$(13)\ A_t = L_{50} - (L_{50} - A_2)\, e^{B(t-2)},\ t \geqslant 2,\ \text{year} \geqslant 1959,$$

and from tables 1 and 3 for the United States we have, in millions of acres,

$$(14)\quad A_t = 53.909 - 20.223\, e^{-0.145\,(t-2)}.$$

By evaluating (14) for any year, beginning with 1959, the time path of irrigated acreage in the United States is approximately as shown by the curve A_{50} in figure 4.

Projections on this hypothesis imply a continued relative shift of irrigated acreage to the Eastern States (11 percent of the U.S. total in the year 2000 compared with 7 percent in 1959). The shift is associated more with continuation of the rapid eastern rate of increase observed between 1939 and 1959 than with relatively greater encouragement of non-Federal irrigation that would favor the East, as was hypothesized in L_{25} above. This hypothesis shows also that if remaining Federal and non-Federal development is only modestly and proportionately constrained in all regions, western irrigation would reach about 75 percent of its maximum economic potential by the year 2000, and the United States would reach 63 percent of its maximum potential.

L_{100}--unconstrained Federal and non-Federal development:

This limit implies that eventual irrigation would approximate the maximum economic limits identified as L_{100} in table 3. On this hypothesis, eventual irrigation in the East could be eight times the 1959 acreage, in the West it could be 80 percent more than the current acreage, and for the United States it could be about 2-1/5 times the current acreage.

As influenced only by past trends and the economic limits defined, the long-run growth function (1) for any region has the form

$$(15) \quad A_t = L_{100} - (L_{100} - A^2) e^{B(t-2)}, \; t \geqslant 2, \text{year} \geqslant 1959.$$

and from tables 1 and 3 we have in millions of acres for the United States

$$(16) \quad A_t = 74.122 - 40.436 \, e^{-0.085\,(t-2)}.$$

Evaluations of (16) generate the time path of irrigated acreage shown as the A_{100} curve in figure 4.

Differential relative rates of growth during 1939-59, and the relative as well as the absolute magnitudes of remaining Federal and non-Federal irrigation potentials in each region, all influence this projection in an undetermined way. But the net result for the year 2000 is still a decided relative shift to the Eastern States, substantial increases in acreage in both the East and the West (175 and 49 percent over 1959, respectively), and an overall national increase to about 53 million acres compared with the 33 million acres computed for 1959 (table 4). Within this procedure, the 6 million acres irrigated in the East at that time would be 34 percent of the East's maximum economic potential, compared with 12 percent in 1959. The 47 million acres in the year 2000 projected for the West would be 83 percent of the maximum, compared with 56 percent in 1959. And the national total of 53 million acres in the year 2000 would be about 72 percent of the national economic limit.

Policies Consistent with Dated Requirements

Here we return briefly to irrigation projections made in various studies cited at the outset. Within stated assumptions concerning future population, the urbanization of farmland, related requirements for farm and nonfarm products or services, and other relevant factors, characteristically these studies have computed corresponding requirements for increasing, decreasing, or perhaps leaving unchanged the extent of crop or pasture irrigation in different regions. The relative and absolute productivity of land, water, or other resources at projected levels of technology and management practices have been considered also, but a point commonly stressed is that the projected patterns of resource use are not economic norms. However, they are seen to identify a number of necessary conditions for optimal resource use, and they have contributed a great deal of information needed by legislators and other policymakers. This raises a question as to whether the research described here can be useful in identifying constraints on eventual irrigation development in different regions or in the country generally that would result in specified acreages being irrigated by specified dates. The foregoing system of regional relations in historical rates of irrigation and remaining Federal and non-Federal development potentials might be adapted to such problems, somewhat as follows:

Assume that 45 million acres of irrigated land have been recommended as the optimal acreage for the United States in 1980. This is roughly midway between the acreages previously calculated for 1980 from the long-run growth function conditioned by modest Federal and non-Federal development (14), and the function (16) unconstrained by any limitations on eventual economic development. The basic growth function (1) is utilized in this case to solve for L, with t-2 = 4.2 for 1980 given, and with A_2 = 33.686 millions of acres for 1959 also known. Our "constant" of 45 million acres is the dashed vertical line in figure 5.

Graphic interpolation is appropriate here, since B varies in (2) not only with L as the limit being sought, but also with A_1 and A_2 as the acreages computed for both 1954 and 1959; respectively, 29.274 and 33.686 million acres. The earlier results of exploring hypothetical policy limits are useful for getting a solution here too, since there is a proportional relation between acreages projected for given dates and the ratio $\left[(L - A_2)/(L - A_1)\right]$. This relation is illustrated in figure 5 for the year 1980, on the basis of three coordinate "observations," identified with the three previously specified limits L_{25}, L_{50}, and L_{100}.

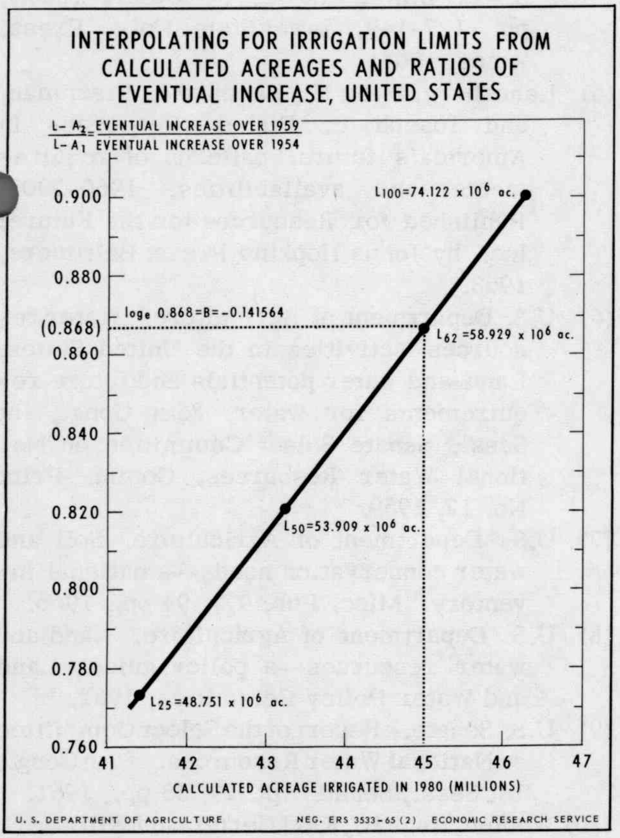

Figure 5

With the additional information in figure 5, the function (1) can be written and solved for L, as follows:

$$(17)\quad 45.000 = L - (L - 33.686)\, e^{-0.141564(4.2)}, \text{ or}$$

$$45.000 = L - (L - 33.686)\,(0.5518), \text{ and}$$

$$L = 26.412/0.448200 = 59 \text{ million acres (approx.).}$$

Thus, the procedure for deriving long-term growth functions that recognize both the inertial effects of historical trends and the damping effects of controlling limits to irrigation suggests that a hypothetical policy aimed at eventual irrigation of about 59 million acres in the United States would result in about 45 million acres being irrigated in the year 1980 (see fig. 4, A' for 1980). The derived limit of 59 million acres implies that about 62 percent of the total remaining Federal and non-Federal potential of 40.4 million acres given in table 2 would be developed eventually; that is, L_{62} as the dashed horizontal in figure 4 is 59 million acres.

Reversing the procedure again yields a continuous growth function or time path of irrigation between the present and 1980, simply by substituting for L the derived limit of 59 million acres in equation (1) as applicable to the United States, and then solving for A_t as desired. Results are the dashed curve A' in figure 4.

Other Applications and Concluding Comments

If one could assess the likelihood over time of various "policy" limits being set on future irrigated acreage, such a probability distribution of irrigation policy could be used to obtain an approximate probability function for dated estimates of irrigated acreage also. In this sense, the growth functions A_{25}, A_{50}, and A_{100} in figure 4 are examples of discrete events to be considered in such derivations.

Pending these derivations one can only say that, if L_{25} and L_{100} are reasonable limits to the actual policy limits set, A_{25} and A_{100} are reasonable limits of actual time paths of acreage, with specific probabilities of occurrence unknown. The seriousness of this and other limitations decreases with a decreasing propensity to project,[3] so there is some justification for using the procedures to obtain short-term intercensus estimates of irrigated acreage that recognize historical trends and possible long-term limits. Thus, advance estimates of the U.S. acreage to be reported from the 1964 Census of Agriculture could be given as the range 36.6-38.5 million acres, with the lower figure being the 1964 value of the A_{25} function in figure 4 and the upper one being an extrapolation of the 1939-59 trend (see table 4). Of course, synoptic observations of regional precipitation, snowpack accumulations, reservoir storage, cost-price relations, and other factors for 1964 would suggest deviations from or lend more validity to such interval estimates.

Another limitation of the analysis is that the hypothetical policy limits have been applied across the board to all regions. Interregional variations in national policy can be expected even if not intended. Their possible effects on regional irrigation development would be interesting to explore also.

A final limitation noted here is that the methods outlined have only partly allowed for pending State and Federal projects for trans-region water diversions and other technological developments, so the given economic limits to irrigation are perhaps too static. Results should be interpreted accordingly, although the methods are quite accommodative to the availability of more and better information.

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[3] To illustrate: For the year 2000 the U.S. projected acreage on A_{25} in figure 4 is about 6 percent under the projection on A_{50} while the projection on A_{100} is 13 percent over the A_{50} projection. Deviations from A_{50} similarly computed for 1980 drop to the range -4 and +7 percent, and for 1964 drop to between -2 and +3 percent (see table 4).

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