Isolating and Measuring Technical Progress and Scale Effect: An Alternative Approach[1]

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Abstract

In this paper, we develop measures of technical progress and scale effect by using directional derivatives and vector algebra. Our approach is distinguishable from earlier ones in that (i) technical progress is measurable and separatable from scale effect irrespective of properties of technical progress in the long-run or properties of returns to scale; (ii) the approach does not require specification of any production or transformation function, thus preclude all the usual problems associated with postulation and estimation of production models; and (iii) technical progress is assumed to be related to inputs, including R&D, which is more appropriate than treating it as an explicit function of time.

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1 Introduction

Technical progress has been a theme of many important studies, notably including Hicks (1965), Solow (1957, 1958, 1961, 1962, 1963), Stigler (1961), Kendrick and Sato (1963), Samuelson (1965), Beckmann and Sato (1969), Sato and Beckmann (1968), Sato (1970, 1980, 1981), Sato and Nôno(1983), Stevenson (1980), Kopp and Smith (1985) and Färe, Grosskopf and Kokkelenberg (1989). An earlier survey was provided by Kennedy and Thirlwall (1972). Initial approach to measuring technical progress by productivity indexes was discarded since they incorporate no causal explanation on the movement of factor productivity. Also, too many sources of error frequently creep into the measurement of movements in total factor productivity (Jorgenson and Griliches 1967). Tinbergen (1942) was the first explicitly to estimate technical progress using aggregate production function method and Solow's study (1957) is recognised as the foundatior of aggregate production function approach to measuring technical progress. The well-known Solow-Stigler controversy centered on the issue of consideration of scale effect, particularly on the existence of and the possibility of quantification of returns to scale effect under technical progress. This controversy was not resolved until Sato (1980), who proved that any technical progress can be separated from and measured together with scale effect, if the production function used is non-holothetic under a given type of technology. Sato's method rests heavily on the specification and/or estimation of production functions and transformation functions, the latter are used to transform actual inputs into what is called effective inputs. These functions must involve time as an independent variable in an attempt to account for technical progress. By so doing, technical progress is assumed to be a smooth and continuous function of time, which is difficult to justify theoretically or empirically, let alone the problems associated with establishing the true transformation and production functions.

The objective of the current paper is to develop an alternative approach to separating technical progress and scale effect by utilising the technique of directional derivative and vector algebra. We also differentiate long-run and short-run neutrality under technical progress in an attempt to explain the contradict empirical results published earlier. The derived measures of technical and scale effects are function-form independent and are operational irrespective of properties of returns to scale and of technology progress in the long run. The measures are easy to compute provided that two or more observations on inputs and output are available.

The basic principle of our approach is illustrated in section 2 using a two-factor production case, where the concepts of long-run and short-run neutrality are suggested. We derive the measures in section 3, assuming a producer is aimed at maximising output. The influence of changes in input prices on factor ratios is considered in section 4, where we show that, with some modifications, the measures developed in section 3 are applicable for profit-maximising producers. Finally, section 5 concludes the paper.

2 Isolation of Technical Process and Scale Effect: Principle and Illustration

It has been accepted that expansion in production, often represented by upright shifts of an isoquant, are basically attributable to two sources: (a) increases in the use of some or all inputs, and (b) technology advances (process innovation, improvement in management and quality of inputs, etc.). The first source may yield scale effect and the second source produces technical effect.

If we let $\mathcal{R}^k$ denote a k-dimensional space which contains all possible combinations of inputs, where k is the number of production factors, then a vector $\mathbf{X} = (x_1, \cdots, x_k)$ from the origin in $\mathcal{R}^k$ can be used to represent a particular input-combination. Clearly, an input-combination, as a vector, is composed of two parts: its magnitude and its direction. In this paper, the magnitude will be represented by the norm of $\mathbf{X}$, denoted by $S = \sqrt{\sum_{i=1}^{k} x_i^2}$ and the direction will be represented by a set of directional cosines, denoted by $\alpha_i = x_i/S$, $i = 1, \cdots, k$. S, to certain extend, indicates the size of inputs, while the direction specifies the input ratios.

As a rational producer, optimal input ratios are always used unless either non-uniform (or non-neutral) technical progress is introduced or input prices change, which causes marginal rate of substitution between any inputs unequal to their price ratios. The impact of price changes on factor ratios is not discussed until section 4. We now can make the following strong, yet reasonable, assumption:

Assumption 1: Non-uniform technical progress and changes in input prices are the only factors which can cause variations among input ratios.

For expository purpose only, we consider a simple two-factor production process as shown in Figure 1. The curves in the figure represent isoquants (physical output or profit) and x_1, x_2 represent inputs.

[Figure 1 here]

Now, suppose a production was initially at the point P_0 on isoquant I_0 and subsequently expanded to P_1 on isoquant I_1. Assuming input price ratios remain the same during the process (the case of price ratio changes will be dealt with in section 4), the expansion must have occurred under non-uniform technical progress. If there was no such technological advance, by assumption 1, the expansion would have been along the line OP_0 and reaches the point P_s for any given increment in inputs, where $OP_s = OP_1$. As shown in the diagram, P_s is on the isoquant I_s. It can be seen that (i) increase in output from P_0 to P_s is due to changes in inputs, which may yield scale effect, and subsequently (ii) increase in output from P_s to P_1 is attributable to technology advance. Our aim is to quantify the scale effect and technical progress when production with k-factors ($k \geq 2$) is expanded.

Two comments are in order: (i) the sequence of determining scale effect and technical effect can be chosen as desired; (ii) the two-factor case, discussed so far, can be easily generalised to the more realistic multi-factor case, as shown in the next section, though a diagrammatical representation becomes impractical.

If technical progress is neutral between two successive time points, our approach would fail to distinguish scale effect from technical progress unless constant returns to scale are pre-assumed. Constant returns to scale is difficult to justify on economic grounds (Stigler 1961). Conversely neutral technology, if existing at all, is rare, at least in the short-run. The empirical evidence of neutral technology provided by Solow (1957) and Salter (1966) needs two qualifications. First, they only considered two-factor production processes. Second, they only looked at long-run effect of technical progress on input ratios. While two factors may change by the same proportion in the long-run. It is difficult to imagine a technology which can alter marginal products of many inputs by the same proportion <u>and</u> in a very short time, say between two adjacent years. In general, when a new technology is introduced, not all production factor can be changed immediately due to institutional limitations, availability of the production factors involved, and time or financial constraints. For example, a uniform technology demands capital expenditure on equipment and suitably qualified labour to increase in the same proportion. While capital can be organised promptly to purchase the equipment, suitably qualified labourer may be difficult to acquire in the short-run. We have not mentioned the requirement on the quantity and quality of other factors. Another example is the case of technology progress in agriculture. It is virtually impossible for any technology to be uniform in terms of land-capital-labour ratios.

3 Measures of Technical and Scale Effect

In this section, we will formally derive the measures of technical progress and scale effect for the case that there are more than two inputs.

3.1 Notations and Definitions

Let $\mathbf{e}_1, \cdots, \mathbf{e}_k$ be the bases of the input space, $\mathcal{R}^k$, then $\mathbf{X} = \sum_{i=1}^k x_i \mathbf{e}_i$ and $\sum_{i=1}^k \alpha_i \mathbf{e}_i$ is a unit vector with a direction the same as that of $\mathbf{X}$. Assuming $\mathbf{Z}$ is any vector in $\mathcal{R}^k$, then the projection of $\mathbf{Z}$ to the direction of a unit vector can be expressed as the inner product of $\mathbf{Z}$ and the unit vector.

Now, define a continuously differentiable production function, $Y = f(\mathbf{X})$, on $\mathcal{R}^k$, then

(a) the directional derivative of $f(\mathbf{X})$ at a point $\mathbf{X}$ along a direction l can be denoted by df/dl, where

$$\frac{df}{dl} = \lim_{\beta \to 0} \frac{f(\mathbf{X} + \beta\mathbf{Z}) - f(\mathbf{X})}{|\beta\mathbf{Z}|}, \tag{1}$$

l is the direction of $\mathbf{Z}$ and β is a real number.

(b) the gradient of $f(\mathbf{X})$ at a point $\mathbf{X}$ is a vector, ∇f, where

$$\nabla f = \left(\frac{\partial f}{\partial x_1}, \cdots, \frac{\partial f}{\partial x_k}\right) \tag{2}$$

(c) provided that ∇f exits, the directional derivatives and gradient defined earlier have the following properties:

Properties

(i) $\frac{df}{dl} = \frac{\nabla f \cdot \mathbf{Z}}{|\mathbf{Z}|}$, where l is the direction of $\mathbf{Z}$;

(ii) at a point $\mathbf{X}$, n, the direction of ∇f, is the normal direction of the isoquant which passes through $\mathbf{X}$;

(iii) $|\nabla f| = \max_l |\frac{df}{dl}|$, where $|\nabla f|$ is the norm of ∇f

3.2 The Technical Effect

Analogs to the two-factor production case, as shown in Figure 1, we assume the initial output is at point P_0, the corresponding input vector is $\mathbf{X}_0 = (x_{01}, \cdots, x_{0k})$ with norm $S_0 = \sqrt{\sum_{i=1}^k x_{0i}^2}$. After changes in inputs, the new output is at point P_1, the corresponding input vector is $\mathbf{X}_1 = (x_{11}, \cdots, x_{1k})$ with norm $S_1 = \sqrt{\sum_{i=1}^k x_{1i}^2}$. The incremental vector is $\Delta\mathbf{X} =$

$X_1 - X_0$. The total increment in output is denoted by $\Delta Y = Y_1 - Y_0$, where $Y_1 = f(X_1)$ and $Y_0 = f(X_0)$. It should be noted that the direction of ΔX must be fixed as from P_0 to P_1. We denote this direction by l. Once l is given, the ending point of vector X_1 must be on the line passing through point P_0 with direction l.

If there is no technical progress, the input will increase along the direction of X_0 from X_0 (P_0) to X_s (P_s), where the incremental vector is ρX_0 and $\rho > 0$ is a real number such that $(1+\rho)S_0 = S_1$. The output at P_s is $Y_s = f((1+\rho)X_0)$. It is clear, as demonstrated in a two-factor case, that the change from P_s to P_1 is due to technical progress. In other words, the movement from P_0 to P_1 can be separated into two steps. The first step is the movement from P_0 to P_s by the amount $\rho|X_0|$ in the direction of vector X_0. The second step is the movement from P_s to P_1 by the amount $|\Delta X_t|$. The magnitudes of input-combination at P_1 and P_s are equal, but the directions of the input vectors are different. Thus, the technical effect is reflected in the vector ΔX_t, while the scale effect is reflected in the vector ρX_0.

Definition 1: Let $Y = f(X)$ be a continuously differentiable production function defined on $\mathcal{R}^k$, the rate of technical progress along the direction l, t_l, at X_0 is defined by the following limitation:

$$t_l = \lim_{P_1 \to P_0} \frac{Y_1 - Y_s}{|\Delta X_t|}$$

where $\Delta X_t = \Delta X - \rho X_0$ and ΔX is the increment vector at X_0 along direction l.

The calculation of t_l is given in Theorem 1 below.

Theorem 1: If the production function possesses up to second order partial derivatives on its domain, the rate of technical effect can be calculated according to

$$t_l = u\frac{df(X_0)}{dl} - v\frac{df(X_0)}{dl_0} \tag{3}$$

where l_0 is the direction of $\mathbf{X}_0$, u and v are given by

$$u = \frac{\sqrt{\sum_{i=1}^k x_{0i}^2}}{\sqrt{\sum_{i=1}^k x_{0i}^2 - (\sum_{i=1}^k \alpha_i x_{0i})^2}}$$

$$v = \frac{\sum_{i=1}^k \alpha_i x_{0i}}{\sqrt{\sum_{i=1}^k x_{0i}^2 - (\sum_{i=1}^k \alpha_i x_{0i})^2}}$$

where $\alpha_i, i = 1, \ldots, k$ are the directional cosines of direction l.

Proof: By definition

$$\begin{aligned} \frac{Y_1 - Y_s}{|\Delta \mathbf{X}_t|} &= \frac{(Y_1 - Y_0) - (Y_s - Y_0)}{|\Delta \mathbf{X}_t|} \\ &= \frac{Y_1 - Y_0}{|\Delta \mathbf{X}|} \frac{|\Delta \mathbf{X}|}{|\Delta \mathbf{X}_t|} - \frac{Y_s - Y_0}{|\rho \mathbf{X}_0|} \frac{|\rho \mathbf{X}_0|}{|\Delta \mathbf{X}_t|} \end{aligned}$$

Clearly, when $P_1 \to P_0$, $|\Delta \mathbf{X}| \to 0$, $\rho \to 0$ and $|\Delta \mathbf{X}_t| \to 0$. Therefore

$$\frac{Y_1 - Y_0}{|\Delta \mathbf{X}|} \longrightarrow \frac{df(\mathbf{X}_0)}{dl}$$

and

$$\frac{Y_s - Y_0}{|\rho \mathbf{X}_0|} \longrightarrow \frac{df(\mathbf{X}_0)}{dl_0}$$

The remaining is to show that

$$\lim_{P_1 \to P_0} \frac{|\Delta \mathbf{X}|}{|\Delta \mathbf{X}_t|} = u$$

and

$$\lim_{P_1 \to P_0} \frac{|\rho \mathbf{X}_0|}{|\Delta \mathbf{X}_t|} = v$$

We know

$$\begin{aligned} \Delta \mathbf{X}_t &= \Delta \mathbf{X} - \rho \mathbf{X}_0 \\ &= |\Delta \mathbf{X}| \sum_{i=1}^k \alpha_i \mathbf{e}_i - \sum_{i=1}^k \rho x_{0i} \mathbf{e}_i \\ &= \sum_{i=1}^k (\alpha_i |\Delta \mathbf{X}| - \rho x_{0i}) \mathbf{e}_i \end{aligned}$$

thus

$$|\Delta \mathbf{X}_t| = \sqrt{\sum_{i=1}^{k}(\alpha_i|\Delta \mathbf{X}| - \rho x_{0i})^2} \tag{4}$$

On the other hand we have

$$\begin{aligned}
\mathbf{X}_1 &= \mathbf{X}_0 + \Delta \mathbf{X} \\
&= \sum_{i=1}^{k}(x_{0i} + \alpha_i|\Delta \mathbf{X}|)\mathbf{e}_i \\
\mathbf{X}_s &= \mathbf{X}_0 + \rho \mathbf{X}_0 \\
&= \sum_{i=1}^{k}(1+\rho)x_{0i}\mathbf{e}_i
\end{aligned}$$

However, it must be true that

$$|\mathbf{X}_1| = |\mathbf{X}_s|$$

i.e.

$$\sqrt{\sum_{i=1}^{k}(x_{0i} + \alpha_i|\Delta \mathbf{X}|)^2} = \sqrt{\sum_{i=1}^{k}(x_{0i} + \rho x_{0i})^2}$$

From the above equation we can obtain

$$\begin{aligned}
|\Delta \mathbf{X}| &= \sqrt{(\sum_{i=1}^{k}\alpha_i x_{0i})^2 + (2\rho + \rho^2)\sum_{i=1}^{k} x_{0i}^2} - \sum_{i=1}^{k}\alpha_i x_{0i} \\
&= \frac{\rho}{2}\frac{(2+\rho)\sum_{i=1}^{k} x_{0i}^2}{\sum_{i=1}^{k}\alpha_i x_{0i}} + O(\rho^2)
\end{aligned}$$

so

$$\frac{|\Delta \mathbf{X}|}{\rho} = \frac{(2+\rho)\sum_{i=1}^{k} x_{0i}^2}{2\sum_{i=1}^{k}\alpha_i x_{0i}} + O(\rho) \tag{5}$$

As $P_1 \to P_0$, $\rho \to 0$, by (4) and (5)

$$\begin{aligned}
\lim_{P_1 \to P_0} \frac{|\Delta \mathbf{X}|}{|\Delta \mathbf{X}_t|} &= \lim_{\rho \to 0} \frac{1}{\sqrt{\sum_{i=1}^{k}(\alpha_i - \frac{x_{0i}}{|\Delta \mathbf{X}|/\rho})^2}} \\
&= \frac{\sqrt{\sum_{i=1}^{k} x_{0i}^2}}{\sqrt{\sum_{i=1}^{k} x_{0i}^2 - (\sum_{i=1}^{k}\alpha_i x_{0i})^2}}
\end{aligned}$$

$$\lim_{P_1 \to P_0} \frac{|\rho \mathbf{X}_0|}{|\Delta \mathbf{X}_t|} = \lim_{\rho \to 0} \frac{\sqrt{\sum_{i=1}^k x_{0i}^2}}{\sqrt{\sum_{i=1}^k (\alpha_i \frac{|\Delta \mathbf{X}|}{\rho} - x_{0i})^2}}$$
$$= \frac{\sum_{i=1}^k \alpha_i x_{0i}}{\sqrt{\sum_{i=1}^k x_{0i}^2 - \left(\sum_{i=1}^k \alpha_i x_{0i}\right)^2}}$$

End of Proof.

Definition 2: Given a continously differentiable production function, $Y = f(\mathbf{X})$, the technical effect, TE, is defined as

$$TE = \lim_{P_1 \to P_0} \frac{t_l |\Delta \mathbf{X}_t|}{f(\mathbf{X}_0 + \Delta \mathbf{X}) - f(\mathbf{X}_0)} \tag{6}$$

The numerator of TE is the increase in output due to technical progress and the denominator is the overall increase in output. Therefore, TE measures the percentage contribution to output growth by technical progress. The value of TE can be obtained according to Theorem 2 given below.

Theorem 2: Let $Y = f(\mathbf{X})$ be a twice continously differentiable production function, then

$$TE = \frac{t_l}{u df(\mathbf{X}_0)/dl} \tag{7}$$

All the notations have their earlier definitions.

Proof: By the definition of TE, we have

$$TE = \lim_{P_1 \to P_0} \frac{t_l \left(|\Delta \mathbf{X}_t| / |\Delta \mathbf{X}|\right)}{\left(f(\mathbf{X}_0 + \Delta \mathbf{X}) - f(\mathbf{X}_0)\right) / |\Delta \mathbf{X}|}$$
$$= \frac{t_l}{u df(\mathbf{X}_0)/dl} \tag{8}$$

End of Proof.

3.3 The Scale Effect

Definition 3: Let $Y = f(\mathbf{X})$ be a continuously differentiable production function, the rate of scale effect corresponding to a change in inputs from $\mathbf{X}_0$ to $\mathbf{X}_s = (1+\rho)\mathbf{X}_0$ is defined as r, where

$$r = \lim_{\rho \to 0} \frac{f((1+\rho)\mathbf{X}_0) - (1+\rho) f(\mathbf{X}_0)}{\rho |\mathbf{X}_0|}$$

and ρ is determined such that $(1+\rho)|\mathbf{X}_0| = |\mathbf{X}_1|$.

In the above definition, $f((1+\rho)\mathbf{X}_0)$ is the output at the point $(1+\rho)\mathbf{X}_0$, which is on the ray from the origin of the coordinate system and passing through the point P_0. Since constant returns to scale implies that output increases at the rate ρ as all inputs increase at the same rate, the numerator in the above equation is the net scale effect caused by the increment in the magnitude of input-combination and the denominator is the net increment in the magnitude of input-combination. It is obvious that if $r > 0$, increasing returns to scale prevails. Conversely, $r < 0$ indicates decreasing returns to scale. When $r = 0$, we have the case of constant returns to scale.

Theorem 3: If the production function, $Y = f(\mathbf{X})$, possesses first order partial derivatives, the rate of scale effect, r, at point $\mathbf{X}_0$ can be expressed as

$$r = \frac{df(\mathbf{X}_0)}{dl_0} - \frac{Y_0}{\sqrt{\sum_{i=1}^{k} x_{0i}^2}} \tag{9}$$

where $\mathbf{X}_0 = (x_{01}, ..., x_{0k})$, l_0 is the direction of $\mathbf{X}_0$ and $Y_0 = f(\mathbf{X}_0)$.

Proof:

$$\begin{aligned}
r &= \lim_{\rho \to 0} \left[\frac{f(\mathbf{X}_0 + \rho \mathbf{X}_0) - f(\mathbf{X}_0)}{\rho |\mathbf{X}_0|} - \frac{f(\mathbf{X}_0)}{|\mathbf{X}_0|} \right] \\
&= \frac{df(\mathbf{X}_0)}{dl_0} - \frac{f(\mathbf{X}_0)}{|\mathbf{X}_0|} \\
&= \frac{df(\mathbf{X}_0)}{dl_0} - \frac{Y_0}{\sqrt{\sum_{i=1}^{k} x_{0i}^2}}
\end{aligned}$$

End of Proof.

Definition 4: Given a continously differentiable production function, $Y = f(\mathrm{X})$, the scale effect, SE, is define as

$$SE = \lim_{P_1 \to P_0} \frac{r|\rho \mathrm{X}_0|}{f(\mathrm{X}_0 + \Delta\mathrm{X}) - f(\mathrm{X}_0)} \tag{10}$$

The numerator of SE is the increase in output due to change in returns to scale and the denominator is the total increase in output. Therefore, SE measures the percentage contribution to output growth by changes in returns to scale. The scale effect can be computed according to the following theorem:

Theorem 4: Let $Y = f(\mathrm{X})$ be a twice continously differentiable production function, then

$$SE = \frac{r\sum_{i=1}^{k} \alpha_i x_{0i}}{S_0 df(\mathrm{X}_0)/dl} \tag{11}$$

where all the symbols have their earlier definitions.

Proof.

$$\begin{aligned} SE &= \lim_{P_1 \to P_0} \frac{\frac{r|\rho \mathrm{X}_0|}{|\Delta\mathrm{X}|}}{\frac{f(\mathrm{X}_0+\Delta\mathrm{X})-f(\mathrm{X}_0)}{|\Delta\mathrm{X}|}} \\ &= \frac{r}{df(\mathrm{X}_0)/dl} \lim_{P_1 \to P_0} \frac{|\mathrm{X}_0|}{|\Delta\mathrm{X}|/\rho} \\ &= \frac{r\sum_{i=1}^{k} \alpha_i x_{0i}}{S_0 df(\mathrm{X}_0)/dl} \qquad (12) \end{aligned}$$

End of Proof.

3.4 Calculation of Directional Derivatives

To compute the scale effect and technical effect or their rates, $df(\mathrm{X}_0)/dl$ and $df(\mathrm{X}_0)/dl_0$ have to be evaluated. To be able to do so, we make the following assumption:

Assumption 2: The direction of production expansion, l, is equivalent to the direction of the gradient of the production function.

With the property (iii) and assumption 2 given above, we have

$$\frac{df(\mathrm{X})}{dl} = |\nabla f| = \sqrt{\sum_{i=1}^{k}\left(\frac{\partial f(\mathrm{X})}{\partial x_i}\right)^2} \tag{13}$$

$$\frac{df(\mathrm{X}_0)}{dl} = \frac{\Delta Y}{|\Delta \mathrm{X}|} \tag{14}$$

where $\Delta Y = Y_1 - Y_0$ and $|\Delta \mathrm{X}| = \sqrt{\sum_{i=1}^{k}(x_{1i} - x_{oi})^2}$. The subscripts 0 and 1 represent the neighbouring observations. Similarly, given assumption 2 and the property (i) above, we can obtain

$$\frac{df(\mathrm{X}_0)}{dl_0} = \nabla f(\mathrm{X}_0)\frac{\mathrm{X}_0}{|\mathrm{X}_0|} \approx \frac{\Delta Y}{|\Delta \mathrm{X}|}\sum_{i=1}^{k}\alpha_{1i}\alpha_{0i} \tag{15}$$

where

$$\alpha_i = \frac{x_{1i} - x_{0i}}{\sqrt{\sum_{i=1}^{k}(x_{1i} - x_{0i})^2}}$$

$$\alpha_{0i} = \frac{x_{0i}}{\sqrt{\sum_{i=1}^{k} x_{0i}^2}}$$

The rational underlying assumption 2 lies in that if resources are limited and a producer aims at output maximisation, he has to continuously adjust their direction of production expansion towards the direction which maximises the increasing rate of output. This direction is equal to the direction of the gradient of his production function.

4 Consideration of Changes in Input Prices

Changes in some input prices may force a producer to alter resource allocation, thus lead to variations among input ratios. The result of input price change is a movement of an input vector along a given isoquant (changes in cost function only change its tangent point, not output level). Although input prices and technology progress may change simultaneously, from a methodological point of view, we can isolate their effects sequentially. The following aims at removing the effect of changes in prices of inputs on factor ratios, thus making the use of the measures developed in section 3 applicable here.

To show our approach diagrammatically, a simplified two-factor production process is depicted in Figure 2. This figure is very similar to figure 1 except that we first allow the initial production move from P_0 to P_0' corresponding to input price changes, and then expand from P_0' to P_1 corresponding to input increases and/or technical progress. If there is no technical progress, the expansion under new price ratios would be along the line OP_0' and reaches P_s. The input vector at P_0' can be denoted by $X_0' = (x_{01}', \cdots, x_{k1}')$. In a two-factor case, P_0' is where the slope of I_0 at P_0' is equal to the ratio of new prices of the two inputs. The P_0' in a multi-dimensional space can be located in the same way. It is clear that once the point P_0' is determined, we can simply substitute X_0' for X_0 and then compute r and t_l according to the procedures developed in section 3. The task in this section is thus to determine the movement from P_0 to P_0'.

[Figure 2 here]

4.1 Determination of the direction of $\overrightarrow{P_0'P_1}$

Let the price of x_i be p_i, maximisation of profit yields

$$\frac{\partial f(X)}{\partial x_j} = p_j \quad \text{without constraints;} \tag{16}$$

$$\frac{\partial f(X)/\partial x_i}{\partial f(X)/\partial x_j} = \frac{p_i}{p_j} \quad \text{with constraints} \tag{17}$$

for $i, j = 1, \cdots, k$. In either case, the j-th directional cosine of the normal direction at P_0', α_j', can be obtained as follows:

$$\alpha_j' = \frac{\partial f(X_0')/\partial x_j}{|\nabla f(X_0')|}$$

$$= \frac{\partial f(X_0')/\partial x_j}{\sqrt{\sum_{i=1}^{k}(\partial f(X_0')/\partial x_i)^2}}$$

$$= \frac{p_j}{\sqrt{\sum_{i=1}^{k} p_i^2}} \tag{18}$$

4.2 Determination of P_0'

Generally speaking, price change is a gradual process, so is the process of input adjustment. Consequently, it seems reasonable to approximate the isoquant, I_0, by a hyperplane that passes through P_0 and is parallel to the tangent-hyperplane at P_0'. The precision of approximation can be improved if the isoquant is approximated by a quadratic or higher order curved surface.

Under assumption 2, the direction of $\overrightarrow{P_0'P_1}$ is equivalent to the direction of I_0 at P_0'. Therefore, the intersection between $\overrightarrow{P_0'P_1}$ and the hyperplane can be used to approximate the point P_0'. The coordinates of the intersection must satisfy the following two equations:

$$\sum_{i=1}^{k} \alpha_i'(x_{0i}'' - x_{0i}) = 0 \tag{19}$$

$$\frac{x_{01}'' - x_{11}}{\alpha_1'} = \frac{x_{02}'' - x_{12}}{\alpha_2'} = \cdots = \frac{x_{0k}'' - x_{1k}}{\alpha_k'} \tag{20}$$

where $x_{0i}'', i = 1, \cdots, k$ are coordinates of the intersection. Solving (19) and (20) simultaneously gives

$$x_{0i}'' = \alpha_i' \sum_{j=1}^{k}(\alpha_j' x_{0j} + \frac{\alpha_j'}{\alpha_1'} x_{11} - x_{1j}) - (\frac{\alpha_i'}{\alpha_1'} x_{11} - x_{1i}), \quad i = 1, ..., k. \tag{21}$$

Obviously, $X_0'' = (x_{01}'', \cdots, x_{0k}'')$ can be used to approximate X_0' or X_0 in the calculation of SE and TE.

5 Concluding Remarks

To our best knowledge, Sato is the only one who successfully separated technical progress from scale effect (Sato and Nôno 1983). We have, in this paper, developed an alternative approach to separating and measuring scale effect and technical progress. The measures derived require little assumptions on the property of returns to scale and on production functions. Apart from that, our method has the advantage of being operational as long as they are two or more input-output observations. The measures are also more accurate than earlier ones as we use directional derivatives rather than ratios of increments in the definitions of the measures. It is, however, unclear to us which assumption is more restrictive: Sato's non-holothetic or our short-run non-uniform technical progress.

References

Beckmann, M.J. and R. Sato (1969), "Aggregate Production Functions and Types of Technical Progress: A Statistical Analyss," *American Economic Review*, 59(1): 88-101.

Binswanger, H.P. (1974), "The Measurement of Technical Change Biases with Many Factors of Production," *American Economic Review*, 64(6): 964-976.

Brown, M. and J. Popkin (1962), "A measure of Technological Change and Returns to Scale," *Review of Economics and Statistics*, 44(4): 402-411.

Färe, R., S. Grosskopf and E. C. Kokkelenberg (1989), "Measuring Plant Capacity, Utilization and Technical Change: a Nonparametric Approach", *International Economic Review*, 30(3): 655-666.

Hicks, J.R. (1965), *Capital and Growth*, Oxford, Clarendon.

Jorgenson, D.W. and Z. Griliches, (1967), "The Explanation of Productivity Change," *Review of Economic Studies*, 34: 249-283.

Kendrick, J.W. and R. Sato (1963), "Factor Prices, Productivity, and Economic Growth," *American Economic Review*, 53(5): 974-1003.

Kennedy, C. (1964), "Induced Bias in Innovation and the Theory of Distribution," *Economic Journal*, 74(295): 541-547.

Kennedy, C. and A. P. Thirlwall (1972), "Surveys in Applied Economics: Technical Progress", *Economic Journal*, 82(325): 11-72.

Kopp, R. J. and V. K. Smith (1985), "The Measurement of Non-neutral Technological Change," *International Economic Review*, 26(1): 135-159.

Salter, W.E.G. (1966), *Productivity and Technical Change*, Cambridge University Press (2nd edition).

Samuelson, P.A. (1965), "A Theory of Induced Innovations Along Kennedy-Weizsäcker Lines," *Review of Economics and Statistics*, 47(4): 343-356.

Sato, R. (1970), "The Estimation of Biased Technical Progress and the Production Function", *International Economic Review*, 11(2): 179-208.

Sato, R. (1980), "The Impact of Technical Change on the Holotheticity of Production Function," *Review of Economic Studies*, 47: 767-776.

Sato, R. (1981), *The Theory of Technical Change and Economic Invariance: Application of Lie Groups*, Academic Press, New York.

Sato, R. and M.J. Beckmann (1968), "Neutral Inventions and Production Functions," *Review of Economic Studies*, 35: 57-66. "An Addendum," 35: 366.

Sato, R. and T. Nôno, (1983), *Invariance Principles and the Structure of Technology*, Springgger-Verlag.

Solow, R. M. (1957), "Technical change and the Aggregate Production Function", *Review of Economics and Statistics*, 39(3): 312-320.

Solow, R.M. (1958), "A Skeptical Note on the Constancy of Relative Shares," *American Economic Review*, 48(4): 618-631

Solow, R.M. (1961), "Comment on Stigler" , *Output, Input and Productivity Measurement: Studies in Income and Wealth Series*, NBER, Princeton University Press: 64-68.

Solow, R.M. (1962), "Technical Progress, Capital Formation and Economic Growth", *American Economic Review Papers and Proceedings*, 52(1): 76-86.

Solow, R.M. (1963), *Capital Theory and the Rate of Return*, North Holland Publishing Co., Amsterdam.

Stevenson, R (1980), "Measuring Technological Bias", *American Economic Review*, 70(1): 162-173.

Stigler, G.J. (1961), "Economic Problems in Measuring Changes in Productivity", *Output, Input and Productivity Measurement: Studies in Income and Wealth Series*, NBER, Princeton University Press: 47-63.

Tinbergen, J. (1942), "Zur Theorie der langfristigen Wirtschaftsentwicklung", *Weltwirschaftliches Archiv.*

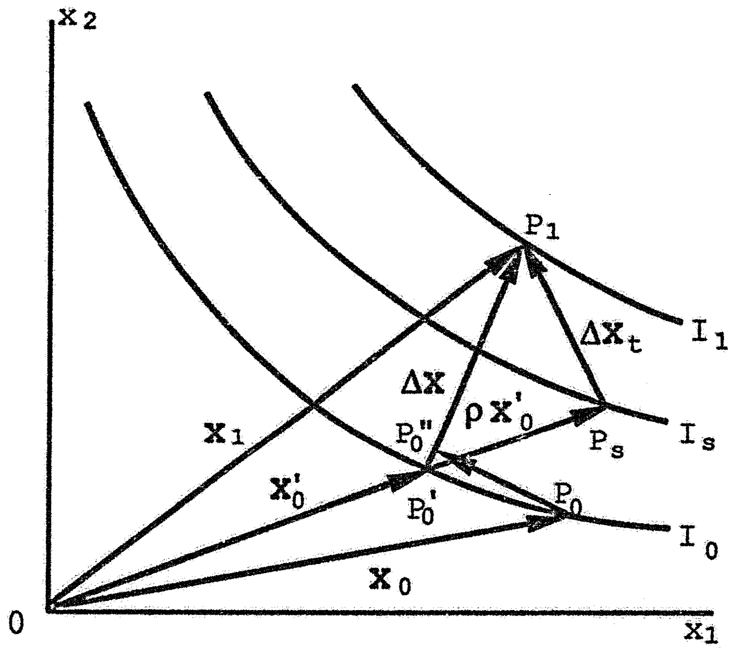

Figure 2

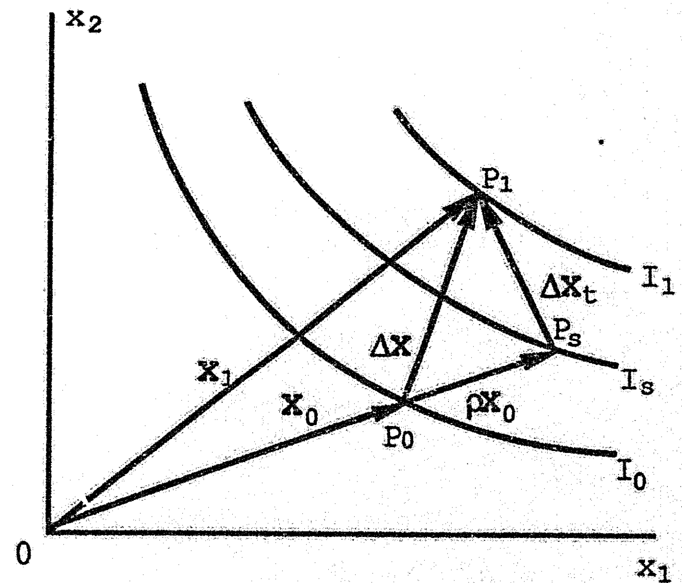

Figure 1