ADDING VALUE TO EXISTING MODELS OF INTERNATIONAL AGRICULTURAL TRADE

Thomas W. Hertel

Everett B. Peterson

and

Marinos E. Tsigas*

* Hertel and Tsigas are associate professor and research associate at Purdue University Peterson is assistant professor at Virginia Polytechnic and State University Hertel is currently on sabbatical leave with the IMPACT Project at Melbourne University He is funded by a Fulbright Senior Research Fellowship This paper was prepared for presentation at the 1991 Australian Agricultural Economics Society Meetings. University of New South Wales, Armidale, Feb 11-14

INTRODUCTION AND MOTIVATION

Numerical results from models of international agricultural trade have recently been in great demand, largely due to the high profile which agriculture has played in the Uruguay Round of the GATT negotiations. In fact, the same group of models has been called upon again and again to analyze the potential implications of liberalizing domestic and trade policies relating to farm and food products. They have ranged from partial equilibrium (e.g., OECD, Roningen and Dixit, Tyers and Anderson, Valdes and Zeitz) to multiple region, general equilibrium (Burniaux, *et al.*; Horridge and Pearce; Parikh, *et al.*; McDonald). The structure of many of these models, and their predictions, are summarized in a recent conference volume cosponsored by the World Bank and the OECD (Goldin and Knudsen, eds.).

With some notable exceptions (to be discussed below), the partial equilibrium models have tended to be multicommodity, numerical generalizations of the familiar, one good, supply-demand model used in introductory economics courses. This simplicity has a number of important virtues. First of all, it requires only information on traded commodity prices and quantities, as well as supply and demand elasticities, in order to make it operational. This has facilitated considerable disaggregation (both by countries and commodities). In some cases it has also permitted researchers to directly estimate the model's parameters (Tyers and Anderson). Both of these features help make the models more useful to policy makers.

A second advantage of the partial equilibrium, supply-demand formulation is that it corresponds directly to a diagrammatic representation of markets which is itself widely understood. Thus, numerical results which may be readily interpreted shifts in, and movements along, supply and demand schedules are easily explained. This has led to widespread use of these partial equilibrium models. Most important, this formulation improves communication and assimilation of model outcomes by policy makers.

By contrast, the general equilibrium trade models have relatively heavy data requirements, since the non-agricultural economy must also be described. Furthermore, they require the modeler to explicitly specify complete production and utility functions for all agents in all regions. In return for this extra effort, the modeler obtains an exhaustive accounting of economic activity and the welfare implications of policy shocks. Also, a wide variety of food and nonfood policies may be imbedded in the model. There are certainly some instances when this extra effort is justified, and it is not the purpose of this paper to pronounce one modeling approach preferable to the other.[1] Rather, the purpose of this paper here is to *bridge the gap* between these two approaches. In so doing, we hope to identify means of "adding value" to both partial and general equilibrium models of agricultural trade.

FOUR QUESTIONS

Many others have wrestled with the problem of relating general and partial equilibrium models. In a theoretical sense the issue is trivial. Any well-defined general equilibrium model can be reduced to a partial equilibrium variant by rendering selected variables exogenous. The opposite is also true. A well-defined partial equilibrium (PE) model may be expanded into its general equilibrium (GE) counterpart by endogenizing all

---

1 See Hertel (1990) for comparison and discussion of the pros and cons of the two modeling approaches.

prices and quantities. The problem of relating these two approaches is really a practical one. In particular, given the way *existing* PE and GE models are specified, how can the two be related?

In this context four important questions arise:

(Q1) To what degree are the matrices of supply/demand elasticities utilized in PE models compatible with the hypotheses of profit or utility maximization?

Here, issues of symmetry, homogeneity and adding-up restrictions arise, as well as more subtle questions regarding the relative magnitudes of elasticities and cost or revenue shares.

(Q2) To what degree can a PE reduction in agricultural output, for example, be related to resources leaving the farm sector altogether? I.e., how large is the contraction (expansion) effect implicit in a given PE model's supply elasticities?

Since factor markets are rarely broken out (some exceptions will be noted below), the answer to this question is unclear. Decomposition of uncompensated supply elasticities into their expansion and substitution (transformation) components would greatly facilitate interpretation of partial equilibrium model productions.

(Q3) When GE models generate vastly different results from their PE counterparts, to what extent can this be attributed to differing *partial equilibrium* assumptions about farm and food sector behavior?

For example, the supply elasticity for corn in most GE models is not explicitly specified. Rather it is a function of the modeler's assumptions about technology and factor mobility. Also, the GE farm level demand elasticity for corn is a function of the price-responsiveness of demand of all agents in the model (Hertel, Ball, Huang and Tsigas). Short of systematically perturbing the model, there is no single demand elasticity which one can compare to the PE model. This leaves great scope for partial equilibrium discrepancies between PE and GE outcomes, which may be erroneously attributed to "general equilibrium feedback effects".

(Q4) Finally, there is the broader issue of data base compatibility. To what degree can the two modeling approaches be built up from similar data bases?

This seems like an obvious means of economizing on data collection time, as well as enhancing the comparability of results from different models, but it has yet to be done on a broad scale. Partial equilibrium models of agricultural trade tend to be built up from supply-utilization tables, whereas general equilibria models begin with a country's input-output table. Merging the two may require a special type of model structure.

There has been considerable work to date which has addressed *parts* of (Q1) - (Q4) for selected models. For example, Horridge and Pearce completed the Tyers and Anderson model by adding a residual "other goods" commodity. They impose symmetry and homogeneity on the supply and demand matrices, thus obtaining a trade CGE model in which economic activity in each region is represented by a production possibility frontier. Ballenger and Krissoff attempted something similar on the demand side of USDA's SWOPSIM model. They too introduced a residual good and imposed homogeneity of degree zero.

A number of authors have addresed (Q1) and (Q2) for selected components of PE trade models. For example Zeitsch introduced a nested constant elasticity of substitution (CES) technology for deriving a complete demand system for livestock industries in the OECD's PE trade model.

Zeitsch also developed a methodology for deriving a complete system of supply and demand elasticities for a multiproduct agricultural industry based on information on a selected subset of parameters. Liapis has implemented this methodology for the U.S. component of USDA's SWOPSIM model. Surry has proposed an alternative methodology for calibrating multiproduct technology in partial equilibrium models of agricultural trade. There have also been attempts to clarify individual relationships among products in parts of these models. For example, Haley has derived restrictions on the multiproduct supply elasticities for several subsectors in the SWOPSIM model.

General equilibrium modelers have also occasionally made an attempt to develop the partial equilibrium structure of their GE models in more details. A good example of this is provided by research on the agricultural sector of the ORANI model (Dixon *et al.*) where econometrically estimated multiproduct technologies are used to summarize supply behavior in each of the model's regions. Hertel, Ball, Huang and Tsigas explore the farm level demand elasticities implicit in a U.S. CGE model for which both supply and preference relationships have been econometrically estimated.

There are many other examples of attempts to address (Q1) - (Q4). What this paper seeks to do is provide a specific methodology for relating *all* the parameters of a representative PE trade model back to explicit preference and technology parameters. This in turn establishes a mapping between partial and general equilibrium trade models which may be exploited, both for purposes of sharing data bases, as well as for comparing model outcomes.

AN OVERVIEW OF A REPRESENTATIVE PE TRADE MODEL

In developing the modeling methodology proposed in this paper, it will be useful to refer to a specific numerical example. Towards this end, we will focus on the United States' component of USDAs SWOPSIM model. There are several reasons choosing this model. First of all, it is publicly available. Consequently, it is also the most widely used and exhaustively documented of the partial equilibrium agricultural trade models (Roningen; Roningen and Dixit; Sullivan, Wainio and Roningen; Gardiner, Roningen and Liu). In addition, the SWOPSIM framework has been used to replicate the results from a number of other widely cited partial equilibrium trade models (Magiera and Herlihy).

Data and Parameter Availability

The basic information employed by the SWOPSIM model includes the following parameters for each region (with our associated notation in parentheses):

- Matrices of aggregate supply and demand elasticities.
- A vector of income elasticities of demand.
- Quantity shares reflecting the relative share of total demand for a given commodity k (q_D^k), going to intermediate use j, i.e., $\alpha_{kj} = q_{Dj}^k)/q_D^k$.

And the following variables:

- Prices for producers (p_S), consumers (p_D),domestic market (p_M)traded commodities (p_T) and the world market (p_W), and the associated policy "wedges."
- Quantities produced (q_S), consumed (q_D) and traded ($q_D = q_S - q_D$).

All of this information for a representative region (the U.S.), is summarized in table 1. It is taken from SWOPSIM's 1989, 3 region "demonstration model" benchmark equilibrium.[2] Similar data are available for a great number of other regions which have been studied with this model.

The elasticities in table 1 were assembled from a variety of sources (Gardiner, Roningen and Liu). They are intended to be consistent across regions in terms of the implied period of adjustment (3-5 years). However, as questions (Q1) and (Q2) have posed, are these elasticities consistent with profit and/or utility maximization? To answer this question, one needs to determine whether they can be related to some underlying set of preferences or technology. If so, one can then use this information about the underlying technology and preferences to add further structure to the model and aid in its interpretation. This additional structure will also assist us in addressing (Q3) and (Q4).

A GENERIC FARM AND FOOD ECONOMY

In order to make any progress on these issues, it is necessary to have some general vision about the structure of the farm and food sector in a generic region. Figure 1 outlines one such "view of the world" which is fairly general, yet can be easily related to the information reported in table 1.

Consumer Demand

In figure 1, consumer preferences within a given region are specified over all consumption items in the form of an aggregate expenditure function. Since most agricultural trade models do not exhaust

---

2 The three regions in this demonstration model are the United States, the European Community, and the rest of the world.

Table 1. Benchmark Data for the United States from the SWOPSIM Model.

E_S: The Matrix of Supply Elasticities

	BF	PK	ML	PN	PE	DM	DB	DC	DP	WB	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Sum
BF	.65	-.01				.02					-.07	-.01			-.01								.57
PK	-.03	1.00		-.01						-.01	-.34	-.06			-.12			-.01					.49
ML			.80								-.29	-.04											.47
PN		-.01		.65	-.02					-.02	-.08	-.01			-.08			-.01					.42
PE				-.05	.55					-.02	-.13	-.02			-.06			-.01					.27
DM	.03					.50					-.05	-.01			-.01								.45
DB						-.19	.43	-.53	.34														.05
DC						-.19	-.11	.44	-.09														.05
DP						-.19	.43	-.53	.34														.05
WB										.60	-.25	-.06		.05			-.03				-.02		.30
CN										-.10	.48	-.01		-.07									.31
CG										-.15	-.04	.60		-.09			-.05						.27
RI													.40										.40
SB										.04	-.14	-.03		.60						-.05	-.01	-.01	.40
SM														-.35	.25	.15							.05
SO														-.35	.25	.15							.03
OS										-.08		-.05					.55			-.08			.34
OM																	-.55	.30	.30				.05
OO																	-.55	.30	.30				.05
CT														-.12			-.06			.65			.47
SU										-.07				-.06							.45		.32
TB														-.06								.25	.19

Prices and Quantities

	P_W	P_N	P_S	P_D	P_T	Q_S	Q_D	Q_T
BF	2544	2579	2809	4675	2543	10560	11139	-579
PK	2054	1367	1451	2730	1367	7224	7587	-363
ML	2316	2394	2610	4788	2386	153	177	-24
PN	1116	1106	1175	2011	1096	9994	9568	426
PE	2029	1150	1209	1909	1140	4012	3940	72
DM	345	295	316	587	240	65850	65850	
DB	3053	2399	2899	3444	1527	570	533	37
DC	3153	3060	3060	4234	1793	2570	2675	-105
DP	2281	3369	3369	3543	1141	390	231	159
WB	171	137	169	193	132	55571	21443	34128
CN	113	94	127	104	94	193788	139251	54537
CG	109	83	110	92	83	30510	23385	7125
RI	320	165	308	322	165	4967	2618	2349
SB	285	206	228	217	206	52704	37055	15649
SM	255	170	170	212	170	23995	19368	4627
SO	433	452	452	904	452	5561	4910	651
OS	933	467	509	480	467	6996	6748	248
OM	204	185	185	231	185	1862	1944	-82
OO	749	461	631	882	441	787	1514	-727
CT	1640	1412	1872	2824	1411	2835	937	1898
SU	371	303	333	604	190	6260	7651	-1391
TB	3636	3583	3854	7166	3583	560	595	-35

E_D: The Matrix of Demand Elasticities

	BF	PK	ML	PN	PE	DM	DB	DC	DP	WB	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SU	TB	Sum
BF	-.70	.05	--	.03																			-.62
PK	.13	-.86	--	.03																			-.70
ML	--	.20	-.70	--																			-.50
PN	.08	.03	--	-.56																			-.45
PE		(-.58)			-.35																		-.35
DM		(.60)				-.17	.01	.04	.01														-.11
DB						--	-.63	--	--														-.63
DC						--	--	-.60	--														-.60
DP						--	--	--	-.65														-.65
WB								(-.32)		-.59	.15	.07	--										-.38
CN								(.33)		.04	-.74	.03	--		.02						.01		-.62
CG										.12	.17	.94	--										-.65
RI										--	--	--	-.25										-.25
SB											(-.20)			-.35	.18	.11	--	--	--				-.06
SM											.07(.22)			--	-.83	--	--	.03	--				-.73
SO														--	--	-.37	--	--	.15				-.22
OS														--	--	--	-.30	.06	.06				-.08
OM														--	.23	--	--	-.96	--				-.73
OO														--	--	.50	--	--	-.69				-.19
CT																(-.19)				-.20			-.20
SU											.09					(.84)					-.24		-.15
TB																						-.20	-.20

n: Income Elasticities

BF	.16
PK	.05
ML	.13
PN	.11
PE	.05
DM	-.07
DB	.10
DC	.33
DP	.30
WB	-.15
CN	-.12
CG	-.08
RI	.14
SB	.22
SM	.30
SO	.10
OS	.17
OM	.30
OO	.05
CT	.40
SU	.04
TB	.22

A: Feed Shares

	SBF	SPK	SML	SPN	SPE	SDM
BF						
PK						
ML						
PN						
PE						
DM						
DB						
DC						
DP						
WB	.05	.03		.07	.04	.03
CN	.21	.25	.01	.10	.08	.15
CG	.24	.27	.01	.11	.08	.16
RI						
SB						
SM	.11	.31		.36	.12	.10
SO						
OS						
OM	.11	.31		.36	.12	.10
OO						
CT						
SU						
TB						

food products, a distinction must be drawn between those food items covered by the model (endogenous food consumption), and those which are not (other food). Typically food products are less income elastic than nonfood items. Furthermore, the household's budget constraint implies some important restrictions across food and nonfood elasticities. Thus it is important to think about the entire budget allocation problem.

Agriculture as a Multiproduct Industry

Agricultural production, as depicted in figure 1, is divided into several parts. The practical rationale for this specification will become clear momentarily. The core of the agricultural technology in this hypothetical region is summarized by an aggregate revenue function. Application of Hotelling's lemma generates product supplies, conditional on the available resource endowment. Partial differentiation of the revenue function with respect to the agricultural endowment generates its shadow value. If this is less than the value of a unit of the resource in the nonagricultural sector, then we will expect a gradual migration of resources from the farm to nonfarm sectors. The degree of resource mobility in a region is governed by an elasticity of transformation (σ^T), which measures the ease with which the region's aggregate resource endowment is shifted between the two sectoral uses.

This specification of agricultural technology highlights the distinction between two components of supply response, namely the substitution effect and the expansion effect. If $\sigma^T = 0$, then the agricultural resource base is fixed, as is likely true in the short run. In this case, agricultural supply response is entirely a function of farmers' willingness to divert resources from one product to another in response to relative price changes. With the exception of joint products such as wool and mutton, and soyoil and soymeal, we expect *a priori* that products will be (net)

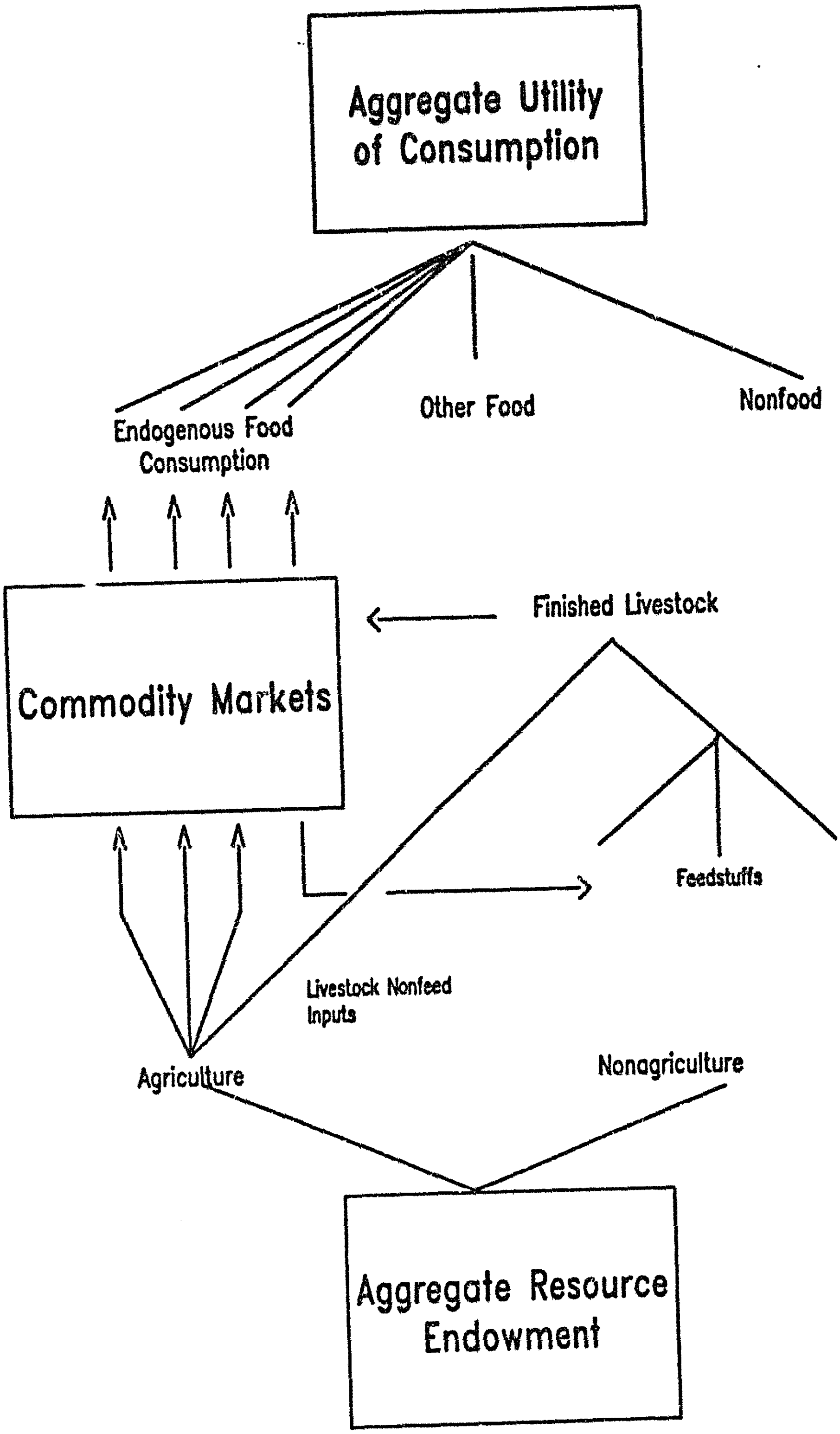

Figure 1. The Structure of a Generic, Regional Economy

substitutes in the short run. Indeed the off-diagonal elements in the N_S matrix at the top of table 1 exhibit this sign pattern. For this reason they will be used to calibrate a revenue function. In the longer run, σ^T is strictly negative, and commodity supply response will include an expansion effect. As σ^T tends to minus infinity, we move to the very long run, where the rental rate on resources in agriculture is determined by the opportunity cost associated with their use in the nonfarm economy. In this case (gross) complementarity is the anticipated outcome [Ball; Hertel (1987)].

Before leaving this discussion of the aggregate agricultural revenue function, it is important to explain why no distinction is drawn between fixed and variable inputs. Clearly there are some factors of production which are responsive to relative prices, even in the very short run. The problem is that table 1 contains no information on variable input usage. (Feed use is an exception and will be discussed momentarily.) While such information is readily obtained for the U.S. and Australia, such is not the case for many regions covered by models of international trade. For this reason we aggregate all inputs into a single endowment. This specification seems adequate for analyzing broad questions of trade policy. Further refinements are easily introduced by replacing the revenue function with a restricted profit function which give rise to a set of variable input demand equations.

Livestock "Assembly" Sectors

The multiproduct treatment of agriculture outlined above has one significant drawback from the point of view of agricultural trade modeling. If feed inputs are subsumed into the maximum revenue function, such that feedgrain supplies are supplies *net* of on-farm use, then there is no possibility for incorporating information about the relative feed intensities of different livestock sectors (the matrix A with elements $\{\alpha_{kj}\}$ in table 1).

Furthermore, it becomes difficult to replicate the sizable (and differential) cross-price relationships between feedstuffs and livestock supplies displayed in table 1. For this reason, we introduce the concept of a "livestock assembly sector".

The role of the livestock assembly sectors in figure one's conceptualization of a regional economy is to combine feed and nonfeed inputs into a finished livestock product. The nonfeed input is derived from the aggregate revenue function and thus competes with other farm activities for the aggregate endowment. In a practical sense, one can think of a short - to medium-run increase in pork production as bidding farm labor and capital away from other livestock and crop activities. The degree to which this can be done will determine the supply response of pork production, given exogenous feed prices. By having a distinct assembly sector for each livestock product, we can incorporate information about relative feed intensities and cross-price elasticities, by commodity. This will be demonstrated below.

Food Processing Activities

A final aspect of the representative region's technology involves the further processing of certain raw products. This is not shown explicitly in figure 1, but it can be thought of as part of the activity occuring in the "commodity markets" block. The model outlined in table 1 has relatively little food processing detail, but it is present in the case of dairy and oilseeds products. In our framework, these activities are handled in a very simple manner. Each of these sectors combines, in fixed proportions, the raw product with a "nonagricultural" input, to produce an composite input. This in turn is used to produce multiple processed outputs. In the case of the oilseed processing sectors, oil and meal are produced in fixed proportions. The same is true of butter and skimmed milk powder.

However, there is nonzero elasticity of transformation between fluid milk, cheese, and the butter/skimmed milk powder composite.

Wholesale/Retail Margins

At this point in the marketing chain, all products are evaluated at producer prices. In order to get to the (higher) consumer prices reported in table 1, a marketing margin must be added. Ideally this would be the outcome of the purchase and resale of the products by wholesale and retail sectors. (For an example of how this might be modeled, see Peterson *et al.*) Lacking detail on these other marketing activities, we adopt a very simple bridge between producer and consumer prices. In particular, we postulate a Leontief marketing technology, whereby the producer good is combined in fixed proportions with resources from the nonfarm sector to produce the consumer product. In this case the nonfarm input requirement per unit of output (measured in dollars) may be shown to simplify to the difference between consumer and producer prices.

CALIBRATION OF PREFERENCES AND TECHNOLOGY: ISSUES AND INSIGHTS

Having outlined the general structure of a representative, partial equilibrium trade model, the issue of calibration needs to be addressed. That is, how can a parameteric link be drawn between figure 1 and table 1? This necessitates specifying a particular functional form to represent technology and preferences, thereafter establishing a mapping from the data in table 1 to the parameters of the underlying revenue and expenditure functions. In the process we hope to generate same insights about the consistency of these data with the basic postulates of economic theory, as well as certain accounting identities.

Calibration of Preferences

Consider in more detail the N_D matrix. This is essentially block-diagonal, with the blocks capturing cross-price relationships among closely related goods. In particular meats, dairy products, grains, and oilseeds all show distinct groupings. Otherwise almost all of the off-diagonal elements in this matrix are zero. This pattern of elasticity entries suggests that some type of nesting of preferences may be useful. For example, consumers could be modeled as allocating expenditures to the meat aggregate, thereupon determining the composition of meat consumption solely on the basis of relative prices within this aggregate. Following the natural pattern of aggregation suggested by the elasticities in the N_D matrix, we arrive at seven separable groups of food items: meats and eggs, dairy products, grains, oilseeds and associated products, cotton, sugar and tobacco.[3] The aggregate own-price elasticity of final demand for each of these composites is given in brackets [•] in table 1. Each of the numbers in parentheses (•) is the implied (constant) elasticity of substitution within each composite.[4]

---

3 Before using these elasticities to calibrate preferences, it was first necessary to extract the price responsiveness in demand which may be attributed to intermediate uses treated elsewhere in our model In particular, intermediate demands by the dairy and oilseed processing sectors and those of the livestock sectors must be netted out of N_D. We do this by weighting the elasticities in each row by the ratio of the own-price elasticity of final demand not shown to the own-price elasticity of total demand. For example, in the case of wheat the ratio is -0.30/-0.59. Now, by summing across the rows within each block and expenditure-share-weighting these sums, we arrive at the aggregate own-price elasticities of final demand for each group. This is reported in brackets [•] below each block. Since intermediate demands are more price-responsive than final demands, and since all of the commodities within the blocks are substitutes, these aggregate elasticities are smaller, on average, than the individual diagonal elements of N_D.

4 The relationship between the (assumed constant) elasticity of substitution in consumption (σ_D) on the one hand, and the compensated (v_{ij}) and uncompensated (ε_{ij}) price elasticities and the income elasticity of demand (η_i) on the other hand, is as follows:

The predominance of zero cross-price elasticities between commodities in different blocks of the demand matrix undoubtedly reflects the absence of existing information on these parameters. However, zero is itself parameter setting! It is also not a very plausible choice. Consider, for a moment, the formula for the uncompensated demand elasticity for good i, given a change in the price of good j:

$$\varepsilon_{ij} = S_j(\sigma_{ij} - \eta_i)$$

where σ_{ij} is the partial elasticity of substitution between i and j, S_j is the budget share of good j and η_i is the income elasticity of demand for good i. Even if there is no direct substitutability between the two goods ($\sigma_{ij} = 0$), the income effect of a change in the price of j will affect the demand for good i. That is, $\varepsilon_{ij} = - S_j \eta_i$. In fact, the only utility function which will generate $\varepsilon_{ij} = 0$ is the Cobb Douglas, whereby $\sigma_{ij} = \eta_i = 1$. But this restriction contradicts all of the other own-price elasticities and income elasticities in table 1. In order to resolve this inconsistency within the N_D matrix, an alternative utility function must be specified.

A more general restriction on preferences, which is compatible with non-unitary income elasticities of demand, is that implied by the Stone-Geary utility function. The resulting Linear Expenditure System

---

$$\sigma_D = S_j^{*-1} v_{ij} = S_j^{*-1} (\varepsilon_{ij} + S_j \eta_i).$$

where S_j is the budget share associate with good j, and S_j^* is the share within the nest. By taking a share-weighted average of the right-hand side of this expression for all *non-zero* off-diagonal elements within a given block in the N_D matrix of table 1, we obtain an "average" degree of substitutability for each of the blocks. These averages are given in parentheses (•) below the aggregate own-price elasticity of demand for each block. The implied elasticity of substitution among meats was raised from 0.38 to 0.60 in order to make the disaggregated elasticities in the compensated matric positive (i.e., net substitutes).

(LES) also includes one free substitution parameter (the so-called "Frisch parameter") which may be used to replicate one of the own-price elasticities of demand. However, there is no guarantee that the other own-price elasticities will even be close in value to those given in table 1. This makes it difficult to compare results across and LES-based model and the type of PE trade model displayed in table 1. Furthermore, since estimates of the own-price elasticity of demand by commodity and region are fairly widely available, it would be a pity to choose a representation of preferences which precludes incorporation of such information.

The LES restricts substitution relationships by virtue of its *explicitly additive* form. That is, it is additive in prices. This is also the case with Houthakker's indirect addilog utility function, as well as the constant elasticity of substitution (CES) utility function which is often employed in CGE models. At the other extreme, are the demand systems which are fully flexible, in the sense that they N(N - 1)/2 free substitution parameters (where n is the number of commodities), such that the utility function may be calibrated to any arbitrary set of elasticities (provided they exhibit the basic properties of symmetry, homogeneity and concavity). An example of this is the translog indirect utility function. From the point of view of applied trade modeling, a major problem with these flexible functional forms is that they have *too many* free parameters. Furthermore, they offer no particular guide to limiting these free parameters to a more manageable subset.

From a practical point of view, it would be attractive to have a representation of preferences which had N free parameters. This is precisely the number of parameters needed to replicate the group-wise own-price elasticities of demand in table 1. Fortunately, Hanoch recognized this gap and filled it in the early 1970's. He introduced a class of *implicitly* additive preference relationships with precisely N substitution parameters (Hanoch, 1975; see also Surry for an agricultural application). One specification is the Constant Ratio Elasticity of Substitution (CRES) utility function and the second is the Constant Difference Elasticity of Substitution (CDE) expenditure function. As it represents a dual approach, the CDE is easier to work with. It also is somewhat more general in the degree of cross-price responsiveness which can be incorporated (Hanoch, 1975, pp. 411-412). Consequently, we have chosen to use a CDE expenditure function to represent preferences, thereby generating off-diagonal elements for N_D.

One likely reason why the CDE representation has not been more widely applied in the last fifteen years is that it is an *implicit* functional form. Specifically, the CDE may be written as follows:

$$G(\mathbf{z}, u) = \sum_{i=1}^{N} B_i u^{e_i b_i} z_i^{b_i} = 1 , \tag{1}$$

where $z_i = p_i/E(\mathbf{p}, u)$, and we require that B_i, $e_i > 0$, $b_i < 1$, with either $0 < b_i < 1$ or $b_i < 0$ for all i. All prices (p_i) are scaled by minimum household expenditures, given prices and utility $[E(\mathbf{p}, u)]$. Thus it is not possible to solve (6) for expenditures as an *explicit* function of $\mathbf{p}$ and u. Also, note that p_j enters both the numerator of z_j and the demoninator of z_i. In economic terms, this implies that the effect of a change in p_j on optimal demands enters both directly through z_j *and* indirectly through a change in the general price index affecting z_i's.

In order to explore the implications of (1) for consumer demand elasticities, it is necessary to derive the conditional demand for good i, by application of Shephard's lemma (and the implicit function theorem) to (1):

$$x_i(p, u) = B_i b_i u^{b_i e_i} z_i^{(b_i - 1)} \Big/ \sum_{k=1}^{N} B_k b_k u^{b_k e_k} z_k^{b_k} . \qquad (2)$$

Hanoch shows that the compensated price elasticities of demand associated with (2) are given by:

$$v_{ij} = s_j \left[\alpha_j - \frac{e_i}{\sum_k s_k e_k} (1-\alpha_i) - \frac{\sum_k s_k e_k \alpha_k}{\sum_k s_k e_k} \right] - \delta_{ij} \alpha_j . \qquad (3)$$

where $\alpha_i = 1 - b_i > 0$, and $\delta_{ii} = 1$ and $\delta_{ij} = 0$ for $i \neq j$. The income elasticities of demand are as follows:

$$\eta_i = \left(\sum_k s_k e_k \right)^{-1} \left[e_i (1 - \alpha_i) + \sum_k s_k e_k \alpha_k \right] + \left(\alpha_i - \sum_k s_k \alpha_k \right) \qquad (4)$$

Since preferences, as summarized in (1) - (4), are specified over *all* goods, it is necessary at this point to supplement the data in table 1 with information on total food and nonfood expenditures in order to obtain S_j for the other food and nonfood categories. We estimate[5] that the 1989 budget share for all food products is 0.13 in the United States, of which

---

5 Based on table 10 (structure of consumption) in the 1989 World Development Report, the private household expenditure share of total food products for the U.S. is 0.13. Thus the nonfood share is 0.87. The expenditure shares of the commodities included in the SWOPSIM model are calculated by multiplying the retail price (P_D) by the quantity consumed (q_D) and dividing by total consumption expenditures. The level of total domestic consumption expenditures is obtained by multiplying total domestic income in the SWOPSIM model by the share of private consumption given in table 9 of the World Development Report (0.66). By subtracting the expenditure shares of the commodities in the SWOPSIM model from .13, one gets the share of other food products (.07664).

0.077 (more than half) is "other food", i.e. products not covered by the SWOPSIM model. This result is somewhat surprising in light of the fact that 85% of the value of all U.S. farm output is covered by the model.

Price and income elasticities for the non-SWOPSIM goods must also be specified. Here we take the simple approach of assigning to the "other food" category the average own-price elasticity of demand for commodities covered by the model. This is ε_{ii} = -0.46349. The income elasticity of demand for other food is assigned a value of 0.4 because this category included many highly processed products which are deemed more income responsive. The income elasticity of demand for nonfood items is derived via Engel aggregation. Finally, the own-price elasticity of demand for nonfood items is rather closely circumscribed by the remaining parameters, and may be left free for the moment.

Calibration of the CDE implicit expenditure function proceeds in three steps (see Hertel *et al.* for an exhaustive discussion of this issue):[6]

Step one: derivation of the substitution parameters $b_i = (1 - \alpha_i)$ from the compensated own-price elasticities of demand and the shares.

Step two: derivation of the expansion parameters (e_i) from the income elasticities of demand, the shares and the substitution parameters.

Step three: derivation of the shift parameters (B_i) from the demand quantities and all of the preceeding information.

In the process of calibrating the CDE preference parameters to the base elasticities, budget shares, and quantities, it is entirely possible that

---

6 While expressions (2) - (4) are nonlinear in the parameters, a series of clever permits steps one to three to be acomplished entirely via linear operations normalizations.

the sign restrictions on these parameters might be violated. Such an occurence implies an inconsistency in the models' parameters, given the maintained hypothesis of utility maximization subject to implicitly additive preferences. This might occur, for instance, if a commodity with a very large budget share were also assigned a relatively large compensated own-price elasticity of demand (v_{ii}). In this case the calibrated α_i would be negative and it would be necessary to reduce the absolute value of v_{ii}. In fact, in the example at hand, this consistency restriction restricts the value of v_{ii} for the nonfood aggregate lie between -0.057 and -0.067.

The extremely small, even negative, values for the income elasticities of demand for many of the food products pose the most severe problem for calibration. They result in numerous negative expansion parameters ($e_i < 0$) which violates the regularity restrictions for the CDE utility function. Consequently, the income elasticities of demand have been adjusted upwards until all e_i's were positive. These revised values are listed in table 2A along with the compensated price elasticities of demand and the calibrated preference parameters. The complete matrix of compensated demand elasticities (v_{ij}) for the aggregate demand system is given in table 2B. Finally, tables 3A and 3B give the full matrices of disaggregate demand elasticities.

Calibration of Agricultural Technology

We assume, for the reasons stated above, that the supply elasticities in the N_S matrix in table 1 are *conditional* on a fixed aggregate factor endowment. Our task is then to calibrate an aggregate agricultural revenue function which will replicate the supply behavior for raw farm products depicted in table 1. At this stage we abstract form the own- and cross-price elasticities involving processed products, which will be handled below. Also, we temporarily ignore the cross-price relationships between

Table 2. Calibration of Aggregate Preferences for the United States

A. The CDE Implicit Expenditure Function

Aggregate Commodity	Step 1			Step 2		Step 3
	s	ν	α	q	e	B
Meats	.0341	-.577	.599	.669	.128	.056
Dairy	.0103	-.317	.318	.351	.027	.013
Grains	.0025	-.204	.204	.25	.037	.003
Oilseeds	.0027	-.189	.189	.234	.035	.003
Cotton	.0009	-.20	.20	.4	.209	.001
Sugar	.0016	-.146	.146	.2	.043	.002
Tobacco	.0014	-.20	.199	.25	.042	.002
Other Food	.0766	-.433	.46	.6	.21	.109
Nonfood	.87	-.063	.584	1.064	1.0	1.40

Total Expenditures = $2.9479 Trillion

B. The Matrix of Aggregated, Compensated Demand Elasticities

	Price								
Commodity	Meats	Dairy	Grains	Oilsds.	Cot.	Sugar	Tob.	Other	Nonfood
Meats	.57716	.0036	.0006	.0006	.0002	.0003	.0003	.0376	.5340
Dairy	.0108	-.3169	-.0001	-.0002	-.0000	-.0002	-.0000	.0160	.2895
Grains	.0069	-.0007	-.2042	-.0005	-.0002	-.0003	-.0002	.0073	.1906
Oilseeds	.0064	-.0009	-.0005	-.1891	-.0002	-.0004	-.0003	.0062	.1774
Cotton	.0068	-.0008	-.0005	-.0006	-.1997	-.0004	-.0003	.0067	.1868
Sugar	.0045	-.0013	-0.0006	-.0007	-.0002	-.1463	-.0003	.0029	.1402
Tobacco	.0068	-.0008	-.0005	-.0006	-.0002	-.0004	-.1997	.0070	.1868
Other Food	.0155	.0019	.0002	.0001	.0001	.0000	.0001	-.4332	.4137
Nonfood	.0208	.0035	.0006	.0006	.0002	.0003	.0003	.0365	-.0626

Table 3A. Disaggregate Compensated Demand Elasticities

	BT	PK	ML	PM	PL	DM	DB	DC	DP	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SK	TB	OF	MF
BT	-0.58815	0.00471	0.00019	0.00437	0.00171	0.00192	0.00022	0.00134	0.00010	0.00026	0.00023	0.00002	0.00007	0.00006	0.00000	0.00033	0.00010	0.00000	0.00010	0.00021	0.00028	0.00033	0.03758	0.53483
PK	0.01185	-0.59529	0.00019	0.00437	0.00171	0.00192	0.00022	0.00134	0.00010	0.00026	0.00023	0.00002	0.00007	0.00006	0.00000	0.00033	0.00010	0.00000	0.00010	0.00021	0.00028	0.00033	0.03758	0.53483
ML	0.01185	0.00471	-0.59591	0.00437	0.00171	0.00192	0.00022	0.00134	0.00010	0.00026	0.00023	0.00002	0.00007	0.00006	0.00000	0.00033	0.00010	0.00000	0.00010	0.00021	0.00028	0.00033	0.03758	0.53483
PM	0.01185	0.00471	0.00019	-0.59353	0.00171	0.00192	0.00022	0.00134	0.00010	0.00026	0.00023	0.00002	0.00007	0.00006	0.00000	0.00033	0.00010	0.00000	0.00010	0.00021	0.00028	0.00033	0.03758	0.53483
PL	0.01185	0.00471	0.00019	0.00437	-0.59879	0.00192	0.00022	0.00134	0.00010	0.00026	0.00023	0.00002	0.00007	0.00006	0.00000	0.00033	0.00010	0.00000	0.00010	0.00021	0.00028	0.00033	0.03758	0.53483
DM	0.00359	0.00223	0.00009	0.00207	0.00081	-0.32304	0.00081	0.00300	0.00036	-0.00005	-0.00005	.00000	-0.00001	-0.00002	0.00000	-0.00009	-0.00003	0.00000	-0.00003	-0.00005	-0.00016	-0.00007	0.01604	0.28955
DB	0.00359	0.00223	0.00009	0.00207	0.00081	0.00718	-0.32940	0.00300	0.00036	-0.00005	-0.00005	.00000	-0.00001	-0.00002	0.00000	-0.00009	-0.00003	0.00000	-0.00003	-0.00005	-0.00016	-0.00007	0.01604	0.28955
DC	0.00359	0.00223	0.00009	0.00207	0.00081	0.00718	0.00081	-0.32521	0.00036	-0.00005	-0.00005	.00000	-0.00001	-0.00002	0.00000	-0.00009	-0.00003	0.00000	-0.00003	-0.00005	-0.00016	-0.00007	0.01604	0.28955
DP	0.00359	0.00223	0.00009	0.00207	0.00081	0.00718	0.00081	0.00300	-0.32985	-0.00005	-0.00005	.00000	-0.00001	-0.00002	0.00000	-0.00009	-0.00003	0.00000	-0.00003	-0.00005	-0.00016	-0.00007	0.01604	0.28955
WH	0.00358	0.00143	0.00006	0.00132	0.00052	-0.00039	-0.00004	-0.00027	-0.00002	-0.21195	0.00553	0.00054	0.00161	-0.00005	0.00000	-0.00026	-0.00008	0.00000	-0.00008	-0.00015	-0.00034	-0.00034	0.00733	0.19065
CN	0.00358	0.00143	0.00006	0.00132	0.00052	-0.00039	-0.00004	-0.00027	-0.00002	0.00620	-0.21281	0.00054	0.00161	-0.00005	0.00000	-0.00026	-0.00008	0.00000	-0.00008	-0.00015	-0.00034	-0.00034	0.00733	0.19065
CG	0.00358	0.00143	0.00006	0.00132	0.00052	-0.00039	-0.00004	-0.00027	-0.00002	0.00620	0.00553	-0.21761	0.00161	-0.00005	0.00000	-0.00026	-0.00008	0.00000	-0.00008	-0.00015	-0.00034	-0.00034	0.00733	0.19065
RI	0.00358	0.00143	0.00006	0.00132	0.00052	-0.00039	-0.00004	-0.00027	-0.00002	0.00620	0.00553	0.00054	-0.21833	-0.00005	0.00000	-0.00026	-0.00008	0.00000	-0.00008	-0.00015	-0.00034	-0.00034	0.00733	0.19065
SB	0.00332	0.00132	0.00005	0.00123	0.00048	-0.00048	-0.00005	-0.00033	-0.00002	-0.00022	-0.00020	-0.00002	-0.00006	-0.77637	0.00000	0.36749	0.10920	0.00000	0.11056	-0.00016	-0.00037	-0.00026	0.00616	0.17737
SM	0.00332	0.00132	0.00005	0.00123	0.00048	-0.00048	-0.00005	-0.00033	-0.00002	-0.00022	-0.00020	-0.00002	-0.00006	0.06807	-0.84444	0.36749	0.10920	0.00000	0.11056	-0.00016	-0.00037	-0.00026	0.00616	0.17737
SO	0.00332	0.00132	0.00005	0.00123	0.00048	-0.00048	-0.00005	-0.00033	-0.00002	-0.00022	-0.00020	-0.00002	-0.00006	0.06807	0.00000	-0.47695	0.10920	0.00000	0.11056	-0.00016	-0.00037	-0.00026	0.00616	0.17737
OS	0.00332	0.00132	0.00005	0.00123	0.00048	-0.00048	-0.00005	-0.00033	-0.00002	-0.00022	-0.00020	-0.00002	-0.00006	0.06807	0.00000	0.36749	-0.73524	0.00000	0.11056	-0.00016	-0.00037	-0.00026	0.00616	0.17737
OM	0.00332	0.00132	0.00005	0.00123	0.00048	-0.00048	-0.00005	-0.00033	-0.00002	-0.00022	-0.00020	-0.00002	-0.00006	0.06807	0.00000	0.36749	0.10920	-0.84444	0.11056	-0.00016	-0.00037	-0.00026	0.00616	0.17737
OO	0.00332	0.00132	0.00005	0.00123	0.00048	-0.00048	-0.00005	-0.00033	-0.00002	-0.00022	-0.00020	-0.00002	-0.00006	0.06807	0.00000	0.36749	0.10920	0.00000	-0.73388	-0.00016	-0.00037	-0.00026	0.00616	0.17737
CT	0.00351	0.00140	0.00006	0.00130	0.00051	-0.00042	-0.00005	-0.00029	-0.00002	-0.00021	-0.00019	-0.00002	-0.00005	-0.00006	0.00000	-0.00031	-0.00009	0.00000	-0.00009	-0.19965	-0.00035	-0.00025	0.00780	0.18485
SK	0.00256	0.00102	0.00004	0.00095	0.00037	-0.00071	-0.00008	-0.00050	-0.00004	-0.00027	-0.00024	-0.00002	-0.00007	-0.00007	0.00000	-0.00039	-0.00012	0.00000	-0.00012	-0.00022	-0.34637	-0.00032	0.00288	0.14816
TB	0.00351	0.00140	0.00006	0.00130	0.00051	-0.00042	-0.00005	-0.00029	-0.00002	-0.00021	-0.00019	-0.00002	-0.00005	-0.00006	0.00000	-0.00031	-0.00009	0.00000	-0.00009	-0.00017	-0.00039	-0.19971	0.00699	0.18481
OF	0.00883	0.00320	0.00013	0.00297	0.00116	0.00100	0.00011	0.00069	0.00005	0.00007	0.00007	0.00001	0.00002	0.00001	0.00000	0.00068	0.00002	0.00000	0.02802	0.00006	0.00001	0.05209	-0.42319	0.61362
MF	0.01081	0.00430	0.00018	0.00399	0.00156	0.00184	0.00021	0.00130	0.00009	0.00025	0.00022	0.00002	0.00004	0.00026	0.00000	0.00031	0.00009	0.00000	0.00010	0.00029	0.00026	0.00032	0.03646	-0.86259

Table 3B. Disaggregate Uncompensated Demand Elasticities

	BT	PK	ML	PM	PL	DM	DB	DC	DP	WH	CN	CG	RI	SB	SM	SO	OS	OM	OO	CT	SK	TB	OF	MF
BT	-0.59997	0.00001	.00000	0.00001	.00000	-0.00177	-0.00020	-0.00123	-0.00009	-0.00048	-0.00043	-0.00004	-0.00012	-0.00013	-0.00000	-0.00068	-0.00020	0.00000	-0.00020	-0.00039	-0.00077	-0.00064	-0.01370	-0.04809
PK	0.00003	-0.59999	.00000	0.00001	.00000	-0.00177	-0.00020	-0.00123	-0.00009	-0.00048	-0.00043	-0.00004	-0.00012	-0.00013	0.00000	-0.00068	-0.00020	0.00000	-0.00020	-0.00039	-0.00077	-0.00064	-0.01370	-0.04809
ML	0.00003	0.00001	-0.60000	0.00001	.00000	-0.00177	-0.00020	-0.00123	-0.00009	-0.00048	-0.00043	-0.00004	-0.00012	-0.00013	0.00000	-0.00068	-0.00020	0.00000	-0.00020	-0.00039	-0.00077	-0.00064	-0.01370	-0.04809
PM	0.00003	0.00001	.00000	-0.39999	.00000	-0.00177	-0.00020	-0.00123	-0.00009	-0.00048	-0.00043	-0.00004	-0.00012	-0.00013	0.00000	-0.00068	-0.00020	0.00000	-0.00020	-0.00039	-0.00077	-0.00064	-0.01370	-0.04809
PL	0.00003	0.00001	.00000	0.00001	-0.60000	-0.00177	-0.00020	-0.00123	-0.00009	-0.00048	-0.00043	-0.00004	-0.00012	-0.00013	0.00000	-0.00068	-0.00020	0.00000	-0.00020	-0.00039	-0.00077	-0.00064	-0.01370	-0.04809
DM	-0.00060	-0.00024	-0.00001	-0.00022	-0.00009	-0.32497	0.00039	0.00366	0.00026	-0.00044	-0.00039	-0.00004	-0.00011	-0.00012	0.00000	-0.00062	-0.00018	0.00000	-0.00019	-0.00036	-0.00071	-0.00058	-0.01084	-0.01544
DB	-0.00060	-0.00024	-0.00001	-0.00022	-0.00009	0.00524	-0.32962	0.00366	0.00026	-0.00044	-0.00039	-0.00004	-0.00011	-0.00012	0.00000	-0.00062	-0.00018	0.00000	-0.00019	-0.00036	-0.00071	-0.00058	-0.01084	-0.01544
DC	-0.00060	-0.00024	-0.00001	-0.00022	-0.00009	0.00524	0.00039	-0.32636	0.00026	-0.00044	-0.00039	-0.00004	-0.00011	-0.00012	0.00000	-0.00062	-0.00018	0.00000	-0.00019	-0.00036	-0.00071	-0.00058	-0.01084	-0.01544
DP	-0.00060	-0.00024	-0.00001	-0.00022	-0.00009	0.00524	0.00039	0.00366	-0.32995	-0.00044	-0.00039	-0.00004	-0.00011	-0.00012	0.00000	-0.00062	-0.00018	0.00000	-0.00019	-0.00036	-0.00071	-0.00058	-0.01084	-0.01544
WH	-0.00083	-0.00033	-0.00001	-0.00031	-0.00012	-0.00177	-0.00020	-0.00123	-0.00009	-0.21222	0.00529	0.00051	0.00154	-0.00012	0.00000	-0.00064	-0.00019	0.00000	-0.00019	-0.00037	-0.00073	-0.00060	-0.01183	-0.02685
CN	-0.00083	-0.00033	-0.00001	-0.00031	-0.00012	-0.00177	-0.00020	-0.00123	-0.00009	0.00392	-0.21286	0.00051	0.00154	-0.00012	0.00000	-0.00064	-0.00019	0.00000	-0.00019	-0.00037	-0.00073	-0.00060	-0.01183	-0.02685
CG	-0.00083	-0.00033	-0.00001	-0.00031	-0.00012	-0.00177	-0.00020	-0.00123	-0.00009	0.00392	0.00529	-0.21763	0.00154	-0.00012	0.00000	-0.00064	-0.00019	0.00000	-0.00019	-0.00037	-0.00073	-0.00060	-0.01183	-0.02685
RI	-0.00083	-0.00033	-0.00001	-0.00031	-0.00012	-0.00177	-0.00020	-0.00123	-0.00009	0.00392	0.00529	0.00051	-0.21680	-0.00012	0.00000	-0.00064	-0.00019	0.00000	-0.00019	-0.00037	-0.00073	-0.00060	-0.01183	-0.02685
SB	-0.00081	-0.00032	-0.00001	-0.00030	-0.00012	-0.00176	-0.00020	-0.00123	-0.00009	-0.00048	-0.00042	-0.00004	-0.00012	-0.77644	0.00000	0.36716	0.10910	0.00000	0.11065	-0.00037	-0.00073	-0.00060	-0.01175	-0.02593
SM	-0.00081	-0.00032	-0.00001	-0.00030	-0.00012	-0.00176	-0.00020	-0.00123	-0.00009	-0.00048	-0.00042	-0.00004	-0.00012	0.06801	-0.84444	0.36716	0.10910	0.00000	0.11065	-0.00037	-0.00073	-0.00060	-0.01175	-0.02593
SO	-0.00081	-0.00032	-0.00001	-0.00030	-0.00012	-0.00176	-0.00020	-0.00123	-0.00009	-0.00048	-0.00042	-0.00004	-0.00012	0.06801	0.00000	-0.47730	0.10910	0.00000	0.11065	-0.00037	-0.00073	-0.00060	-0.01175	-0.02593
OS	-0.00081	-0.00032	-0.00001	-0.00030	-0.00012	-0.00176	-0.00020	-0.00123	-0.00009	-0.00048	-0.00042	-0.00004	-0.00012	[illegible]	[illegible]	0.36716	-0.72534	0.00000	0.11065	-0.00037	-0.00073	-0.00060	-0.01175	-0.02593
OM	-0.00081	-0.00032	-0.00001	-0.00030	-0.00012	-0.00176	-0.00020	-0.00123	-0.00009	-0.00048	-0.00042	-0.00004	-0.00012	[illegible]	0.00000	0.36716	0.10910	-0.84444	0.11065	-0.00037	-0.00073	-0.00060	-0.01175	-0.02593
OO	-0.00081	-0.00032	-0.00001	-0.00030	-0.00012	-0.00176	-0.00020	-0.00123	-0.00009	-0.00048	-0.00042	-0.00004	-0.00012	0.06801	0.00000	0.36716	0.10910	0.00000	-0.73504	-0.00037	-0.00073	-0.00060	-0.01175	-0.02593
CT	-0.00354	-0.00141	-0.00006	-0.00131	-0.00051	-0.00262	-0.00030	-0.00183	-0.00013	-0.00065	-0.00058	-0.00006	-0.00017	-0.00017	0.00000	-0.00091	-0.00027	0.00000	-0.00027	-0.29801	-0.00098	-0.00082	-0.02366	-0.16115
SK	-0.00097	-0.00039	-0.00002	-0.00036	-0.00014	-0.00181	-0.00021	-0.00126	-0.00009	-0.00049	-0.00043	-0.00004	-0.00013	-0.00013	0.00000	-0.00069	-0.00020	0.00000	-0.00021	-0.00040	-0.14430	-0.00061	-0.01345	-0.03304
TB	-0.00091	-0.00036	-0.00002	-0.00034	-0.00013	-0.00179	-0.00020	-0.00125	-0.00009	-0.00048	-0.00043	-0.00004	-0.00013	-0.00013	0.00000	-0.00068	-0.00020	0.00000	-0.00021	-0.00040	-0.00078	-0.20707	-0.01217	-0.02649
OF	-0.00237	-0.00102	-0.00004	-0.00093	-0.00037	-0.00231	-0.00026	-0.00161	-0.00012	-0.00058	-0.00052	-0.00005	-0.00015	-0.00015	0.00000	-0.00083	-0.00025	0.00000	-0.00023	-0.00044	-0.00093	-0.00078	-0.47917	-0.10838
MF	-0.00798	-0.00318	-0.00013	-0.00295	-0.00115	-0.00400	-0.00045	-0.00279	-0.00020	-0.00092	-0.00082	-0.00008	-0.00036	-0.00034	0.00000	-0.00129	-0.00020	0.00000	-0.00029	-0.00076	-0.00141	-0.00122	-0.04505	-0.08795

crops and livestock. These will be reflected in the livestock "assembly" technology. This leaves us with a set of own-price elasticities, and a "sprinkling" of cross-price elasticities, for fifteen farm commodities, including six livestock products: beef, pork, mutton and lamb, poultry meat, poultry eggs, dairy, and nine crops: wheat, corn, coarse grains, rice, soybeans, oilseeds, cotton, sugar and tobacco. For the sake of completeness, a residual "other farm products" sector must also be added, to reflect those farm products not treated in the model, but which also draw on the aggregate agricultural resource endowment. This gives a total of sixteen commodities.

The previous section's discussion of interactions between functional form and calibration is also quite pertinent to the problem of calibrating $R(\mathbf{p}, v)$ to the information in table 1. While there are a number of cross-price elasticities reported in this table, by far the majority of potential entries are absent. There simply is not enough information to calibrate a fully flexible functional form for the revenue function. This leads us quite naturally once again to the CDE.

The CDE revenue function is analogous to (1) and may be obtained by redefining $z_i = p_i/R(\mathbf{p}, v)$, where v now measures the level of the agricultural endowment. Also, convexity requires that the substitution parameter (b_i) must now be strictly greater than (rather than less than) one, so that $\alpha_i = 1 - b_i < 0$. We will also assume that, in the aggregate, agricultural technology exhibits constant returns so scale. That is, a doubling of v would double all farm supplies, at constant prices. This restriction implies that all of the expansion parameters in (1) now equal to one i.e. $e_i = 1$. When combined with the assumption that factors are paid their marginal value product, the revenue function collapses to the following price possibility frontier (Hertel *et al* .):

$$\sum_{j=1}^{M} B_j \,(p_j/w_A)^{b_j} = 1 \,. \tag{5}$$

The implicit CDE maximum revenue function may be calibrated to a vector of own-price elasticities of supply in the manner outlined above. (Step two maybe omitted, since all expansion parameters have been restricted to equal one.)[7] Once again, we are missing data for the residual "other agriculture" sector. As on the consumption side, we will assign to this last commodity the share-weighted average own-price elasticity of supply for the fifteen endogenous goods. The estimate of other agricultural sales is obtained by deducting endogenous farm receipts from an independent estimate of total farm receipts for the benchmark period. Table 4 summarizes the two calibration steps employed in fitting R(p, v) to the information in table 1. Remember that we are implicitly assuming that these calibrated elasticities do not embody an expansion effect.

Calibration of the Livestock Assembly Sectors

The postulated framework outlined in figure 1 gives us a sharp lens with which to focus on the feed livestock interactions, as implied by table 1. If we assume that the livestock assembly sectors are perfectly

---

[7] A relevant question is whether or not we should have some additional "nesting" of individual groups of commodities. That is, the supply of some of these farm products separable from that of others? From a farm level perspective there would seem to be some likely candidates among the crops. For example, a farmer engaged in wheat-soybean double-cropping is likely to view the two crops as a "package," with net returns to this package being traded off against other activities. This hypothesis is reflected in the positive cross-price elasticity (a complementary relationship) between these two products in table 1. However, from a sector-wide perspective, this type of separability seems more difficult to justify. Soybeans are also grown in annual rotation with corn. In other cases these crops are grown on a continuous basis. Thus it is not clear where the line should be drawn in such a nesting strategy. Consequently, we simply calibrate R(p, v) to all sixteen disaggregate farm commodity supply elasticities sumultaneously.

Table 6. Calibration of an Aggregate Agricultural Revenue Function for the United States

Commodity	Step 1			Step 2
	s	ν	α	B
Primary Factors Supplied to Livestock Assembly Sectors				
Beef & Veal	.1727	.557	.697	.160
Pork	.0365	.453	.466	.039
Mutton & Lamb	.0017	.468	.468	.002
Poultry, Meat	.0580	.45	.471	.062
Poultry, Eggs	.0207	.319	.320	.025
Dairy	.1204	.426	.469	.129
Wheat	.0626	.60	.644	.06
Corn	.1635	.48	.572	.164
Other Course Grains	.0224	.60	.615	.022
Rice	.0102	.40	.402	.011
Soybeans	.0801	.60	.659	.076
Other Oilseeds	.0237	.55	.563	.024
Cotton	.0329	.65	.675	.031
Sugar	.0139	.45	.455	.015
Tobacco	.0144	.25	.249	.018
Other Ag	.1664	.508	.616	.162

Total Domestic Ag Revenue = $150.10677 billion.

competitive and operate under locally constant returns to scale, then the own-price elasticity of livestock supply (η_{SS}) and the cross-elasticity with respect to the feed price (η_{SF}) may both be expressed in terms of structural parameters of the CGE model. In particular:

$$\eta_{SS} = (\upsilon_{LL} + c_F \sigma) c_L^{-1} \tag{6}$$

$$\eta_{SF} = (\eta_{SS} + \sigma) c_F \tag{7}$$

where υ_{LL} is the elasticity of supply of the nonfeed livestock input with respect to its own-price, c_L is the cost share of this input, $c_F = (1 - c_L)$ is the cost share of feed, and σ is the elasticity of substitution between feed and nonfeed inputs. These equations may be solved to express υ_{LL} and σ in terms of the remaining parameters, all of which may be extracted from table 1.

Since there are multiple values of η_{SF} for any given livestock product, there are also potentially multiple estimates of σ. Indeed this is the case, as may be seen from table 5. These diverse estimates imply that different feed types substitute differentially for the nonfeed input. Given the lack of evidence on this latter, we prefer to invoke the separability assumption such that the optimal feed mix is invariant to the price of nonfeed inputs. In this case there is a unique value of σ. The bottom row in that table reports the aggregated estimate of σ, which is the value used in calibrating the model. The elasticity of primary factor supply (υ_{LL}) may then be derived from σ, c_F, c_L and η_{SS}. These are reported in table 4.

In addition to the livestock supply elasticities, the model in figure 1 also implies a particular structure for the feed demand elasticities:

$$\eta_{DD} = - c_L \sigma = - \eta_{DL} . \tag{8}$$

where η_{DD} is the output constant, own-price elasticity of demand for feed in production of a given livestock product, and η_{DL} is the cross-price elasticity with respect to the livestock (nonfeed) input. This leads to two important points about conditional feed demand equations in the model underlying table 1. The first point is that $\eta_{DL} = 0$. Thus there is no feed-livestock price interaction. This in turn implies the absence of substitutability between feed and livestock inputs (i.e. $\sigma = 0$). But this contradicts the evidence as presented in table 5. A second point is that when cost shares and σ vary across livestock types, as they do in this case, then the price elasticities of derived demand will also vary. Rather than one demand equation for corn, there really should be six -- one for each livestock type.

Finally, there is the issue of substitution among alternative feedstuffs. This may be readily accommodated in the conceptual framework outlined in figure 1. There, illustrative substitutability among three feedtypes is shown. These might be aggregates of grains and proteins, for example. More detailed nesting structures are also possible and have been built into the OECD's MTM trade model. (See also Hertel and Tsigas for an example of this type of nesting.) Unfortunately, the data is not available in table 1 to calibrate the livestock sectors in this degree of detail.

Calibration of Oilseed Processing Sectors:

There are several key modeling issues that arise when defining processing sector activities to be imbedded in a trade model. From a technological point of view it seems plausible to argue that industries such as soybean crushing will be characterized by locally constant returns to scale, in the neighborhood of a competitive equilibrium. This follows logically from the fact that optimal plant size is small, relative to most domestic markets, and entry and exit is relatively easy (Diewert).

Table 5: Implied Elasticities of Substitution Between Feed and Nonfeed Inputs $\hat{\sigma} = (-\eta_{0j}/c_j) - \eta_{00}$

	Livestock Type					
Feedstuff	BF	PK	ML	PM	PE	DM
WH	.267	.411	-	.267	.226	.205
CN	.072	.111	.088	.072	.061	.055
CG	.072	.111	.088	.072	.061	.056
SM	.162	.25	-	.162	.137	.125
OM	.162	.25	-	.162	.137	.125
All Feed	.089	148	.088	.124	.091	.070

However, unless transportation costs are taken into account, a model based on such technology will tend to give extreme solutions if both the raw input (i.e., soybeans) and the output (soyoil and soymeal) are tradeable. a slight change in the relative profitability of soybean crushing in one region (for example due to an export tax on soybeans) would encourage complete specialization of that activity in the favored region.

Of course the reason that this type of specialization does not occur in reality is the presence of transport costs. In order to overcome this limitation, while avoiding the introduction of spatial considerations into this model, we introduce fixed factor in the soybean crushing industry which is completely immobile. This causes the industry production function to exhibit decreasing returns to scale in the remaining inputs. In equilibrium, the imputed return to this fixed factor represents the economic rents which accrue to domestic producers as a consequence of being proximate to the domestic market. Only when these returns evaporate will the domestic industry shut-down. Conversely, some of the benefits of a favorable policy (such as an export tax on the raw input), will be capitalized into the fixed factor, reflecting the enhanced rents accruing to domestic producers under such a policy.

Formally, we postulate a generic model for each agricultural processing sector, which is characterized by: (a) locally constant returns to scale in *all* inputs, (b) no substitution between raw agricultural and other inputs, (c) a constant transformation elasticity among the m outputs ($\sigma_T \leq$)0, and (d) where necessary, a fixed factor which substitutes for other nonagricultural inputs according to $\sigma_S > 0$.

In the particular case of oilseeds, we assume $\sigma_T = 0$ and $\sigma_S = 1$. The nonagricultural input is then apportioned between fixed and variable components in order to achieve the desired supply response. However, based on the U.S. data in table 1, the cost and revenue information implied by those prices and quantities is incompatible with zero profits. In particular, total expenditures by the oilseed crushing industries, at market prices, exceed revenue at producer prices! (This difference is very large in the case of other oilseeds.) Thus in order to have nonzero values for c_V and c_F, costs must be lowered or revenues raised. Here, we lower costs until $c_A = 0.50$ which is roughly the value of oilseed purchases by the processing sector in the U.S. input-output table. This yields estimates of $c_V = 0.084$ and $c_F = 0.416$, based on the supply elasticities in table 1. A similar approach is applied in the case of the other oilseeds sector.

Calibration of Dairy Processing Sectors

In the case of dairy products, there are several important differences with oilseed crushing. First of all, the agricultural input--fluid milk--is not generally traded. This obviates the need for a fixed input to prevent specialization in dairy processing. The second distinction arises from the fact that the mix of processed products can be altered in response to changes in the relative prices of (e.g.) cheese and butter.

Taking into account the nonzero elasticity of transformation among processed dairy products ($\sigma_T < 0$) and the inelastic domestic supply of raw milk (with elasticity η_{AA}), we can solve the underlying system of equations to obtain the elasticity of transformation as a function of the own-price elasticity of supply for the processed product (η_{jj}) and the raw product (η_{AA}), as well as the cost share of the raw product in the processing sector (c_A) and the revenue share of the j^{th} processed product (r_j):

$$\sigma_T = (\eta_{jj} - \eta_{AA}\, c_A^{-1})\,(r_j - 1)^{-1} \leq 0. \quad (9)$$

Since r_j and c_A are both less than one, we must have $\eta_s < c_A\, \eta_{jj} < \eta_{jj}$. This makes intuitive sense. Processed products involve nonagricultural as well as raw milk inputs. Thus we expect the processing sector to be more price responsive than the primary product sector, provided nonagricultural inputs (e.g., labor and capital) are in perfectly elastic supply. Indeed it would seem plausible to assume that additional processing facilities can be brought "on-line" over the 3-5 year time period envisioned for this model's solution.

In light of these observations it is rather striking that the own-price elasticity of supply for raw milk (η_{AA}) in table 1 is larger than η_{jj} for each of the processed dairy products. Perhaps this is due to the fact that no distinction is drawn between raw and processed fluid milk. In our model we force all raw milk to pass through the processing sector. Since (31) is infeasible, given the information in table 1, we simply adopt the value of η_{AA} provided there (0.50), thereupon (arbitrarily) choosing a value of σ_T = -1.0.

Completing the Model

Thus far we focussed on procedures for calibrating technology and preferences for commodities which are endogenous to SWOPSIM. However, in this calibration process we have also generated: (i) a supply of non-SWOPSIM farm products (other agriculture), (ii) a demand for non-SWOPSIM food products (other food), and (iii) a demand for nonfood products. With a few naive assumptions, it is possible to complete this model.

First, consider the food system as a whole. If we assume a unique mapping from "other agriculture" to "other food," then completion of the model's treatment of the food system requires us to impute a pattern of net trade and marketing margins such that world supplies of these residual products equal world demands, and consumer expenditures and farm receipts match their initial equilibrium values. If one assumes that the "other agriculture" marketing margin is constant across regions, then it may be obtained by simple deducting farm receipts for "other agriculture" from consumer expenditures on "other food," and dividing by the supply world agricultural output, as in equation (3). Having equated world expenditure and world receipts, net trade for any region is simply the difference between demand (expenditure divided by the retail price), and supply in each of the regions. These in turn will sum to zero.

The nonfood economy is balanced by noting that each region must be on its budget constraint (when international borrowing and transfers are accounted for). These capital flows may be modeled explicitly, or we may abstract from them, permitting them to be subsumed in the nonfood transactions. We do the latter, so that by setting the nonagricultural resource endowment to equal total income minus agricultural revenues, balance of payments equilibrium is assured. Indeed, a pattern of net trade in nonagricultural products exactly offsets the balance of trade in agriculture.

MODEL IMPLEMENTATION AND SOLUTION

The model outlined in the previous section is implemented using GEMPACK, a suite of software developed at the IMPACT Project and designed to support applied general equilibrium modeling (Codsi and Pearson). While GEMPACK has historically been oriented exclusively towards solving linearized models, a recent prototype version of this software now solves nonlinear general equilibrium problems. This is the version which we employ. The problem is still written down in its linearized form, but the addition of a set of parameter updating equations [e.g., equations (3) and (4)], permits the nonlinear solution to be obtained after successive iterations of the model.

There are three blocks of equations in the model. The first set of equations describe the behavior of individual sectors. Since some of the sectors in this model are multiproduct in nature, the simplest approach is to permit every sector to produce multiple products. CET production possibilities may be handled as a special case of the CDE in which all transformation parameters are equal. (The CDE revenue function collapses to an identity in the case of a single product sector.) Thus the three sets of equations in the first block are industry supply equations, industry demand equations (input-output separability is assumed), and a zero profit condition.

The second block contains equations which are region-specific. Up to this point we have said nothing about the number of regions involved. Since the generic structure of each regional economy is the same, and since GEMPACK is designed to handle large-dimensioned problems it is rather indifferent to the total number of regions. This is determined by the availability of data and parameters. We are currently working with both three and ten region models. However, USDA has published documentation on more than thirty regions, so that there is considerable potential for strategic disaggregation.

Each region in the model has three sets of equations which most hold in equilibrium. First of all, since there is only one representative household per region, there is a set of region-specific final demand equations. One argument in these equations is regional income which must be computed in a separate equation which takes account of primary factor payments and all taxes and subsidies accruing to, or paid by, the region in question. Finally, non-tradeable commodity markets must achieve equilibrium within each region. In this particular model this includes markets for: all primary factors (including the livestock inputs to the livestock assembly sectors), fluid milk, and all consumer goods. (Only producer goods are traded. Once the wholesale/retail margin is applied to a producer good, it must be consumed domestically).

The final block of equations capture the international trade linkages in the model. They include market clearing conditions for all but one of the tradeable commodities, as well as price linkage equations translating world prices into domestic prices, and domestic prices into producer and consumer prices. All taxes and subsidies are applied here. Finally there is a dummy equation which evaluates excess demand for the one tradeable commodity which was omitted from the market-clearing conditions. According to Walras' Law this must equal zero. Since any sort of logical or computational error will generally lead to a violation of Walras' Law, this offers a valuable consistency check on the entire model.

SUMMARY AND CONCLUSIONS

The purpose of this paper has been to build a bridge between existing partial equilibrium models of international agricultural trade, and their general equilibrium counterparts. In particular, a set of procedures has been outlined which permit us to calibrate a complete set of production and utility functions, based almost entirely on data available in USDA's SWOPSIM regional data files. In the process of performing these calibration exercises, a number of insights are obtained which suggest methods of altering and improving this valuable data base.

For example, by invoking the assumption of implicitly additive consumer preferences, we are able to generate a complete matrix of uncompensated price elasticities of demand based on the own-price and income elasticities available in the SWOPSIM file. In the process of doing this for the U.S., we found some evidence that the income elasticities of demand were inconsistent with the other model parameters. In particular, they required upward adjustment in order to satisfy regularity properties on the utility function, given the existing budget shares and price elasticities (and the maintained hypothesis of implicit additivity).

The calibration exercise also points to the need to reexamine same of the price indices embodied in this data base. At the individual sector level, a number of sectors exhibit remarkably small, or even negative value-added, given stated prices. More generally, when all SWOPSIM commodities are evaluated at producer prices, they account for about 85% of U.S. farm sales, yet at consumer prices they account for less than half of U.S. food expenditures. Finally, a detailed examination of the feed-livestock complex suggests a number of possibilities for "adding value" to this partial equilibrium model of agricultural trade.

Having calibrated this particular general equilibrium model, it may be simulated in a number of different "modes". First of all, by fixing exogenously the prices of all non-SWOPSIM commodities, as well as the aggregate agricultural resource endowment, and regional income, it is possible to imitate the solution of the SWOPSIM model. The advantage of this particular partial equilibrium formulation is its specification in terms of fundamental preference and technology parameters. Thus it is easy to examine, for example, the sensitivity of the model's solution to the degree of livestock-feed substitution. Alternatively, the implications of varying the mobility of resources between the farm and nonfarm sectors may be examined.

It is also possible to endogenize *all* agricultural production and food consumption and examine the degree to which policy changes affect the demand for agricultural resources in *all* uses. Finally, by closing the model with respect to the production and consumption of nonfood commodities, we are able to endogenize the determination of regional income, along with the terms of trade effects engendered by individual regions balance of payments constraints. This in turn provides a much more comprehensive accounting of the welfare effects of multilateral liberalization of agricultural trade.

REFERENCES

Ball, V.E. "Modelling Supply Response in a Multiproduct Framework", *American Journal of Agricultural*

Burniaux, J.M. and J. Waelbroeck, "The Common Agricultural Policy of the European Community and the World Economy: A General Equilibrium Analysis", in the IIASA Conference Proceedings, 1989.

Codsi, G. and Pearson, K.R. (1988) *Developing and Implementing Large Economic Models Using GEMPACK, a General Purpose Software Suite*, Impact Preliminary Working Paper No. IP-39, Univerity of Melbourne, Melbourne.

Dixon, P.B., B.R. Parmenter, J. Sutton, and D.P. Vincent (1982) *ORANI: A Multisectoral Model of the Australian Economy*. Amsterdam: North Holland Press.

Gardiner, Walter H., Vernon O. Roningen, and Karen Liu (1989) Elasticities in the TReade Liberalization Database, ERS, Staff Report No. AGES 89-20, May.

Goldin, I. and O. Knudsen (1990) "The Implications of Agricultural Trade Liberalization for Developing countries." in I. Goldin and O. Knudsen (eds.) *Agricultural Trade Liberalization: IMplications for Developing countries*, Paris: OECD (forthcoming).

Haley, Steven L. (1988) Joint Products in the Static World Policy Simulation Model (SWOPSIM) Modelling Framework, Staff Report AGES 881024, ERS, USDA, Nov.

Hanoch, G. (1975) "Production and Demand Models with Direct or Indirect Implicit Additivity," *Econometrica*, 43: 395-419.

Hertel, T.W. (1990) "General Equilibrium Analysis of Agriculture: What Does in Contribute?" *Agricultural Economics Research* (forthcoming in the summer issue).

Hertel, T.W., "Estimating Substitution and Expansion Effects: Comment", *American Journal of Agricultural Economics*, 69(1), 1987.

Hertel, T.W., E.B. Peterson, P.V. Preckel, Y. Surry and M.E. Tsigas (1991) "Implicit Additivity as a Strategy for Restricting the Parameter Space in CGE Models", *Economic and Financial Computing*, forthcoming.

Hertel, T.W., V.E. Ball, K.S. Huang and M.E. Tsigas (1989) *Computing General Equilibrium Farm Level DEmand Elasticities for Agricultural Commodities*, AES Research Bulletin No. 988, Purdue University.

Horridge, M. and D. Pearce, "Modeling the Effects on Australia of Interventions in World Agricultural Trade", IMPACT Preliminary Working Paper No. OP-65, University of Melbourne, 1988.

Krissoff, Bary, and Nicole Ballenger (1989) "Agricultural Trade Liberalization in a Multi-Sector Model: Implications for Argentina, Brazil, and Mexico," pp. 67-71 in Bruce Greenshields and Margot Bellamy, eds., Government Intervention in Agriculture, IAAE Occasional Paper No. 5.

Liapis, Peter (1990) Incorporating Inputs in the Static World Policy Simulation Model (SWOPSIM), ERS, USDA, Tech. Bulletin No. 1780, June.

Magiera, S.L., and M.T. Herrlihy "A Technical Comparison of Trade Liberalization Model Results," Draft Manuscript, Economic Research Service, U.S. Department of AGriculture, Washington, DC, 1988.

McDonald, B.J. (1990) "A General Equilibrium Analysis of World Agricultural Policy Liberalization," USDA/ERS/ATAD Staff Report, Washington, D.C.: ERS/USDA (forthcoming).

OECD (1990) *Modeling the Effects of Agricultural Policies*, Economic Studies Special Issue, Winter, 1989-90, Paris.

Organization for Economic Co-operation and Development (OECD), *National Policies and Agricultural Trade*, Paris OECD 1987.

Parikh, K.S., G. Fischer, K. Frohberg, and O. Gulbrandsen *Towards Free Trade in AGriculture*. Laxenburg, Austria: Martinus Nijhoff Publishers, 1988.

Pearson, K. (1990) UPDATE and multi-step version of TABLO, mimeo 21p. October.

Peterson, E.B., T.W. Hertel and P.V. Preckel (1990) "The Farm-Retail Price Spread Revisited: A General Equilibrium Perspective," Staff Paper, Department of Agricultural Economics, Purdue University, West Layafette, IN.

Ronigen, V.O. and P.M. Dixit (1989) *Economic Implications of Agricultural Policy Reform in Industrial Market Economies*. USDA Staff Report No. AGES 89-36, Washington, DE.C.

Roningen, V.O., "A Static World Policy Simulation (SWOPSIM) Modelling Framework", Staff Report AGES860625, Economic Research Service, US Department of Agriculture, Washington, DC, July 1986.

Sakai, Y. (1974 "Substitution and Expansion Effects in Production Theory: The Case of Joint Production," *Journal of Economic Theory*, 9: 255-274.

Sullivan, J., J. Wainio, and V. Roningen, "A Database for Trade Liberalization Studies", Staff Report AGES89-12, Economic Research Service, US Department of Agriculture, Washington, DC, March 1989.

Surry, Y. (1988) "A Note for an Alternative Methodology to Add Input Demand Equations to the OECD MTM Model," unpublished manuscript, International Trade Policy Division, Agriculture Canada, Ottawa.

Tyers, R. and K. Anderson (1986) "Distortions in World Food Markets: A Quantitative Assessment," background paper for the *World Development Report, Washington, D.C.: The World Bank.*

Valdes, A. and J. Zietz (1980) *Agricultural Protection in OECD Countries: Its Costs to Less-Developed Countries.* Research Report 21, Washington, D.C.: IFPRI.

Zeitsch, J. (1989) "Adding Input Demands to Partial Equilibrium AGricultural Trade Models," paper presented at the thirty-third conference of the Australian Agricultural Economics Society, Christchurch, New Zealand, February, 7-9.