A HYDROLOGICAL-ECONOMIC MODELLING APPROACH TO DRYLAND SALINITY IN WESTERN AUSTRALIA

Miss. J. Gomboso & Dr. G. Hertzler
University of Western Australia[1]

Paper Presented to:

AUSTRALIAN AGRICULTURAL ECONOMICS SOCIETY
35th Annual Conference
Armidale, New South Wales, 11-14 February, 1991

1 This research has been supported by a postgraduate research scholarship from the Rural Industries Research and Development Corporation.

A HYDROLOGICAL-ECONOMIC MODELLING APPROACH TO DRYLAND SALINITY IN WESTERN AUSTRALIA

The aim of this paper is to describe the development of a steady-state and transient groundwater flow model which simulates the current agricultural situation and a range of alternative farm management options, that could be employed to control groundwater levels and the redistribution of salt throughout a severely affected catchment in Western Australia. Such a study must account for not only the dynamic but spatial variations in recharge rate, discharge rate, salt flow and groundwater levels throughout the catchment.

Optimal economic use of land resources over time requires an evaluation of the net present value of alternative agricultural practices. This requires an examination of the effects of altering the current clearing rate on future agricultural production and income. The primary objective of the study is the maximisation of net farm income in the long-run throughout the catchment, by identifying and internalising the externalities associated with dryland salt over time. The study also examines the concept of common property, resulting from the flow of recharge and dispersal of saline discharge across farm boundaries.

1. OUTLINE OF RESEARCH

Before management strategies can be altered within a catchment to ensure that the maximum Net Present Value of production is obtained from agriculture in the long-run, hydrological modelling studies are needed to quantify the causal relationships between variables, such as land use, rainfall, clearing and dryland salinity and resultant groundwater levels and salt levels. This can be done by using a groundwater model that provides information on future groundwater levels and identifies key recharging and discharging areas. Results of this hydrology model may then be incorporated into a Common Property Resource model to evaluate the potential gains in Net Present Value of farm income from a co-ordinated catchment effort to control dryland soil salinity in the North Stirlings Land Conservation District. Knowledge of these hydrological relationships play an important role in contributing valuable scientific information to the policy process of formulating the appropriate mix of management options for treating the salinity problem.

There is clearly an underdeveloped niche of integrating environmental and economic models.

Relying solely on economic models to manage a salt-affected catchment without incorporating complex hydrological functional relationships, cannot be expected to yield catchment predictions that would be as accurate as those that would prevail under a modelling strategy that incorporated both economics and an extensive hydrological framework. Errors in predicting groundwater fluctuations resulting from incorrect estimation of the hydrological responses of the environment to variations in vegetation across the catchment, may increase over time, and therefore will cause problems in using models for long term predictions of aquifer response.

This is an area of environmental economics that is yet to be empirically formulated. No hydrological-economic model, that takes into account both the dynamic and spatial characteristics of dryland salinity, and determines the optimal combination of vegetation communities that will maximise the return to agriculture both in the long-run and in the current period, currently exists. Such a study then, would contribute greatly to the understanding of integrated economic-hydrological catchment management.

2. COMMON PROPERTY THEORY AND ENVIRONMENTAL MANAGEMENT

Defining Common Property

An understanding of the concept of Common Property plays an important role in the development of a more adequate analysis of the economics of the environment *(Quiggin, 1986)*. The traditional understanding of a Common Property Resource is that of free access for its utilisation. For example, *Miller (1982)* describes Common Property Resources as those resources which are owned by everyone and therefore owned by no one.

As explained by *Hardin (1968)* in "Tragedy of the Commons" the exploitation of a common resource often results in a divergence between social and private values as a result of the emergence of a range of externalities, including the depletion of the resource. In the case of dryland salinity, individuals seek to maximise their utility via the exploitation of productive agricultural land, but the costs of that additional exploitation are shared by all users (both current and future) and society as a whole. Thus there is a substantial gain to the individual at a relatively small personal cost *(Anderson & Thampapillai, 1990)*.

To quote *Hardin (1968)*:

> "*Therein is the tragedy*". Each man is locked into a system that compels him to degrade the land even further, without limit - in a world that is limited. "*Freedom in a commons brings ruin to all*".

The term Common Property, in Hardin's context, is used as if it were synonymous for limited or open access. *Quiggin (1986, 1988)* and *Ciriacy-Wantrup and Bishop (1975)* criticise the traditional understanding of common property resources and point out that this is a contradiction in terms since the term property is being used for something which is not property at all. They argue that the term Common Property implies the existence of ownership and collective property rights - or common ownership. According to *Quiggin (1986)*, Common Property structures involve well defined rights of exclusion and use.

The focus of this paper, however, will be on the *traditional* understanding of Common Property, which is based on the assumption that owners of limited and open access property do not co-operate.

The exploitation of a common resource (such as productive agricultural land) often results in a divergence between social and private values. Market forces fail to provide incentives which encourage land-users to take into account the full repercussions of their land-use practices. This conflict of interest between private profit makers and the community at large may be attributable to:

(i) the externalities (or external costs) associated with the private use of the land resource (or spillover effects associated with outcomes of private decisions, whereby costs or returns are not borne totally by the individual);
(ii) the possible divergence between private and social rates of discount;
(iii) the side effects of government policy;
(iv) information deficiency on the part of land users; and
(v) the potential irreversibility of the land degradation process. A specific concern in the management of natural resources associated with agriculture is that some of the so-called renewable resources are becoming increasingly scarce or degraded, and transferred into the category on non-renewable resources in the event of over-exploitation or mis-management *(Wills 1987; Anderson & Thampapillai 1990)*.

Common Property and Dryland Salinity

Whilst salt accession occurs predominantly in the lower areas of a catchment, the clearance of deep-rooted vegetation from agricultural areas in upslope recharge areas reduces the rate of evapo-transpiration, with the consequence of raising the level of the watertable and transporting soluble salts to areas of restricted drainage, and downslope discharge areas. The phenomenon of the clearance of deep-rooted vegetation leading to dryland salinity exhibits a number of characteristics which indicate the presence of market failure" *(Hodge, 1982,* *p185).* With respect to dryland salinity the utilisation of common resources results in the generation of three types of externalities:

(i) costs borne by downstream water-users;
(ii) losses in agricultural production associated with dryland salt; and
(iii) loss of unique flora and fauna species, and species diversity *(Quiggin 1986; Hardin 1968).*

The side-effects of clearing and agricultural land-use practices may be unilateral (where the increase in salinity generated by one farmer has no adverse effects on that farmer) or reciprocal (where a given farmer is both a polluter and a victim, generating increased saline seepage through clearing, and suffering, in turn, from the activities of other farmers or his own actions) *(Quiggin, 1986).* The externalities that result from clearing are thus not confined to individual properties.

The optimal extent of clearing will ultimately depend upon the nature of the physical environment (including soil-type, climate, topography and drainage) and the interactions between physical variables, relative prices and technology *(Hodge, 1982).* From the above discussion, it is suggested that the current level of clearing that has occurred in the absence of control is likely to exceed the socially optimal level.

Difficulties involved in achieving economic efficiency arise because the decisions to clear or revegetate, made by individuals and the community, are linked by the physical characteristics of the groundwater hydrology. All parties acknowledge that their agricultural practices are affecting a common aquifer, but in the absence of institutions that permit collective decisions about groundwater management, each individual or group correctly assumes that neighbours will place private interests first. The result, under situations of scarcity may be socially non-optimal groundwater accession created by externalities, the source of which lies in the producer's ability to influence the level of the watertable, and transmit costs to other areas of the catchment. *Thus the watertable represents a form of common property (Hodge 1982; Lemoine & Gotsch 1985).*

Common Property Resource Economics and Hydrological-Economic Modelling

A watertable depth that permits a socially optimum clearing and revegetation level and agricultural production mix, requires both spatial efficiency and temporal efficiency. The two objectives are connected by the fact that lateral and horizontal groundwater flows are governed by complex physical phenomena and do not occur instantaneously, but may vary from only a few years, to several decades, depending on the nature of the physical environment *(Hodge 1982; Lemoine & Gotsch 1985)*.

> "The development of models to assist with groundwater management has been slow because of the inability to include, in the same model, optimisation algorithms that allocate water on the basis of its economic value across space and through time, and simulations of the movement of water in the aquifer as a result of stresses imposed by pumping or recharge. Most modelling efforts to date have proceeded either by interacting between independent optimization of the economic and hydrology models or by using simulation (as opposed to optimising) techniques." *(Lemoine & Gotsch, 1985, p294)*.

Mathematical models of groundwater flow are beneficial to the management of agricultural systems as they allow us to approximate the components of the hydrological processes (including infiltration, surface water flow, drainage and evapotranspiration) and provide a mechanistic description of the flow of water and salts in and through the soil profiles under different vegetation regimes. Due to the topography and hydrogeological aspects of the catchment, variations in vegetation are expected to have a significant affect on salinity inducement and future productive capacity. This is evident in the North Stirlings Land Conservation District (the study site) from the speed with which the severity of secondary salinity has developed as a result of over-clearing during the late 1950's to early 1970's. In addition the location of vegetation communities should have a significant impact on groundwater fluctuations. Hence both vegetation-type and planting location are important factors influencing the long-run steady-state optimal production mix.

The study will test the hypothesis that, as a result of the Common Property nature of dryland salinity, over-clearing in the North Stirling Lands Conservation District will result in a less-than-optimal level of agricultural production in the long-run. Altering the vegetation-type from annual to perennial pastures, using crop species that transpire more water (for example, barley instead of wheat) and revegetating with high water-using trees on recharge areas and salt-tolerant trees on saline discharge areas, is expected to lower groundwater tables and result in a steady state production mix in which the present value of net farm income is maximised in the long-run.

3. THE HYDROLOGICAL MODEL

Literature Review

Hydrology models of catchments may be divided into four main types: *Stochastic*

Catchment Models, Deterministic Models, Process Models and *Conceptual Models (Mein, 1977).*

Stochastic Catchment Models are those which aim to produce an output which has certain statistical properties, and the output is derived from the input via some probability function. *Deterministic Models* have no stochastic component. For a given input, the output is entirely predictable. These can be sub-divided further into Conceptual Models and Process Models. *Conceptual Models* are taken to be those whose structure makes no attempt to represent the movement of water throughout the catchment. Only the catchment input and output have an physical meaning. *Process Models* refer to those in which some effort is made to simulate the hydrologic component of a catchment. Examples of process models include: Stanford Watershed Model *(Crawford and Linsley, 1966)*, USDA Hydrograph Laboratory Model *(Comer and Henson, 1976)*, the Boughton Model *(Boughton, 1966, 1968)*, the Monash Model *(Porter and McMahon, 1971)* and the AWRC Representative Basin Model *(Chapman, 1970).*

Not all hydrological models examine groundwater flow in its entirety. Some models deal only with specific components of the hydrological cycle (such as infiltration, runoff and drainage). *Rainfall-runoff models* include the Boughton, Stanford Watershed, and Monash models mentioned previously; *Flood and runoff estimation models* include the Clarke-Johnstone unit hydrograph approach *(Johnstone and Cross, 1949)* and the Runoff-Routing Model of Laurenson *(Mein et al, 1974)*; *Evapotranspiration models* by Sedgley (1979); *Infiltration models* include those by *Green & Ampt (1911), Philip (1957), Morel-Seytoux (1975)*, and *Smith & Parlange (1978)*; and *Saltflow models* include those by *Hillel (1977), Manoel & Laing (1975), Jones & Watson (1980)* and *Smith (1980).*

Most models of groundwater flow assume steady state conditions, but more recent models, including those by *Bresler (1972), Smith (1980)* and *Ghassemi et al (1989A, 1989B)* model the hydrology of catchments in their *transient state.*

Hydrology Modelling

The aim of the hydrology model is to develop a steady-state and transient groundwater flow model which simulates the current agricultural situation and a range of alternative farm management options, that could be employed to control groundwater levels and the redistribution of salt throughout a severely affected catchment in Western Australia. Such a model will account for not only the dynamic but spatial variations in recharge rate, discharge rate, salt flow and groundwater levels throughout the catchment.

The model is based on an optimal control model of wildlife migration developed by Hertzler (1990) to determine the degree of common property for the change in the directed movement (or flux) of wildlife over an environment. It is also based on groundwater flow models developed by *Ghassemi et al (1989A, 1989B)* which simulates a range of management options to control groundwater levels and the flow of salts to the River Murray.

The model estimates horizontal groundwater flow on a farm under appropriate assumptions regarding the distributions for transmissivities and storativities throughout the aquifer and its flow boundary conditions. Recharge occurs mainly from rainfall. Discharge in the area occurs as a result of the clearing of land for agriculture, causing a continuing rise in groundwater levels, increased evaporation and hence salting of land and water resources.

The flow of water in the aquifer will be simulated with transient and steady state models. The mathematical formulation for two dimensional partial differential equation to be employed in the estimation of groundwater flow is derived in the Appendix as:

$$\partial W_{ft}/\partial t = \{2Z(\rho_f)\,[\,T_t(\,W_{nt}/A_n - W_{ft}/A_f\,) + ru_t\,] + Q_t\}\,A_f/S \qquad \ldots(1)$$

Here the transmissivity (T) and storativity (S) represent physical aquifer properties in space, ($\partial W_{ft}/\partial t$) is the change in groundwater depth on the farm over time, Z is a cumulative probability which varies from zero to one-half; ρ_f is the radius of the farm; W_{nt} represents the watertable depth on neighbouring farms; A_n is the area of neighbouring farms; A_f is the area of the farm; r is the surface slope of the farm; u is an exogenous source affecting groundwater movement; and Q is the source/sink term on the farm. Note that the temporal derivative $\partial W_{ft}/\partial t$ is zero in the steady-state case.

The equation shows that water will enter the farm if:

(i) the average watertable depth on neighbouring farms is greater than the average watertable depth on the farm;

(ii) the watertable is higher upslope in the catchment than on the farm; or

(iii) there are exogenous sources present which affect the groundwater flow.

Firstly, groundwater depth links the decisions of the farmer to decisions of his neighbours. If the average watertable depth on upslope and downslope neighbouring farms is greater than the groundwater on the farm being modelled, there will be a positive rise in groundwater depth towards the farm. Conversely, if surrounding groundwater levels are lower than that on the farm, there will be a decline in groundwater over time. The speed with which groundwater will be redistributed over time depends on several factors including: soil texture; type of clay present; organic matter content; depth of wetting front; and the presence of impervious layers in the profile.

Secondly, groundwater movement may change over time as a result of agricultural/ engineering techniques employed in upslope recharge areas. Increased surface water accession in upslope areas raises watertables and transports soluble salts to downslope discharge areas and onto the farm. Conversely, the introduction of high water-using crops and pastures and tree species will decrease the amount of water recharging into the common aquifer and hence lower watertables downslope. This type of groundwater movement depends largely on climate, topography, hydro-geology, soil type, and external practices (such as engineering or agricultural modifications to the land).

Thirdly, the watertable will change over time in the presence of exogenous factors. The specific combination of surface agricultural practices and the surface and sub-surface engineering techniques employed can both affect changes in the piezometric head of farms. "From a physical point of view, evapo-transpiration can be viewed as a stream flowing from a *source* of limited capacity and of variable potential, namely the reservoir of soil moisture, to a *sink* of virtually unlimited capacity (through the variable evaporative potential) - the atmosphere." *(Hillel, 1971., p206)*.

The amount and rate of water uptake by plants depends on the ability of roots to absorb water from the soil and the ability of the soil to supply and transmit water toward the roots at a rate sufficient to meet transpiration requirements. These in turn, depend on the properties of plants, soil and micrometeorological conditions.

Both *transmissivity* and *storativity* are important aquifer properties that should be accounted for when modelling horizontal groundwater flow within an aquifer. A direct concern in groundwater studies is the amount of water released from storage (or added to it) per unit horizontal area of the aquifer and per unit decline (or rise) of piezometric head. When the watertable of an aquifer declines, water is released from storage. When there is a rise in the watertable depth within an aquifer, storativity rises. *Transmissivity* may be defined as the rate of flow of water at the prevailing water temperature through the entire thickness of the saturated part of the aquifer per unit hydraulic gradient *(Bear, 1979; Heath & Trainer, 1968)*. The correct estimation of transmissivity and storativity coefficients are an important component in the computation of piezometric heads throughout an aquifer in steady state and transient modes.

The model is based on an optimal control model of wildlife migration developed by Hertzler (1990) to determine the degree of common property for the change in the directed movement (or flux) of wildlife over an environment. It is also based on groundwater flow models developed by *Ghassemi et al (1989A, 1989B)* which simulates a range of management options to control groundwater levels and the flow of salts to the River Murray.

The model estimates horizontal groundwater flow on a farm under appropriate assumptions regarding the distributions for transmissivities and storativities throughout the aquifer and its flow boundary conditions. Recharge occurs mainly from rainfall. Discharge in the area occurs as a result of the clearing of land for agriculture, causing a continuing rise in groundwater levels, increased evaporation and hence salting of land and water resources.

The flow of water in the aquifer will be simulated with transient and steady state models. The mathematical formulation for two dimensional partial differential equation to be employed in the estimation of groundwater flow is derived in the Appendix as:

$$\partial W_{ft} / \partial t = \{ 2Z(\rho_f) \left[T_t \left(W_{nt} / A_n - W_{ft} / A_f \right) + r u_t \right] + Q_t \} A_f / S \qquad \ldots(1)$$

Here the transmissivity (T) and storativity (S) represent physical aquifer properties in space, ($\partial W_{ft} / \partial t$) is the change in groundwater depth on the farm over time, Z is a cumulative probability which varies from zero to one-half; ρ_f is the radius of the farm; W_{nt} represents the watertable depth on neighbouring farms; A_n is the area of neighbouring farms; A_f is the area of the farm; r is the surface slope of the farm; u is an exogenous source affecting groundwater movement; and Q is the source/sink term on the farm. Note that the temporal derivative $\partial W_{ft} / \partial t$ is zero in the steady-state case.

The equation shows that water will enter the farm if:

(i) the average watertable depth on neighbouring farms is greater than the average watertable depth on the farm;

(ii) the watertable is higher upslope in the catchment than on the farm; or

(iii) there are exogenous sources present which affect the groundwater flow.

Firstly, groundwater depth links the decisions of the farmer to decisions of his neighbours. If the average watertable depth on upslope and downslope neighbouring farms is greater than the groundwater on the farm being modelled, there will be a positive rise in groundwater depth towards the farm. Conversely, if surrounding groundwater levels are lower than that on the farm, there will be a decline in groundwater over time. The speed with which groundwater will be redistributed over time depends on several factors including: soil texture; type of clay present; organic matter content; depth of wetting front; and the presence of impervious layers in the profile.

Secondly, groundwater movement may change over time as a result of agricultural/ engineering techniques employed in upslope recharge areas. Increased surface water accession in upslope areas raises watertables and transports soluble salts to downslope discharge areas and onto the farm. Conversely, the introduction of high water-using crops and pastures and tree species will decrease the amount of water recharging into the common aquifer and hence lower watertables downslope. This type of groundwater movement depends largely on climate, topography, hydro-geology, soil type, and external practices (such as engineering or agricultural modifications to the land).

Thirdly, the watertable will change over time in the presence of exogenous factors. The specific combination of surface agricultural practices and the surface and sub-surface engineering techniques employed can both affect changes in the piezometric head of farms. "From a physical point of view, evapo-transpiration can be viewed as a stream flowing from a *source* of limited capacity and of variable potential, namely the reservoir of soil moisture, to a *sink* of virtually unlimited capacity (through the variable evaporative potential) - the atmosphere." *(Hillel, 1971., p206).*

The amount and rate of water uptake by plants depends on the ability of roots to absorb water from the soil and the ability of the soil to supply and transmit water toward the roots at a rate sufficient to meet transpiration requirements. These in turn, depend on the properties of plants, soil and micrometeorological conditions.

Both *transmissivity* and *storativity* are important aquifer properties that should be accounted for when modelling horizontal groundwater flow within an aquifer. A direct concern in groundwater studies is the amount of water released from storage (or added to it) per unit horizontal area of the aquifer and per unit decline (or rise) of piezometric head. When the watertable of an aquifer declines, water is released from storage. When there is a rise in the watertable depth within an aquifer, storativity rises. *Transmissivity* may be defined as the rate of flow of water at the prevailing water temperature through the entire thickness of the saturated part of the aquifer per unit hydraulic gradient *(Bear, 1979; Heath & Trainer, 1968).* The correct estimation of transmissivity and storativity coefficients are an important component in the computation of piezometric heads throughout an aquifer in steady state and transient modes.

Finally, the equation of motion specified above takes into account the degree of common property within the catchment through the variable Z. It is the movement of water from farm to farm which causes the common property problem. Given $0 \leq Z \geq 0.5$, when there are a large number of farms within the catchment, Z is large and the degree of common property is high. If the catchment comprises only a few farms, Z is close to zero, and the management of the farm would be closer to that which would prevail under a socially managed situation. Hence the number of farms within a catchment affects the degree of common property and affects the producer's ability to influence the level of the watertable and transmit costs to other farms within the catchment.

The above equation of motion describes groundwater fluctuations when the catchment is in a transient state. If $\partial W_{jt} / \partial t$ is equal to zero, the environment is at a steady-state equilibrium, in which recharge entering the groundwater system equals discharge, there is no change in net movement of groundwater from neighbouring properties onto the farm, and consequently there is no change in watertable level over time.

The aim of the proceeding section is to incorporate this equation of motion into a dynamic optimisation model as a constraint to the system. The objective of the model is to derive the long-run steady state equilibrium by determining the agricultural production mix and combination of engineering method that should be employed to maximise net farm income in the catchment. By modelling also the transient environment, the optimal approach path available to achieve the steady-state objective can be determined.

4. OPTIMAL CONTROL MODEL OF DRYLAND SALINITY

Groundwater is a natural resource which may rise or fall, depending on ground-surface practices and affect crop production. It may affect downstream waterways, increase dryland salinity in discharge areas and lead to loss of unique flora and fauna species. By including information on the movement of groundwater, it is possible to obtain an optimal spatial and temporal catchment management strategy. In this section, a dynamic optimisation model will be developed.

A farmer manages a portion of land in the catchment and influences groundwater fluctuations. His effect on the aquifer depends on the topographical, climatic and hydro-geological characteristics of his farm. He seeks to maximise his private value of farm output which equals the net present value of agricultural production minus the costs of farming:

$$j(W_{f0}) = Max_Q \int_0^{\infty} e^{-\delta t} [pg(W_{ft}, Q_t) - c(Q_t)]\, dt \qquad \text{... (2)}$$

where j is the net present value of the farm; W_{f0} is the water table depth on the farm at t = 0; Q is a source/sink term representing recharge and discharge; δ is the discount rate; p is a vector of prices for all cropping, pastoral and tree-planting activities; farm output, X, is a function of watertable depth (W_{ft}) and the rate of water removal from exogenous factors (including agronomic and engineering techniques - Q_t), such that $X = g(W_{ft}, Q_t)$; c are the variable costs of agricultural production; and W_{ft} is the watertable depth on the farm at time t. Each farmer faces a similar decision problem.

The state variable in this optimal control model is the groundwater level on the farm (W_{ft}). Once this variable is identified, the size of the recharge and discharge areas can be determined. The different pasture and crop types, and tree species that will grow on these areas can also be determined. Thus W_{ft} determines the potential and actual carrying capacity of the farm. The decision (or control) variable in this model is the source/sink term, Q. It represents the inflow of water to the aquifer (resulting from rainfall) and outflows or reductions in the groundwater volume as a result of agronomic and engineering practices.

The objective function is maximised subject t the equation of motion (1). The right hand side of Equation (1) is the change in groundwater depth over time on the farm. Multiplying this difference equation by an imputed cost, gives the total user-cost to agriculture of groundwater rises. Subtracting total user-cost from static profits, which represents total revenue from crops, pastures and trees minus total cost of inputs, gives a dynamic measure of profit at time t.

$$\begin{aligned}\pi [W_{ft}, \lambda_t] = \; & [pg(W_{ft}, Q_t) - c(Q_t)] \\ & + \lambda_t \left\{ \{2Z(\rho_f) [T_t (W_{nt} / A_n - W_{ft} / A_f) + ru_t] + Q_t \} A_f / S \right\} \qquad \text{... (3)}\end{aligned}$$

where π is dynamic profit at time t and λ is the costate variable or marginal user-cost of groundwater fluctuations which is negative. This co-state provides a single measure of opportunity-cost of increasing the water-table now rather than managing the land for future use. The multiplier has two components:

(i) The effect an individual farmer has in reducing groundwater tables in the catchment before the area of saltland, and annual reclamation costs, are increased exponentially[2]; and

(ii) The loss of future benefits due to less productive agricultural land on the farm.

π is the current-value Hamiltonian and λ is the current-value costate. Neither is discounted; both are denominated in dollars at time t. Because the costate captures the effect of current decisions on the future, maximising the Hamiltonian in each time period is equivalent to maximising the net present value of the farm in Equation (2) subject to the change in the groundwater depth in Equation (1).

Optimisation is characterised by first-order conditions with respect to the source/sink term, groundwater depth, marginal user-cost plus an initial condition on groundwater depth and a terminal condition on marginal user-cost.

$$\partial\pi/\partial Q_t = 0 = p\partial g/\partial Q_t - \partial c/\partial Q_t - \lambda_t T_t A_f/S\,;\ 0 \le t \qquad \text{... (4a)}$$

$$-\partial\pi/\partial W_{ft} = \dot{\lambda}_t - \delta\lambda_t = -p\partial g/\partial W_{ft} + \lambda_t\,[\,2Z(\rho f)T_t/S\,] \qquad \text{... (4b)}$$

$$\partial\pi/\partial\lambda_t = \{2Z(\rho f)\,[\,T_t\,(W_{nt}/A_n - W_{ft}/A_f) + ru_t\,] + Q_t\}\,A_f/S_t \qquad \text{... (4c)}$$

$$W_{f0} = \text{given} \qquad \text{... (4d)}$$

$$\lim e^{-\delta t}\,\lambda_t = 0; \qquad \text{... (4e)}$$

Condition (4a) equates marginal value product to marginal factor costs from recharge or discharge, plus marginal user-cost. In the case of open-access, with many small farms in the catchment, the scarcity rent due to groundwater would be dissipated, the individual farmer would places no value at all on conservation and, hence, the marginal user-cost would be zero. The clearing/revegetation process would be based on decisions in which current profits were maximised, with no regard for the future. In the case of partially limited access, with a few large farms in the catchment, the marginal user-cost would be greater than zero, but less than the full rent due. In either case, the diminishing level of the shadow price, as compared to optimal social management, means that the resource will be degraded more rapidly than is socially optimal. Further, the present value of returns will be lower. This illustrates a general principal, that competitive exploitation of a common

2 Results of Western Australian studies undertaken by *Peck and Hurle (1973)*, and *Loh and Stokes (1981)*, like *Greig and Devonshire (1981)*, show an exponential relationship between salinity increase and forest clearing.

property resource tends to be economically inefficient and anti-conservationist *(McKelvey, 1980)*.

The marginal user-cost is defined by equation (4b) which can be rearranged into a form similar to equation (4a).

$$0 = p\partial g \,/\partial W_{ft} \,/\, [\delta + 2Z(p_f)\, T_t \,/S - \dot{\lambda} \,/\lambda] - \lambda_t \qquad \text{... (4b')}$$

The marginal value product of the watertable is capitalised at an appropriate discount and equated to marginal user-cost. Thus marginal user-cost is the present value of the damage caused in the future by increasing the watertable today. Because the marginal value product of water is negative, marginal user-cost is also negative. The appropriate discount rate equals the rate of interest plus the marginal change in watertable, minus the rate of growth in the marginal user-cost.

The marginal change in watertable determines the degree of rent dissipation. It equals twice the cumulative probability Z times transmissivity per unit of storativity, $T_t \,/S$. If the farm encompasses the entire catchment, probability Z goes to zero, the marginal change is zero, the marginal user-cost is as large in magnitude as possible and no rent is dissipated. If the farm is very small, Z goes to one-half and $2Z$ goes to one. The transmissivity per unit of storativity, the rate at which water moves in response to a hydraulic head, adds to the interest rate in discounting the future. However, transmissivity would have to be infinite for the future to be completely discounted and the marginal user-cost to be driven to zero. Thus the textbook case of open-access and complete dissipation of rent will never occur. Groundwater will always be a limited access resource and farmers will dissipate some but not all of the rent.

The change in watertable on the farm is reproduced in the first-order condition (4c). Watertable increases if the watertable on neighbouring farms is greater, if a source recharges the groundwater, and also if the farm is downslope from farms with a higher watertable. The farmer's choice of agronomic practices affects the increase in watertable from neighbouring farms and the source of recharge on his farm, but groundwater flowing downslope is exogenous to his decisions. Rent is indirectly dissipated through higher watertables on the farm but the only recourse a farmer has to stop the incursion of water from upslope is to buy the farms above his or lobby for government intervention.

5. *CONCLUSION*

Changes in groundwater flux across farm boundaries caused by clearing, agronomic and engineering practices is a root cause of common property. Yet spatial movement of groundwater has not been extensively modelled in economic models or studies of common property.

Economic models have avoided modelling the spatial aspects of groundwater by comparing open access with exclusive access. Government policies are based on this comparison. But the degree of common property almost always lies in between the two extremes and economic models and policies should begin to include the spatial aspects of groundwater flow.

Using the information generated by agricultural and hydrologic modules, models of private optimisation and social optimisation may be compared. The private optimisation model assumes that present practices will continue over the model's time horizon and the choice of agricultural and engineering techniques employed will remain uncontrolled. The social optimisation or "collective action" model "captures all common pool externalities of the intertemporal and spatial allocation of water. It reflects not only the future value of groundwater stock for each period, and each sub-area, but explicitly, through the equation of motion, takes into account physical hydrological interactions as well" *(Lemoine and Gotsch, 1985, p314).*

BIBLIOGRAPHY

Anderson, J.R., and Thampapillai, D.J., (1990)., "Soil Conservation: Project and Policy Intervention in Developing Countries"., World Bank Staff Paper., PPR Policy Research Series., World Bank, Washington DC (in press).

Bear, J., (1979)., "Hydraulics of Groundwater"., McGraw-Hill Book Company, New York.

Boughton, W.C., (1968)., "A Mathematical Catchment Model for Estimating Runoff"., Journal of Hydrology (N.Z.)., Vol. 7, No. 2., pp 75-100.

Boughton, W.C., (1966)., "A Mathematical Model for Relating Runoff to Rainfall with Daily Data"., The Institution of Engineers, Australia., Civil Engineering Transactions., April., CE8., pp83-97.

Bresler, E., (1972)., "Control of Soil Salinity"., in Hillel, E., (Ed)., Optimising the Soil Physical Environment Toward Greater Crop Yield"., Academic Press, New York., pp 101-32.

Ciriacy-Wantrup, S.V., and Bishop, R., (1975)., "Common Property as a Concept in Natural Resource Policy"., Natural Resource Journal., Vol. 15., pp 713-27.

Crawford, N.H., and Linsley, R.K., (1966)., "Digital Simulation in Hydrology: Stanford Watershed Model IV"., Technical Report No. 39., Department of Civil Engineering, Stanford University.

Ghassemi, F., Jakeman, A.J., and Thomas, G.A., (1989A)., "Estimation of Vertical Recharge Via Mathematical Models of Groundwater Flow"., in Sharma. M.L., (Ed)., Groundwater Recharge., Proceedings of the Symposium on Groundwater Recharge., Mandurah., 6-9 July., pp 139-155.

Ghassemi, F., Jakeman, A.J., and Thomas, G.A., (1989B)., "Ground-Water Modeling for Salinity Management: An Australian Case Study"., Groundwater., Vol. 27, No. 3., May-June., pp 384-392.

Ghassemi, F., Jakeman, A.J., Thomas, G.A., and Daamen, C., (1986)., "Salinity Management in the Mildura Area and the Use of Mathematical Models"., Hydrology and Water Resources Symposium., Brisbane., 25-27 November., The Institute of Engineers, Australia., National Conference Publication No. 86/13., pp 96-102.

Green, W.H., and Ampt, G.A., (1911)., "Studies on Soil Physics: I. Flow of Air and Water Through Soils"., *Journal of Agricultural Science.*, Vol. 4., pp1-24.

Greig, P.J., and Devonshire, P.G., (1981)., "Tree Removal and Saline Seepage in Victorian Catchments: Some Hydrologic and Economic Results"., Australian Journal of Agricultural Economics., Vol. 25, No. 2., pp 134-148.

Hardin, G.J., (1968)., "The Tragedy of the Commons"., Science., Vol. 162 (3859)., pp 1243-8.

Heath, R.C., and Trainer, F.W., (1968)., "Introduction to Ground-Water Hydrology"., John Wiley and Sons, Inc., New York.

Hertzler, G., (manuscript under review)., "Migration and the Degree of Common Property for a Natural Resource"., University of Western Australia.

Hertzler, G., (1990)., "Migration as a Cause of Common Property"., Paper Submitted to the Annual Meeting of the American Agricultural Economics Association, University of British Columbia, Vancouver, Canada., August 4-6.

Hillel, D., (1977)., "Computer Simulation of Soil-Water Dynamics: A Compendium of Recent Work"., International Development Research Centre, Ottawa, Canada.

Hillel, D., (1971)., "Soil and Water: Physical Principles and Processes"., Academic Press, New York.

Hodge, I., (1982)., "Rights to Cleared Land and the Control of Dryland-Seepage Salinity"., Australian Journal of Agricultural Economics., Vol. 26, No. 3., pp 185-201.

Johnstone, D., and Cross, W.P., (1949)., "Elements of Applied Hydrology"., Ronald Press.

Jones, M.J., and Watson, K.K., (1980)., "Solute Movement in Soil Under and Intermittent Water Application Regime"., Hydrology and Water Resources Symposium., Adelaide., The Institute of Engineers, Australia., National Conference Publication No. 80/9., pp 1-5.

Lemoine, P.H., and Gotsch, C.H., (1985)., "Water Resource Development in the Salinas Valley: Economics and Hydrology in a Closed Control Model"., Food Research Institute Studies., Vol. 19, No. 3., pp 293-333.

Loh, I.C., and Stokes, R.A., (1981)., "Predicting Stream Salinity Changes in South-Western Australia"., in Holmes, J.W., and Talsma, T., (ed's)., "Land and Stream Salinity., Proceedings of an International Seminar and Workshop., Perth, Western Australia., November 1980., Elsevier Scientific Publishing Company, Amsterdam., pp 227-254.

McKelvey, R., (1980)., "Common Property and the Conservation of Natural Resources"., in Levin, S.A., and Hallam, T.G., (ed's)., "Applied Mathematical Ecology"., Springer-Verlag, Berlin., pp 58-80.

Manoel, P.J., and Laing, I.E., (1975)., "A Flow and Salt Transport Simulation System for the River Murray in South Australia"., Hydrology Symposium., Armidale., The National Committee on Hydrology of the Institute of Engineers, Australia., National Conference Publication No. 75/3., pp 51-55.

Mein, R.G., (1977)., "Mathematical Modelling of Catchments"., Hydrology Symposium., Brisbane., 28-30 June., The Hydrology of Northern Australia., The Institute of Engineers, Australia., National Conference Publication No. 77/5., pp 146-149.

Mein, R.G., Laurenson, E.M., and McMahon, T.A., (1974)., "Simple Nonlinear Model for Flood Estimation"., Proc. A.S.C.E., Journal of Hydrology Div., Volume 100., HY 11., November., pp 1507-18.

Miller, R.L., (1982)., "Intermediate Microeconomics - Theory, Issues, Applications"., Second Edition., McGraw-Hill International Book Co.

Morel-Seytoux, J.J., (1976)., "Derivation of Equations for Rainfall Infiltration"., Journal of Hydrology., Vol. 12., pp 754-62.

Peck, A.J., and Hurle, D.H., (1973)., "Chloride Balance of Some Farmed and Forested Catchments in South Western Australia"., Water Resources Research., Vol. 9, No. 3., pp 648-57.

Philip, J.R., (1957)., "The Theory of Infiltration: 1: The Infiltration Equation and its Solution."., *Soil Science.*, Vol. 83., pp345-57.

Porter, J.W., and McMahon, T.A., (1971)., "A Model for the Simulation of Streamflow from Climatic Records"., Journal of Hydrology., Vol. 13, No. 4., pp 297-324.

Quiggin, J., (1988)., "Murray River Salinity - An Illustrative Model"., American Journal of Agricultural Economics., Vol. 70, No. 3., pp 635-645.

Quiggin, J., (1986)., "Common Property, Private Property and Regulation - The Case of Dryland Salinity"., Australian Journal of Agricultural Economics., Vol. 30., No. 2-3., pp 103-117.

Sedgley, R.A., (1979)., "A Simple Evapotranspiration Model for Use in Assessing Hydrological Changes in Catchments in Response to Changing Land Use"., Hydrology and Water Resources Symposium., Perth., 10-12 September., National Committee on Hydrology and Water Resources of the Institute of Engineers, Australia., National Conference Publication No. 79/10., pp 123-127.

Smith, R.E., (1980)., "Mathematical Simulation of Water and Sediment Flow Processes on the Catchment Surface"., Hydrology and Water Resources Symposium., Adelaide., The Institute of Engineers, Australia., National Conference Publication No. 80/9., pp 167-171.

Smith, R.E., and Parlange, J. Y., (1978)., "A Parameter-Efficient Hydrologic Infiltration Model"., Water Resources Research., Vol. 14., pp 533-38.

Wills, I., (1987)., "Resource Degradation on Agricultural Land: Information Problems, Market Failures and Government Intervention"., Australian Journal of Agricultural Economics., Vol. 31, No. 1., pp 45-55.

APPENDIX:
HYDROLOGY FOR AN ECONOMIC MODEL OF DRYLAND SALINITY

Hydrology at a Point

Assume the catchment is circular with radius ρ_C and the surface of the catchment is tilted going downward from right to left:

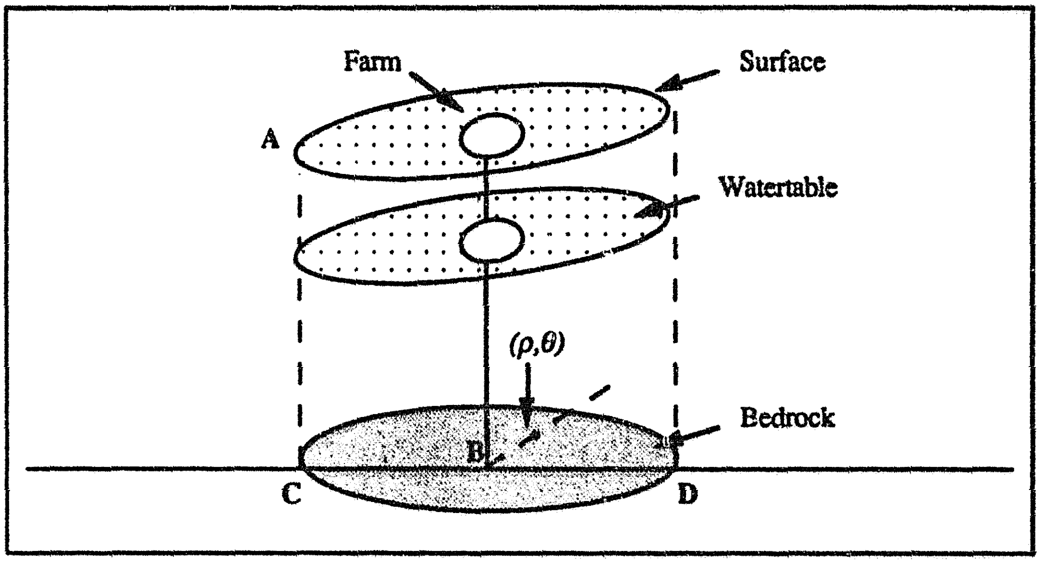

Figure 1: Schematic of a Circular Catchment

This approximates upslope and downslope areas in a catchment which drains at point A. Centred on the surface of the catchment is a farm of radius ρ_f. Below the surface is a watertable and below that is bedrock.

Using polar co-ordinates centred at the origin, *B*, then point (ρ,θ) is described by radius ρ and angle θ from the axis connecting points *C and D*. The slope of the surface along axis *C* to *D* is *r* and the slope perpendicular to the axis is zero.

Flux is the water moving from right to left past point (ρ,θ) along a line through the origin per unit of time.

$$F = T\, \partial W / \partial \rho \qquad \text{... (1)}$$

where T is transmissivity equal to the permeability of the aquifer (k) multiplied by the depth from the surface to the aquifer (m); W is the depth of the watertable and $\partial W / \partial \rho$ is the hydraulic head.

If the hydraulic head increases to the right of point (ρ,θ), water moves to the left at rate T which varies over the catchment due to the slope of the surface.

By law of conservation of mass, flux from right to left past point (ρ,θ) must equal the increase over time in watertable at all points on the left.

$$F = \int_{-\rho_c}^{\rho} S\, (\partial W / \partial t)\, d\rho_l \qquad \text{... (2)}$$

where S is the storativity of water stored per unit of volume in the aquifer; and ρ_l is a radius to the left of point (ρ,θ).

Differentiating with respect to ρ shows that the change in watertable over time at point (ρ,θ) is the change in flux.

$$S\, (\partial W / \partial t) = \partial F / \partial \rho = T\, (\partial^2 W / \partial \rho^2) + \partial T / \partial \rho\, (\partial W / \partial \rho) \qquad \text{... (3)}$$

Therefore the watertable changes over time at point (ρ,θ) if the watertable is non-linearly distributed and the hydraulic head changes, or if changes in transmissivity are combined with a hydraulic head.

There may also be sources or sinks of water at each point due to infiltration or drainage. This augments the change in watertable at point (ρ,θ).

$$\partial W / \partial t = [\, T\, (\partial^2 W / \partial \rho^2) + (dT / \partial \rho)\, (\partial W / \partial \rho) + Q\,] / S \qquad \text{... (4)}$$

This is a typical formulation for hydrological modelling. However, there are two problems which make it inappropriate for an economic analysis. *First* it models the change in watertable at a point in a large catchment. However, the catchment is limited in size and managed by farms comprising many points. The movement of water from farm to farm causes the common property problem. *Second*, it is a partial differential equation in time and causes the common property problem. *Second,* it is a partial differential equation in time and space and it must be converted to an ordinary differential equation to be included in a maximisation by usual methods.

Hydrology on a Farm

For a given point within a finite circular catchment, *Hertzler ("Migration and the Degree of Common Property for a Natural Resource"., manuscript under review),* has shown that flux past the point is:

$$F = 2Z(\rho) [\, {}^{1}/_{2}\, \sigma^2 (\partial W / \partial \rho) + W\, r \cos\theta \,] \qquad \ldots (5)$$

where σ^2 is the variance of water movement and is equivalent to transmissivity, T; r is the slope of the catchment surface; and Z is one-half the probability that water moving along a line through the origin will not reach a border of the catchment minus one-half the probability that water will be stopped at the border.

Cumulative probability Z modifies flux for a finite catchment. In a small catchment, $2Z$ is near zero, and in a large catchment, $2Z$ goes to one.

The change in watertable over time at the point becomes:

$$\begin{aligned} \partial W / \partial t = \; & 2(\partial Z / \partial \rho) [\, {}^{1}/_{2}\, \sigma^2 (\partial W / \partial \rho) + W\, r \cos\theta \,] / S \\ & + 2Z [\, {}^{1}/_{2}\, \sigma^2\, \partial^2 W / \partial \rho^2 + r \cos\theta \,] / S \end{aligned} \qquad \ldots (6)$$

This differs from changes over time in an infinite aquifer because probability Z becomes smaller near the boundaries of the catchment.

The change in watertable over a farm is the change in watertable along each radius passing through the farm, summed over all radii.

$$\partial W_f / \partial t = \int_0^{2\pi} \rho_f \int_0^{\rho_c} \partial W / \partial t \;\; d\, d\theta \qquad \ldots (7)$$

where W_f is the watertable on the farm. Given a quadratic approximation to the distribution of the watertable over the farm, the change in the watertable integrates to:

$$\partial W_f / \partial t = 2Z(\rho_c) [\, \sigma^2 [\, \eta_3(t) + \eta_4(t) \,] + r\, \eta_1(t) \,] A_f / S \qquad \ldots (8)$$

where η_3 and η_4 are second-order coefficients for the curvature of the watertable distribution; η_1 is the first-order coefficient for the slope of the watertable along the axis; and A_f is the area of the farm.

Coefficients η_3, η_4, and η_1 are functions of time making this an ordinary differential equation. Notice that the slope of the watertable perpendicular to the axis, η_2, does not enter into the equation.

Finally, Hertzler has shown how to convert coefficients η_3, η_4, and η_1 into observable watertables on the farm and on neighbouring farms in the catchment.

$$\begin{aligned} \partial W_f / \partial t = & \Big\{ 2Z\,(\rho_f) \Big[\sigma^2 \{ (W_u + W_d) / A_n - W_f / A_f \} \, 4\pi / A_c \\ & + \rho \, (W_u - W_d)\, {}^3/_4\, \pi^{1.5} / (A_c^{1.5} - A_f^{1.5}) \Big] + Q \Big\} \, A_f / S \qquad \ldots (9) \end{aligned}$$

where W_u is the watertable on neighbouring farms upslope; W_d is the watertable on neighbouring farms downslope; A_n is the area of neighbouring farms; A_f is the area of the catchment; and Q is a source or sink.

Equation (9) may be simplified further to Equation (1) in the text.