SPATIAL EQUILIBRIUM MODELLING OF TRADE IN QUASI-HOMOGENEOUS COMMODITIES: THE CASE OF WOOL

by

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Paper contributed to the 35th Annual Conference of the Australian Agricultural Economics Society,
12 - 14 February, 1991, University of New England, Armidale, N.S.W.

Spatial Equilibrium Modelling of Trade in Quasi-homogeneous Commodities: The Case of Wool[†]

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Abstract

As a part of the effort in building a spatial model of the international wool industry, the suitability of the standard competitive spatial equilibrium model is analysed. Commodity and trade aspects of the perceived international wool industry are found not to conform with some principles and assumptions of the standard spatial equilibrium model. These are: the principle of commodity homogeneity, arbitrage conditions and uni-directional trade between a pair of regions. Considering these and two additional aspects of fixed technical coefficients in processing and constant export distribution shares, a quadratic multi-commodity spatial model for trade in quasi-homogeneous woollen commodities is specified. A simplified example is presented and solved.

Introduction

The international wool industry is complex. Wool undergoes several stages of processing from when it is shorn up to when the transformed consumer products are retailed to consumers. In between these value-adding processes of wool growers, scourers, spinners, weavers and various manufacturers, wool and woollen commodities are traded within and between nations.

To date most research into wool trade has been centred either on demand for the unprocessed product (for example, Simmons et al. 1987), or supply of the same product on an individual country basis. Work by Carland (1977) is somewhat an exception in that he studied the wool processing chain by analysing the Japanese textile industry.

Given this background of little research into world trade in wool and woollen products, the purpose of this paper is to provide an outline of a model that will allow analysis of the structure of processing and trade in the international wool industry and the consequences of changes in this structure for the Australian wool industry. Such a model will permit assessment of the impacts that changes in specific parts of the international wool industry may have on the rest of the

[†] Material in this paper is based on research supported by the Wool Research and Development Fund. The assistance of the fund is gratefully acknowledged. The research has been supervised by Gordon MacAulay, on leave from the University of New England and currently with the Australian Bureau of Agricultural and Resource Economics. His assistance and support during the preparation of this paper is much appreciated.

industry. A modification of the standard spatial equilibrium model is outlined in this paper to properly allow for trade share relationships which seem to be involved in the international trade in wool.

Spatial Equilibrium Theory

Spatial equilibrium theory evolved from formulations of linear allocation models of commodities by mathematicians such as Koopmans (1949) and Dantzig (1951). Such modelling was made possible by the development of mathematical programming methods. It was during this time when Enke (1951) and Samuelson (1952) provided for the definition, formulation and solution of a standard spatial equilibrium problem. Subsequently, Takayama and Judge (1964 and 1971) formulated the spatial equilibrium problem as a quadratic programming model and used the Wolfe (1959) simplex method to obtain solutions for a wide range of single as well as multiple commodity problems. Since then these models have been widely used in applied research.

Key aspects and principles of the standard spatial equilibrium model that are of specific concern in this paper can be highlighted using a simple two region, single commodity diagrammatic model as in figure 1. In the trade section of the figure, excess supply for region 1 is derived as a horizontal difference between the supply and demand functions of that region. Since supplies from that region can only be sold to region 2 after transportation, the secondary excess supply function (ES_1+t_{12}) is derived by adding transportation costs to ES_1. An identical procedure is used to derive the excess demand functions for region 2.

Equilibrium in this model is achieved where the primary excess demand function for region 2 equals the derived excess supply function for region 1. This results in the equilibrium traded quantity (x_{12}), where the primary excess supply function for region 1 equals the derived excess demand function for region 2 ($ED_2 - t_{12}$). Equilibrium price levels that relate to each primary function of the two regions in this trade section then determine the levels of domestic supply (x_1 and x_2) and demand quantities (y_1 and y_2) in regions 1 and 2. It can be shown that equilibrium solutions in the above model are obtained where a function representing the area between the excess demand and supply functions is at maximum. Additional conditions associated with this maximum are that these solutions satisfy the supply and demand functions in the two regions and that the price differential between them is less than or equal to the transfer costs (t_{12}). This is the mathematical version of the spatial model displayed in figure 1.

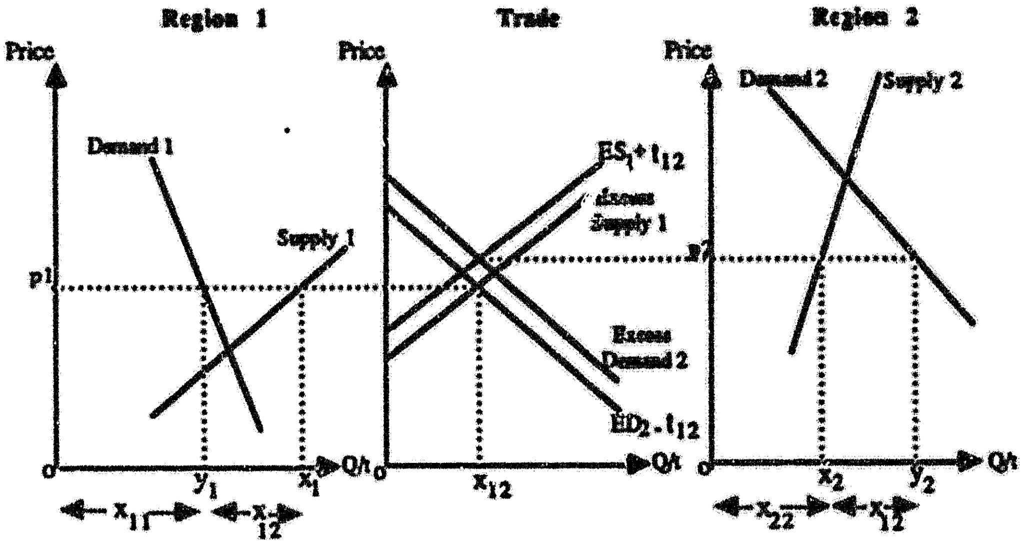

Figure 1. Representation of a simplified two region spatial equilibrium model of trade in a homogeneous commodity.

Three aspects of the standard spatial model are of primary interest in this paper. First is the principle of *homogeneity*. Basic to the model is the assumption that the commodity being traded between the regions is homogeneous in the sense that the commodity from any one region is a perfect substitute for the commodity from any other region in the model. In other words, the elasticity of substitution between the produce of any two regions is one. Even in multi-commodity formulations, for example Takayama and Judge (1971), the assumption of homogeneity is strictly maintained as no trade is allowed between commodities.

Secondly, the 'law of one price' is assumed between trading regions. This principle is represented by the *arbitrage conditions* in the mathematical model, which require the price differential between the importing region and the exporting region to be no more than the unit transfer costs between the two regions.

A third aspect of interest is the *direction of trade*. Any pair of regions in the standard spatial model cannot simultaneously export the same commodity to each other. In Enke's (1951) electric analogue, '... by reversing the usual but artificial electrical convention that current flows from high to low voltages ...', this aspect was modelled as a restriction on the current (analogous to commodities) to flow only from low to high voltages (prices).

It is argued in this paper that these three assumptions of the standard model are unsuitable to represent the nature of the international trade in wool. Bearing in mind that this paper is aimed at providing an outline of a model for the international wool industry, the emphasis given in the paper will be on the differences of the model for quasi-homogeneous commodities from that of the standard model along with a discussion of the information necessary to formulate an appropriate model.

Aspects of the International Wool Industry

The model of the international wool industry is based on the following approach. Woollen commodities are defined according to the major stages of processing at which significant international trading occurs. *Greasy wool* is considered as a primary commodity from which *scoured wool* is then produced. This is processed into *wool sliver*, which is further used to produce *wool yarn*. Wool yarn is then processed into either *wool fabrics*, *wool carpets* or *hand-knitting yarn*. Wool fabrics are used to produce either *woollen wovenwear*, *woollen knitwear* or *woollen blankets*. Of course, some blending of wool with other fibres often takes place from the yarn processing stage onwards. For this reason blended woollen commodities are assumed to be 'woollen' if they contain at least 50 per cent wool. Regions considered to engage in production and trade for purposes of the model are Australia, New Zealand, South Africa, Argentina, Uruguay, the United States, Japan, the Soviet Union, China, the United Kingdom, West Germany, Italy, France, Belgium and the rest of the world.

Quasi-homogeneity

Each of the woollen commodities just defined is not internationally homogeneous. For example, greasy wool from South Africa is not a perfect substitute for New Zealand greasy wool. Generally, the coarser fibre from New Zealand is suited for carpet manufacture while finer South African 'combing' wool is useful for high grade apparel like suitings (D'Arcy, 1986). Angel, Beare and Zwart (1989) showed that fibre length, diameter and vegetable content account for most of the variation in greasy wool prices within Australia and New Zealand. These characteristic differences feed through to higher levels of processing, compounded with blending and design differences between firms in different countries. As a result, for a commodity like wool yarn which is commonly traded internationally, the differences in characteristics of aggregate production from different countries make such commodities quasi-homogeneous. This point of view is in line with Armington's (1969) theory of demand for products distinguished by place of production. He argued that the assumption of merchandise supplied by different countries being perfect substitutes in international trade would appear to be neither realistic nor attractive theoretically. For the purposes of the model it will suffice to formally define a quasi-homogeneous commodity as one including varieties which are seemingly similar but are imperfect substitutes, with a less than unitary elasticity of substitution between any pair of them. Econometric results from the on-going research show that different suppliers (of say greasy wool) face different elasticities of demand in the same (world) market, which is sufficient to support the maintained hypothesis of imperfect substitutability.

Arbitrage conditions

For trade in a non-homogeneous good it does not seem reasonable to impose the law of one price simply because regional average price differentials are not entirely dependent on transfer costs. Quasi-homogeneous goods provide 'slightly' different levels of utility satisfaction to the consumer, hence an Italian woollen suit may command a different price from that of an equivalent English suit even within the same market. For example, according to statistics on the unit values for traded items in 1987, the average unit f.o.b. value for wool yarn in the United Kingdom was A$13,592 and the corresponding c.i.f. values for France and Italy were A$18,982 and A$6,616 respectively. Although the United Kingdom exports wool yarn to both France and Italy, it is unlikely that such price differentials can be simply accounted for by transfer costs. It should also be noted that trade flows from a higher priced region (United Kingdom) to a lower priced region (Italy) does not conform to the arbitrage conditions outlined earlier. As a result of all this, the only requirement relating to arbitrage conditions in quasi-homogeneous commodity modelling is that a particular price differential for shipment from one supplying region to another be equal to transfer costs. Under these conditions, supplies from a given region have different supply prices.

Direction of trade

Continuing with the above example, even though the United Kingdom exported 440 tonnes of wool yarn to France and 700 tonnes of wool yarn to Italy in 1987, France and Italy also exported 1,432 and 557 tonnes respectively of wool yarn to the United Kingdom. This of course is made possible by the fact that wool yarns from these countries have different compositions of woollen and worsted yarns, spun from different qualities of top and carded sliver. Such cases of simultaneous trading in the defined commodities between pairs of countries are common in the international wool industry (see forthcoming Wool Research Reports).

Statistics that have been compiled on the international wool industry also reveal two more aspects that are useful in formulating an appropriate spatial model for the industry. These are the relatively fixed nature of the input-output relationships in wool processing when measured in physical terms and the relatively constant export distribution shares for the defined woollen commodities.

Fixed technical coefficients

The validity of the assumption of fixed technical relationships between woollen inputs and outputs was tested by Carland (1977) in his study of the Japanese wool industry. Such an assumption means that for a given country the amount of a woollen input, say wool yarn, that is required per unit of output, say of woollen fabric, is a constant established over a number of years. Such a parameter could be significantly changed by major changes in industrial textile design and technology. However, for a particular country this could be expected to change over a number of years rather than at a rapid rate. This assumption has been validated econometrically for all stages of wool processing in different countries.

Constant supply distribution shares

Export shares were calculated for the trade flows in wool and woollen products between a number of countries (see forthcoming Wool Research Reports for details). For each woollen commodity produced by a given country, production was split into export and domestic consumption components. The ratio of each of these components to total production was then calculated. These ratios were found to be fairly constant for the years 1980 to 1988. Similarly, the export distribution shares were calculated as a ratio of exports to a designated destination divided by total exports. These calculations were done for each commodity in each producing country over the 1980 - 87 period. These shares were also found to be surprisingly constant over the period, exceptions being export shares to the Soviet Union and China and for trade flows where the average value of such shares was less than 1 per cent. A graphical example is presented in Figure 2.

The criterion used in assessing these shares was the coefficient of variation (the ratio of standard deviation to the mean). The value for the coefficient of variation which turned out to be generally well under 0.5. In fact, where large export shares were involved (say around 10 per cent), the coefficient of variation was very small, generally under 0.2. This indicates very stable export shares. Studies in international trade which are based on trade shares measured as exports to a particular country as a share of total supplies suggest that such shares are related to relative prices. For example, Blandford (1988) reported relative prices to be a significant determinant of export shares for agricultural exports from the United States when using quarterly data, but this relationship disappeared when using annual data. The central point is that, although the analysis is of a different type of share (different from those commonly used in trade share models), it is possible that export distribution shares are dependent upon relative prices. However, the observed lack of variation in annual relative prices for internationally traded wool could be the reason for the stability of the calculated export shares. In any case, for purposes of the current

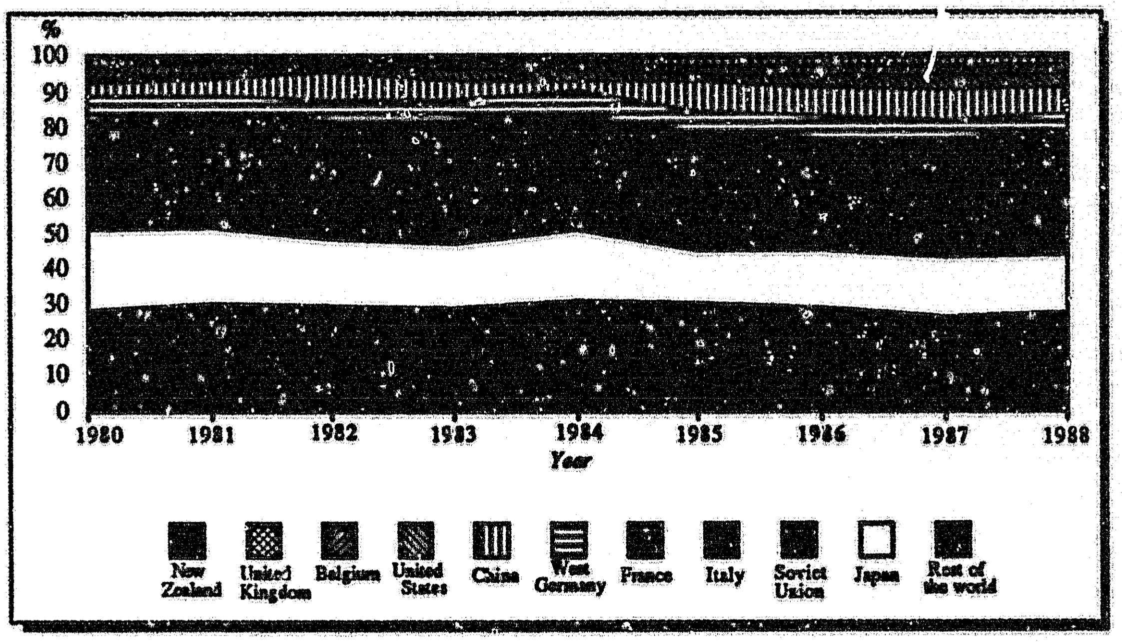

Figure 2. Percentage distributions of Australia's greasy wool exports (1980-88)

modelling effort, it seems reasonable to assume that annual output of a defined woollen commodity of a given country is distributed in constant proportions among the buying countries.

Given the above five special characteristics of the international wool industry, an attempt will be made in the remainder of this paper to formulate an appropriate spatial equilibrium model that incorporates a suitable set of assumptions.

A Spatial Model for Quasi-homogeneous Commodities

In its mathematical form, a net revenue formulation of a standard spatial equilibrium model consists of the following: an objective function expressed as the sum of all the demand revenues less the sum of all supply costs and transportation costs and a set of constraints, which require that the demand and supply equations be satisfied and a set of arbitrage conditions that require that spatial price differentials are less than or equal to the transfer costs. The essential modification for the quasi-homogeneous commodity case where the supply is quasi-homogeneous and fixed export supply shares are observed is to break a regions supply down in export supply functions. Assuming that the coefficients of the supply function which applies to the whole of regions supply also apply to the trade shares then the export supply functions can be derived by weighting the overall supply function by the trade share weights. Thus, if the supply function for region i is

(1) $x_i = \theta_i + \gamma_i p^i$,

where x_i is the supply quantity for region i, and p^i is the supply price for region i, and θ_i and γ_i are intercept and slope parameters then the export supply function for the flow from region i to region j, x_{ij}, will be

(2) $x_{ij} = w_{ij}(\theta_i + \gamma_i p_{ij})$,

where w_{ij} is the export share for the trade flow between regions i and j including the flow from the domestic supply to the domestic demand and p_{ij} is the price for the particular commodity quality supplied from region i to meet region j's demand. The export shares for a given region should sum to 1.0.

The demand function for a particular region i may be defined as

(3) $y_i = \alpha_i - \beta_i p_i$,

where y_i is the quantity demanded in region i, p_i is the demand price for region i and α_i and β_i and intercept and slope coefficients for the linear demand relationship.

The relevant vectors and matrices of coefficients are defined below where the t_{ij} represent the transfer costs between regions i and j, x_i is the supply quantity in region i, y_i is the demand quantity in region i, x_{ij} is the quantity shipped from region i to region j, ρ_i is the non-negative demand price in region i, ρ^{ij} is the non-negative supply price for the quantity supplied as a share of supply x_i from region i to region j and ω_{ij} is the supply distribution share of supply x_i shipped from region i to region j.

$$\alpha = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \cdot \\ \cdot \\ \cdot \\ \alpha_n \end{bmatrix}, \quad \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \cdot \\ \cdot \\ \cdot \\ \theta_n \end{bmatrix}, \quad T = \begin{bmatrix} t_{11} \\ t_{12} \\ \cdot \\ \cdot \\ \cdot \\ t_{nn} \end{bmatrix},$$

$$(n \times 1) \qquad (n \times 1) \qquad (n^2 \times 1)$$

$$B = \begin{bmatrix} \beta_1 & & & 0 \\ & \beta_2 & & \\ & & \ddots & \\ 0 & & & \beta_n \end{bmatrix}, \quad \Gamma = \begin{bmatrix} \gamma_1 & & & 0 \\ & \gamma_2 & & \\ & & \ddots & \\ 0 & & & \gamma_n \end{bmatrix},$$

$$(n \times n) \qquad\qquad (n \times n)$$

$$G_y = \begin{bmatrix} 1 & & & 1 & & & & 1 & & \\ & 1 & & & 1 & & \cdots & & 1 & \\ & & \ddots & & & \ddots & & & & \ddots \\ & & 1 & & & 1 & & & & 1 \end{bmatrix},$$

$$(n \times n^2)$$

$$G_x = \begin{bmatrix} -1\ -1 \ldots -1 & & & \\ & -1\ -1 \ldots -1 & & \\ & & \ddots & \\ & & & -1\ -1 \ldots -1 \end{bmatrix},$$

$$(n \times n^2)$$

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad X = \begin{bmatrix} x_{11} \\ x_{12} \\ \vdots \\ x_{nn} \end{bmatrix}.$$

$$(n \times 1) \qquad (n \times 1) \qquad (n^2 \times 1)$$

$$\rho_y = \begin{bmatrix} \rho_1 \\ \rho_2 \\ \vdots \\ \rho_n \end{bmatrix} \geq 0 \quad \text{and} \quad \rho_X = \begin{bmatrix} \rho^{11} \\ \rho^{12} \\ \vdots \\ \rho^{nn} \end{bmatrix} \geq 0$$

$$(n \times 1) \qquad\qquad (n^2 \times 1)$$

$$W = \begin{bmatrix} \omega_{11} & & & 0 \\ & \omega_{12} & & \\ & & \ddots & \\ 0 & & & \omega_{nn} \end{bmatrix}$$

$$(n^2 \times n^2)$$

The basic structure of the net social revenue formulation of the spatial problem has been outlined by Takayama and Judge (1971). In the formulation of the export share model outlined below it has been assumed that the problem is a 'regular' problem in which the irregular cases of negative supply and demand prices or negative demand or supply quantities can be assumed not to occur (for details see Takayama and Judge 1971). This assumption merely simplifies the algebra and reduces the problem size but has no impact on the general nature of the problem.

The maximisation problem for the quasi-homogeneous spatial equilibrium model is expressed as Problem 1.

Problem 1: Find $(\rho_{\bar{y}}' \ \rho_{\bar{X}}' \ \bar{X}') \geq 0'$ that maximizes

(4) $$Z = (\rho_y' \ \rho_X' \ X') \left\{ \begin{bmatrix} \alpha \\ -WG'_y\theta \\ -T \end{bmatrix} + \begin{bmatrix} -B & & -G_y \\ & -(I \otimes \Gamma)W & I \\ G'_y & -I & \end{bmatrix} \begin{bmatrix} \rho_y \\ \rho_X \\ X \end{bmatrix} \right\}$$

subject to

(5) $$\begin{bmatrix} \alpha \\ -WG'_y\theta \\ -T \end{bmatrix} + \begin{bmatrix} -B & & -G_y \\ & -(I \otimes \Gamma)W & I \\ G'_y & -I & \end{bmatrix} \begin{bmatrix} \rho_y \\ \rho_X \\ X \end{bmatrix} \leq 0$$

(6) and $(\rho_y' \ \rho_X' \ X') \geq 0'$

The objective function for this problem consists of the revenue from the consumption of the good in each region less the supply costs less the transfer costs. This objective function is then subjected to the constraints that the demand functions must be satisfied, the $n^2 \times n^2$ share weighted supply functions must be satisfied and the spatial pricing conditions satisfied. These constraints can be written as follows:

(7) $\alpha - B\rho_y - G_y X \leq 0$,

and for each of the trade flows between regions i and j

(8) $\omega_{ij}\theta_i - \omega_{ij}\gamma_i \rho^{ij} + x_{ij} \leq 0$,

(9) $\rho_i - \rho^{ij} \leq t_{ij}$.

The essential difference between the standard spatial equilibrium model and the trade share model is that there is a specified supply function for each specific trade flow.

The existence of an optimal solution to this problems depends on the problem being a well behaved convex programming problem. As Takayama and Judge (1971) have shown, the standard model is well-behaved and since only linear constraints have been added in this model the same will hold true in this case also. The first order conditions for a optimum solution to the problem can be obtained by differentiating the Lagrangian function ϕ for Problem 1 with respect to each of the solution variables. These conditions must hold at the optimal solution values for the primal and dual variables.

The Lagrangian function is:

$$(10)\quad \phi = (\rho_y'\ \rho_X'\ X')\left\{\begin{bmatrix}\alpha \\ -WG'_y\theta \\ -T\end{bmatrix} + \begin{bmatrix}-B & & -G_y \\ & -(I\otimes\Gamma)W & I \\ G'_y & -I & \end{bmatrix}\begin{bmatrix}\rho_y \\ \rho_X \\ X\end{bmatrix}\right\}$$

$$-\begin{bmatrix}\lambda_1 \\ \lambda_2 \\ \lambda_3\end{bmatrix}'\left\{\begin{bmatrix}\alpha \\ -WG'_y\theta \\ -T\end{bmatrix} + \begin{bmatrix}-B & & -G_y \\ & -(I\otimes\Gamma)W & I \\ G'_y & -I & \end{bmatrix}\begin{bmatrix}\rho_y \\ \rho_X \\ X\end{bmatrix}\right\} ,$$

and the first-order conditions are:

$$(11)\quad \frac{\partial\bar{\phi}}{\partial\bar{P}} = \begin{bmatrix} \alpha \\ -WG'_y\theta \\ -T \end{bmatrix} + \left\{ \begin{bmatrix} -B & & -G_y \\ & -(I\otimes\Gamma)W & I \\ G'_y & -I & \end{bmatrix} + \begin{bmatrix} -B & & G'_y \\ & -(I\otimes\Gamma)W & -I \\ -G_y & I & \end{bmatrix} \right\} \begin{bmatrix} \bar{\rho}_y \\ \bar{\rho}_X \\ \bar{X} \end{bmatrix}$$

$$- \begin{bmatrix} \bar{\lambda}_1 \\ \bar{\lambda}_2 \\ \bar{\lambda}_3 \end{bmatrix}' \begin{bmatrix} -B & & -G_y \\ & -(I\otimes\Gamma)W & I \\ G'_y & -I & \end{bmatrix} \leq 0 \text{ and } \frac{\partial\bar{\phi}}{\partial\bar{P}}\bar{P} = 0 \ ,$$

$$(12)\quad \frac{\partial\bar{\phi}}{\partial\bar{D}} = -\begin{bmatrix} \alpha \\ -WG'_y\theta \\ -T \end{bmatrix} - \begin{bmatrix} -B & & -G_y \\ & -(I\otimes\Gamma)W & I \\ G'_y & -I & \end{bmatrix} \begin{bmatrix} \bar{\rho}_y \\ \bar{\rho}_X \\ \bar{X} \end{bmatrix} \geq 0 \text{ and } \frac{\partial\bar{\phi}}{\partial\bar{D}}\bar{D} = 0 \ ,$$

and

$$(13)\quad (\bar{P}, \bar{D}) \geq 0 \ ,$$

where P is the vector of primal variables and D the vector of dual variables or Lagrangian multipliers. Thus

$$P = (\rho_y' \, \rho_X' \, X')$$

and

$$D = (\lambda_1' \lambda_2' \lambda_3') \ .$$

A special property of this class of problem is that at the optimum P = D which follows from the special skew-symmetric nature of the square matrices shown in equation (11). Also, at the optimum the value of the primal part of the objective function equals the dual part so that the overall value of the objective function is zero.

The first-order conditions indicated by equations (11) and (12) can be given a market interpretation. The D vector can be interpreted as the shadow values on the set of constraints with λ_1 as the vector of demand prices, λ_2 as the vector of supply prices and λ_3 as the vector of interregional flows or shipments. At the optimum this set will match the set P of primal variables.

The first set of conditions (11) require that at the optimum and when the demand price for a region is positive then the demand function must be exactly satisfied while when the demand price is zero then the demand relationship is satisfied as an inequality. The associated complementary slackness condition contained in (11) requires that either the demand price is zero or equation (7) above is strictly equal to zero implying that the demand quantity less the quantity calculated using the equilibrium price in the demand function must equal zero. In practical terms this means that with a non-negative price that the demand function must be satisfied as an equality.

In a similar way to the demand function, the market-share weighted supply functions must be satisfied for non-negative supply prices associated with each trade flow and if one of the prices is zero then the relationship in (8) will be satisfied as an inequality. In a standard setting, with each supplying region having a trade share for each demand region it would normally be expected that trade would occur and the supply function would hold as an equality.

Also, the arbitrage conditions in equation (11) require that when a trade flow is non-negative that the arbitrage condition must hold as an equality. If a particular trade flow is zero then the arbitrage conditions may be satisfied as an inequality.

A Sample Problem

To illustrate the solution of the model, a small sample problem is used. Assume three regions each engaged in production and trade of a commodity. Qualitatively, the commodity produced by a given region is seemingly similar but a imperfect substitute for another region's product, that is, it is a quasi-homogeneous product.

Supply of the commodity in each region is represented by a linear function as follows:

$$x_1 = -1 \quad + 16\,p^1$$
$$x_2 = -1.2 + 6\,p^2$$
$$x_3 = -1.5 + 5\,p^3$$

Each region is faced by a linear demand function as follows:

$$y_1 = 80 - 2.0\,p_1$$
$$y_2 = 40 - 1.5\,p_2$$
$$y_3 = 60 - 2.0\,p_3$$

The constant supply distribution shares for each region's product are as follows:

$$\omega_{11} = 0.2 \quad \omega_{12} = 0.3 \quad \omega_{13} = 0.5$$
$$\omega_{21} = 0.1 \quad \omega_{22} = 0.4 \quad \omega_{23} = 0.5$$
$$\omega_{31} = 0.1 \quad \omega_{32} = 0.5 \quad \omega_{33} = 0.4$$

Transfer costs between regions are as follows:

$$t_{11} = 0 \quad t_{12} = 2 \quad t_{13} = 2$$
$$t_{21} = 2 \quad t_{22} = 0 \quad t_{23} = 1$$
$$t_{31} = 2 \quad t_{32} = 1 \quad t_{33} = 0$$

Given this information, the problem is formulated using equations (4) and (5), which result in the matrix structure i, Table 1. Notice that the signs in the objective function (equation (1)) are reversed because RAND QP minimizes.

The optimal solution for the sample problem is presented in Table 2.

Two outstanding departures from the solutions of the standard spatial model can be noticed from Table 2. These are that the two way flows of the commodity between a pair of regions is indicated and that the prices differ for supply from the same supplying region. These price differences reflect the fact that a different quality of product can be purchased from a given region and that these different qualities are determined according to the requirements of the demanding region. The demanding region is assumed to purchase a given quality mix from the particular supplying region.

Also apparent from the solution in Table 2 is the observation that for each shipment path the demand price is equal to t: supply price plus transfer cost (the arbitrage conditions are satisfied for each trade flow and can be satisfied in both directions).

Table 1 Tableau for Constant Export Share Model

	DP1	DP2	DP3	SPX11	SPX12	SPX13	SPX21	SPX22	SPX23	SPX31	SPX32	SPX33	X11	X12	X13	X21	X22	X23	X31	X32	X33	
DP1	2																					
DP2		1.5																				
DP3			2																			
SPX11				3.2																		
SPX12					4.8																	
SPX13						8																
SPX21							0.6															
SPX22								2.4														
SPX23									3													
SPX31										0.5												
SPX32											2.5											
SPX33												2										
OBJ	-80	-40	-60	0.2	0.3	0.5	0.12	0.48	0.6	0.15	0.75	0.6	0	2	2	2	0	1	2	1	0	
RDP1	-2												-1			-1			-1			≤-80
RDP2		-1.5												-1			-1			-1		≤-40
RDP3			-2												-1			-1			-1	≤-60
RPX11				-3.2									1									≤ 0.2
RPX12					-4.8									1								≤ 0.3
RPX13						-8									1							≤ 0.5
RPX21							-0.6									1						≤ 0.12
RPX22								-2.4									1					≤ 0.48
RPX23									-3									1				≤ 0.6
RPX31										-0.5									1			≤ 0.15
RPX32											-2.5									1		≤ 0.75
RPX33												-2									1	≤ 0.6
RX11	1			-1																		
RX12		1			-1																	≤ 2
RX13			1			-1																≤ 2
RX21	1						-1															≤ 2
RX22		1						-1														
RX23			1						-1													≤ 1
RX31	1									-1												≤ 2
RX32		1									-1											≤ 1
RX33			1									-1										

Table 2. Solution for Constant Export Share Model

=OPTIMAL SOLUTION

OBJECTIVE TOTAL	=	L	*	LINEAR (PX)	+	QUADRATIC (X'QX)
0.00	=	1.00	*	-1385.89	+	1385.89

PRIMAL VARIABLES SOLUTION NAME	VALUE	DUAL VARIABLES SOLUTION NAME	VALUE
DP1	12.97	RDP1	12.97
DP2	4.52	RDP2	4.52
DP3	5.15	RDP3	5.15
SPX11	12.97	RPX11	12.97
SPX12	2.52	RPX12	2.52
SPX13	3.15	RPX13	3.15
SPX21	10.97	RPX21	10.97
SPX22	4.52	RPX22	4.52
SPX23	4.15	RPX23	4.15
SPX31	10.97	RPX31	10.97
SPX32	3.52	RPX32	3.52
SPX33	5.15	RPX33	5.15
X11	41.71	RX11	41.71
X12	12.37	RX12	12.37
X13	25.73	RX13	25.73
X21	6.70	RX21	6.70
X22	11.32	RX22	11.32
X23	13.06	RX23	13.06
X31	5.64	RX31	5.64
X32	9.54	RX32	9.54
X33	10.91	RX33	10.91

Concluding Remark

In the paper, a spatial model for trade in quasi-homogeneous commodities has been specified as a quadratic mathematical programming problem. A simplified sample problem was formulated and solved using RAND-QP (Cutler and Pass, 1971), a quadratic programming package. As in a standard net social revenue spatial equilibrium model, the optimal solution to the primal-dual problem has been shown to give a zero value for the objective function.

This quasi-homogeneous problem is readily expanded to take into account the transformation of a raw material to a processed product using constant transformation coefficients and following the activity analysis formulation of Takayama and Judge (1971). In this way the problem of the analysis of the wool processing sector can readily be included in such a model provided that estimates can be made of the transformation coefficients and the cost of processing. Where there is considerable uncertainty about these data sensitivity analysis can be carried out.

In an international trade context exchange rates are important. The quasi-homogeneous commodity version of the spatial equilibrium model is readily adapted to include exchange rates (MacAulay 1978), as well as, tariffs (both fixed or ad valorem), quantitative restrictions and a variety of other policy instruments. Further, if it were necessary to use non-linear demand, supply and/or transfer functions these could be incorporated and solved (if a solution exists) using a nonlinear solver such as MINOS (Takayama and MacAulay 1990). Following this nonlinear approach it would also seem to be possible to develop formulations which would incorporate a dependence of the market shares on the endogenously determined prices. Issues of the existence and uniqueness of solution may, however, then need to be considered on a case by case basis. A further adaption of the model which may be appropriate in some instances is the combination of trade shares for some regions as the determinants of the trade flows with competitive determination of flows for other regions. As well, there would seem to be no reason why the model would not function equally well with import trade shares as fixed rather than the export shares as outlined in this paper.

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