# Normative Supply Functions by Linear Programming Procedures

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**In many analyses of profit maximization, attention is directed to large herds and specialization in milk production. Why then do some farmers persist in keeping small dairy herds even when prices for Grade B milk become quite low? This article shows that even with relatively low prices for milk the inclusion of the small dairy herd in the combination of enterprises may still provide the maximum profit from the resources available on the farm. At higher levels of milk prices, it becomes profitable to increase the number of cows and the consequent quantity of milk produced. The various levels of prices and corresponding production of milk constitute a supply function for the farm. This paper illustrates the supply function by utilizing the resources found on a typical 160-acre farm in northeastern Iowa with the full-time labor of two men and $10,250 in capital available. With the aid of variable-price programming, optimum combinations of enterprises for maximizing profits were computed, starting with milk prices so low (92 cents per hundredweight for grade B milk) that no milk cows would be included in the organization. With the price of milk increased to $1.01, a 13-cow herd would maximize profits while the small poultry enterprise would be dropped. With the price of milk at $2.38, a 24-cow herd would be the most profitable and the fall pig enterprise would be dropped. Thus at each level of milk prices, the farmer would have an optimum organization of cows, hogs, and poultry for profit maximization.**

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Variations in the basic methods of linear programming have been developed recently that extend its usefulness in handling analysis in which such things as resources, factor costs, and product prices vary.[1] Application of the new analytical tools is greatly advanced by the use of electronic computers, which reduce both time and cost of computations.

The purpose and procedures of variable-price programming is the subject of this article. In a variable-price programming analysis for a given resource situation and a particular product, prices for which production plans for the farm should change to maximize net farm income are computed. Thus the scale of prices revealed by the analysis constitutes a normative supply function. The procedures for estimating normative supply functions are illustrated by data taken from the results of a study on the opportunities for making adjustments on dairy farms in northeastern Iowa.[2]

Variable-price programming derives a supply function that can be formalized as follows: $q_A = f(P_1, P_2, P_a, \ldots P_n, R_1, R_2 \ldots R_n, C_1, C_2 \ldots C_n)$ in which $q_A$=quantity of $A$ produced ($P_A$ varied)
$P_1$ to $P_n$=prices of factors and products at the farm level
$R_1$ to $R_n$=the fixed resources of the farm (firm)
$C_1$ to $C_n$=coefficients of production on the farm in all production alternatives considered.

The quantity of product A produced is not considered as simply a function of the price of product A. The supply function also considers alternative products that may be produced with the given resources, at specified prices for the factors, and the technology of the farm under consideration. The optimum farm production plans derived for each price level are subject to the restrictions imposed and the alternatives offered. Any number of alternative products and techniques of production can be considered, with each combination of techniques included giving a different optimum plan.

[1] Cf. Heady, Earl O., and Candler, Wilfred V., *Linear Programming Methods.* Iowa State College Press, Ames, 1958, Ch. 8.

[2] Additional details will appear in a forthcoming bulletin of the Iowa Agricultural Experiment Station which will deal with research on adjustments to meet changes in prices and improve incomes on dairy farms in northeastern Iowa (an application of programming procedures in deriving supply responses and imputed resource values).

## The Nature of the Results

The supply function thus obtained is normative in the sense that it indicates what a farmer would plan to produce if he intended to maximize profits. It is not predictive in the sense that it will explain what he actually will produce.

Production plans for optimum allocation of resources from the viewpoint of society could be obtained by including all possible off-farm opportunities, as well as on-farm opportunities for the use of the farm's resources. But if it is known that the farm operator will not consider off-farm use of his own labor or capital or certain on-farm opportunities, these alternatives can be deleted and the result will be a more "realistic" supply function. Thus by including only the alternatives that the farm operator is likely to consider, the resulting supply function becomes, so to speak, "less normative" and "more realistic." The programming procedure still gives a profit-maximizing plan within the given alternatives. To the extent that farmers do not maximize profits within that framework, the results of this technique are never predictive.

## Application to a Typical Farm

The typical 160-acre dairy farm in northeastern Iowa includes 116 acres of rotation land and 30 acres of permanent pasture. The plans presented here are based on a labor supply of two full-time men and a capital supply of $10,250 which can be used for annual crop expenses, livestock investments, or annual livestock expenses. The farm machinery for crops is considered to be given and adequate for any of the possible cropping programs offered. Building resources include space for 36 litters of spring pigs, 16 litters of fall pigs, 175 hens, and 35 dairy cows.

The cropping alternatives include three rotations consisting of corn, oats, and meadow, with variations of from 25 to 50 percent of the land in meadow. The soil type and topography of the area require a minimum of 25-percent meadow for sustained crop yields. Two levels of fertilization are also offered as alternatives.

Livestock opportunities include production of whole milk to be sold as grade B milk at a production level of 9,500 pounds per cow; spring pigs with 7.4 pigs weaned per litter, fall pigs with 7.3 pigs weaned per litter; and a small poultry enterprise of average production. The alternative of buying corn was included at a price of $1.35 p[illegible] bushel.

The plans shown are the ones that maximize income. No other use of the resources would produce this amount of net income under the assumptions. The programming utilizes all available resources as long as net income can be increased. Whether the farmer would choose the particular plan which maximizes net income would depend upon his propensity for income. However, there might be alternative plans which would produce slightly less income, say $200 less, with a great deal less work, say with 10 fewer cows. The operator might decide to utilize the alternative plan rather than do the work of caring for 10 extra cows for the $200 extra income. Frequently, farmers carry on an enterprise even though it is not as profitable as others at the moment, possibly anticipating higher returns at some later time. Dairying is often one of these because cow herds are difficult to establish or to increase quickly.

## Effect of Varying the Price of Milk

Table 1 presents the results of varying the milk price from zero to $3.02 per hundredweight wit[illegible] all other prices held constant. In this instance, the price for hogs was fixed at $17.98 per hundredweight. The price of $3.02 for milk was the upper limit for the latest plan developed.

As noted in table 1, income remains the same for at least two plans at some point in the price range. This is the point at which the farm manager can be indifferent between two plans in maximizing profit. In some instances, three or more such alternatives are available (for example, plans 1, 2, and 3 of table 2). However, as the price moves in any direction from this point, only one plan remains optimum. Likewise, when dairy is included in the optimum plan, income for any one plan varies as the price of milk varies.

At $0.93 per hundredweight of milk, dairy begins to compete with crop fertilization for capital resources. At $1.01, the dairy enterprise also draws resources from the poultry enterprise. When 13 cows are included, production of forage is expanded at the expense of corn production. Fertilizer is the most economical way to do this, but it also becomes necessary to change the rotation

TABLE 1.—*Variable milk prices—hog price constant*

| Plan No. | Range in milk price per cwt. | Cropping program | | Dairy cows | Litters of pigs | | Laying hens | Corn purchased | Range in net income |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | Spring | Fall | | | |
| | *Dollars* | *Rotation* [1] | *Acres* | *Number* | *Number* | *Number* | *Number* | *Bushels* | *Dollars* |
| 1 | 0. 00–0. 92 | $CCOM_o$<br>$CCOM_f$ | 101<br>15 | 0 | 36 | 16 | 175 | 1, 628 | 7, 439 |
| 2 | . 93–1. 00 | $CCOM_o$ | 116 | 3 | 36 | 16 | 175 | 1, 898 | 7, 440– 7, 457 |
| 3 | 1. 01–1. 38 | $CCOM_o$ | 116 | 13 | 36 | 16 | 37 | 2, 396 | 7, 458– 7, 927 |
| 4 | 1. 39–2. 14 | $CCOM_o$<br>$CCOM_f$ | 103<br>13 | [2] 13 | 36 | 16 | 0 | 2, 280 | 7, 928– 8, 895 |
| 5 | 2. 15–2. 37 | $CCOM_o$<br>$CCOMM_f$ | 106<br>10 | 14 | 36 | 16 | 0 | 2, 437 | 8, 896– 9, 203 |
| 6 | 2. 38–3. 02 | $CCOM_o$<br>$CCOMM_f$ | 30<br>86 | 24 | 36 | 5 | 0 | 2, 067 | 9, 204–10, 682 |

[1] In the rotations C=corn, O=oats, M=meadow. Rotation subscripts, $_o$=no fertilizer, $_f$=fertilized.

[2] The increase in dairy cows in this step was fractional and was not recorded when results were rounded to whole numbers.

to provide sufficient forage. A price of $2.38 for milk is required to put dairying on the same profit level as fall pigs. At some price above $3.02, the dairy enterprise would begin to draw resources from spring pigs. Hence, a dairy enterprise selling grade B milk at $2.68 per hundredweight cannot compete with spring pigs at a price of $17.98 per hundredweight for hogs.

With this combination of resources and production alternatives, it appears that production of milk is relatively profitable at a low milk price. This results primarily because alternative uses of labor and forage in such enterprises as beef cattle were not offered. In each of these plans, not all available labor in any period is utilized; thus labor has an opportunity cost of zero. Hence under the assumptions, any use of the farmer's labor which in combination with other unused resources will produce a return above variable costs will add to the income. Therefore, as long as the price is sufficient to satisfy the farm operator, he will milk cows even if whole milk is as low as $1.01 per hundredweight. This is his only opportunity, among those considered, to use his labor and other unused resources. He can earn this much or nothing. As a result, many operators work for very low returns per hour with some enterprises.

If labor were evaluated at $1.50 per hour, the results would be somewhat different than those shown in table 1. Ordinarily, a farmer would not add 3 cows or 13 cows for an income of $1 or $19 or as in the area of $2.15 to $2.38 per hundredweight for milk—10 cows for $308. This paper is not advocating that farmers do this, only pointing out how a farmer might react to obtain more income under the assumptions of the study. The plans do show, however, what a farmer would produce if his sole objective was to maximize profits.

Similar considerations would be applicable to the data in table 2. A farmer would not likely wish to do all the work necessary to keep 14 sows (in place of keeping 3 milk cows) for $50 extra profit.

Table 2.—*Variable hog prices—milk price constant*

| Plan No. | Range in hog price per cwt. | Cropping program | | Dairy cows | Litters of pigs | | Laying hens | Corn purchased | Range in net income |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | Spring | Fall | | | |
| | *Dollars* | *Rotation* [1] | *Acres* | *Number* | *Number* | *Number* | *Number* | *Bushels* | *Dollars* |
| 1 | 0.00–12.95 | CCOMM$_f$<br>CCOM$_f$ | 113<br>3 | 33 | 0 | 0 | 175 | 0 | 7, 301 |
| 2 | 12.96 | CCOMM$_f$<br>CCOM$_f$ | 103<br>13 | 30 | 14 | 0 | 175 | 0 | 7, 351 |
| 3 | 12.97–15.10 | CCOMM$_f$<br>CCOM$_f$ | 90<br>26 | 26 | 31 | 0 | 175 | 1, 257 | 7, 352– 8, 269 |
| 4 | 15.11–15.49 | CCOMM$_f$<br>CCOM$_f$ | 86<br>30 | 25 | 36 | 0 | 60 | 1, 464 | 8, 270– 8, 491 |
| 5 | 15.50–17.44 | CCOMM$_f$<br>CCOM$_f$ | 83<br>33 | 24 | 36 | 2 | 0 | 1, 570 | 8, 492– 9, 549 |
| 6 | 17.45–19.26 | CCOMM$_f$<br>CCOM$_f$ | 86<br>30 | 24 | 36 | 5 | 0 | 2, 067 | 9, 550–10, 621 |
| 7 | 19.27–19.41 | CCOMM$_f$<br>CCOMM$_o$ | 23<br>93 | 22 | 36 | 11 | 0 | 2, 946 | 10, 622–10, 724 |
| 8 | 19.42–19.52 | COMM$_f$<br>CCOMM$_o$ | 7<br>109 | 21 | 36 | 13 | 0 | 3, 192 | 10, 725–10, 800 |

[1] See table 1 for meaning of letters of rotation.

## Effects of Varying the Price of Hogs

Table 2 presents the results of varying the price of hogs with all other prices held constant. In this instance, the price of milk is fixed at $2.68 per hundredweight. Plan 3 of table 2 indicates that hog production would increase from zero to 31 litters of spring pigs at a price of $12.97 per hundredweight. The dairy enterprise would decrease by 7 cows, and the cropping program would change to some extent. The hog-corn price ratio here is only 10 to 1.

Plan 5 indicates that the price of hogs must rise to $15.50 before fall hogs will be produced, and then only 2 litters will be included. Higher prices for hogs increase production by very little. Hence, a supply curve concave upward with decreasing elasticity of supply with respect to hog prices is indicated.

## Stability of Production

These plans show that under certain conditions relatively large changes in the farm organization and production program need to be made to conform to profit maximization, given a change in the price of one particular product. However, the increase in net income as a result of the change in production may not be substantial. In tables 1 and 2, the ranges of net income given are the result of the variation in price. For example, as shown in table 2, plan 6 has a range of net income of $1,071 owing to a price range of $1.81, or a change in net income of $5.92 for each 1-cent change in the price of hogs. If, for example, plan 6 was held until the price rose to $19.41 (the upper limit of plan 7), the resulting net income would be $10,680, or only $44 under the maximum profit net income of $10,724. This $44 can be obtained only by a considerable change in the cropping rotation, decreasing cow numbers by 2, increasing litters of fall hogs by 6, and purchasing an additional 879 bushels of corn. The farmer may well conclude that $44 is not worth the effort.

## Assumption as to Mobility of Resources

In linear programming, it is assumed that capital and other resources can be moved freely from one enterprise to another without any difficulty but only within the combinations specified by the production plans. Outside this framework of resources and assumptions, the resources are quite fixed. This is a condition that may be completely true only in the long run. For example, it might be fairly easy to reduce the size of a dairy enterprise by selling some cows, but adding cows of the same type might be difficult. A dairyman might prefer not to buy stock because of uncertainty as to disease or other problems, and it would take a number of years to raise the additional cows.

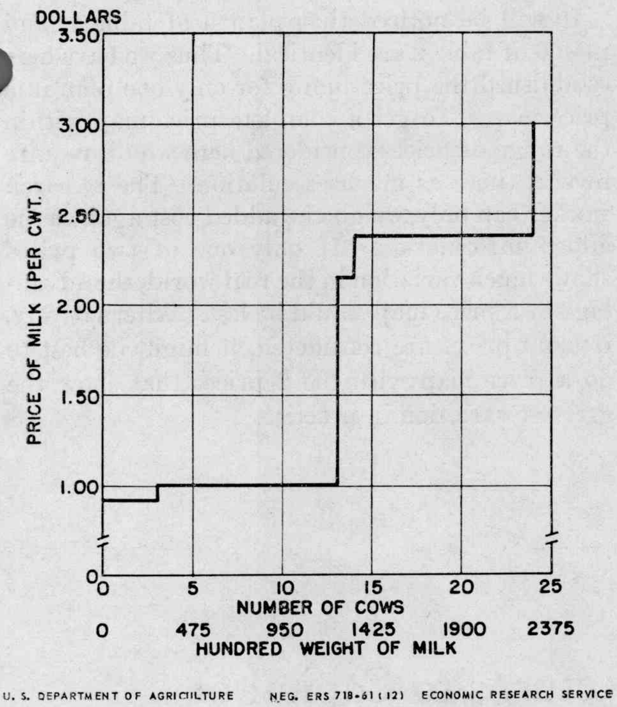


FIGURE 1.—Supply function for milk.

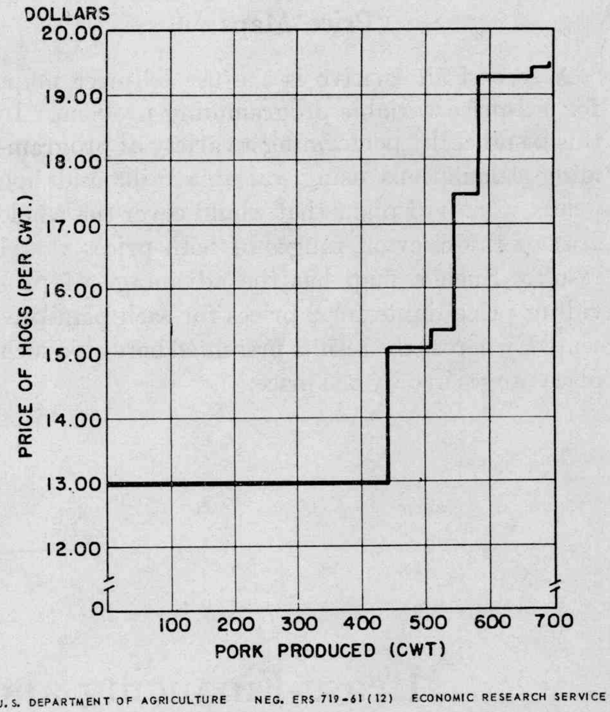


FIGURE 2.—Supply function for pork.

Thus the plans developed here would apply to a situation in which the prices are those expected to hold over a period of time—long enough to permit the adjustments to take place. For short-run price variations, farmers may make simple adaptation such as changing feed rations or levels of feeding.

## Graphic Presentation

Figures 1 and 2 present the results in graphic form. The resulting supply curves are of a "stair-step" nature. The stairstep characteristic results from a finite number of alternatives, and rigid resource restrictions used in the programming calculations. The number of "steps and corners" is a function of the number of alternatives and restricting resources.

Although a finite number of major production alternatives is available to any one farmer, different activities can be represented through variations in hog or dairy rations, breeds, timing of production, levels of fertilization, equipment arrangements, and so on. Similarly, the resources available could be subdivided into labor available at many different periods, capital available at different periods, or many other schemes to result in many more resource restrictions. Including more activities and more restrictions will give a normative supply function with more and smaller "steps."

However, the nature of supply functions of individual farms is more nearly represented by the type presented here than by a continuous regression function. Ordinarily, farmers change their patterns of production only for fairly large changes in expected prices and then in a discrete manner. Few if any individuals make adjustments in the sense of a continuous function. Such a continuous regression function would typify an aggregate supply situation, as the steps in the individual supply functions would occur at different prices because of differences in resources and coefficients of production. Also, the effect of any individual change would be almost unnoticeable in the typical aggregate supply functions found in agriculture.

Other alternative methods of presenting results for practical use by individual farmers and extension personnel could be used. One possibility is a production possibility curve with linear segments. However, a 2-dimensional diagram allows an analysis of only 2 products and is thus rarely useful in presenting actual programming results.

## Price Maps

A second alternative is the use of price maps for a 2-price variable programming problem. In this instance, by performing a variety of programming calculations using variable milk and hog prices, a map of plans that would cover the whole area of the relevant ranges of both prices would result. Such a map has the advantage of providing price limits for 2 prices for each plan presented, whereas the results presented here give such price ranges in only one price.

It will be noticed that plan 6 of table 1, and plan 6 of table 2 are identical. Thus we have here established the price limits for only one plan in a price map. To get a complete price map within the range of prices considered here would require several times as much calculation. The research worker can only weigh the added cost against the added information. If only one of two prices shows much variation in the real world, the advantage of a price map would be less. Alternatively, if more prices are considered, it might be best to do a price map with the 2 prices that show the greatest variation in practice.

# Lease-Financing and Returns to Capital of Food Marketing Firms

*By Stephen J. Hiemstra*

**The changing structure of the food marketing system focuses attention on competitive relationships existing in food industries. Conventional measures of profits, such as profit rates on stockholders' equity and on total assets, often are used to appraise the performance of these industries. This article points out some of the shortcomings of these profit ratios, giving special emphasis to the impact of lease-financing. Many retail chains use leases to finance their long-term capital needs, but few food processors do. When a firm finances its capital by leasing rather than by ownership or mortgage, the firm's net profit and its stockholders' equity each represent a larger percentage of total assets. As leased assets do not appear on the balance sheet, they represent implicit leverage. The following estimates present value of leased assets for a group of large food processing and retailing firms by capitalizing rental obligations. Total assets plus leased assets give total capital supplied by owners, creditors, and lessors. Gross returns to this capital are nearly equivalent for processors and retailers in spite of inequalities in conventional profit ratios. Some implications of this finding are considered.**

IN COMPARING relative profitability of resources engaged in different firms or industries, dollar profits must be taken as a ratio of all capital responsible for generating those profits.[1] Both equity and creditor capital must be included, since distribution of income consistent with ownership rights is not the purpose of the comparison. Similarly, returns to both equity and creditor capital must be computed on total capital.

Stockholders' equity represents only one part of the total bundle of resources. Total assets include a greater share of the bundle because they include debtor as well as equity capital. Next in this progression is the addition of assets leased or hired. This final aggregation more nearly approximates the capital upon which profits and interest are accrued than does either stockholders' equity or total assets. The interest component of rental payment and total interest paid must be added to profits before taxes in obtaining gross returns to capital.

[1] "Profits" in this report are defined by the principles of conventional accounting. Profit data were taken from secondary sources, without adjustment.