The Calculation of Research Benefits with Linear and Non-linear Specifications of Demand and Supply Functions

Introduction

Market models have been used extensively for measuring the level and the distribution of benefits from R&D-induced supply shifts. In those models the supply and demand schedules were assumed to be either linear or non-linear with constant elasticity (NLCE), and the R&D-induced supply shifts were assumed to be either parallel or non-parallel (convergent or pivotal shifts). Comparisons of research benefits occurring with linear parallel and linear non-parallel shifts in supply were provided by Lindner and Jarrett (1978), Rose (1980), Wise and Fell (1980), and Norton et al (1981). Comparisons of research benefits for linear and non-linear specifications of the demand and supply curves have not been provided. The main aim in this paper is to provide such comparisons.

Both the linear and the NLCE frameworks have frequently been used in empirical work on evaluation of research benefits. The NLCE pivotal supply shift framework, for instance, has been adopted by Peterson (1967), Ayer and Schuh (1972), Akino and Hayami (1975), Flores-Moya et al. (1978), Nagy and Furtan (1978), Wise (1981) and Zentner and Peterson (1984). The linear pivotal supply shift framework can be found for instance in Lindner and Jarrett (1978), Rose (1980), Mclean (1982), Wise (1984) and Norton et al. (1987). Comparison of research benefits for linear and NLCE frameworks provides an understanding of the differences in results caused by model specification. This understanding is important in choosing between frameworks for estimating economic benefits from research, and in interpreting results. The key point is that if a particular model specification is a better description of reality, then use of the alternative model specification can cause overestimation (or underestimation) of returns to research.

Comparison of economic benefits from research with linear and NLCE supply curves requires specification of the type of supply shift resulting from research. This is not a straightforward task. The problem is that there is no NLCE counterpart for some specifications of shifts in linear supply curves. The best example of this is the parallel shift in a linear supply curve, a specification used by several researchers (e.g. Rose, 1980; Edwards and Freebairn, 1984). Researchers using NLCE supply curves, on the other hand, appear not to have used parallel shifts in supply due to research. This may be due to an absence of analytical techniques for accurately specifying a parallel shift of the NLCE supply curve and the function of the new supply curve since a NLCE supply curve normally passes through the origin.

Since our aim is to compare estimated research benefits with linear and NLCE supply curves, it is important to set the analysis up so that other factors affecting the size of research benefits, including the nature of the supply shift, are identical for the linear and NLCE supply curves. In order to do this, the approach we have taken is to confine the comparison to estimates of research benefits resulting from corresponding pivotal shifts in linear and NLCE supply curves.

In the following sections, we outline the approach and then present an illustration comparing research benefits using linear and NLCE specifications of the demand and supply curves.

A comparison of research benefits for linear and non-linear pivotal supply shift frameworks

In this section, we outline the procedures used for comparing the level and the distribution of research benefits using linear and non-linear constant elasticity specifications of the demand and supply curves. For the purpose of comparison, we fix the following conditions: the supply shifts are pivotal for both cases; shifts in supply due to research, measured vertically as cost reductions at the initial equilibriums, are identical for the two cases; price is identical for the two cases at the intial equilibrium, as is quantity; and price elasticities of demand are identical for the two cases at the initial equilibrium, as are price elasticities of supply.

The linear inverse demand curve is represented by $P = a - \alpha Q$. Note that $\alpha = \frac{P}{\eta Q}$, where η denotes the own price elasticity of demand at the initial equilibrium. The linear inverse supply curve in the absence of research is represented by $P = b + \beta Q$. Again, $\beta = \frac{P}{eQ}$, where e is the own price elasticity of supply at the initial equilibrium. The linear supply curve with research is denoted by $P' = b + \beta' Q'$ where $\beta' = \frac{P(\frac{1}{e} - k)}{Q}$ and where k represents the proportionate vertical shift in the supply curve ($0 < k < 1$). The constant elasticity demand curve is represented by $P = AQ^{-\sigma}$ where σ represents the price flexibility of demand. The constant elasticity supply curve without research is $P = BQ^{\gamma}$ (γ is the price flexibility of supply) and the constant elasticity supply curve with research is $P' = (1 - k)BQ^{\gamma}$. The equilibrium price and quantity withou the supply shift are P and Q, and the equilibrium price and quantity with the supply shift are P' and Q'.

Comparison of the gross annual research benefits (GARB) is performed for area OAB (or area $O'AC$)[1] in Figure 1(a) and area OEF in Figure 1(b). The distribution of research benefits between producers and consumers with the linear and NLCE specifications is also compared.

Four combinations of the linear and NLCE demand and supply curves are identified. These are set out as follows:

- case 1: linear demand and supply curves ($D_L S_L$),
- case 2: NLCE demand and linear supply curves ($D_N S_L$),
- case 3: linear demand and NLCE supply curves ($D_L S_N$),
- case 4: NLCE demand and supply curves ($D_N S_N$).

Let TS be the change in total economic surplus, CS be the change in consumer surplus, PS be the change in producer surplus, and subscripts 1, 2, 3 and 4 denote cases 1, 2, 3, and 4 respectively. The formula for calculating the level and the distribution of research benefits for case 1 (Figure 1a) is expressed as:

$$TS_1 = 1/2kPQ + 1/2kP(Q' - Q), \tag{1}$$

$$CS_1 = 1/2(P - P')(Q + Q'), \tag{2}$$

$$PS_1 = TS_1 - CS_1. \tag{3}$$

The 'with research' equilibrium point for case 1 can be determined using the alternative formulae developed in the appendix[2]:

$$P' = P[1 - \frac{ke}{(e + \eta - ke\eta)}], \tag{4}$$

$$Q' = Q[1 + \frac{ke\eta}{(e + \eta - ke\eta)}]. \tag{5}$$

The formula for calculating research benefits with NLCE demand and supply (case 4) is expressed as:

$$TS_4 = \int_0^Q S(dQ) - \int_0^Q S'(dQ) + \int_Q^{Q'} D(dQ) - \int_Q^{Q'} S'(dQ), \tag{6}$$

$$CS_4 = \int_{P'}^{P} D(dP), \tag{7}$$

$$PS_4 = TS_4 - CS_4, \tag{8}$$

where $Q' = \exp^{-\frac{\ln(1-k)}{\sigma+\gamma}}$ and $P' = (Q')^{-\sigma}$.

Similarly, changes in research benefits for the intermediate cases (i.e. case 2 and 3) can be calculated using the above approaches.

Results

For illustrative purposes, the initial equilibrium prices and quantities were set at unity, k was set at 0.1 (so the absolute shift is $0.1P$), and a range of demand and supply elasticities was used for comparison. The results for these four cases are tabulated in Table 1.

For supply elasticities less than about unity, the values of GARB calculated for case 1 ($D_L S_L$) are higher than those calculated for case 4 ($D_N S_N$). For elasticities of supply greater than about unity, however, the values of GARB are larger for case 4 than case 1. The magnitudes of the differences in GARB for these two polar cases are larger the smaller the values for the supply elasticities (e.g. a difference of about 142% for $e = 0.25$ and about

16% for $e = 0.75$) (see Table 2). The differences in GARB are very small when the price elasticity of supply approaches unity. The values of GARB for case 2 ($D_N S_L$) are similar to those for case 1 ($D_L S_L$) for each combination of η and e, while the values of GARB for case 3 ($D_L S_N$) correspond closely to those for case 4 ($D_N S_N$).

For each of the four cases, the values of GARB are insensitive to the value of the price elasticity of demand. The values of GARB calculated using the $D_L S_L$ and $D_N S_L$ specifications (cases 1 and 2) are not sensitive to the choice of supply elasticity (see footnote 1 for explanation). However, the values of GARB calculated using the $D_L S_N$ and $D_N S_N$ specifications (cases 3 and 4) are extremely sensitive to changes in supply elasticity. The explanation for this is as follows: for $e < 1$, the NLCE supply curves slope upwards at an increasing rate (convex supply curves) whereas for $e > 1$, the NLCE supply curves slope upwards but at a decreasing rate (concave supply curves). Given an identical size of cost reduction at the initial equilibrium, the 'with research' and the 'without research' supply curves for $e < 1$ lie closer together (i.e. the mean vertical distance between the two supply curves is smaller), resulting in smaller values of GARB relative to the linear supply curves (see Figure 2(a)). The 'with research' and the 'without research' for $e > 1$ lie further from each other (i.e the mean vertical distance between the two supply curves is greater) resulting in larger values of GARB relative to the linear supply curves (see Figure 2(b)).

Consumers' benefits from research are similar for each of the four cases. The significant differences in GARB between cases 1 and 2 on the one hand and cases 3 and 4 on the other reflect differences in benefits to producers. Producers can lose for each of the four cases when the demand for a commodity is price inelastic[3] (i.e. $|\eta| < 1$), and with nonlinear supply (cases 3 and 4) producers gain from research only when $|\eta| > 1$. This contrasts with a linear parallel shift in supply when producers always gain (unless demand is perfectly inelastic). For each of the four cases producers' gain from research increase (or losses decrease) with increases in η and fall with increases in e.

Conclusions

The major finding reported in this note is that the values of GARB calculated using the linear pivotal supply shift model (cases 1 and 2) are substantially larger than those calculated using the constant elasticity pivotal supply shift model (cases 3 and 4) when the price elasticity of supply for the commodity is significantly lower than unity. However, when the elasticity of commodity supply is greater than one, the values of GARB calculated using the linear supply shift framework are considerably smaller than those calculated using the constant elasticity supply shift framework. An important implication of this finding is as follows: if the constant elasticity specification is a better description of reality, the linear pivotal supply shift model for evaluating research benefits leads to marked overestimation of research benefits when the commodity supply is inelastic, but to considerable underestimation when supply is elastic.

It is debatable whether the real world situation corresponds more closely to constant elasticity or to constant slope. In our view, commodity supply curves in agriculture are more likely to take the constant elasticity form because the marginal costs of production tend to be forced up at an increasing rate with increases in production. Longrun diseconomies of scale and limitations in resources are possible reasons for this. Furthermore, both the shortrun and the longrun supply for many rural commodities have often been reported to be in the inelastic range (less than unity) (e.g. Vincent et al., 1982; Tweeten, 1970). With inelastic supply at the initial equilibrium, a linear supply curve passes through the negative fourth quadrant. An extrapolation of the linear inelastic commodity supply curve to the negative fourth quadrant (i.e. production of the commodity at undefined 'negative' prices) is unrealistic. This problem is avoided by the use of constant elasticity supply curves; these curves pass through the origin. Our analysis, therefore, leads us to the view that the use of a non-linear, constant elasticity specification of the supply curve and a pivotal shift due to research is usually preferable to use of a linear supply curve with pivotal shift.

FOOTNOTES

1. For $e < 1$, the gross annual research benefit equals area $O'AC$ and for $e > 1$, the gross annual research benefit equals area OAB. For identical cost reductions, however, the values of the gross annual research benefits are unaffected by changes in supply elasticity notwithstanding that, for $e < 1$, the supply curve passes through the fourth quadrant negative euclidean space.

2. Past models used the equations of Pinstrup-Andersen, Ruiz de Londoño and Hoover (1976) (where $P' = P[1 - \frac{ke}{(e+\eta)}]$ and $Q' = Q[1 + \frac{ke\eta}{(e+\eta)}]$) for deriving the post-innovation (or 'with research') equilibrium point (i.e. point B or C in Figure 1(a)), but these equations, as pointed out by Rose (1980), are strictly correct only if the supply shift is parallel. For the linear pivotal supply shift model, the actual equations for determining the 'with research' equilibrium point have been developed in this paper (i.e. equations 4 and 5). When research benefits calculated using the past approach were recalculated using the actual approach, using the assumed parameter values set out in this note, the values of GARB were observed to be only marginally higher (see Table 3 in the Appendix). The difference was found to be approximately 0-3% with demand and supply elasticities in the range 0.1 and 5.0. A larger difference, however, is found in the calculated distributions of research benefits. For instance, recalculations showed that consumers' share of research benefits calculated from the past approach is smaller than those calculated from the actual approach by 10-30% with larger values of η and e (see Table 3 in Appendix).

3. Our results are consistent with the major finding reported in Miller et al. (1988).

That paper focussed on the effects of supply shifts on producers' surplus. For supply and demand curves that are linear or power functions, a small downward pivot of the supply curve increased producers' surplus only if the equilibrium point was far enough into the elastic region of the demand curve, and any downward pivot of the supply curve decreased producers' surplus if the equilibrium point was in the inelastic region of the demand curve

REFERENCES

AKINO, M., and Y. HAYAMI. 'Efficiency and equity in public research: rice breeding in Japan's economic development.' Amer. J. Agr. Econ. 57(1975):1-10.

AYER, H.W., and G.E. SCHUH. 'Social rates of return and other aspects of agricultural research: The case of cotton research in São Paulo, Brazil.' Amer. J. Agr. Econ. 54(1972):557-69.

EDWARDS, G.W., and J.W. FREEBAIRN. 'The gains from research into tradable commoditities.' Amer. J. Agr. Econ. 66(1984):41-49.

FLORES-MOYA, P., R.E. EVENSON, and Y. HAYAMI. 'Social returns to rice research in the Philippines: domestic benefits and foreign spillover.' Econ. Develop. and Cultur. Change 26(1978):591-607.

LINDNER, R.K., and F.G. JARRETT. 'Supply shifts and the size of research benefits.' Amer. J. Agr. Econ. 60(1978):48-56.

McLEAN, I.W. 'The demand for agricultural research in Australia 1870-1914.' Aust. Econ. Pap. 21(1982):294-308.

MILLER, G.Y., J.M. ROSENBLATT, and L.J. HUSHAK. 'The effects of supply shifts on producers' surplus.' Amer. J. Agr. Econ. 70(1988):886-91.

NAGY, J.G., and W.H. FURTAN. 'Economic costs and returns from crop development research: The case of rapeseed breeding in Canada.' Can. J. Agr. Econs. 26(1978):1-14.

NORTON, G.W., and J.S. DAVIS. 'Evaluating returns to agricultural research: a review.' Amer. J. Agr. Econ. 63(1981):686-99.

NORTON, G.W C. VICTOR, and C. POMAREDA. 'Potential benefits of agricultural research an tension in Peru.' Amer. J. Agr. Econ. 69(1987):247-57.

PETERSON, W.L. 'Returns to poultry research in the United States.' J. Farm Econ. 49(1967):656-69.

PINSTRUP-ANDERSEN, P., N.R. DE LONDOÑO, and E. HOOVER. 'The impact of increasing food supply on human nutrition: implication of commodity priorities in agricultural research and policy.' Amer. J. Agr. Econ. 58(1976):131-42.

ROSE, F. 'Supply shifts and the size of research benefits: comment.' Amer. J. Agr. Econ. 62(1980):834-7.

WISE, W.S. 'The theory of agricultural research benefits.' J. Agr. Econ. 32(1981):147-56.

TWEETEN, L.G. 'Foundations of farm policy.' Lincoln: University of Nebraska press. 1970.

VINCENT, D.P., A.A. POWELL, and P.B. DIXON. 'Changes in supply of agricultural products.' Agriculture in the Australian economy, ed. D.B. Williams. Sydney university press, 1982.

WISE, W.S. 'The shift of cost curves and agricultural research benefits.' J. Agr. Econ. 35(1984):21-30.

WISE, W.S., and E. FELL. 'Supply shifts and the size of research benefits: comment.' Amer. J. Agr. Econ. 62(1980):838-40.

ZENTNER, R.P., and W.L. PETERSON. 'An economic evaluation of public wheat research and extension expenditures in Canada.' Can. J. Agr. Econ. 32(1984):327-53.

APPENDIX

Derivation of Equations 4 and 5 in text

Prior to the supply shift, the inverse demand and supply curves can be specified as

$$P = a - \alpha Q$$

$$P = b + \beta Q.$$

With the pivotal linear supply shift, the new demand and supply schedules become

$$P' = a - \alpha Q'$$

$$P' = b + \beta' Q'$$

where $\beta' = \frac{P(1-k)-b}{Q} = \frac{P(\frac{1}{\epsilon}-k)}{Q}$. The initial equilibrium quantity is $Q = \frac{a-b}{\alpha+\beta}$ and the new equilibrium quantity is $Q' = \frac{a-b}{\alpha+\beta'}$, thus,

$$Q' - Q = \frac{a-b}{\frac{P(1-k)-b}{Q} + \alpha} - \frac{a-b}{\alpha+\beta}.$$

Given that $a = P(1+\frac{1}{\eta})$, $b = P(1-\frac{1}{\epsilon})$, $\alpha = \frac{P}{\eta Q}$, $\beta = \frac{P}{\epsilon Q}$ and $\beta' = \frac{P(1-k)-P(1-1/\epsilon)}{Q}$, and by substitution and simplification,

$$Q' = Q[1 + \frac{k\epsilon\eta}{(\epsilon + \eta - k\epsilon\eta)}].$$

Using similar approach P' can be derived and is expressed as follow,

$$P' = P[1 - \frac{ke}{(e + \eta - ke\eta)}].$$

Table 1: Comparison of the level and the distribution of research benefits for the 4 combinations of demand and supply curves

		Case 1 ($D_L S_L$)			Case 2 ($D_N S_L$)			Case 3 ($D_L S_N$)			Case 4 ($D_N S_N$)		
η	e	CS	PS	TS	CS	PS	TS	CS	PS	TS	CS	PS	TS
		$	$	$	$	$	$	$	$	$	$	$	$
	0.01	0.0500	0.0000	0.0500	0.0500	0.0000	0.0500	0.0513	-0.0503	0.0010	0.0513	-0.0503	0.0010
	0.25	0.0963	-0.0462	0.0501	0.0963	-0.0462	0.0501	0.0964	-0.0763	0.0201	0.0964	-0.0763	0.0201
0.01	0.75	0.0988	-0.0488	0.0501	0.0988	-0.0488	0.0501	0.0988	-0.0559	0.0429	0.0988	-0.0559	0.0429
	1.00	0.0992	-0.0491	0.0501	0.0992	-0.0491	0.0501	0.0991	-0.0491	0.0501	0.0991	-0.0491	0.0501
	2.00	0.0997	-0.0496	0.0501	0.0997	-0.0496	0.0501	0.0996	-0.0329	0.0667	0.0996	-0.0329	0.0667
	0.01	0.0039	0.0462	0.0501	0.0039	0.0462	0.0501	0.0041	-0.0030	0.0010	0.0041	-0.0030	0.0010
	0.25	0.0510	0.0003	0.0506	0.0510	0.0000	0.0510	0.0517	-0.0310	0.0207	0.0517	-0.0310	0.0207
0.25	0.75	0.0772	-0.0262	0.0510	0.0772	-0.0260	0.0512	0.0767	-0.0329	0.0438	0.0767	-0.0329	0.0438
	1.00	0.0825	-0.0315	0.0510	0.0825	-0.0313	0.0513	0.0817	-0.0307	0.0510	0.0817	-0.0306	0.0511
	2.00	0.0919	-0.0408	0.0511	0.0920	-0.0407	0.0513	0.0905	-0.0227	0.0678	0.0904	-0.0226	0.0678

	0.01	0.0010	0.0491	0.0501	0.0010	0.0491	0.0501	0.0010	0.0000	0.0010	0.0010	0.0000	0.0010
	0.25	0.0206	0.0304	0.0510	0.0206	0.0312	0.0518	0.0211	0.0000	0.0211	0.0211	0.0000	0.0211
1.00	0.75	0.0458	0.0065	0.0522	0.0458	0.0077	0.0535	0.0452	-0.0001	0.0451	0.0452	0.0000	0.0452
	1.00	0.0540	-0.0014	0.0526	0.0541	-0.0001	0.0539	0.0527	-0.0001	0.0526	0.0527	0.0000	0.0527
	2.00	0.0740	-0.0204	0.0536	0.0740	-0.0204	0.0547	0.0703	-0.0002	0.0701	0.0702	0.0000	0.0702
	0.01	0.0001	0.0500	0.0501	0.0001	0.0501	0.0501	0.0001	0.0010	0.0010	0.0001	0.0010	0.0010
	0.25	0.0013	0.0500	0.0513	0.0013	0.0512	0.0525	0.0013	0.0200	0.0213	0.0013	0.0200	0.0213
20.00	0.75	0.0041	0.0499	0.0539	0.0041	0.0536	0.0576	0.0040	0.0429	0.0468	0.0040	0.0429	0.0468
	1.00	0.0055	0.0497	0.0553	0.0056	0.0547	0.0603	0.0053	0.0500	0.0553	0.0053	0.0500	0.0553
	2.00	0.0124	0.0488	0.0611	0.0125	0.0586	0.0711	0.0105	0.0664	0.0770	0.0105	0.0665	0.0770

Negative (-) indicates a loss in surplus.

Table 2: Differences in values of GARB[a] calculated using the linear (case 1) and the NLCE (case 4) specifications

Supply Elasticity	Linear demand Linear supply $	NLCE demand NLCE supply $	Difference in GARB %
0.01	0.05005	0.00104	4700
0.25	0.05102	0.02107	142
0.50	0.05172	0.03512	47
0.75	0.05224	0.04515	16
1.00	0.05264	0.05268	0
1.25	0.05294	0.05853	-11
1.50	0.05319	0.06322	-19
2.00	0.05357	0.07024	-31
5.00	0.05455	0.08780	-61

[a] Major finding from Table 1, using $\eta = 1$.

Table 3: Comparisons of the differences in the values of the level and the distributions of research benefits calculated using the past and the alternative linear pivotal supply shift frameworks

		Gain to Consumers			Gain to Producers			Aggregate Gain		
η	e	PA ($)	AA ($)	Diff[a](%)	PA ($)	AA ($)	Diff[a] (%)	PA ($)	AA ($)	Diff[a] (%)
	0.01	0.0500	0.0500	0.00	0.0000	0.0000	0.00	0.0500	0.0500	0.00
0.01	0.50	0.0981	0.0982	-0.10	-0.0480	-0.0481	-0.20	0.0501	0.0501	0.00
	2.00	0.0996	0.0997	-0.10	-0.0495	-0.0496	+0.20	0.0501	0.0501	0.00
	0.01	0.0039	0.0039	0.00	0.0462	0.0462	0.00	0.0501	0.0501	0.00
0.25	0.50	0.0672	0.0684	-1.75	-0.0164	-0.0175	+5.15	0.0509	0.0509	0.00
	2.00	0.0899	0.0919	-2.18	-0.0387	-0.0408	+5.15	0.0511	0.0511	0.00
	0.01	0.0005	0.0005	0.00	0.0496	0.0496	0.00	0.0501	0.0501	0.00
2.00	0.50	0.0204	0.0213	-4.23	0.0316	-0.0308	+2.60	0.0520	0.0521	0.19
	2.00	0.0525	0.0586	-10.41	-0.0025	-0.0031	+2.60	0.0550	0.0556	-1.08

	0.01	0.0000	0.0000	0.00	0.0500	0.0500	0.00	0.0501	0.0501	0.00
20.00	0.50	0.0025	0.0026	-3.85	0.0499	0.0499	0.00	0.0524	0.0525	0.19
	2.00	0.0099	0.0124	-20.16	0.0491	0.0488	+0.61	0.0591	0.0611	-3.27

[a] + means the past approach (PA) yields larger values than the actual approach (AA).
- means PA yields lower values than AA.

In this paper, alternative formulae are developed for calculating the 'with research' price and quantity equilibrium for the linear and pivotal supply shift framework used for assessing research benefits. Differences in results obtained using the past and the present approaches (see footnote 3 for details) are shown in Table 3.

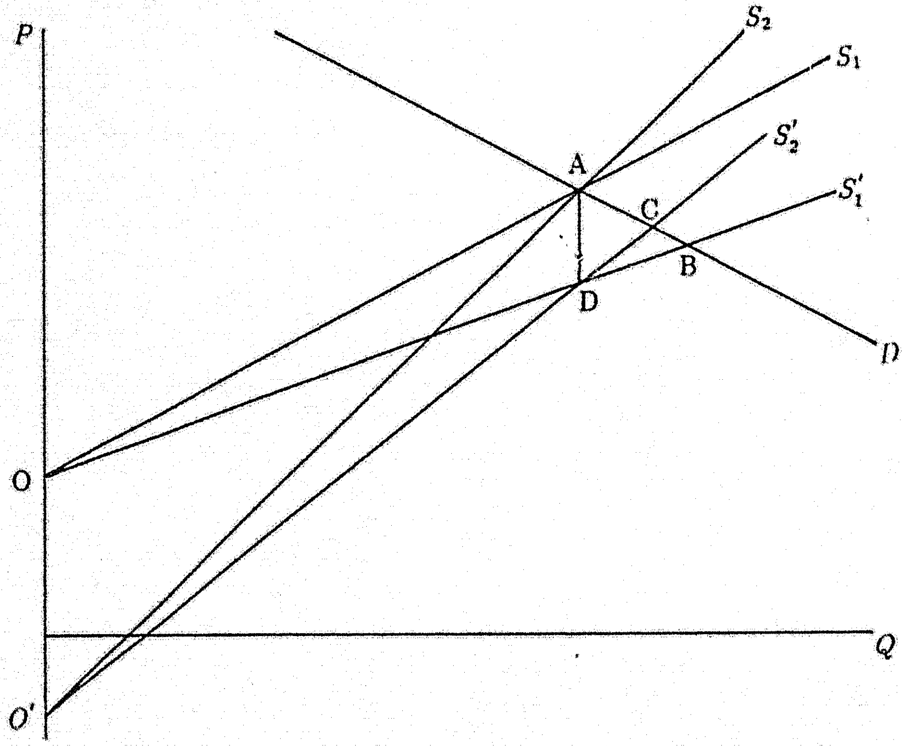

Figure 1(a) Effect of research with linear demand and supply curves.

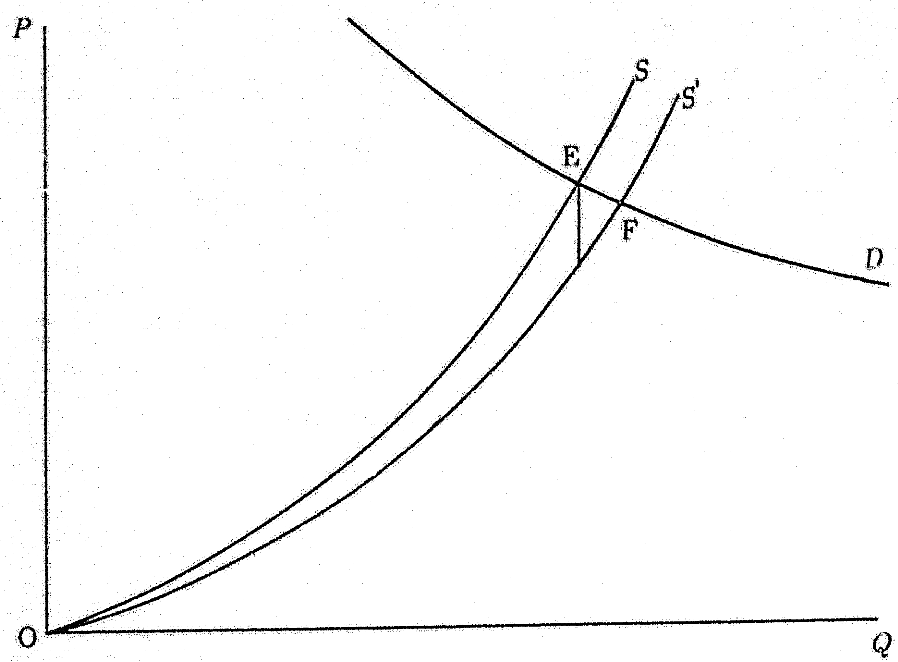

Figure 1(b). Effect of research with non-linear constant elasticity demand and supply curves.

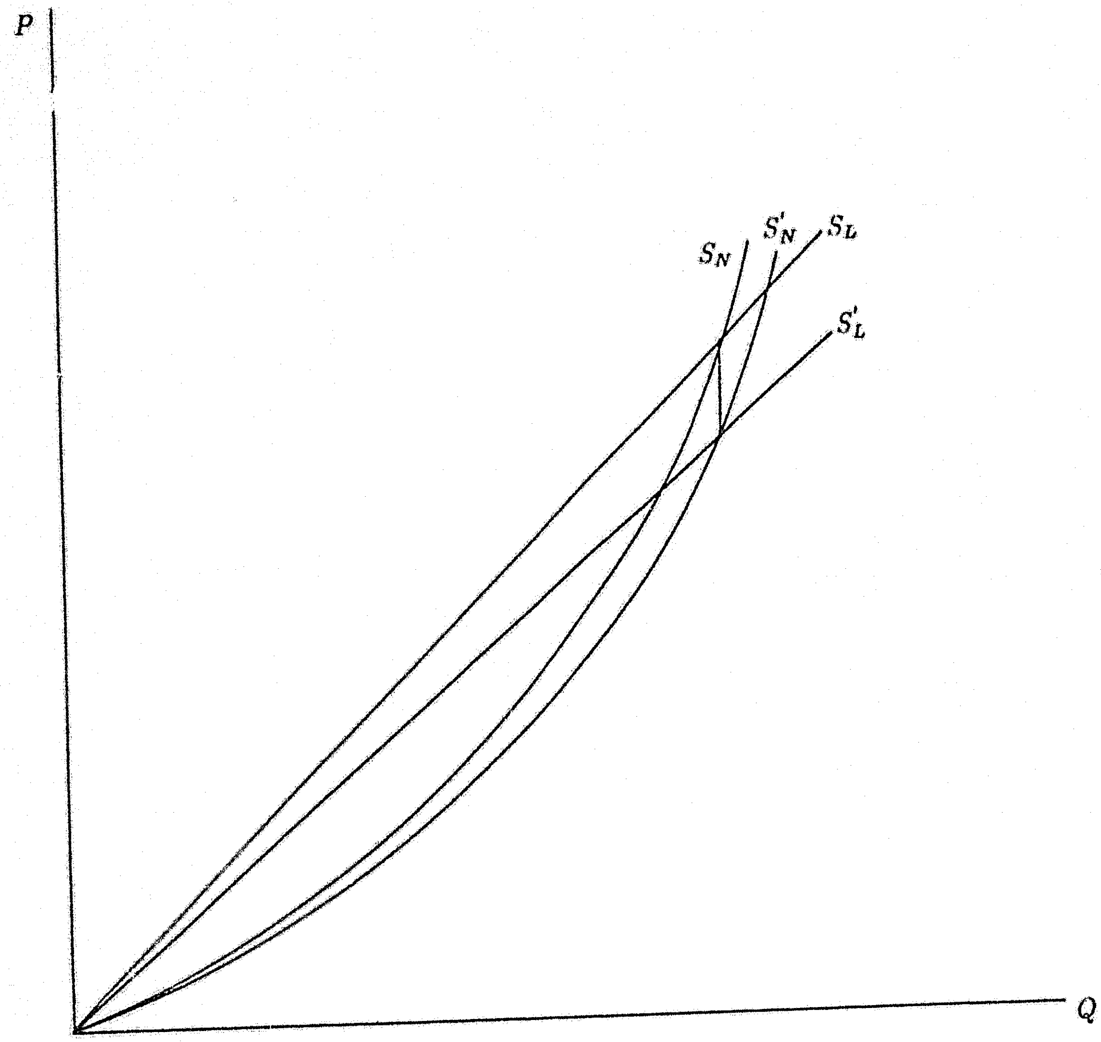

Figure 2(a). The behaviour of the NLCE supply curves relative to the linear supply curves for $e < 1$.

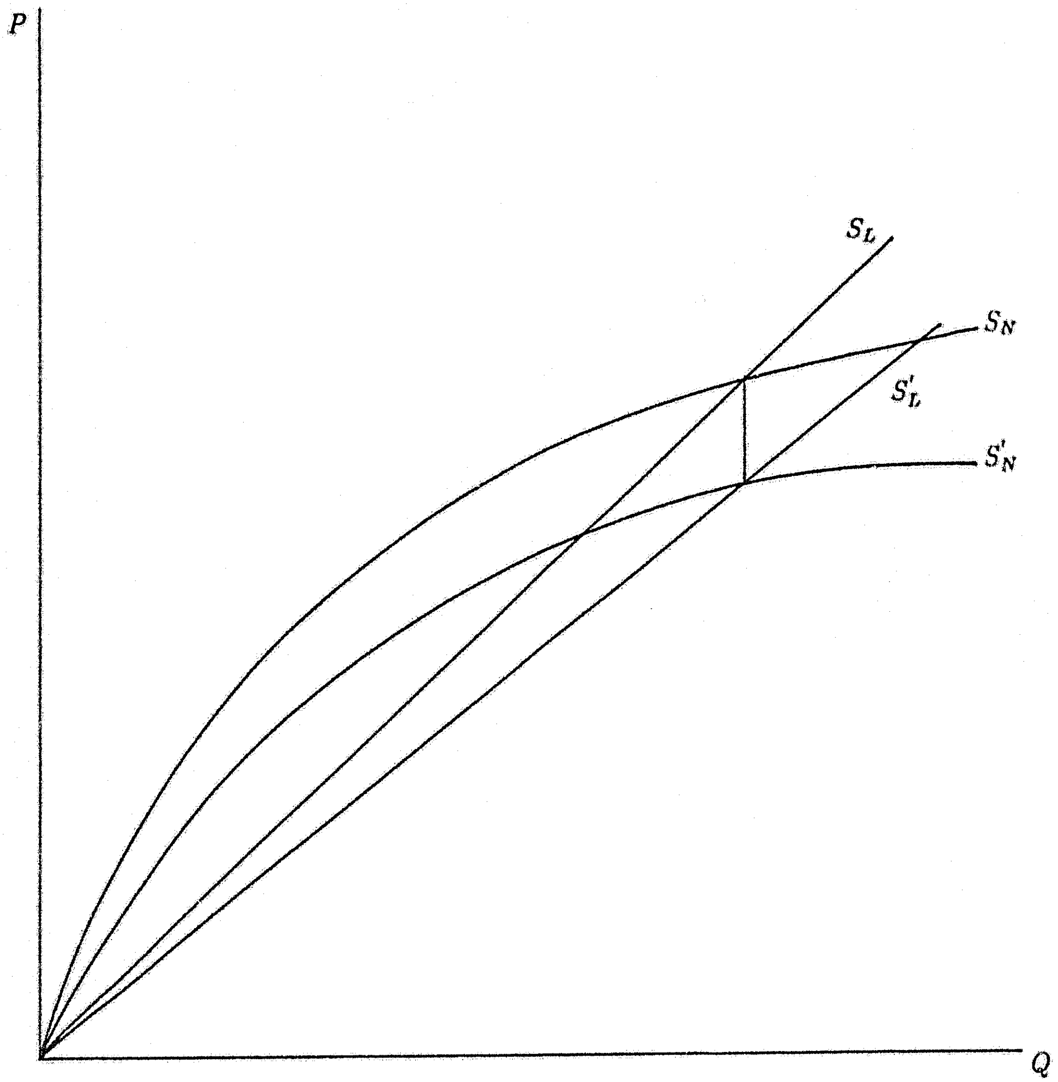

Figure 2(b). The behaviour of the NLCE supply curves relative to the linear supply curves for $e > 1$.